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# PRIMORDIAL NUCLEOSYNTHESIS: A CRITICAL COMPARISON OF THEORY AND OBSERVATION

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## ABSTRACT

Primordial nucleosynthesis is reexamined in the context of a detailed comparison of theory and observation. A new argument is presented to show how the observed abundances of D and $^3$He can be used to derive a *lower* bound to the nucleon density. In concert with the previously known upper bound from D alone, we define a conservative ("safe bet") range for the nucleon-to-photon ratio: $\eta = (3\text{–}10) \times 10^{-10}$. New observations of $^7$Li are consistent with the abundances of D and $^3$He and help us to define a reasonable ("best bet") range: $\eta = (4\text{–}7) \times 10^{-10}$. In either of these ranges the predicted and observed abundances of D, $^3$He, and $^7$Li are all in concordance. The upper bounds correspond to $\Omega_N \leq 0.14\text{–}0.19$, and we conclude that nucleons fail to close the universe by at least a factor of 5–7. We review the recent observational data on $^4$He and conclude that there is complete consistency between the predicted abundance of $^4$He and those of the other light elements. In particular, for the standard model ($N_\nu = 3$, $10.4 \leq \tau_{1/2} \leq 10.8$ minutes) we find that $Y_p \leq 0.25$ for $\eta \lesssim 7 \times 10^{-10}$. We predict a lower bound to the $^4$He mass fraction of $Y_p \geq 0.24$ if $N_\nu \geq 3$ and $\tau_{1/2} \geq 10.4$ minutes. If $\nu_\tau$ is not light (i.e., $m \gtrsim 1$ MeV), $N_\nu \geq 2$, and $Y_p$ can be as low as 0.226; if $Y_p$ is unambiguously determined to be less than this value, the standard model will be in trouble. Only one additional light, two-component neutrino species ($N_\nu = 4$) is marginally permitted: for $N_\nu = 4$, $\tau_{1/2} \geq 10.4$ minutes, and $\eta \geq 3 \times 10^{-10}$, $Y_p \geq 0.253$.

*Subject headings:* cosmology — early universe — nucleosynthesis

## I. INTRODUCTION

The almost universal acceptance of the standard (i.e., simplest) hot big-bang model (Friedmann-Robertson-Walker cosmology) rests, in large part, on the success of this model in accounting for the abundances of the light elements, particularly helium-4 and deuterium. The pioneering calculations of Peebles (1966) and Wagoner, Fowler, and Hoyle (1967) demonstrated that an epoch of nucleosynthesis, occurring within the first few minutes of the evolution of the universe, would produce a large amount of $^4$He ($Y \approx 0.25$; $Y = {}^4$He mass fraction) as well as an abundance of D now known to be comparable to that observed in our Galaxy today (D/H $\approx$ few $\times 10^{-5}$). To date, there are no other astrophysical scenarios for producing such large amounts of both $^4$He and D. In addition, it was noticed that astrophysically interesting amounts of $^3$He and $^7$Li were also produced during primordial nucleosynthesis. As a result of the general agreement between the predictions of the standard model and the observed abundances of the light elements (for a recent comparison see Olive *et al.* 1981, hereafter OSSTY), big-bang nucleosynthesis has been exploited as a probe of the early universe. As such it has placed useful and important constraints on both particle physics and cosmology. For example, primordial nucleosynthesis has been used to infer an upper limit to the energy density at early epochs (Shvartsman 1969), which leads to constraints on the number of light ($m \lesssim 1$ MeV) neutrino (or, other light) species (Steigman, Schramm, and Gunn 1977; Yang *et al.* 1979, hereafter YSSR; OSSTY; Szalay 1981), on the number of intermediate-mass ($m \approx 0.1\text{–}10$ MeV) neutrino species (Kolb and Scherrer 1982), and on the number of superweakly interacting particles (Steigman, Olive, and Schramm 1979; Olive, Schramm, and Steigman 1981). From the perspective of cosmology, primordial nucleosynthesis serves to constrain the universal density of nucleons (Wagoner, Fowler, and Hoyle 1967; Reeves *et al.* 1973; Gott *et al.* 1974; YYSR; OSSTY) and to limit the amplitude of density inhomogeneities and anisotropy during the early universe (Wagoner 1973; Gisler, Harrison, and Rees 1974; Epstein and Petrosian 1975; Barrow 1976; Matzner and Rothman 1982). For a review of this earlier work, see Schramm and Wagoner (1977).

To provide the proper context for the new results to be presented in this article, we shall briefly review the conclusions reached in previous work. The abundances of $^4$He and D are the primary probes of primordial nucleosynthesis. The mass fraction of $^4$He produced primordially, $Y_p$, is an increasing function of the nucleon-to-photon ratio $\eta$ ($= n_N/n_\gamma$), the neutron half-life $\tau_{1/2}$, and the number of light species present, parameterized by the number of equivalent neutrino species $N_\nu$ ($N_\nu$ provides a measure of the "speedup" of the universal expansion rate; for sufficiently fast expansion, $Y_p$ eventually decreases with increasing $N_\nu$ [see OSSTY]). For an *upper* limit to $Y_p$ of 0.25 and *lower* limits to $\tau_{1/2}$ (10.4 minutes) and $\eta$ ($2 \times 10^{-10}$ as inferred from the dynamics of binaries and small groups of galaxies), one obtains an *upper* limit to the number of light neutrino species (OSSTY): $N_\nu \leq 4$. As Kolb and Scherrer (1982) have shown, intermediate-mass neutrinos contribute one unit or slightly more than one unit to $N_\nu$ depending on their mass. If, however, one entertains the possibility that something other than nucleons (e.g., massive neutrinos, gravitinos) dominates the mass on scales of binaries and small groups (BSG), then $\eta$ may be smaller than the lower bound of $2 \times 10^{-10}$. From the mass contained within the luminous parts of galaxies (Holmberg radius) a more certain but less restrictive lower bound to the nucleon

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abundance may be obtained: $\eta > 0.3 \times 10^{-10}$; in this case there is no constraint to $N_\nu$ from primordial nucleosynthesis (OSSTY). One of the main results of this article is to remedy this situation by deriving a new, restrictive lower bound to $\eta$ from primordial nucleosynthesis alone.

The abundance of D produced primordially decreases very rapidly with increasing $\eta$. The requirement that big-bang production account for the minimum observed abundance of deuterium, $(D/H)_o > 1 \times 10^{-5}$, leads to the constraint that $\eta < 10^{-9}$ (YSSR). The fraction of the critical density contributed by nucleons, $\Omega_N$, is related to the nucleon abundance $\eta$ by

$$\Omega_N = 3.53 \times 10^{-3} h_o^{-2} (T_o/2.7\ \mathrm{K})^3 \eta_{10}\ , \tag{1}$$

where the present value of the Hubble parameter is $H_0 = 100 h_o$ km s$^{-1}$ Mpc$^{-1}$ ($\frac{1}{2} < h_o < 1$), $T_o$ is the present temperature of the microwave background (2.7 K $< T_o <$ 3.0 K), and $\eta_{10} = 10^{10}\eta$. The upper bound on $\eta_{10}$ ($\lesssim 10$) from the deutrium abundance leads to the upper limit $\Omega_N \lesssim 0.2$. Thus, nucleons alone fall a factor of at least 5 short of providing the mass density needed if the universe is to be closed ($\Omega_o > 1$).

In the last few years there have been new observations which provide data of value in our attempts to infer the primordial abundances of the light elements. The quantity and quality of the $^4$He data, in particular, have improved significantly (see references in § IV). Observations of $^3$He in several galactic H II regions (Wilson, Rood, and Bania 1983) are helping to clarify the issue of galactic versus primordial production. Very recently, Spite and Spite (1982*a*, *b*) have reported observations of $^7$Li in halo and old disk stars which may provide a measure of the primordial abundance of $^7$Li. In view of the potential significance of the new data, we have carried out a thorough reexamination of primordial nucleosynthesis with several goals in mind. Our primary aim is to test the consistency of the standard hot big-bang model. That is, do the predicted abundances of the light elements agree with the primordial abundances inferred from current observations? We answer this question affirmatively and then proceed to deal with more specific issues in greater detail. In particular, we use the observed abundances of D and $^3$He to establish a *lower* bound to the nucleon abundance $\eta$ (based solely on primordial nucleosynthesis). From D and $^7$Li we are able to place upper bounds on $\eta$. Our very conservative ("safe bet") range for $\eta$ is (3–10) $\times 10^{-10}$, while our reasonable ("best bet") range is (4–7) $\times 10^{-10}$. The restrictive range for $\eta$ allowed by the results of primordial nucleosynthesis has important consequences for cosmology and particle physics; that there *is* an allowed range establishes the concordance of the standard model. Since the standard model does provide a good fit to the observed abundances, we are able to investigate the effects of deviations from the standard model. The new lower bound on $\eta$ permits us to reestablish a firm upper limit to the number of light, two-component neutrino species: $N_\nu \leq 4$. We also constrain the amplitude of isothermal density perturbations (fluctuations in $\eta$) during the epoch of primordial nucleosynthesis: $\delta\eta/\eta < O(2)$. Finally, keeping in mind the existing uncertainties in the abundances derived from the observational data, we delimit the range of the abundances for which the predictions of big-bang nucleosynthesis are in concordance with the observations.

The outline of this article is as follows. In § II we briefly review the physics of big-bang nucleosynthesis with the purpose of elucidating the dependences of the predicted abundances on the various parameters ($\eta$, $\tau_{1/2}$, $N_\nu$). In § III we concentrate on the abundances of D, $^3$He, and $^7$Li. Here we derive the "standard" upper bound on $\eta$ from the lower bound to D/H, and we find a more restrictive upper bound when we also consider $^7$Li. Most importantly, we use D plus $^3$He to provide an upper bound to the primordial abundance of D. This leads to the new *lower* bound to $\eta$. In § IV we review the recent observational work relating to the primordial abundance of $^4$He, both the values derived and their uncertainties. We show that the predicted and inferred abundances of $^4$He agree, and we obtain an upper bound to $N_\nu$. In § V we discuss the consistency of the standard model. The cosmological consequences are outlined in § VI, and in § VII we derive limits to isothermal density perturbations. Our results and conclusions are summarized in § VIII. Uncertainties in the numerical code for calculating the primordial abundances are discussed in Appendix A. In Appendix B we discuss the survival of $^3$He in the course of galactic evolution.

## II. REVIEW OF PRIMORDIAL NUCLEOSYNTHESIS

During the early evolution of the universe all interactions proceed sufficiently rapidly (compared with the universal expansion rate) to ensure that thermal equilibrium is established (even among weakly interacting particles). For this reason (equilibrium) we may begin our discussion of big-bang nucleosynthesis relatively late in the early evolution of the universe. For example, when $t \approx 10^{-2}$ s, the temperature is $\sim 10$ MeV, and the universe is dominated by a relativistic gas of photons, electron-positron pairs, and light neutrinos. At such relatively late times and low temperatures the cosmology and the physics are well understood. For example, the average interparticle separations exceed $\sim 10$ fm. This relativistic gas is contaminated by a trace amount of nucleons ($\eta \approx 10^{-10}$ to $10^{-9}$); the internucleon separation exceeds $\sim 10^4$ fm. At a temperature $T \approx 10$ MeV the neutral-current weak interactions (for the *e*-neutrino the charged-current interactions also contribute),

$$e^+ + e^- \leftrightarrow \nu_i + \bar{\nu}_i \quad (i = e, \mu, \tau, \ldots)\ , \tag{2}$$

are sufficiently rapid to ensure that all light ($m \ll T$) neutrino species are in thermal equilibrium ($T_\nu = T_e = T_\gamma$). The charged-current weak interactions,

$$p + e^- \leftrightarrow n + \nu_e\ , \quad n + e^+ \leftrightarrow p + \bar{\nu}_e\ , \quad n \leftrightarrow p + e^- + \bar{\nu}_e\ , \tag{3}$$

maintain chemical equilibrium between neutrons and protons, and

$$n/p = \exp(-\Delta m/T)\ . \tag{4}$$

Although the rate for neutrons and protons to collide and form deuterons is rapid,

$$n + p \leftrightarrow d + \gamma\ , \tag{5}$$

the equilibrium abundance of deuterons is very small [$n_D/n_N \propto \eta \exp(2.2\ \mathrm{MeV}/T)$] because the background density of photons is enormous. The weak binding of the deuteron results in a bottleneck to nucleosynthesis until the number of photons capable of dissociating the deuteron becomes sufficiently small. Since the number of such photons per nucleon varies as $\eta^{-1} \exp(-2.2\ \mathrm{MeV}/T)$, nucleosynthesis is delayed until $T \approx 0.1$ MeV; for smaller $\eta$ the bottleneck persists to a lower temperature.

The light neutrinos decouple from the other particles at $T \approx$ few MeV when the weak rates (2) become slow compared with the universal expansion rate ($T \approx 3.5$ MeV for $\nu_\mu$, $\nu_\tau$, ..., and $T \approx 2$ MeV for $\nu_e$). Subsequently, the "free" gas of neutrinos cools adiabatically as the universe expands. At a slightly lower temperature (which depends on the strength of the charged-current weak interaction [3]), interaction rates (3) become slow compared with the universal expansion rate, and reactions (3) can no longer maintain the neutron-to-proton ratio at its equilibrium value (4). At this point the neutron-to-proton ratio is said to "freeze out" although, in fact, the ratio continues to decrease but at a rate much slower than if equilibrium were maintained. When $T \lesssim m_e/3$, the electron-positron pairs annihilate, heating the photons but *not* the already decoupled neutrinos, so that

$$\left(\frac{T_\nu}{T_\gamma}\right)^3_{T \ll m_e} = \frac{4}{11}\left(\frac{T_\nu}{T_\gamma}\right)^3_{T \gg m_e} = \frac{4}{11} . \qquad (6)$$

Finally, for $T \lesssim 0.1$ MeV (with the precise value of $T$ depending logarithmically on $\eta$), the density of photodissociating photons is sufficiently small to permit the buildup of deuterium. As soon as the deuterium abundance becomes significant, further $n$, $p$, and $d$ reactions occur leading to the synthesis of $^3$H, $^3$He, $^4$He, and a small amount of $^7$Li.

The gaps at mass-5 and mass-8 (no stable nuclei) provide bottlenecks to the synthesis of heavier elements. Furthermore, as the temperature decreases it is increasingly (i.e., exponentially!) difficult to penetrate the Coulomb barriers in nuclear collisions. As a result, most of the neutrons which were present when nucleosynthesis began in earnest find themselves incorporated into the most tightly bound light nucleus, $^4$He, with trace abundances of D, $^3$He, and $^7$Li being produced.

With the above review as background we can now understand how the predicted abundances depend on the various parameters: $\eta$, $\tau_{1/2}$, $N_\nu$. Let us first consider $^4$He. Since most neutrons are incorporated into $^4$He, the primordial abundance (by mass) $Y_p$ is essentially determined by the neutron-to-proton ratio at the time nucleosynthesis starts: $Y_p \approx 2n/p(1 + n/p)^{-1}$. The $n/p$ ratio is determined by the competition between the weak interaction rate (which varies as $\tau_{1/2}{}^{-1}$) and the universal expansion rate. For example, for a longer half-life, interactions (3) will freeze out at a higher temperature, when there are more neutrons relative to protons (see eq. [4]), leading to the production of more $^4$He. In Figure 1, we show the predicted primordial abundance of $^4$He for three choices of the neutron half-life (Christensen *et al.* 1972; Kugler, Paul, and Trinks 1978, 1979; Wilkinson 1979; Byrne *et al.* 1980; Byrne 1982).

When the universe is radiation dominated, the expansion rate is determined by the effective number of relativistic degrees of freedom:

$$t(\mathrm{s}) = 2.42 g_{\mathrm{eff}}{}^{-1/2} T_{\mathrm{MeV}}{}^{-2} , \qquad (7)$$

where

$$g_{\mathrm{eff}} = \sum_B g_B(T_B/T_\gamma)^4 + \tfrac{7}{8} \sum_F g_F(T_F/T_\gamma)^4 . \qquad (8)$$

In equation (8), $g_B(g_F)$ are the number of boson (fermion) helicity states, and $T_B(T_F)$ are the temperatures ($\leq T_\gamma$) of the various species. For example, with three two-component neutrino species we have (for $T > m_e$) $g_{\mathrm{eff}} = 2 + \frac{7}{8}(4 + 3 \times 2) = 43/4$. It is clear from equations (7) and (8) that the addition of new, light particles increases $g_{\mathrm{eff}}$ and,

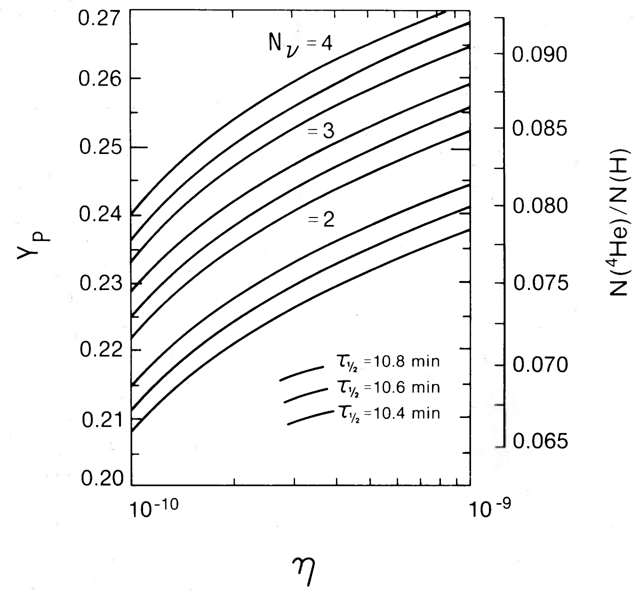


FIG. 1.—The abundance of $^4$He (by mass and by number) as a function of the nucleon-to-photon ratio ($\eta$) for $N_\nu = 2, 3, 4$ species of light, two-component neutrinos and for three choices for the neutron half-life ($\tau_{1/2} = 10.4$, 10.6, 10.8 minutes).

therefore, leads to a faster expansion. A faster expansion results in an earlier freezing out of the weak interactions, leaving behind more neutrons, which results in more $^4$He. The effect of $N_\nu = 2$, 3, 4 neutrino species is also shown in Figure 1.

Finally, it should be noted that although the predicted abundance of $^4$He is rather insensitive to the nucleon abundance $\eta$, $Y_p$ does increase slowly with increasing $\eta$. Basically, a larger $\eta$ means that the "deuterium bottleneck" can be overcome earlier, at a higher temperature, when the neutron-to-proton ratio is higher. To summarize (see Fig. 1), the primordial abundance of $^4$He depends on $\eta$, $\tau_{1/2}$, and $N_\nu$, increasing with increasing values of each of these parameters. An upper limit (from observational data) to $Y_p$, coupled to lower limits to any of the three parameters, will yield an upper limit to the third. We will exploit this in § IV to obtain upper limits to $\eta$ and to $N_\nu$.

Now let us turn to D and $^3$He. Since $^4$He is the most tightly bound of the light nuclei, nuclear reactions tend to burn D and $^3$He to $^4$He. The surviving abundances of these elements depend on the competition between the reaction rates (which depend on $\eta$) and the expansion rate. The higher the nucleon abundance, the faster D and $^3$He are destroyed. The predicted primordial abundances (by number relative to hydrogen) of D and $^3$He are shown in Figure 2 as a function of $\eta$. A faster expansion rate will permit more D and $^3$He to survive. Since the effect of changing the expansion rate is equivalent to a very small change in $\eta$ (see YSSR), we show the results in Figure 2 for $N_\nu = 3$ and $\tau_{1/2} = 10.6$ minutes. The rapid decrease in D/H with increasing $\eta$ means that a *lower* limit to the primordial abundance of D will lead to a firm *upper* limit to $\eta$. If we can bound D/H from above [in fact, in § III we will bound (D + $^3$He)/H from above], we may obtain a *lower* bound to $\eta$. This will be discussed in § III.

Finally, consider the synthesis of $^7$Li. As mentioned earlier, the gaps at mass-5 and mass-8 inhibit the production of nuclei heavier than $^4$He. Nevertheless, some $^7$Li is synthesized; see

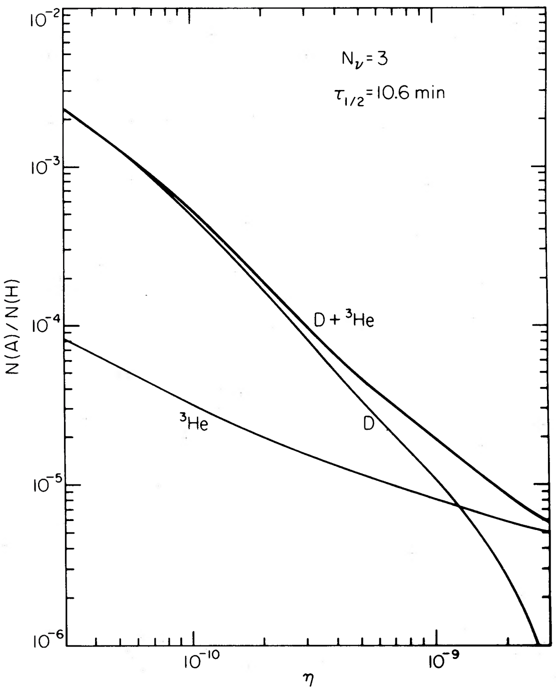


FIG. 2.—The abundance (by number relative to H) of D, $^3$He, and their sum as a function of $\eta$ for $N_\nu = 3$ and $\tau_{1/2} = 10.6$ minutes.

Figure 3. For low nucleon abundance ($\eta \lesssim 3 \times 10^{-10}$), $^7$Li is produced mainly via $^4\mathrm{He} + {}^3\mathrm{H} \rightarrow {}^7\mathrm{Li} + \gamma$. For higher nucleon abundance, most of the $^7$Li comes from the decay of $^7$Be produced via $^3\mathrm{He} + {}^4\mathrm{He} \rightarrow {}^7\mathrm{Be} + \gamma$. Here too, the effect of a speedup in the expansion rate is indistinguishable from a slight change in $\eta$ (see YSSR). The primordial production of $^7$Li is shown in Figure 3 for $N_\nu = 3$, $\tau_{1/2} = 10.6$ minutes. We will return to $^7$Li in § III when we use the new results of Spite and Spite (1982*a*, *b*) to constrain $\eta$.

To facilitate the comparison between theory and observation, Table 1 provides a detailed summary of the predicted abundances of D, $^3$He, and $^7$Li for $1 \leq \eta_{10} \leq 20$.

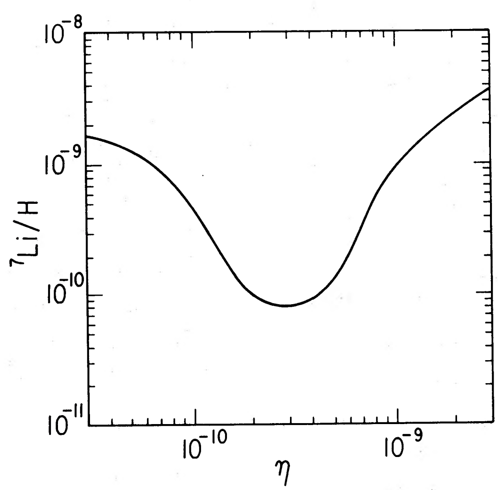


FIG. 3.—The abundance (by number relative to H) of $^7$Li as a function of $\eta$ for $N_\nu = 3$ and $\tau_{1/2} = 10.6$ minutes.

TABLE 1

PRIMORDIAL ABUNDANCES OF D, $^3$He, AND $^7$Li

| $10^{10}\eta$ | $10^5$(D/H) | $10^5$(D + $^3$He)/H | $10^{10}$($^7$Li/H) |
|---|---|---|---|
| 1 | 49 | 53 | 4.4 |
| 1.5 | 25 | 28 | 1.8 |
| 2 | 16 | 18 | 1.1 |
| 3 | 8.1 | 9.7 | 0.76 |
| 4 | 5.1 | 6.5 | 1.0 |
| 5 | 3.6 | 4.8 | 1.7 |
| 6 | 2.7 | 3.8 | 2.7 |
| 7 | 2.1 | 3.1 | 3.9 |
| 8 | 1.7 | 2.6 | 5.3 |
| 9 | 1.4 | 2.3 | 6.9 |
| 10 | 1.1 | 2.0 | 8.6 |
| 15 | 0.48 | 1.2 | 17 |
| 20 | 0.23 | 0.87 | 25 |

## III. DEUTERIUM, HELIUM-3, AND LITHIUM-7

Since the barely bound deuteron is so easily destroyed, it is hard to find an astrophysical site, other than the big bang, where it can be produced in its observed abundance (Epstein, Lattimer, and Schramm 1976). It is generally agreed, therefore, that the observed abundance of D provides a *lower* limit to the primordial abundance.

Observations of deuterated molecules in the atmosphere of Jupiter provide a possible estimate of the protosolar deuterium abundance. As is well known from interstellar studies, chemical fractionation may distort the true D/H ratio; observations of deuterated molecules in the atmospheres of the giant planets may provide more information about the chemistry and physics of planetary atmospheres and less about protosolar abundances. A possible way to minimize the model-dependent fractionation effects is to concentrate on those molecules which contain most of the H and most of the D. Using observations of the $P_4(1)$ line of HD and the $S_4(1)$ line of $H_2$, Trauger *et al.* (1973) derived: $(\mathrm{D/H})_{\mathrm{Pre}\odot} = (2.1 \pm 0.4) \times 10^{-5}$. McKellar, Goetz, and Ramsey (1976) used new laboratory parameters for the $P_4(1)$ line to revise the Trauger *et al.* (1973) value to $(\mathrm{D/H})_{\mathrm{Pre}\odot} = (5.6 \pm 1.4) \times 10^{-5}$. Then Trauger, Roesler, and Mickelson (1977) remeasured the $P_4(1)$ line and found $(\mathrm{D/H})_{\mathrm{Pre}\odot} = (5.1 \pm 0.7) \times 10^{-5}$. The Trauger, Roesler, and Mickelson (1977) observations were reanalyzed by Combes and Encrenaz (1979), who compared HD with $CH_4$ and, after making a necessary—and uncertain—correction for the solar C/H ratio, derived $(\mathrm{D/H})_{\mathrm{Pre}\odot} = (1.3 \pm 1.0) \times 10^{-5}$. More recently, Encrenaz and Combes (1982) have revised their previous result to $(\mathrm{D/H})_{\mathrm{Pre}\odot} = (1.2\text{–}3.1) \times 10^{-5}$. A still more model-dependent, recent estimate by Kunde *et al.* (1982) is based on $CH_3D$: $(\mathrm{D/H})_{\mathrm{Pre}\odot} = 3.6^{+1.0}_{-1.4} \times 10^{-5}$.

An indirect estimate of the protosolar abundance of deuterium can be inferred from the meteoritic and solar wind data on $^3$He (to be discussed below) (Black 1971, 1972; Geiss and Reeves 1972). Using this approach, the abundance relative to $^4$He is found to be $(\mathrm{D}/{}^4\mathrm{He})_{\mathrm{Pre}\odot} = (2.8 \pm 0.8) \times 10^{-4}$. If we adopt a lower bound to the $^4$He abundance, $({}^4\mathrm{He/H})_\odot \geq 0.08$, then $(\mathrm{D/H})_{\mathrm{Pre}\odot} \geq 1.6 \times 10^{-5}$ is derived. This result is consistent with the estimates from deuterated molecules on Jupiter, and, taken together, all the solar system data suggest a protosolar abundance in excess of $1\text{–}2 \times 10^{-5}$.

Ultraviolet absorption studies (York and Rogerson 1976; Vidal-Madjar *et al.* 1977; Vidal-Madjar *et al.* 1983) have led to a determination of the deuterium abundance in the interstellar gas in the vicinity ($\lesssim 1$ kpc) of the Sun. Although an average abundance $\langle D/H \rangle_{ISM} = 2 \times 10^{-5}$ has been derived, individual abundance determinations span the range $(D/H)_{ISM} \approx (\frac{1}{4}\text{–}4) \times 10^{-5}$ (see the discussion in YSSR and Bruston *et al.* 1981). Since the weak deuterium lines are in the wings of the much stronger hydrogen lines, complex structure in the interstellar medium (e.g., many clouds at various velocities) and in the circumstellar medium (Laurent 1983) may complicate the analysis. York (1983) notes that for the "cleanest" lines of sight there is no evidence for a variation in the deuterium abundance, and he suggests that $\langle D/H \rangle_{ISM} = 2 \times 10^{-5}$ and $(D/H)_o \geq 1 \times 10^{-5}$ are reliable inferences from the present data. We note that the solar system and interstellar estimates of $D/H \approx 2 \times 10^{-5}$ are consistent. To perhaps err on the side of caution, we shall adopt as a *lower* limit to the present (or presolar) abundance $(D/H)_o \geq 1 \times 10^{-5}$. On the assumption that the present abundance provides a lower limit to the primordial production (no post-big-bang production of D), we may derive (see Fig. 2 and Table 1) a safe-bet upper limit to the nucleon abundance:

$$(D/H)_p \geq (D/H)_o \geq 1 \times 10^{-5} \Rightarrow \eta_{10} \leq 10 \,. \tag{9}$$

Note that if we had adopted the best-bet value of $(D/H)_p \gtrsim 2 \times 10^{-5}$, the limit on the nucleon abundance would have been $\eta_{10} \lesssim 7$.

Gas-rich meteorites, lunar soil, and the foil placed on the Moon by the *Apollo* astronauts provide samples of solar wind–implanted $^3$He. Since deuterium was burned to helium-3 during the Sun's approach to the main sequence, solar wind $^3$He is the sum of presolar D and $^3$He. From gas-rich meteorites, Black (1972) found $[(D + {}^3He)/{}^4He]_{Pre\odot} = (3.9 \pm 0.3) \times 10^{-4}$. Analyzing $^3$He implanted on the lunar foil, Geiss and Reeves (1972) concluded that $[(D + {}^3He)/{}^4He]_{Pre\odot} = (4.3 \pm 0.3) \times 10^{-4}$. These results are consistent with the abundance derived from spectroscopic studies of $^3$He in the wings of $^4$He in quiescent solar prominences (Hall 1975): $[(D + {}^3He)/{}^4He]_{Pre\odot} = (4 \pm 2) \times 10^{-4}$. We therefore adopt for an estimate of the presolar abundance of D plus $^3$He

$$[(D + {}^3He)/{}^4He]_{Pre\odot} = (4.1 \pm 0.2) \times 10^{-4} \,. \tag{10}$$

In contrast to the gas-rich meteorites, carbonaceous chondrites provide a sample of the primitive solar nebula in which the deuterium is unprocessed. Several components of $^3$He are found in carbonaceous chondrites; that with the smallest $^3He/^4He$ ratio is identified with the presolar $^3$He. Anders, Heymann, and Mazor (1970) find $(^3He/^4He)_{Pre\odot} = (1.25 \pm 0.76) \times 10^{-4}$, a result consistent with Black's (1972) estimate of $(^3He/^4He)_{Pre\odot} = (1.5 \pm 1.0) \times 10^{-4}$. For our safe-bet estimate of the presolar abundance of $^3$He we adopt

$$(^3He/^4He)_{Pre\odot} = (1.3 \pm 0.6) \times 10^{-4} \,. \tag{11}$$

From equations (10) and (11) we may derive an indirect estimate of the presolar abundance of deuterium:

$$(D/^4He)_{Pre\odot} = (2.8 \pm 0.8) \times 10^{-4} \,. \tag{12}$$

To convert the above abundance estimates relative to $^4$He to abundances relative to H, we need to know the solar abundance of $^4$He. Direct measurements of $^4$He line intensities in solar prominences (Heasley and Milkey 1978) yield $y_\odot = (^4He/H)_\odot = 0.10 \pm 0.025$. Indirect estimates of $y_\odot$ follow from a comparison of solar models with observations. Bahcall *et al.* (1982) find[5] $Y_\odot = 0.25 \pm 0.01$ ($3\,\sigma$); this corresponds to $y_\odot = 0.086 \pm 0.005$. This estimate agrees with the result obtained by Christensen-Dalsgaard and Gough (1980) from a detailed comparison of the observed periods of solar surface oscillations with model calculations. If we assume that $(^4He/H)_\odot$ lies in the range $0.08 \lesssim y_\odot \lesssim 0.10$, we find from equations (10)–(12) that

$$[(D + {}^3He)/H]_{Pre\odot} \lesssim 4.3 \times 10^{-5} \,, \tag{13a}$$

$$[^3He/H]_{Pre\odot} \lesssim 1.9 \times 10^{-5} \,, \tag{13b}$$

$$[D/H]_{Pre\odot} \gtrsim 1.6 \times 10^{-5} \,. \tag{13c}$$

We now use these estimates to place an *upper* limit on the primordial abundance of D plus $^3$He and, hence, a *lower* limit to $\eta$ (see Fig. 2 and Table 1). Since deuterium is easily destroyed, the primordial abundance may exceed the present abundance (or, the presolar abundance) by a large factor. Although models of galactic evolution (Truran and Cameron 1971; Audouze and Tinsley 1976) suggest astration of no more than a factor of 2–3, large destruction may be possible (Pagel 1982). In stars, deuterium is burned to $^3$He, which is more difficult to destroy. Some $^3$He will survive stellar processing; the observed $^3$He therefore provides a constraint on the primordial abundance of deuterium (i.e., too much primordial D will result in too much D and/or $^3$He in the presolar nebula). In Appendix B we discuss the survival of $^3$He in some detail. If $g$ is the fraction of $^3$He which survives processing through stars, then (see eq. [B3′])

$$\left(\frac{D + {}^3He}{H}\right)_p \leq \left(\frac{D}{H}\right)_{Pre\odot} + \frac{1}{g}\left(\frac{^3He}{H}\right)_{Pre\odot} = \left(\frac{D + {}^3He}{H}\right)_{Pre\odot} + \left(\frac{1}{g} - 1\right)\left(\frac{^3He}{H}\right)_{Pre\odot} \,. \tag{14}$$

Using the upper limits from equations (13a) and (13b), we find

$$\left(\frac{D + {}^3He}{H}\right)_p \leq \left(2.4 + \frac{1.9}{g}\right) \times 10^{-5} \,. \tag{15}$$

From the analysis in Appendix B we are led to a safe-bet lower limit to $g$ of $g \geq \frac{1}{4}$; a best-bet lower limit is $g \geq \frac{1}{2}$. For the safe-bet value of $g \geq \frac{1}{4}$,

$$[(D + {}^3He)/H]_p \leq 10 \times 10^{-5} \Rightarrow \eta_{10} \geq 3 \,, \tag{16}$$

whereas for the best-bet value of $g \geq \frac{1}{2}$,

$$[(D + {}^3He)/H]_p \leq 6.2 \times 10^{-5} \Rightarrow \eta_{10} \geq 4 \,. \tag{17}$$

If the arguments concerning the survival of $^3$He outlined in Appendix B are correct, it is expected that stellar production of $^3$He in the last $4.6 \times 10^9$ yr has enhanced the interstellar abundance relative to the presolar abundance (Rood, Steigman, and Tinsley 1976). Recent observations of the $^3He^+$ hyperfine line in three H II regions (W3, W51, and W43) by Wilson, Rood, and Bania (1983) are in apparent agreement with this prediction.

Finally, let us turn to the observations of $^7$Li. Until very recently our information concerning $^7$Li was limited to observations of meteorites (Mason 1979), the interstellar medium (Vanden Bout *et al.* 1978), and the atmospheres of some

[5] The helium-4 abundance by mass $Y$, is related to the $^4$He/H ratio $y$ by $Y = 4y(1 + 4y)^{-1}(1 - Z)$, where $Z$ is the heavy-element abundance by mass. For the Sun we have assumed $Z_\odot = 0.02$.

Population I stars (Zappala 1972; Duncan 1981). All these observations are consistent with a present abundance of $(^7\mathrm{Li/H})_o$ within a factor of 2 of $1 \times 10^{-9}$. Recently, Spite and Spite (1982*a*, *b*) have observed $^7$Li in the atmospheres of 12 halo and old disk stars. From their data Spite and Spite derive an abundance

$$(^7\mathrm{Li/H})_{\mathrm{Pop\ II}} = (1.12 \pm 0.38) \times 10^{-10} . \qquad (18)$$

If we refer to Figure 3, we see that this estimate for the primordial (~Population II) abundance puts us in the valley of the $^7$Li/H versus $\eta$ curve: $2 \lesssim \eta_{10} \lesssim 5$. It is, however, possible that there was some astration of $^7$Li prior to the formation of these stars:

$$\left(\frac{^7\mathrm{Li}}{\mathrm{H}}\right)_{\mathrm{Pop\ II}} = f_7 \left(\frac{^7\mathrm{Li}}{\mathrm{H}}\right)_p ; \quad f_7 \leq 1 . \qquad (19)$$

But, since deuterium is more fragile than lithium, D too would be astrated from its primordial value:

$$\left(\frac{\mathrm{D}}{\mathrm{H}}\right)_o \approx \left(\frac{\mathrm{D}}{\mathrm{H}}\right)_{\mathrm{Pre}\odot} = f_2 \left(\frac{\mathrm{D}}{\mathrm{H}}\right)_p ; \quad f_2 \leq f_7 . \qquad (20)$$

Combining equations (19) and (20), we find a bound on the primordial ratio of $^7$Li to D:

$$\left(\frac{^7\mathrm{Li}}{\mathrm{D}}\right)_p \leq \frac{(^7\mathrm{Li/H})_{\mathrm{Pop\ II}}}{(\mathrm{D/H})_o} \lesssim 1.5 \times 10^{-5} . \qquad (21)$$

From Figures 2 and 3 and Table 1, it follows that $\eta_{10} \lesssim 7$. In obtaining this best-bet limit to $\eta$ we have compared the lithium abundance in old stars with the deuterium abundance at present or in the presolar nebula. We therefore expect the inequality $f_2 \leq f_7$ to hold not only because deuterium is more fragile, but also because we have allowed more time for its destruction. On the other hand, as we discuss in Appendix A, uncertainties in the key rates, which determine the predicted abundance of $^7$Li, lead to uncertainties in the abundance of a factor of 2. If we allow for no astration of $^7$Li ($f_7 = 1$), but allow the predicted abundance to be uncertain by a factor of 2, we also obtain the limit $\eta_{10} \lesssim 7$.

So, to summarize, from observations of D, $^3$He, and $^7$Li, limits to the primordial abundances of these elements have been derived, and these have led to upper ($\eta_{10} \lesssim 7$–10) and lower ($\eta_{10} \gtrsim 3$–4) limits to the ratio of nucleons to photons.

## IV. PRIMORDIAL HELIUM-4

Of the elements produced during primordial nucleosynthesis, helium-4 provides the potentially most stringent test of the consistency of the standard model and is the most powerful probe of the conditions in the early universe. There are several reasons for the special position occupied by $^4$He. Next to hydrogen, helium is the most abundant element in the universe. The large observed abundance of $^4$He cannot be accounted for by stellar production alone. It is a significant achievement of the standard model that primordial nucleosynthesis can account for the observed abundance. Furthermore, since $^4$He abundances may be determined with relatively high precision, very detailed comparisons between theory and observation are possible. $^4$He thus differs markedly from the other light elements, whose abundances are known, at best, only to within a factor of 2. In addition, data on the abundance of $^4$He exist not only for our own Galaxy, but for many nearby and distant galaxies as well. There is, therefore, the possibility that the observational data may lead to an accurate estimate of the primordial abundance of $^4$He.

There are, of course, some problems. Since $^4$He is synthesized in stars, some of the observed $^4$He is not primordial. Because $^4$He is not easily destroyed, the observed abundances may only provided an upper limit to the primordial abundance ($Y_o \geq Y_p$). Since an accurate estimate of $Y_p$ is required, the crucial problem is to derive $Y_p$ from the data on $Y_o$. In the attempt to infer from $Y_o$, different astrophysical objects offer different advantages and pose different problems. For example, nearby, bright galactic H II regions can be resolved spatially, and line strengths can be measured to high accuracy. However, galactic H II regions are significantly contaminated with stellar debris; thus some of the observed $^4$He has been synthesized in stars. In contrast, the $^4$He is more nearly primordial in "gassy" galaxies which have low abundances of the heavy elements. However, most such objects are distant and unresolved; what is observed may be the superposition of many H II regions. Such circumstances complicate the analysis necessary for an accurate determination of the $^4$He abundance.

If $^4$He is to provide a probe of cosmology and particle physics, an accurate estimate of the helium abundance $y_p = (\mathrm{He/H})_p$ is required. Before discussing the theoretical implications of the observational data, we will attempt to critically review the present observational situation, paying particular attention to the uncertainties. Previously, the observational situation was reviewed by YSSR and OSSTY. More recently, in an excellent review of the abundances of the light elements, Pagel (1982) has presented a comprehensive and critical discussion of $^4$He. There has been a general consensus that all current data are consistent with a primordial abundance no greater than $Y_p \lesssim 0.25$ ($y_p \lesssim 0.083$).[6] Here, we focus on two issues: (i) Can a primordial abundance be derived from the data? (ii) If so, what are the uncertainties associated with such an estimate?

Pagel (1982) has discussed the estimates of $Y_p$ derived from stellar observations and notes, "... the uncertainties just about cover the range of interest [$0.2 < Y_p < 0.3$] which is encouraging in a way, but is of no help in estimating $Y_p$." We have nothing more to add regarding stellar determinations of $^4$He and refer the reader to Pagel (1982).

In Table 2 we have gathered together the *average* $^4$He abundance determined from emission lines from H II regions in a wide variety of objects; the data covers a modest range in the heavy-element abundance. For objects with roughly "galactic" abundance [$\mathrm{O/H} \approx (4 \pm 2) \times 10^{-4}$], the helium abundance is in the range $Y \approx 0.30 \pm 0.02$. We note that from this perspective, the solar helium abundance ($0.23 \lesssim Y_\odot \lesssim 0.27$; see § III) seems anomalously small. This hint and the scatter in Table 2 suggest that the estimate, $\Delta Y \approx 0.02$, might be too conservative. It does seem clear from Table 2 that the helium abundance is generally lower where the oxygen abundance is lower: for $\mathrm{O/H} \lesssim 10^{-4}$, $Y \approx 0.25 \pm 0.02$.

To study this trend further we have collected in Table 3 helium abundances determined from observations of individual H II regions with oxygen abundances which are estimated to be about an order of magnitude below solar ($\mathrm{O/H} \lesssim 7 \times 10^{-5}$). Even with this restricted group the scatter is large, $0.22 \lesssim Y \lesssim 0.30$. For the data in Table 3, $\langle y \rangle = 0.086$ ($\langle Y \rangle = 0.26$). From Tables 2 and 3 it is clear that all the $^4$He abundance

[6] For the primordial abundance ($Z_p = 0$) the number ratio and the mass fraction are related by $Y_p = 4y_p(1 + 4y_p)^{-1}$.

TABLE 2

AVERAGE HELIUM ABUNDANCES

| Object | $10^4\langle O/H \rangle$ | He/H ($Y$)[a] | Reference |
|---|---|---|---|
| ⟨Orion⟩ ................ | 5.6 | 0.100 (0.29) | PTP III |
| | 4.2 | 0.101 (0.29) | Dufour 1975 |
| | 4.2 | 0.114 (0.31) | Hawley 1978 |
| ⟨Gal. H II reg.⟩ ........ | 4.0 | 0.117 (0.32) | Hawley 1978 |
| | 2.6 | 0.127 (0.34) | Talent and Dufour 1979 |
| | 2.5 | 0.097 (0.28) | Peimbert, Torres-Peimbert, and Rayo 1978 |
| ⟨LMC⟩................. | 3.8 | 0.084 (0.25) | PTP I |
| | 2.7 | 0.103 (0.29) | Dufour 1975 |
| ⟨SMC⟩................. | 1.1 | 0.078 (0.24) | PTP II |
| | 1.1 | 0.094 (0.27) | Dufour 1975 |
| | 1.0 | 0.081 (0.24) | Dufour and Harlow 1977 |
| ⟨DELG⟩[b] .............. | 1.4 | 0.092 (0.27) | French 1980 |
| | 1.1 | 0.080 (0.24) | Lequeux *et al.* 1979 |
| | 1.0 | 0.082 (0.25) | Kunth 1982 |
| | 0.95 | $\geq$0.082[c] ($\geq$0.25) | Kinman and Davidson 1981 |
| | 0.58 | 0.078 (0.24) | Tully *et al.* 1981 |

[a] $Y = 4y(1 + 4y)^{-1}$, $y$ = He/H.

[b] DELG = extragalactic H II regions, dwarf emission-line galaxies, blue compact galaxies, etc.

[c] Kinman and Davidson 1981 make no corrections for neutral helium.

determinations are compatible with $Y = 0.26 \pm 0.04$. Although this result is reassuring, smaller uncertainties are required if significant constraints on cosmology and particle physics are to be derived from primordial nucleosynthesis. The results presented in Tables 2 and 3 represent averages over many H II regions and/or parts of H II regions in several galaxies. To obtain a better sense of the range in $y$ and the uncertainty in $y$, it is valuable to consider the data in more detail.

It cannot be emphasized too strongly that abundances are not observed, emission lines are. There are numerous steps to be taken, each involving uncertainties, in going from the observational data to the derived abundances. The strengths of various recombination lines of $He^+$ and $He^{++}$ are measured and compared with those of $H^+$ to derive $y^+$ and $y^{++}$. In making this comparison one relies on the accuracy of standard star observations to obtain relative line fluxes, and in addition, a correction (usually small) for reddening must be made. To derive the total helium abundance, a correction for neutral helium must also be made; the fact that the He and H Strömgren zones may be of unequal size introduces uncertainty into this correction. For example, selective absorption by dust within the H II region and/or line blanketing in the exciting star(s) can reduce the $He^+$ zone relative to the $H^+$ zone. The $He^+$ zone will be smaller than the $H^+$ zone if the effective temperature of the ionizing radiation is less than $\sim 37 \times 10^3$ K. In these cases the measured value of $y^+$ will also depend on the beam size and the position in the H II region where the observation is made (Shaver *et al.* 1983). Since these effects tend to reduce the observed value of $y^+$, Shaver *et al.* (1983) suggest that the *upper envelope* of the $y^+$ versus O/H distribution may provide a better estimate of the helium abundance. These problems of deriving accurate abundances from accurate observational data are exacerbated if the emission nebulae are inhomogeneous and are ionized by radiation from a distribution (in space as well as in spectral type) of stars. In general, they are. There is ample evidence in the literature which suggests that accurate abundances are difficult to obtain.

By studying nearby, bright, large H II regions, many of the difficulties mentioned above may be mitigated. The exciting stars may be identified; position-by-position reddening and ionization corrections may be made; abundances derived from several different lines may be intercompared. Even so, problems still persist. For example, Hawley (1978) has studied 14 galactic H II regions in detail. Even within a given H II region he finds

TABLE 3

HELIUM ABUNDANCES IN H II REGIONS OF LOW METAL ABUNDANCE

| Object | $10^4$O/H | He/H ($Y$) | Reference |
|---|---|---|---|
| POX 120 ............ | 0.71 | 0.090 (0.26) | Kunth 1982 |
| POX 105 ............ | 0.70 | 0.077 (0.24) | Kunth 1982 |
| Mrk 600 ............. | 0.69 | $\geq$0.098[a] ($\geq$0.28) | Kinman and Davidson 1981 |
| Mrk 36 .............. | 0.69 | 0.089 (0.26) | French 1980 |
| DDO 64 ............. | 0.68 | $\geq$0.083[a] ($\geq$0.25) | Kinman and Davidson 1981 |
| PHL 293B ........... | 0.60 | 0.095 (0.28) | French 1980 |
| VII Zw 403 .......... | 0.58 | 0.078 (0.24) | Tully *et al.* 1981 |
| POX 186 ............ | 0.53 | 0.081 (0.25) | Kunth 1982 |
| A1228+12........... | 0.44 | $\geq$0.095[a] ($\geq$0.28) | Kinman and Davidson 1981 |
| CG 1116+51........ | 0.43 | 0.107 (0.30) | French 1980 |
| TOL 65 .............. | 0.34 | 0.079 (0.24) | Kunth 1982 |
| I Zw 18 .............. | 0.15 | 0.076 (0.23) | Lequeux *et al.* 1979 |
| I Zw 18 .............. | 0.14 | $\geq$0.069[a] ($\geq$0.22) | Kinman and Davidson 1981 |

[a] Kinman and Davidson 1981 make no correction for neutral He.

variations in $y^+$ and $y$ (corrected for neutral He) which far exceeds the internal uncertainties. For example, for position 1 in M16, Hawley (1978) finds $y^+ = 0.079$ and derives $y = 0.100$, whereas for position 2: $y^+ = 0.063$ and $y = 0.154$. For M20, he finds $y_1{}^+ = y_2{}^+ = 0.068$, whereas $y_1 = 0.134$ and $y_2 = 0.110$. For galactic H II regions the ionization correction is often large (and, clearly, uncertain). If these same H II regions were distant and unresolved, "average" ionization and reddening corrections would have to be made. From Hawley's work it is clear that such average corrections are uncertain.

Hawley's results are borne out by other investigations. For example, in their study of the Orion nebula, Peimbert and Torres-Peimbert (1977, hereafter PTP III) have data from 12 different positions within the nebula. They find a wide spread in $He^+$ ($0.060 < y^+ < 0.090$). A significant and variable ionization correction reduces the spread in He ($0.091 < y < 0.109$, $0.27 < Y < 0.30$). Similar dispersion is found by Talent and Dufour (1979) in their study of four galactic H II regions, by Dufour (1975) in his study of SMC and LMC H II regions, and by Dufour and Harlow (1977) in their study of 10 SMC H II regions. Furthermore, Dufour and Harlow's (1977) data show that different lines lead to different estimates of $y^+$; the dispersion is larger than would be expected on statistical grounds alone. This is also found by Rayo, Peimbert, and Torres-Peimbert (1982, hereafter RPTP) in M101. Indeed, the large dispersion of estimates for $y^+$ from different lines leads Tully *et al.* (1981), in their study of VII Zw 403, to conclude, "... the scatter in abundance estimates from the individual lines is significantly greater than what was anticipated based on the internal errors ...." Tully *et al.* (1981) discuss a number of reasons why the helium abundance may be underestimated and, echoed by Shaver *et al.* (1983), note that, "in the absence of very detailed spatial coverage on well-resolved emission regions, a better estimate of the helium abundance may come from the *upper envelope* in the helium versus metal abundance plots." For VII Zw 403, Tully *et al.* (1981) derive a helium abundance $Y = 0.24 \pm 0.04$; with such a large uncertainty, this measurement is of no cosmological use.

It is often claimed that many of the sources of uncertainty which affect the derivation of the helium abundance from the observational data are reduced or become negligible if one concentrates on emission nebulae with low heavy-element abundance. Ionization and reddening corrections are reduced for objects where line blanketing, selective absorption, etc., are less important. However, it must be noted that the "low" metal abundance emission nebulae studied so far (with the exception of I Zw 18, the most metal deficient H II region yet discovered) have roughly an order-of-magnitude higher "metal" abundance than the globular clusters; clearly, we are not yet seeing primordial material. If a precision of greater than 5%–10% or so is desired ($\Delta Y \leq \pm 0.01$–$0.02$), ionization corrections cannot be neglected (Tully *et al.* 1981; Kunth 1982). Since most low-$Z$ emission nebulae are unresolved, the potential advantages they offer are, to some extent, compensated for by the effects of unresolved inhomogeneities. The dispersion in helium abundances derived for different H II regions with comparable, but low, abundances of the heavy elements lends some support to this suggestion. For example, Lequeux *et al.* (1979) find similar metal abundances for II Zw 40, II Zw 70, and IC 10(1) ($Z \approx 0.2\ Z_\odot$) but find helium abundances which range from $y = 0.074$ ($Y = 0.23$) to $y = 0.084$ ($Y = 0.25$).

Even if very accurate estimates of the present helium abundance were available, one would still have to account for the contamination due to stellar production since the big bang. We have already noted that the abundance of $^4$He is somewhat lower where the heavy-element (usually oxygen) abundance is lower. If a linear correlation between $Y$ and $Z$ could be established, the relation could be extrapolated to $Z = 0$ to obtain the primordial abundance $Y_p$. The present observational situation is murky at best. As already noted, there is a dispersion in $Y$ even for objects with similar values of $Z$. As a result, most studies of $Y$ versus $Z$ (or O/H) yield a scatter diagram. Evidence for a $Y$ versus $Z$ correlation ($\Delta Y/\Delta Z \approx 3$) is found by Peimbert and Torres-Peimbert and their collaborators (Peimbert and Torres-Peimbert 1974, 1976, hereafter PTP I, PTP II; Peimbert, Torres-Peimbert, and Rayo 1978; Lequeux *et al.* 1979). Although French (1980) claimed evidence for a $Y$ versus $Z$ correlation ($\Delta Y/\Delta Z \approx 3.2$), his data were contaminated by an anomalously low value of $Y = 0.17$ for I Zw 18.[7] In contrast, many other observers find no evidence in their data to support a linear $Y$ versus $Z$ correlation (Dufour 1975; Smith 1975; Hawley 1978; Talent and Dufour 1979; Kinman and Davidson 1981; Kunth 1982; Shaver *et al.* 1983). For example, although Peimbert, Torres-Peimbert, and Rayo (1978) find $\Delta Y/\Delta Z = 2.0$ from a study of galactic H II regions, Shaver *et al.* (1983), in a recent study of galactic H II regions, find no evidence for a correlation and place the limit at $\Delta Y/\Delta Z \leq 0.8$.

It must be kept in mind that very accurate data are required if a $Y$ versus $Z$ correlation is to be revealed. For example, Kunth's (1982) data are perhaps the most accurate and extensive to date, and although he finds no $Y$ versus $Z$ correlation, his limit, $\Delta Y/\Delta Z = 1.2 \pm 3.2$, cannot entirely exclude a trend in $Y$ versus $Z$. At present then, the observational situation is ambiguous. We do expect that $Y$ will increase with $Z$ (stars *do* produce $^4$He), and the data in Tables 2 and 3 do suggest that "solar" metal abundance H II regions are enhanced in $^4$He relative to "low" metal abundance H II regions. However, the uncertainties in $Y$ are apparently still too large to conclude that there exists a convincing linear relation between $Y$ and $Z$.

There is little succour to be found in turning to theory. Early models of galactic evolution (which ignored mass loss) found little helium enrichment. For example, Arnett (1978) concluded that $\Delta Y/\Delta Z \approx 0.73$. In contrast, Dearborn and Trimble (1978) found that the helium enrichment increased dramatically ($\Delta Y/\Delta Z \approx 2.0$–$3.3$) when mass loss was included. Similarly, Chiosi (1979) and Chiosi and Caimmi (1979) derive $\Delta Y/\Delta Z \approx 1.8$–$3.3$ from models including mass loss. However, these results are contradicted by Maeder (1980), who finds, for evolution with mass loss included, $\Delta Y/\Delta Z \approx 0.73$–$0.77$. The discrepancy is apparently due to different choices for the initial mass function (IMF) (cf., discussion at the ESO Workshop on Primordial Helium, Munich, 1983 February). Clearly, there are large uncertainties in the theory too (e.g., the IMF, the evolution of very low $Z$ stars, metal-enhanced star formation, etc.).

All the data of which we are aware are consistent with a universal abundance of helium in the range $Y = 0.26 \pm 0.04$. We should be pleased that the helium abundance is known so well (uncertainties as small as 15% are extremely rare for the estimates of the abundance of any other element) and reassured

[7] According to French (1981), the helium abundance obtained for I Zw 18 was in error, being too low because of contamination from a night-sky line.

that the abundance derived from observations falls nicely in the range predicted by primordial nucleosynthesis (see Fig. 1). The difficulties arise if one attempts a more detailed comparison between the predicted (primordial) and observed abundances. For such a comparison to yield significant constraints requires an accurate ($\Delta Y \lesssim 0.01$) estimate of the primordial abundance $Y_p$. Although we have found no convincing evidence for a linear $Y$ versus $Z$ relation, we do believe that the estimates of $Y$ for low-$Z$ objects provide a reasonable estimate of the upper limit to $Y_p$. From the data assembled in Table 2 it is not unreasonable to assert that $Y_p < 0.26$.

It is, however, much more difficult to bound $Y_p$ from below. Most of the low estimates of $Y_p$ (<0.22–0.23) are derived from extrapolating an assumed $\Delta Y$ versus $\Delta Z$ relation. As already emphasized, this procedure is suspect. There are also individual objects for which a low value of $Y$ has been derived. I Zw 18 is one example. However, disregarding French's (1980) result, different observers have derived $0.22 < Y(\text{I Zw 18}) < 0.25$. Thus we are hesitant to base a determination of $Y_p$ on the observations of a single object. Another case in point is the recent study of M101 (RPTP). RPTP studied five H II regions and the nucleus of M101. Only for three of the H II regions were the data of sufficiently high quality to permit them to derive a helium abundance. However, they found that different helium lines yield different abundances $y^+$; thus, half the lines were not considered because they might be strongly affected by stellar emission or absorption. To determine the appropriate ionization correction factor (to account for $y^0$) they compared their observations with Stasinska's (1980) models. Only for NGC 5471 did they find any fit, and then they derived $y = 0.0721 \pm 0.0009$ ($Y = 0.224 \pm 0.002$). We suspect that their estimate of the uncertainty is artificially small. Indeed, under similar circumstances in their study of VII Zw 403, Tully *et al.* (1981) concluded that the spread in derived abundances was more representative of the true uncertainty than the formal internal errors. For comparison, Smith (1975) found for NGC 5471, $y \approx 0.074$–$0.078$ ($Y \approx 0.23$–$0.24$). Clearly, caution is suggested in dealing with any estimate of the primordial helium abundance derived from the study of one object.

Recently, Kunth (1982) has studied 13 blue compact galaxies with $Z \lesssim 0.2\, Z_\odot$. For this sample Kunth finds $0.23 < Y < 0.27$ and concludes that there is no evidence for a $Y$ versus $Z$ correlation. From a weighted average of his own data he derives a *primordial* abundance $Y_p = 0.245 \pm 0.003$. Allowing for 3 $\sigma$ uncertainties, Kunth's data suggest $0.236 \leq Y_p \leq 0.254$.

In previous studies of the helium abundance (OSSTY; YSSR; Pagel 1982) it was concluded that $Y_p \lesssim 0.25$. Nothing in the foregoing discussion leads to a contradiction of this conclusion. From the currently accumulated data, $Y_p \gtrsim 0.23$ is reasonable, but $Y_p$ as low as 0.22 may also be consistent with the data. We find no convincing evidence that $Y_p$ is less than 0.22. Keeping these conclusions (especially the uncertainties) in mind, we will now compare the derived primordial abundance with the theoretical predictions and discuss the implications.

We shall utilize Figure 1 for our comparisons between theory and observation. Figure 1 has been corrected for the systematic decrease of 0.003 in $Y$ calculated recently by Dicus *et al.* (1982). To facilitate these comparisons we present the relevant numerical data in Tables 4 and 5. Recall that in § III we used the data on D, D + $^3$He, and $^7$Li to constrain the nucleon abundance to the range $3$–$4 \lesssim \eta_{10} \lesssim 7$–$10$. Consistency among D, $^3$He, $^7$Li, and $^4$He requires that we limit our attention to this range. The following conclusions follow from Figure 1 and Tables 4 and 5 (and the assumption that $\tau_{1/2} \geq 10.4$ minutes).

1. If, indeed, $\eta_{10} \gtrsim 3$, then $N_\nu = 4$ is permitted only if $Y_p \gtrsim 0.253$. Although we believe that $Y_p \lesssim 0.25$, to exclude $N_\nu = 4$ requires third-decimal place accuracy in the adopted upper limit to $Y_p$.

2. The mass fraction $Y_p \leq 0.25$ is consistent with $N_\nu \leq 3$ provided that $\eta_{10} \leq 7.2$. Note, however, if we use Kunth's

TABLE 4

Constraints on $\eta$ from Upper Limits to $Y_p$ for $\tau_{1/2} \geq 10.4$ Minutes

| $Y_p \leq$ | $N_\nu$ | $\eta_{10} \leq$ |
|---|---|---|
| 0.26 ............... | 2 | [a] |
| | 3 | [a] |
| | 4 | 6 |
| 0.25 ............... | 2 | [a] |
| | 3 | 8 |
| | 4 | 2 |
| 0.24 ............... | 2 | [a] |
| | 3 | 3 |
| | 4 | <2 |
| 0.23 ............... | 2 | 4 |
| | 3 | <2 |
| | 4 | <1 |
| 0.22 ............... | 2 | <2 |

[a] A more restrictive constraint, $\eta_{10} < 10$, follows from the requirement that $(D/H)_p > 1 \times 10^{-5}$.

TABLE 5

Lower Limit to $Y_p$ as a Function of $\eta$ for $\tau_{1/2} \geq 10.4$ Minutes

| $\eta_{10}$ | $N_\nu$ | $Y_p \geq$ [a] |
|---|---|---|
| 2 ................. | 2 | 0.221 |
| | 3 | 0.235 |
| | 4 | 0.247 |
| 3 ................. | 2 | 0.226 |
| | 3 | 0.240 |
| | 4 | 0.253 |
| 4 ................. | 2 | 0.230 |
| | 3 | 0.244 |
| | 4 | 0.256 |
| 5 ................. | 2 | 0.232 |
| | 3 | 0.246 |
| | 4 | 0.259 |
| 6 ................. | 2 | 0.234 |
| | 3 | 0.248 |
| | 4 | 0.260 |
| 7 ................. | 2 | 0.235 |
| | 3 | 0.250 |
| | 4 | 0.261 |
| 8 ................. | 2 | 0.236 |
| | 3 | 0.251 |
| | 4 | 0.263 |
| 9 ................. | 2 | 0.237 |
| | 3 | 0.252 |
| | 4 | 0.264 |
| 10 ................. | 2 | 0.238 |
| | 3 | 0.252 |
| | 4 | 0.265 |

[a] These values for $Y_p$ are accurate to ±0.002.

(1982) 3 $\sigma$ upper limit, $Y_p \leq 0.254$, $N_\nu \leq 3$ is allowed for all $\eta_{10} \leq 10$; $N_\nu = 4$ is just barely consistent ($\eta_{10} \geq 3$) with this limit.

3. For $\eta_{10} \geq 3$ and $N_\nu \geq 3$, $Y_p \geq 0.24$. If future observations were to imply that $Y_p$ is less than 0.24, the standard model would remain consistent only if $N_\nu < 3$. This is possible if the $\tau$-neutrino is heavy (> 10 MeV) and unstable.

4. Since $e$- and $\mu$-neutrinos are known to be light, $N_\nu \geq 2$. For $\eta_{10} \geq 3(4)$ and $N_\nu \geq 2$, $Y_p \geq 0.226(0.230)$, so that inconsistency would arise if future observations yield $Y_p \lesssim 0.22$–0.23.

## V. CONSISTENCY OF THE STANDARD MODEL

The observed abundances of deuterium and helium-4 (the two elements which require a primordial origin) differ by some four orders of magnitude. Big-bang nucleosynthesis predicts a unique relationship (depending weakly on the assumed values of $\tau_{1/2}$ and $N_\nu$) between the primordial abundances of D (or D plus $^3$He) and $^4$He. This connection is shown in Figure 4, where the anticorrelation between $^4$He and D + $^3$He is clearly seen. The predicted primordial abundances *do* differ by some four orders of magnitude. Indeed, for $\tau_{1/2} = 10.6 \pm 0.2$ minutes and $N_\nu = 3 \pm 1$, the predicted abundances of D, $^3$He, $^4$He, and $^7$Li are in excellent agreement with the observational data (see Fig. 5). This success is a significant achievement of the standard, hot big-bang model.

As Figure 4 illustrates, consistency among the abundances of D, $^3$He, and $^4$He permits us to constrain $N_\nu$ to $N_\nu \leq 4$. This is to be compared with the analysis presented in OSSTY, where it was noted that uncertainties in bounding $\eta$ from below might permit $N_\nu > 4$. The constraint $N_\nu \leq 4$ has been reestablished here by requiring that D + $^3$He not be overproduced primordially; this requirement corresponds to the constraint $\eta_{10} \gtrsim 3$–4.

There are claims in the literature that the standard model is inconsistent with current data (Stecker 1980, 1981; Rana 1982; Gautier and Owen 1983). We believe that such conclusions have been reached by considering a carefully selected subset of the available data (e.g., see the discussion in Olive and Turner 1981) and by ignoring the uncertainties in the abundances derived from the observational data. We have already noted that the standard model with $N_\nu = 3$ is consistent provided

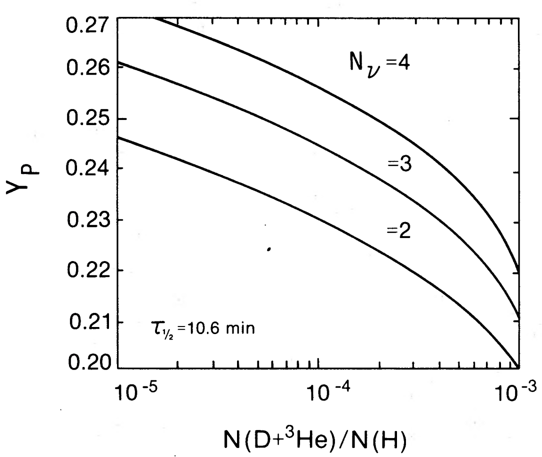


FIG. 4.—The predicted abundance by mass of $^4$He ($Y_p$) vs. the predicted abundance (by number relative to H) of D plus $^3$He for $N_\nu = 2, 3, 4$ ($\tau_{1/2} = 10.6$ minutes).

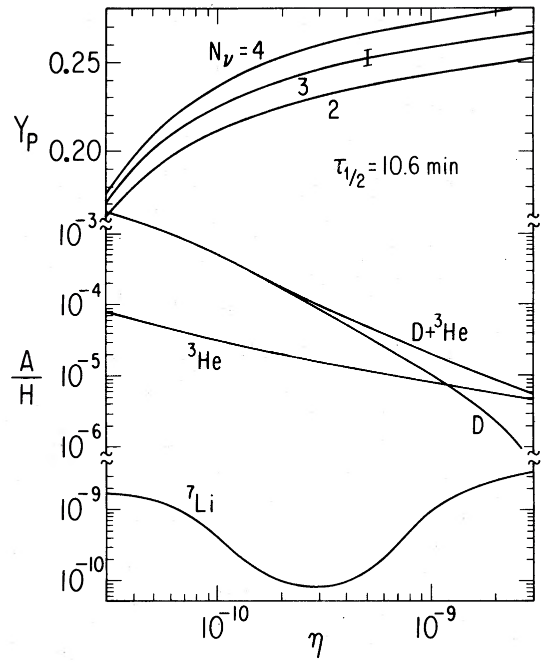


FIG. 5.—The predicted primordial abundances of $^4$He (by mass), D, $^3$He, and $^7$Li (by number relative to H) as a function of $\eta$ for $\tau_{1/2} = 10.6$ minutes; for $^4$He the predictions for $N_\nu = 2, 3, 4$ are shown, and the size of the "error" bar shows the range in $Y_p$ which corresponds to $10.4 < \tau_{1/2} < 10.8$ minutes. Note the changes in the abundance scales.

that $Y_p \geq 0.24$; if $\nu_\tau$ is heavy ($N_\nu = 2$), then consistency remains for $Y_p \geq 0.22$–0.23. From our review of the observational literature we find that $Y_p$ may be as large as 0.25. In a very recent discussion of the data presented by Kunth (1982), Kunth and Sargent (1983) reaffirm Kunth's estimate of the primordial abundance $Y_p = 0.245 \pm 0.003$ and note that this result is consistent, within the errors, with all previous high-quality results.

The data at present suggest that $Y_p \gtrsim 0.23$–0.24. What options would be available if future observations should lead to the conclusion that $Y_p \lesssim 0.22$? One possibility has already been addressed by OSSTY. If the lower bound to $\eta$ derived from the abundances of D + $^3$He is ignored, the universal abundance of nucleons may be quite small: $\eta_{10} \gtrsim 0.1$–0.3 (OSSTY). For such a low nucleon-to-photon ratio, primordial nucleosynthesis of $^4$He yields a low abundance: $Y_p \gtrsim 0.11$–0.15. The restrictive bound on $\eta$ ($\gtrsim 3 \times 10^{-10}$) imposed by D and $^3$He may be avoided by entertaining the possibility that a generation (Population III) of exotic stars might succeed in destroying D *and* $^3$He without the concomitant overproduction of $^4$He or the heavy elements. Or, perhaps, the current data, being restricted to the solar system and the Galaxy, are not a fair sample of "universal" abundances. Extragalactic observations of (or, limits to) D and $^3$He would be very valuable.

A more radical option would be to modify the standard cosmology. For example, neutrino degeneracy (Wagoner, Fowler, and Hoyle 1967; Beaudet and Goret 1976; Yahil and Beaudet 1976; David and Reeves 1980) will change the relative abundances of D, $^3$He, and $^4$He. Anisotropy may be another panacea. Although small anisotropy will increase the $^4$He abundance by speeding up the expansion rate (Hawking and Tayler 1966; Thorne 1967; Barrow 1976), Matzner and Rothman (1982) suggest that larger anisotropy may decrease the abundance of the $^4$He without changing that of D significantly.

In extremis, of course, the standard cosmology could be discarded in favor of more exotic models such as a cold (or tepid) bang (Rees 1978) or even a matter-antimatter symmetric cosmology (Stecker 1980). Indeed, Population III objects have been proposed (Wagoner 1969; Talbot and Arnett 1971; Negroponte, Rowan-Robinson, and Silk 1981; Tarbet and Rowan-Robinson 1982; Bond, Arnett, and Carr 1984) as the source of the observed (pregalactic) helium. Although these hypothetical objects are capable of producing large amounts of helium, the predicted yield depends on a large number of adjustable parameters such as the IMF, stellar rotation, and magnetic fields. In general, it is not possible to produce deuterium in its observed abundance in such Population III objects. It is, therefore, our view that such proposals are extremely ad hoc and have little to recommend themselves when compared with the simplicity of big-bang nucleosynthesis and the success of the standard model in fitting the observed abundances of the light elements with so few free parameters.

## VI. CONSTRAINTS FROM PRIMORDIAL NUCLEOSYNTHESIS

Our analysis has used the observational data on the abundances of D, $^3$He, and $^7$Li to derive bounds to the nucleon abundance. For convenience, these constraints are summarized in Table 6, and all the predicted abundances are shown together in Figure 5. The nucleon contribution to the total mass density, measured by the density parameter $\Omega_N$, is related to the nucleon abundance $\eta$ by equation (1). For $\frac{1}{2} \leq h_o \leq 1$ and $2.7 \leq T_o \leq 3.0$ K,

$$0.0035\eta_{10} \leq \Omega_N \leq 0.019\eta_{10} . \quad (22)$$

From Table 6, it is a "safe bet" that the nucleon abundance is in the range $3 \lesssim \eta_{10} \lesssim 10$. For this safe-bet range, $0.011 \lesssim \Omega_N < 0.19$. If we accept the inferences based on the recent observations of $^7$Li (Spite and Spite 1982*a*, *b*), the upper limit may be reduced to $\eta_{10} \lesssim 7$; this limit is consistent with the average deuterium abundances which suggest that $(\mathrm{D/H})_p \gtrsim 2 \times 10^{-5}$. The lower limit to $\eta$ may be increased somewhat if $^3$He proves more resilient, as recent estimates (Brunish and Truran 1984) suggest; for $[(\mathrm{D} + {}^3\mathrm{He})/\mathrm{H}]_p < 6 \times 10^{-5}$, $\eta_{10} > 4$. Our best-bet range for $\eta$, based on the current data, is $4 \lesssim \eta_{10} \lesssim 7$. For the best-bet range, $0.014 \lesssim \Omega_N \lesssim 0.14$.

It is clear that the constraints from primordial nucleosynthesis point to a low (nucleon) density universe ($\Omega_N \lesssim$ 0.14–0.19). How do our bounds on $\Omega_N$ compare with other estimates of the universal mass density? Dynamical estimates rely on measurements of the mass (direct and indirect) and the light on various scales. A critical mass-to-light ratio follows from the average luminosity density. The recent deep survey of Kirshner *et al.* (1983) yields a value $(M/L)_c \approx 1200h_o$ (solar units). By comparing the mass and light on various scales, the average mass-to-light ratio is sought. This dynamical approach relates $\Omega_o$ to $\langle M/L \rangle$,

$$\Omega_o = \langle M/L \rangle / \langle M/L \rangle_c \approx (1200h_o)^{-1} \langle M/L \rangle . \quad (23)$$

For the inner, luminous parts of galaxies, $(M/L)_{\mathrm{Gal}} \approx (8\text{–}20)h_o$ (Faber and Gallagher 1979), so that $\Omega_{\mathrm{Gal}} \approx 0.007\text{–}0.017$. It is clear that nucleons can account for all the mass inferred from the dynamics of the luminous parts of galaxies. To see if nucleons are capable of accounting for all the mass in the universe requires a good estimate of the average mass-to-light ratio. It is expected that dynamics on the largest scales will provide the data necessary to infer $\langle M/L \rangle$ and $\Omega_o$.

Although past studies of large-scale dynamics have tended to favor a moderately high density, more recent analyses seem to favor lower values. For example, from an application of the "cosmic virial theorem," Davis, Geller, and Huchra (1978) find $0.2 \leq \Omega_o \leq 0.7$, and Peebles (1979), using the data of Kirshner, Oemler, and Schechter (1979), obtains $0.2 \leq \Omega_o \leq 0.6$. A more recent study of clusters by Press and Davis (1982), however, permits a lower density, $\Omega_o \geq 0.07$, although they conclude that values as high as $\Omega_o \approx 0.6$ cannot be excluded. Studies of the Virgocentric flow have followed the same pattern. Davis *et al.* (1980) find $0.3 \leq \Omega_o \leq 0.5$, and Davis and Huchra (1981) claim $0.4 \leq \Omega_o \leq 0.5$, whereas Yahil, Sandage, and Tammann (1980) find $0.03 \leq \Omega_o \leq 0.2$, and Aaronson *et al.* (1982) derive $0.06 \leq \Omega_o \leq 0.13$. On the largest scale yet probed—superclusters—Ford *et al.* (1981) find, by modeling the dynamics, a preferred range $0.06 \leq \Omega_o \leq 0.16$.

As we have already noted, the nucleon contribution to the present universal mass density may be as large as $\Omega_N \approx 0.14$ (for $\eta_{10} = 7$) or even $\Omega_N \approx 0.19$ (for $\eta_{10} = 10$). Unless it is reliably established that $\Omega_o > 0.2$, nucleons alone may account for the bulk of the mass of the universe. Only for $\Omega_o > 0.2$ are massive neutrinos or other exotic particles required (Schramm and Steigman 1981).

Even so, there are other hints that nucleons may not be the whole story. The problem has to do with the growth of perturbations in a low-density universe. Assume for the moment that nucleons do dominate the current mass density, and that there are three species of neutrinos which are massless or so light as to be relativistic during the epochs to be considered. The energy density in nucleons is

$$\epsilon_N = \Omega_N \epsilon_c = 37(T_o/2.7\ \mathrm{K})^3 \eta_{10}\ \mathrm{eV\ cm^{-3}} , \quad (24)$$

and that in relativistic particles is $\epsilon_R = \epsilon_\gamma + \epsilon_\nu = \epsilon_\gamma[1 + 3 \times \frac{7}{8}(T_\nu/T_\gamma)^4] \approx 0.42(T_o/2.7\ \mathrm{K})^4\ \mathrm{eV\ cm^{-3}}$. In equation (24), $\epsilon_c = 1.1 \times 10^4 h_o^2\ \mathrm{eV\ cm^{-3}}$ is the critical energy density. The universe is "radiation dominated" for epochs whose redshift $z$ exceeds $z_{\mathrm{eq}}$, where

$$1 + z_{\mathrm{eq}} = \epsilon_N/\epsilon_R = 89(2.7\ \mathrm{K}/T_o)\eta_{10} . \quad (25)$$

Since $\eta_{10} \lesssim 7\text{–}10$, for a nucleon-dominated universe, $z_{\mathrm{eq}} \lesssim 600\text{–}900 \lesssim z_{\mathrm{rec}} \approx 10^3$. A nucleon-dominated universe is radiation dominated at recombination. After decoupling and before $z_{\mathrm{eq}}$, perturbations only grow by a factor of 0(2.5) (Mészáros 1974). On the other hand, perturbations in the linear regime cease growing when $z < z_*$, where $1 + z_* = \Omega_o^{-1}$. Thus, in a nucleon-dominated universe perturbations must become nonlinear ($\delta\rho/\rho > 1$) before $z_*$ ($z > z_*$). Here

$$1 + z_* = \Omega_N^{-1} = 283h_o^2(2.7\ \mathrm{K}/T_o)^3 \eta_{10}^{-1} . \quad (26)$$

TABLE 6

SUMMARY OF CONSTRAINTS

| Abundance Constraint | Bound on $\eta$ |
|---|---|
| $\mathrm{D/H} \gtrsim 1 \times 10^{-5}$ | $\eta_{10} \lesssim 10$ |
| $\mathrm{D/H} \gtrsim 2 \times 10^{-5}$ | $\eta_{10} \lesssim 7$ |
| $^7\mathrm{Li/D} \lesssim 1.5 \times 10^{-5}$ | $\eta_{10} \lesssim 7$ |
| $^7\mathrm{Li/H} \approx 1 \times 10^{-10}$ | $2 \lesssim \eta_{10} \lesssim 5$ |
| $(\mathrm{D} + {}^3\mathrm{He})/\mathrm{H} \lesssim 6 \times 10^{-5}$ | $\eta_{10} \gtrsim 4$ |
| $(\mathrm{D} + {}^3\mathrm{He})/\mathrm{H} \lesssim 10 \times 10^{-5}$ | $\eta_{10} \gtrsim 3$ |

Between $z_*$ and $z_{eq}$, growth is linear $[\delta\rho/\rho \propto (1+z)^{-1}]$, so that the perturbations which are to become nonlinear before $z_*$ must be large at $z_{eq}$:

$$\left(\frac{\delta\rho}{\rho}\right)_{eq} \gtrsim \frac{1+z_*}{1+z_{eq}} = 3.2\left[\left(\frac{h_o}{\eta_{10}}\right)\left(\frac{2.7\ \mathrm{K}}{T_o}\right)\right]^2 \gtrsim \frac{0.65}{\eta_{10}{}^2}\,. \quad (27)$$

Even if we allow $\eta_{10} \approx 10(\Omega_N \lesssim 0.2)$, we infer $(\delta\rho/\rho)_{eq} \gtrsim 6.5 \times 10^{-3}$. Allowing for a factor of 0(2.5) growth between decoupling and $z_{eq}$, it follows that $(\delta\rho/\rho)_{dec} \gtrsim 2.6 \times 10^{-3}$. Adiabatic perturbations of this magnitude are not reflected in the microwave radiation (Wilson and Silk 1981*a*, *b*; Peebles 1981; Wilkinson 1982).

Finally, we turn to a consideration of the entropy of the universe. For three species of light neutrinos, the specific entropy density $s/k$ is related to the number density of photons by $s/k = 7.04n_\gamma$; each additional light-neutrino species would increase $s/k$ by $1.1n_\gamma$. Our bounds on the nucleon abundance, $3\text{–}4 \leq \eta_{10} \leq 7\text{–}10$, restrict the baryon number–to–specific entropy ratio, $kn_B/s \sim 0.14\eta$, to the interval

$$(4\text{–}6) \times 10^{-11} \leq kn_B/s \leq (10\text{–}14) \times 10^{-11}\,. \quad (28)$$

If the universal expansion has been adiabatic, and if baryon number is conserved (likely to be an excellent approximation for $kT \ll 10^{14}$ GeV), then the baryon-to-entropy ratio is a constant of the expansion. If the universal baryon asymmetry is produced during the very early evolution of the universe ($t \approx 10^{-34}$ s, $kT \approx 10^{14}$ GeV), then this ratio is a measure of the asymmetry which must be generated in order to account for the present ratio of nucleons to photons. Since entropy could have been produced since the epoch of baryogenesis, this ratio represents the *minimum* asymmetry which must have been produced.

Because our constraints on $\eta$ and $kn_B/s$ are, more precisely, constraints on $\eta$ and $kn_B/s$ during the epoch of nucleosynthesis, we can use the presently observed baryon-to-entropy ratio to constrain entropy production between the epoch of nucleosynthesis and the present epoch:

$$S(\mathrm{now})/S(\mathrm{nuc}) = [kn_B/s(\mathrm{nuc})]/[0.14\eta(\mathrm{now})]\,, \quad (29)$$

where $S$(now), $S$(nuc) are, respectively, the entropy of the universe now and at the epoch of big-bang nucleosynthesis. In order to obtain a bound on the amount of entropy production since primordial nucleosynthesis, i.e., $S$(now)/$S$(nuc), we need a *lower* bound on $\eta$(now) which does not involve primordial nucleosynthesis. Since nucleons certainly dominate the mass-to-light ratio in the solar neighborhood, $\eta(\mathrm{now}) > 0.14 \times 10^{-10}$ (OSSTY), and a conservative conclusion is that $S(\mathrm{now}) < (50\text{–}70)S(\mathrm{nuc})$, since $kn_B/s(\mathrm{nuc}) < (10\text{–}14) \times 10^{-11}$. It is most likely that the inner, luminous parts of galaxies are also dominated by nucleons, so that $\eta(\mathrm{now}) > 0.3 \times 10^{-10}$ (OSSTY). This limit also receives support from studies of the X-ray emitting gas in many rich clusters of galaxies. With this less conservative but still reasonable limit to $\eta$(now), and with the previous limits to $kn_B/s$(nuc), we find $S(\mathrm{now}) < (24\text{–}33)\,S(\mathrm{nuc})$. [To summarize, the standard model can tolerate at most a factor of $O(24\text{–}70)$ entropy production since nucleosynthesis. In fact, if the mass of binaries and small groups of galaxies is dominated by baryons, implying that $\eta \gtrsim 2 \times 10^{-10}$, then only a factor of $O(5)$ increase in the entropy can be tolerated.]

We conclude this section by noting the amusing consequences of a somewhat unconventional interpretation of our results. Equation (1) may be thought of as relating the microwave temperature $T_o$ to the nucleon density $(\Omega_N h_o{}^2)$ and the nucleon-to-photon ratio $\eta$:

$$T_o = 17.7(\Omega_N h_o{}^2 \eta_{10}{}^{-1})^{1/3}\ \mathrm{K}\,. \quad (30)$$

For $0.01 < \Omega_N h_o{}^2 < 1$ and $3 < \eta_{10} < 10$, we would "predict" for the present temperature of the relic photon background $1.8 < T_o < 12$ K.

## VII. LIMITS TO INHOMOGENEITIES IN THE EARLY UNIVERSE

We have seen that primordial nucleosynthesis in the standard model yields abundances consistent with bounds to the primordial abundances inferred from current observations. What of departures from the standard model? In this section we relax the assumption of perfect homogeneity and investigate the constraint which primordial nucleosynthesis places upon the amplitude of isothermal density perturbations: $\delta = \delta\eta/\bar{\eta}$. The effect of adiabatic perturbations has previously been analyzed by Gisler, Harrison, and Rees (1974).

To constrain $\delta$ we will utilize the abundances of D, $^3$He, and, most importantly, $^4$He. In general, the helium abundances averaged over many regions with differing local values of $\eta$, $\langle Y_p \rangle$, exceeds the helium abundance corresponding to the average nucleon abundance $\bar{\eta}$, $Y_p(\bar{\eta})$ (see Fig. 6). Therefore, to obtain an upper limit to $\delta$, the minimum value of $\langle Y_p \rangle$ predicted for a given value of $\bar{\eta}$ and $\delta$ must be compared with the upper limit to $\langle Y_p \rangle$ derived from observations, and thus we shall use $\tau_{1/2} = 10.4$ minutes and $N_\nu = 3$.

Our treatment of the effects of inhomogeneities on primordial nucleosynthesis follows that of Epstein and Petrosian (1975). Define the volume distribution of nucleon abundance, $f(\eta)$, so that $f(\eta)d\eta$ is the fraction of space for

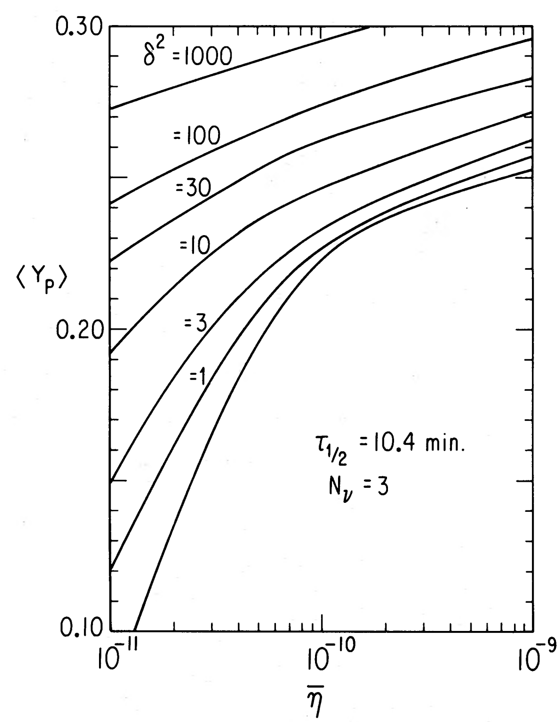


FIG. 6.—The average primordial abundance of $^4$He (by mass), $\langle Y_p \rangle$, as a function of $\bar{\eta}$ for a range of density perturbations $\delta$ ($\equiv \delta\eta/\bar{\eta}$). Since we are interested in the *minimum* value of $\langle Y_p \rangle$, we have chosen $\tau_{1/2} = 10.4$ minutes. The results are shown for $N_\nu = 3$.

which $\eta$ is in the interval $(\eta, \eta + d\eta)$. For simplicity, Epstein and Petrosian (1975) chose

$$f(\eta) \propto \eta^{a-1} \exp(-a\eta/\bar{\eta}) . \quad (31)$$

For the above choice, the perturbations are

$$\delta^2 = (\delta\eta/\bar{\eta})^2 = (\langle\eta^2\rangle - \bar{\eta}^2)/\bar{\eta}^2 = a^{-1} . \quad (32)$$

Fluctuations in $\eta$ would, on sufficiently small scales, have been smoothed by nucleon diffusion. At the epoch of nucleosynthesis the diffusion scale corresponds to a (nucleonic) mass scale of $O(10^{-20} M_\odot)$. On larger scales, fluctuations will not have been erased prior to nucleosynthesis, and the synthesis of the elements would have preceeded independently in each fluctuation volume, with the various yields determined by the local value of $\eta$.

The relevant abundances are derived from observations of regions (e.g., the local interstellar medium, galactic and extragalactic H II regions, metal-poor galaxies) which, at the epoch of nucleosynthesis, contained many diffusion volumes; indeed, they contain many horizon volumes as well. As a result, the data refer to well-mixed volumes and, therefore, provide average abundances

$$\langle X_i\rangle = \int_0^\infty X_i(\eta)\eta f(\eta)d\eta \Big/ \int_0^\infty \eta f(\eta)d\eta . \quad (33)$$

With the form adopted for $f(\eta)$,

$$\langle X_i\rangle = \frac{1}{a!}\int_0^\infty X_i(a^{-1}\bar{\eta}z)z^a e^{-z}dz . \quad (34)$$

For $\eta \gtrsim 2 \times 10^{-10}$, $N_\nu = 3$, and $\tau_{1/2} = 10.4$ minutes, the predicted abundance of $^4$He is well fitted by

$$Y_p(\eta) \approx 0.23 + 0.01 \ln \eta_{10} . \quad (35)$$

Using equation (35) in equation (34) yields

$$\langle Y_p\rangle - Y_p(\bar{\eta}) \approx 0.01 \left[\frac{d}{da}(\ln a!) - \ln a\right] . \quad (36)$$

In the limit $\delta^2 = a^{-1} \gg 1$, this reduces to

$$\langle Y_p\rangle - Y_p(\bar{\eta}) \approx 0.01 \ln \delta^2 . \quad (37)$$

For $\eta \lesssim 2 \times 10^{-10}$, equation (35) no longer provides an adequate fit to $Y_p$. However, for $\delta^2 \gg 1$, most of the contribution to $\langle Y_p\rangle$ is from those regions with $\eta > 2 \times 10^{-10}$ (because of the weighting factor in eq. [34]). In this case ($\eta < 2 \times 10^{-10}$, $\delta^2 \gg 1$), $\langle Y_p\rangle$ can still be approximated as

$$\langle Y_p\rangle \approx 0.23 + 0.01 \ln(\bar{\eta}_{10}\delta^2) . \quad (38)$$

The integral in equation (34) for $\langle Y_p\rangle$ has been done numerically, and the results as a function of $\bar{\eta}$ and $\delta^2$ are shown in Figure 6.

Now consider the abundances of D and $^3$He, keeping in mind that observations place an upper limit on the sum of the primordial abundances. Define $X_{23} \equiv (\mathrm{D} + {}^3\mathrm{He})/\mathrm{H}$; for $0.2 < \eta_{10} < 30$, $X_{23}(\eta)$ is well represented by

$$X_{23}(\eta) \approx 5.0 \times 10^{-4}\eta_{10}{}^{-1.4} . \quad (39)$$

Because of the rapid increase of $X_{23}$ with decreasing values of $\eta$, for $\delta^2 \gg 1$, small values of $\eta$ will contribute very significantly to $\langle X_{23}\rangle$. However, for $\eta_{10} < 0.2$, the dependence of $X_{23}$ on $\eta$ changes, becoming much less steep. We have numerically integrated equation (34), and the results for

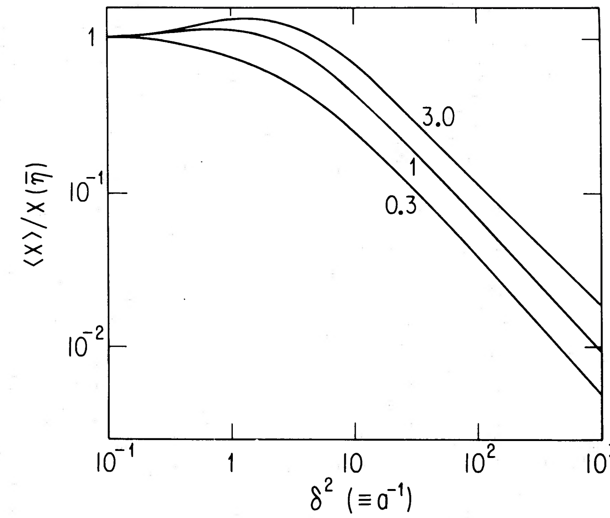


FIG. 7.—The ratio of the average abundance (by number relative to H) of D plus $^3$He ($\langle X_{23}\rangle$) to the D plus $^3$He abundance evaluated at the average nucleon-to-photon ratio [$X_{23}(\bar{\eta})$] as a function of the amplitude of the density perturbation $\delta$ ($\equiv \delta\eta/\bar{\eta}$). The results are shown for three values of the average nucleon-to-photon ratio: $\bar{\eta}_{10} = 0.3, 1, 3$.

$\langle X_{23}\rangle/X_{23}(\bar{\eta})$ as a function of $\delta^2$ are shown in Figure 7 for $\bar{\eta} = 0.3, 1, 3.0$. Note that for small values of $\delta^2$, $\langle X_{23}\rangle/X_{23}(\bar{\eta})$ is nearly independent of $\bar{\eta}$ and slightly greater than 1. In contrast, for large values of $\delta^2$, $\langle X_{23}\rangle/X_{23}(\bar{\eta})$ is *less* than 1 and depends on $\bar{\eta}$. For $\delta^2 \gg 1$ and $0.3 \lesssim \eta_{10} \lesssim 3$, $\langle X_{23}\rangle/X_{23}(\bar{\eta}) \approx 3\bar{\eta}_{10}{}^{1/2}(\delta^2)^{-0.85}$.

With the results presented in Figures 6 and 7, the observed abundances of D, $^3$He, and $^4$He can be used to constrain $\delta$. First consider $^4$He and the requirement that $\langle Y_p\rangle < 0.25$; of course, corresponding constraints will follow from different choices for $\langle Y_p\rangle_{\max}$. To use $^4$He alone requires that we adopt a *lower* limit to $\bar{\eta}$; from OSSTY we take $\bar{\eta} > 3 \times 10^{-11}$. Using Figure 6, $\delta^2 < 50$ follows from the requirements that $\langle Y_p\rangle < 0.25$ and $\bar{\eta} > 3 \times 10^{-11}$. If we had required $\langle Y_p\rangle < 0.24$, we would find $\delta^2 < 25$ for $\bar{\eta}_{10} > 0.3$; in contrast, for $\langle Y_p\rangle < 0.25$ and $\bar{\eta}_{10} > 0.1$, $\delta^2 < O(100)$.

Now let us consider the effect of imposing the further requirement that D + $^3$He not be overproduced; recall that $\langle X_{23}\rangle_p \lesssim (0.6\text{–}1.0) \times 10^{-4}$. Above, only considering $^4$He and insisting that $\langle Y_p\rangle \le 0.25$ and $\bar{\eta}_{10} \ge 0.3$, we found that $\delta^2 \approx 50$ is allowed. For these values of $\bar{\eta}$ and $\delta^2$ we find, from Figures 2 and 7, $\langle X_{23}\rangle \approx 1.7 \times 10^{-4}$; D + $^3$He is overproduced! By referring to Figures 2, 6, and 7 (and Table 1), we find that the joint constraints $\langle Y_p\rangle \le 0.25$ and $\langle X_{23}\rangle_p \le 1 \times 10^{-4}$ can only be satisfied for $\bar{\eta}_{10} > 3$ and $\delta^2 < 3$. For $\langle Y_p\rangle \le 0.25$ and $\langle X_{23}\rangle_p \le 6 \times 10^{-5}$, even tighter restrictions are obtained: $\bar{\eta}_{10} > 4$, $\delta^2 < 2$.

To summarize, if we ignore the lower bound on $\bar{\eta}$ imposed by D + $^3$He and only require $\langle Y_p\rangle < 0.25$, we find that for $\bar{\eta}_{10} > 0.1(0.3)$, $\delta < 14(7)$; for $\langle Y_p\rangle < 0.24$, these limits become $\delta < 9(5)$. More restrictive bounds are obtained when the limits from D + $^3$He are included. For $\langle Y_p\rangle < 0.25$ and $\langle X_{23}\rangle_p < (0.6\text{–}1.0) \times 10^{-4}$, it follows that $\bar{\eta}_{10} > 3\text{–}4$ and $\delta < 1.4\text{–}1.7$. Recently, Barrow and Morgan (1983) have also used the abundances of D and $^4$He to constrain $\delta$; they find $\delta < O(3)$.

## VIII. SUMMARY AND CONCLUSIONS

If the full potential of primordial nucleosynthesis to test the standard model and to probe the early evolution of the universe is to be exploited, a careful and detailed comparison between theory and observation is required. A critical confrontation of the predicted abundances with the presently available data has been the goal of this paper. The standard model (isotropic and homogeneous Friedmann-Robertson-Walker cosmology; conventional particle physics with three species of light, two-component neutrinos) emerges from this comparison with flying colors. Consistent upper and lower bounds to the nucleon abundance are obtained ($\eta_{10} < 7$–10, $\eta_{10} > 3$–4), and a restrictive limit to the number of light neutrino flavors is derived: $N_\nu \lesssim 4$ ($N_\nu = 4$ is excluded if $Y_p < 0.250$). Although small-scale deviations from homogeneity cannot be excluded, the amplitude of such deviations is restricted to $\delta\eta/\eta < O(2)$.

With the possible exception of $^7$Li, the accuracy of the predicted abundances of the light elements far exceeds that of the primordial abundances inferred from currently available data. Clearly, more—and more accurate—observational data coupled to more severe constraints on chemical evolution (astration, stellar production, etc.) would be very valuable. The conclusions of this study are no more certain than the data on which they are based. If the ranges we have used for the abundances should prove to be unduly restrictive, then our bounds, also, will be too constraining. On the other hand, unless the central values for the abundances change dramatically (by an amount larger than the currently adopted uncertainty), our confirmation of the concordance of the standard model and observations will be unaltered.

A major result of our investigation is that upper limits to the presolar abundances of deuterium and helium-3 provides an upper limit to the sum of the primordial abundances of these elements. If the fraction of $^3$He which survives stellar processing in the material returned to the interstellar medium exceeds $\frac{1}{4}$ (see Appendix B), then $[(D + {}^3He)/H]_p < 10 \times 10^{-5}$, implying $\eta_{10} > 3$. If this fraction exceeds $\frac{1}{2}$, then $[(D + {}^3He)/H]_p < 6 \times 10^{-5}$, so that $\eta_{10} > 4$.

The presolar and interstellar estimates of the abundance of deuterium suggest that the primordial abundance exceeds $(D/H)_p > (1$–$2) \times 10^{-5}$. This leads to an upper bound to $\eta$: $\eta_{10} < 7$–10. The recent data (Spite and Spite 1982*a*, *b*) on $^7$Li in Population II stars suggest $\eta_{10} < 7$ and are consistent with $2 < \eta_{10} < 5$.

We cannot help but notice that the upper and lower bounds on $\eta$ seem to be converging. For $\eta_{10} = 5$, the predicted primordial abundances, $(D/H)_p = 3.6 \times 10^{-5}$, $(^3He/H)_p = 1.2 \times 10^{-5}$, and $(^7Li/H)_p = 1.7 \times 10^{-10}$, are remarkably consistent with the observational data. For $\eta_{10} = 5$ and $N_\nu = 3$, the predicted abundance of $^4$He ranges from $Y_p(\tau_{1/2} = 10.4) = 0.246$ to $Y_p(\tau_{1/2} = 10.8) = 0.253$; for $N_\nu = 2$, $Y_p < 0.238$; for $N_\nu = 4$, $Y_p > 0.259$.

Nucleons alone fail to close the universe by at least a factor of 5. For $3$–$4 < \eta_{10} < 7$–10, the nucleon contribution to the universal mass density lies in the range $0.011$–$0.014 < \Omega_N < 0.14$–$0.19$. Although a nucleon-dominated universe is a low-density universe, nucleons can account for the mass inferred from dynamics of the luminous parts of galaxies ($\Omega_{Gal} \approx 0.01$); they may also account for the dark mass inferred on larger scales.

We have come a long way in utilizing primordial nucleosynthesis to test the standard model and to provide constraints on crucial parameters in cosmology ($\eta$) and particle physics ($N_\nu$). Further progress in testing the standard model and in constraining the allowed range of parameters relies on progress in observational and theoretical astronomy and in laboratory experiments. From the nuclear physics laboratories, reduced uncertainties in the neutron half-life and in the rates of the reactions $^4$He($^3$He, $\gamma$)$^7$Be and $^7$Li($p$, $\alpha$)$^4$He would be very valuable. High-energy physics experiments may lead to interesting constraints on $N_\nu$ from the measured width of the $Z^0$. [The width of the $Z^0$ boson can be used to determine the number of neutrino species lighter than $O$(10 GeV).] From the observational side, more and better data on the abundances of the light elements as well as an improved understanding of possible evolutionary effects would be very valuable.

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## APPENDIX A

## UNCERTAINTIES IN THE CALCULATED ABUNDANCES

If primordial nucleosynthesis is to serve as a stringent test of the standard model and as a powerful probe of the early universe, it is important to reexamine the precision of the predictions. This is particularly important if, as is anticipated, the accuracy to which the abundances of the light elements are determined improves. We have already emphasized that the predicted yields are most sensitive to the values of $\eta$, $N_\nu$, and $\tau_{1/2}$. Here we consider their dependence on the adopted values of relevant reaction rates.

Since Wagoner (1973) last updated his code, there have been some changes in a few of the relevant reaction rates (Fowler, Caughlan, and Zimmerman 1975; Caughlan and Fowler 1980). In Table 7 we list those rates which differ from Wagoner (1973). We have incorporated these changes in the code and find relative changes of less than 1% in the primordial abundances of D, $^3$He, and $^4$He. In contrast, the yield of $^7$Li has increased by approximately a factor of 3 (OSSTY). The reason for this large change is as follows: Nucleosynthesis begins in earnest when the temperature drops to $O$(0.1 MeV). First, deuterium is formed via $n(p, \gamma)d$, and then tritium by $d(d, p)t$. The tritium is converted to $^4$He by $t(d, n)^4$He, and at the same time, small amounts of $^7$Li are produced via $^4$He($t$, $\gamma$)$^7$Li. Since 1973, the rate for $t(d, n)^4$He decreased by a factor of $\sim 3$ in the relevant temperature range. This decrease results in more $t$ surviving for synthesis into $^7$Li. In addition, among those reactions which destroy $^7$Li, the rate for $^7$Li($\alpha$, $\gamma$)$^{11}$B decreased by $\sim 20\%$, resulting in more $^7$Li surviving.

How sensitive are the predicted yields of primordial nucleo-

TABLE 7

New Reaction Rates

| Reaction | Analytic Fit to the Reaction Rate[a] |
|---|---|
| $^3H(d, n)^4He$ | $8.09 \times 10^{10} T^{-2/3} \exp[-4.524T^{-1/3} - (T/0.120)^2] \times (1 + 0.092T^{1/3} + 1.80T^{2/3} + 1.16T + 10.52T^{4/3} + 17.2T^{5/3}) + 8.73 \times 10^8 T^{-2/3} \exp(-0.523/T)$ |
| $^3He(n, p)^3H$ | $7.06 \times 10^8 (1 - 0.15T^{1/2} + 0.098T)$ |
| $^3He(\alpha, \gamma)^7Be$ | $5.40 \times 10^6 T^{-2/3} \exp(-12.826T^{-1/3})(1 + 0.033T^{1/3} - 0.350T^{2/3} - 0.080T + 0.056T^{4/3} + 0.033T^{5/3})$ |
| $^3He(d, p)^4He$ | $6.67 \times 10^{10} T^{-2/3} \exp[-7.181T^{-1/3} - (T/0.315)^2] \times (1 + 0.058T^{1/3} - 1.14T^{2/3} - 0.464T + 3.08T^{4/3} + 3.18T^{5/3}) + 4.36 \times 10^8 T^{-1/2} \exp(-1.72/T)$ |
| $^4He(2\alpha, \gamma)^{12}C$ | $2.79 \times 10^{-8} T^{-3} \exp(-4.403/T) + 1.35 \times 10^{-8} T^{-3/2} \exp(-24.811/T)$ |
| $^6Li(p, \gamma)^7Be$ | $5.87 \times 10^5 T^{-2/3} \exp(-8.413T^{-1/3})$ |
| $^6Li(n, \alpha)^3H$ | $2.54 \times 10^9 T^{-3/2} \exp(-2.53/T) + 1.68 \times 10^8 [1 - 0.261(1 + 49.18T)^{-3/2}]$ |
| $^6Li(p, \alpha)^3He$ | $3.80 \times 10^{10} T^{-2/3}(1 + 0.095T)^{-5/6} \exp[-8.41T^{-1/3}(1 + 0.095T)^{1/3}] + 5.97 \times 10^9 T^{-3/2} \exp(-18.30/T)$ |
| $^6Li(\alpha, \gamma)^{10}B$ | $1.27 \times 10^7 T^{-2/3} \exp[-18.79T^{-1/3} - (T/1.327)^2] \times (1 + 0.022T^{1/3} + 1.00T^{2/3} + 0.156T + 1.39T^{4/3} + 0.549T^{5/3}) + 3.18 \times 10^3 T^{-3/2} \exp(-3.484/T) + 1.54 \times 10^4 T^{-1} \exp(-7.212/T)$ |
| $^7Li(p, \alpha)^4He$ | $8.04 \times 10^8 T^{-2/3}(1 + 0.049T^{1/3} + 0.230T^{2/3} + 0.079T - 0.027T^{4/3} - 0.023T^{5/3}) \exp[-8.471T^{-1/3} - (T/30.07)^2] + 1.07 \times 10^{10} T^{-3/2} \exp(-30.443/T) + 1.54 \times 10^6 T^{-3/2} \exp(-4.479/T)$ |
| $^7Li(\alpha, \gamma)^{11}B$ | $2.26 \times 10^8 T^{-2/3} \exp[-19.161T^{-1/3} - (T/0.991)^2] \times (1 + 0.022T^{1/3} + 0.382T^{2/3} + 0.058T + 0.571T^{4/3} + 0.221T^{5/3}) + 1.51 \times 10^3 T^{-3/2} \exp(-3.03/T) + 3.45 \times 10^4 T^{-1/2} \exp(-5.35/T)$ |
| $^7Be(n, p)^7Li$ | $6.76 \times 10^9 (1 - 0.903T^{1/2} + 0.218T)$ |
| $^7Be(p, \gamma)^8B$ | $4.09 \times 10^5 T^{-2/3} \exp(-10.262T^{-1/3}) + 3.30 \times 10^3 T^{-3/2} \exp(-7.306/T)$ |
| $^7Be(\alpha, \gamma)^{11}C$ | $2.12 \times 10^8 T^{-2/3} \exp[-23.212T^{-1/3} - (T/0.658)^2] \times (1 + 0.018T^{1/3} + 0.228T^{2/3} + 0.029T + 0.548T^{4/3} + 0.175T^{5/3}) + 7.27 \times 10^4 T^{-3/2} \exp(-6.53/T) + 1.23 \times 10^5 T^{-1} \exp(-9.742/T)$ |
| $^9Be(p, \alpha)^6Li$ | $2.11 \times 10^{11} T^{-2/3} \exp[-10.36T^{-1/3} - (T/0.520)^2] \times (1 + 0.040T^{1/3} + 1.09T^{2/3} + 0.307T + 3.21T^{4/3} + 2.30T^{5/3}) + 4.51 \times 10^8 T^{-1} \exp(-3.046/T) + 6.70 \times 10^8 T^{-3/4} \exp(-5.160/T)$ |
| $^9Be(p, d)2^4He$ | $2.11 \times 10^{11} T^{-2/3} \exp[-10.36T^{-1/3} - (T/0.520)^2] \times (1 + 0.040T^{1/3} + 1.09T^{2/3} + 0.307T + 3.21T^{4/3} + 2.30T^{5/3}) + 5.79 \times 10^8 T^{-1} \exp(-3.046/T) + 8.50 \times 10^8 T^{-3/4} \exp(-5.800/T)$ |
| $^9Be(p, \gamma)^{10}B$ | $1.33 \times 10^7 T^{-2/3}(1 + 0.04T^{1/3} + 1.52T^{2/3} + 0.428T + 2.15T^{4/3} + 1.54T^{5/3}) \exp[-10.359T^{-1/3} - (T/0.846)^2] + 9.64 \times 10^4 T^{-3/2} \exp(-3.445/T) + 2.72 \times 10^6 T^{-3/2} \exp(-10.62/T)$ |
| $^{10}B(p, \gamma)^{11}C$ | $4.61 \times 10^5 T^{-2/3} \exp[-12.062T^{-1/3} - (T/4.402)^2] \times (1 + 0.035T^{1/3} + 0.426T^{2/3} + 0.103T + 0.281T^{4/3} + 0.173T^{5/3}) + 1.93 \times 10^5 T^{-3/2} \exp(-12.041/T) + 1.14 \times 10^4 T^{-3/2} \exp(-16.164/T)$ |
| $^{10}B(p, \alpha)^7Be$ | $1.26 \times 10^{11} T^{-2/3} \exp[-12.062T^{-1/3} - (T/4.402)^2] \times (1 + 0.035T^{1/3} - 0.498T^{2/3} - 0.121T + 0.300T^{4/3} + 0.184T^{5/3}) + 2.59 \times 10^9 T^{-1} \exp(-12.26/T)$ |
| $^{11}B(p, \gamma)^{12}C$ | $4.62 \times 10^7 T^{-2/3}(1 + 0.035T^{1/3} + 3.00T^{2/3} + 0.723T + 9.91T^{4/3} + 6.07T^{5/3}) \exp[-12.095T^{-1/3} - (T/0.239)^2] + 7.89 \times 10^3 T^{-3/2} \exp(-1.733/T) + 9.68 \times 10^4 T^{-1/5} \exp(-5.617/T)$ |
| $^{11}B(p, \alpha)2^4He$ | $2.59 \times 10^{11} T^{-2/3} \exp[-12.10T^{-1/3} - (T/2.02)^2] \times (1 + 0.035T^{1/3} + 1.22T^{2/3} + 0.295T + 2.15T^{4/3} + 1.32T^{5/3}) + 7.40 \times 10^6 T^{-3/2} \exp(-1.733/T) + 8.14 \times 10^9 T^{-3/2} \exp(-7.177/T) + 1.71 \times 10^9 T^{-2/3} \exp(-12.70/T)$ |

Note.—Those rates not listed in this table either have not changed since Wagoner 1973 or do not noticeably change our calculated abundances. These new rates are from Fowler, Caughlan, and Zimmerman 1975 and Caughlan and Fowler 1980.

[a] The rates are $N_A\langle\sigma V\rangle$ in $cm^3\ s^{-1}\ g^{-1}$; the temperature is in units of $10^9$ K.

TABLE 8

THE SENSITIVITY OF THE YIELDS OF PRIMORDIAL NUCLEOSYNTHESIS TO UNCERTAINTIES (ESTIMATED 1 $\sigma$) IN NINE KEY REACTION RATES

| | Adopted Rate | $\eta_{10} = 1$ | | | $\eta_{10} = 10$ | | |
|---|---|---|---|---|---|---|---|
| Reaction | Nominal Rate | $X_2$ | $X_3$ | $X_7$ | $X_2$ | $X_3$ | $X_7$ |
| $p(n, \gamma)d$ | 1.1 | 93 | 101 | 91 | 99 | 101 | 109 |
| | 0.9 | 108 | 101 | 110 | 102 | 99 | 90 |
| $d(d, n)^3$He | 1.1 | 96 | 103 | 101 | 95 | 101 | 105 |
| | 0.9 | 104 | 97 | 99 | 106 | 99 | 94 |
| $d(d, p)t$ | 1.1 | 96 | 95 | 102 | 95 | 98 | 100 |
| | 0.9 | 104 | 105 | 98 | 105 | 102 | 100 |
| $t(d, n)^4$He | 1.1 | 100 | 100 | 91 | 100 | 100 | 100 |
| | 0.9 | 100 | 100 | 110 | 100 | 100 | 100 |
| $^3$He$(d, p)^4$He | 1.1 | 100 | 100 | 100 | 100 | 99 | 100 |
| | 0.9 | 100 | 100 | 100 | 100 | 101 | 100 |
| $^4$He$(t, \gamma)^7$Li | 1.1 | 100 | 100 | 110 | 100 | 100 | 100 |
| | 0.9 | 100 | 100 | 90 | 100 | 100 | 100 |
| $^4$He$(^3$He$, \gamma)^7$Be | 2.0 | 100 | 100 | 102 | 100 | 100 | 196 |
| | 1.29 | 100 | 100 | 101 | 100 | 100 | 128 |
| | 1.07 | 100 | 100 | 100 | 100 | 100 | 107 |
| | 0.86 | 100 | 100 | 99 | 100 | 100 | 86 |
| | 0.5 | 100 | 100 | 98 | 100 | 100 | 51 |
| $^7$Li$(p, \alpha)^4$He | 2.0 | 100 | 100 | 38 | 100 | 100 | 99 |
| | 0.5 | 100 | 100 | 274 | 100 | 100 | 101 |
| $^7$Be$(n, p)^7$Li | 1.13 | 100 | 100 | 100 | 100 | 100 | 94 |
| | 0.94 | 100 | 100 | 100 | 100 | 100 | 103 |
| | 0.75 | 100 | 100 | 100 | 100 | 100 | 115 |

NOTE.—The yields of D, $^3$He, and $^7$Li are normalized to 100 for the nominal values of the rates. For these calculations, $N_\nu = 3$ and $\tau_{1/2} = 10.6$ minutes were adopted. In all cases, $Y_p$ changes by less than 1%.

synthesis to possible future changes in the reaction rates? In an attempt to answer this question quantitatively, we varied nine of the key reaction rates by a 1 $\sigma$ uncertainty (adopted from Caughlan and Fowler 1980). For $Y_p$ the relative change was less than 1%, and the relevant changes in D and $^3$He were less than 10%. In contrast, changes in the predicted abundance of $^7$Li as large as a factor of 2 were found when the rates for $^7$Li$(p, \alpha)^4$He and $^4$He$(^3$He$, \gamma)^7$Be were varied by 1 $\sigma$. More recent data on this latter reaction (Harris *et al.* 1983) suggest a central rate close to the central rate we have adopted. Fowler (1983) prefers a rate 1.07 times our adopted rate and estimates the 1 $\sigma$ uncertainty to be $\pm 20\%$. For completeness, we have in Table 8 shown the results for the "new" and the "old" standard rates and for the "new" and the "old" estimated uncertainties. To understand this result, note that for large values of $\eta(>3 \times 10^{-10})$ $^7$Li is produced mainly via $^4$He$(^3$He$, \gamma)^7$Be followed by $^7$Be beta-decaying to $^7$Li. For $\eta > 3 \times 10^{-10}$, doubling this rate (the estimated 1 $\sigma$ uncertainty) doubles the yield of $^7$Li. For smaller values of $\eta(<3 \times 10^{-10})$, $^7$Li is mainly produced directly by $^4$He$(t, \gamma)^7$Li. In this regime ($\eta < 3 \times 10^{-10}$), the amount of $^7$Li which survives is most sensitive to the rate of the reaction $^7$Li$(p, \alpha)^4$He, which has a large uncertainty. As Beaudet and Reeves (1983) have noted, the $^7$Li yield is also sensitive to the rate of the $^7$Be$(n, p)^7$Li reaction. At high values of $\eta$ the $^7$Be converted to $^7$Li will be destroyed and will not, therefore, be available to free decay to $^7$Li later—when the $^7$Li can survive. In contrast to Beaudet and Reeves (1983), who assign a factor of 2 uncertainty to this reaction, Fowler (1983) estimates an uncertainty no bigger than $\pm 20\%$ about a "new" standard value which is 6% below that in Table 7. The effect on the $^7$Li yield of the "new" rate and its uncertainty is shown in Table 8. In Table 8 we summarize the sensitivity of the predicted abundances of D, $^3$He, and $^7$Li to the nine key reactions.

Recently, Dicus *et al.* (1982) (see also Cambier, Primack, and Sher 1983) have calculated the corrections to the predicted abundance of $^4$He resulting from radiative, Coulomb, and finite temperature effects in the rates for the weak reactions: $n \leftrightarrow p + e^- + \bar{\nu}_e$, $n + e^+ \leftrightarrow p + \bar{\nu}_e$, $n + \nu_e \leftrightarrow p + e^-$. For the following range of parameters, $\eta_{10} = 0.3$–30, $N_\nu = 2$–10, $\tau_{1/2} =$ 10.1–11.1 minutes, they find a *systematic decrease* in $Y_p$ of $\sim 0.003$; they also find relative changes of about 1%–2% in the abundances of D, $^3$He, and $^7$Li. The values of $Y_p$ presented in this paper *include* the $-0.003$ correction of Dicus *et al.* (1982).

In summary, for a specific choice of values of $\eta$, $N_\nu$, and $\tau_{1/2}$, the calculated abundance of $^4$He should be accurate to $\sim 1\%$ ($\Delta Y \approx \pm 0.002$), those of D and $^3$He should be accurate to $\sim 10\%$. The predicted abundance of $^7$Li is not more reliable than a factor of 2; for $^7$Li, then, the accuracy of the observational data may exceed that of the predicted abundance.

## APPENDIX B

### THE SURVIVAL OF $^3$He DURING GALACTIC EVOLUTION

Deuterium is burned to $^3$He via D$(p, \gamma)^3$He whenever the temperature exceeds $\sim 6 \times 10^5$ K. Since pre-main-sequence collapse is convective, sufficiently high temperatures will be reached to convert any primordial deuterium to helium-3 in the material that has been cycled through stars. Helium-3 can be destroyed by $^3$He$(^3$He$, ^4$He$)2p$ and $^3$He$(\alpha, \gamma)^7$Be. In addition, however, $^3$He is produced during proton-proton burning by the sequence of reactions $p + p \rightarrow \mathrm{D} + e^+ + \nu_e$, $\mathrm{D} + p \rightarrow {}^3\mathrm{He} + \gamma$. At high temperatures, as the $^3$He production and destruction rates drive the $^3$He abundance toward its equilibrium value, the $^3$He abundance will be very small. At low temperatures, however, the time to achieve equilibrium exceeds the lifetime of the star, and the $^3$He abundance can be large. For temperatures above $\sim 7 \times 10^6$ K, $^3$He destruction will be faster than $^3$He production. Above $\sim 10^8$ K, $^4$He is burned to $^{12}$C and $^{16}$O by the $3\alpha$ reaction and by $^{12}$C$(\alpha, \gamma)^{16}$O. The following general picture emerges: Virtually all the prestellar D is burned to $^3$He either during the pre-main-sequence collapse or in those zones of the star with temperatures which exceed $\geq 6 \times 10^5$ K (since the base of the outer convective zones of low-mass stars is at least this hot, D is burned to $^3$He even in low-mass stars); $^3$He will survive (and even be *produced* by *p-p* burning) in those zones where $T \lesssim 7 \times 10^6$ K; for those zones where $7 \times 10^6 \lesssim T \lesssim 10^8$ K, $^3$He is burned to $^4$He; for those zones where $T > 10^8$ K, $^4$He is converted to $^{12}$C and $^{16}$O.

Since *some* $^3$He will survive stellar processing, an estimate of the fraction that survives, $g$, will enable us to place an upper limit on the primordial abundance of D plus $^3$He. To quantify this effect, suppose that $f$ is the fraction of the interstellar gas, at any time $t$, that has never been through stars (more precisely, the fraction that has not been at temperatures hotter than $6 \times 10^5$ K).

Then, the primordial abundance of D is related to that at time $t$ by

$$(\mathrm{D/H})_t = f(\mathrm{D/H})_p\,. \tag{B1}$$

Since some $^3$He survives stellar processing, the abundance of $^3$He at time $t$ is

$$(^3\mathrm{He/H})_t \geq f(^3\mathrm{He/H})_p + g(1-f)[(\mathrm{D} + {}^3\mathrm{He})/\mathrm{H}]_p\,. \tag{B2}$$

The inequality in equation (B2) arises from the neglect of any newly synthesized $^3$He from $p$-$p$ burning. Equations (B1) and (B2) may be combined to yield

$$\left(\frac{\mathrm{D} + {}^3\mathrm{He}}{\mathrm{H}}\right)_p \leq \left(\frac{\mathrm{D}}{\mathrm{H}}\right)_t + \frac{1}{g(^3\mathrm{He/H})_t} - \left(\frac{1-g}{g}\right) f \left(\frac{^3\mathrm{He}}{\mathrm{H}}\right)_p\,. \tag{B3}$$

Since the last term on the right-hand side of equation (B3) is negative (and, in general, small), we may reinforce the inequality by neglecting it,

$$\left(\frac{\mathrm{D} + {}^3\mathrm{He}}{\mathrm{H}}\right)_p \leq \left(\frac{\mathrm{D}}{\mathrm{H}}\right)_t + \frac{1}{g(^3\mathrm{He/H})_t}\,. \tag{B3$'$}$$

Although the constraint on the primordial abundances in equation (B3′) is weaker than that in equation (B3), equation (B3′) permits us to avoid specifying the model-dependent quantity $f$. An estimate of (or, lower limit to) $g$ permits us to bound from above the primordial abundance of D plus $^3$He. In constraining $g$ from below, we will proceed from more general to more model (of galactic chemical evolution) dependent results.

The models of Iben (1967$a$, $b$) and Rood (1972) show that low-mass stars ($M < 2\,M_\odot$) are net producers of $^3$He. Of the material these stars return to the interstellar medium, more has been processed at low temperatures ($< 7 \times 10^6$ K) than at high temperatures. These results were used by Rood, Steigman, and Tinsley (1976) to predict that the $^3$He abundance should have increased since the formation of the solar system. Detections of $^3\mathrm{He}^+$ by Wilson, Rood, and Bania (1983) in several H II regions seem to support this prediction and provide some confidence in the models on which it was based. For stars less massive than 8 $M_\odot$, Iben and Truran (1978) find a relationship between the abundance of $^3$He returned to the interstellar medium and the D plus $^3$He abundance in the gas from which the star formed (their eq. [18]),

$$(^3\mathrm{He/H})_f = 0.7[(\mathrm{D} + {}^3\mathrm{He})/\mathrm{H}]_i + 18 \times 10^{-5} M^{-2}\,. \tag{B4}$$

In equation (B4), $M$ is the mass of the star ($M < 8$) in units of the solar mass. Thus, for low-mass stars, we may conclude that $g \gtrsim 0.7$.

Now let us turn to high-mass stars ($8 < M < 100$). The massive star models (with mass loss) computed by Dearborn *et al.* (1978) were examined to determine the survival of $^3$He. According to Dearborn (1983), for stars in this mass range, more than one-fourth (by mass) of the star is in a zone which has never been hotter than $7 \times 10^6$ K. Using the Snow and Morton (1976) mass loss rates, at least this much mass is lost prior to the traverse of the supernova shock wave. For massive stars then, we conclude that $g \gtrsim \frac{1}{4}$. This conclusion is supported by the recent calculations (without mass loss) of Brunish and Truran (1984), who find for $10 < M < 50$, $g \approx 0.25$.

Since we are ignorant of the IMF for the earliest generations of stars (Population III), our first estimate of the survival of $^3$He should be on a star-by-star basis. For "normal" stars with $M < 100$, the model calculations discussed above suggest that $g > \frac{1}{4}$. Unless exotic stars are postulated for early galactic evolution, we consider this constraint to be conservative.

Most IMFs give greater weight to low-mass stars in the contribution to material returned to the interstellar medium. For a Salpeter (1955) IMF, Brunish and Truran (1984) find for a single generation of stars ($0.3 < M < 50$)

$$(^3\mathrm{He/H})_f \approx 1 \times 10^{-5} + 0.5[(\mathrm{D} + {}^3\mathrm{He})/\mathrm{H}]_i\,. \tag{B5}$$

If most of the interstellar material has, on average, been cycled through at most one generation of stars (i.e., the fraction which has been through several generations is small), then it is likely that $g \gtrsim \frac{1}{2}$. This conclusion receives some support when we consider the buildup of $^4$He and the heavier elements in the course of galactic chemical evolution.

Of the material returned by stars, some has come from regions which are sufficiently cool that $^3$He survives; in the rest of the material, $^4$He and heavier elements have been produced. Since observations constrain the amount of $^4$He and heavier nuclei synthesized in the course of galactic evolution, we may derive a limit to the destruction of $^3$He. For the fraction by mass of $^4$He (heavier nuclei) produced from the matter which has been cycled through stars, write $\Delta Y_*(\Delta Z_*)$. The abundances at time $t$ are

$$Y_t = Y_p + (1-f)\Delta Y_*\,; \qquad Z_t = (1-f)\Delta Z_*\,. \tag{B6}$$

The amount of stellar-produced $^4$He is $\Delta Y_t = Y_t - Y_p = (1-f)\Delta Y_* = Z_t(\Delta Y_*/\Delta Z_*)$. Note that $\Delta Z_*$ and $\Delta Y_*$ could be large (e.g., $\Delta Y_* + \Delta Z_* \approx 1$) but $\Delta Y_t$ and $Z_t$ small if $1-f$, the fraction of interstellar matter which has been cycled through stars, is small ($f \approx 1$). Now, recall that $g$ is the fraction of the matter cycled through stars in which $^3$He survives so that

$$g \approx 1 - (\Delta Y_* + \Delta Z_*) = 1 - (1-f)^{-1}(\Delta Y_t + Z_t)\,. \tag{B7}$$

In the above it has been assumed that where $^3$He has been destroyed, all the material has been burned completely to $^4$He and/or heavier nuclei. Indeed, stellar models indicate that stars spend sufficient time on the main sequence that those regions which are hot enough to burn away the $^3$He will convert any initial hydrogen to $^4$He and beyond. However, to allow for the possibility of incomplete burning (i.e., only partial burning to $^4$He and beyond in those zones where $^3$He has been destroyed) we write

$$g \approx 1 - (1 + \alpha)(\Delta Y_* + \Delta Z_*) = 1 - (1 - f)^{-1}(1 + \alpha)(\Delta Y_t + Z_t) . \tag{B8}$$

Equation (B8) may be rewritten to relate the unknown quantities $f$ and $g$ to the potentially observable quantities $\Delta Y_t$ and $Z_t$:

$$f + (1 - f)g = 1 - (1 + \alpha)(\Delta Y_t + Z_t) . \tag{B9}$$

If we take the sum of equations (B1) and (B2), we find

$$\left(\frac{\mathrm{D} + {}^3\mathrm{He}}{\mathrm{H}}\right)_t \geq [f + (1 - f)g]\left(\frac{\mathrm{D} + {}^3\mathrm{He}}{\mathrm{H}}\right)_p , \tag{B10}$$

so that

$$\left(\frac{\mathrm{D} + {}^3\mathrm{He}}{\mathrm{H}}\right)_t \geq [1 - (1 + \alpha)(\Delta Y_t + Z_t)]\left(\frac{\mathrm{D} + {}^3\mathrm{He}}{\mathrm{H}}\right)_p . \tag{B11}$$

At present or at the time of the formation of the solar system, it is likely that $\Delta Y + Z \approx 0.05$–$0.1$. As a result, we expect (unless $\alpha$ is large) that the sum of the primordial abundances of D and $^3$He is close to the present (or presolar) sum of abundances.

To summarize, some $^3$He survives the evolution of all standard stars ($M < 100$). Even for a pregalactic generation of stars, star by star (independent of the IMF), we expect that $g > \frac{1}{4}$. Low-mass stars preserve more of their initial $^3$He and may even be net producers of $^3$He. For most IMFs, which give significant weight to the low-mass end, we expect that $g > \frac{1}{2}$. Indeed, even more $^3$He might survive than these estimates suggest. Since $^4$He and heavier nuclei are produced in those zones of a star where $^3$He is destroyed, the present abundances of $^4$He and the heavier nuclei may constrain even more severely the destruction of $^3$He. For $\Delta Y_0 + Z_0 < 0.1$ and $\alpha < 1$, more than 80% of the primordial D plus $^3$He should be present today.

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