THE ASTROPHYSICAL JOURNAL, **281**: 67–75, 1984 June 1 © 1984. The American Astronomical Society. All rights reserved. Printed in U.S.A

## MASS LOSS IN A AND B SUPERGIANTS AND THE EXTRAGALACTIC DISTANCE SCALE

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### ABSTRACT

Samples of B5 and A0 stars in the Large Magellanic Cloud (LMC) demonstrate the existence of tight correlations between luminosity and equivalent widths in the H $\alpha$  and H $\beta$  lines. The H $\alpha$  line is in emission for stars brighter than  $M_v = -7$ , and this easily identifiable feature should be detectable at the distances of nearby groups of galaxies. The correlations imply that mass loss in A and B supergiants is strongly dependent on luminosity and therefore on stellar mass.

Similar samples of stars in the Small Magellanic Cloud (SMC) show systematically smaller H $\alpha$  emission and more scatter in the relationships between luminosity and H $\alpha$  line strengths than were found for the LMC stars. There is independent evidence that mass-loss rates are smaller in the SMC than in the LMC, and this fact probably accounts for the lower emission at H $\alpha$  in the SMC stars. The differences between the samples in the two clouds may be caused by differences in stellar chemical composition.

Subject headings: cosmology — galaxies: Magellanic Clouds — stars: early-type — stars: emission-line — stars: mass loss — stars: supergiants

### I. MOTIVATION

Supergiant stars of spectral types A and B are among the most luminous visual objects in galaxies. They are abundant in systems actively forming stars. Often enough, they lie in relatively unconfused and unreddened environments because they are old enough to have migrated from their places of birth. If they participate in multiple systems, they are likely to dominate their companions in visual luminosity at this moment in their evolution. These kinds of stars are easy to identify because of a pronounced Balmer jump and strong Balmer absorption lines. In sum, if one is looking for a standard candle to address the problem of distances to nearby galaxies, it would be hard to identify a more desirable candidate than A–B supergiants.

It is known that these stars possess luminosity-dependent characteristics. For one thing, surface gravity is lower in a higher luminosity star. Consequently, the absorption line wings of the Balmer lines, and hence the equivalent widths, decrease in strength with increasing luminosity. A number of methods that utilize this characteristic have been used to provide distances within our own Galaxy and to the Magellanic Clouds. For example, there had been direct measurements of the equivalent widths of the Hy line (Petrie 1953, 1965; Hutchings 1966; Balona and Crampton 1974; Crampton 1979; Azzopardi 1981), and calibration of a narrow-band index at  $H\beta$  (Crawford 1958; Westerlund, Danziger, and Graham 1963; Osmer 1973). Measurements of the cumulative effects of absorption in the higher order Balmer lines (the BCD method named after Barbier, Chalonge, and Divan: Chalonge and Divan 1973; Divan 1973) also can be used to infer absolute magnitudes.

If observations of the Balmer lines are to be used to obtain the distances of stars in galaxies beyond the Magellanic Clouds, they will necessarily be restricted to stars of very high luminosity. There are very few such stars in our own Galaxy that are relatively unreddened and that have accurately known

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distances. To obtain a large enough sample to calibrate luminosity effects among the brightest stars, we must go to the Magellanic Clouds. These galaxies contain hundreds of suitable stars at common distances and with little reddening. The differences between the Large and Small Cloud can be exploited to test for metallicity factors. We are looking for a method that is useful at distance moduli between 24 and 29 mag, where the *brightest* A–B stars will have visual magnitudes between 15 and 20 mag. The luminosity-dependent characteristics will have to be gross features, easily discerned at low dispersions.

With these ideas in mind, we observed 142 A–B stars in the LMC and 85 in the SMC, over a range in luminosity from -9.5 to -5 mag. Since the most promising luminosity effect was expected to be associated with the surface gravity dependence, our initial observations focused on the Balmer decrement from H $\gamma$  through to the Balmer discontinuity. This strategy proved to be a mistake. There certainly are luminosity dependencies in the higher order Balmer absorption lines, but there is something better.

We had been worrying about the effects of emission from stellar winds on the Balmer absorption profiles, as mass loss is a common characteristic of supergiant stars. If there is an upper envelope to bolometric luminosities for spectral types more advanced than B2, as appears to be empirically established (Humphreys and Davidson 1979), then the highest luminosity stars might have to lose a great deal of mass to evolve under the envelope. To monitor this effect, we observed 70 stars in our LMC sample and 43 stars in the SMC sample at H $\alpha$  and H $\beta$ .

We found very strong luminosity dependencies in both these lines. With increasing luminosity, the core of the H $\alpha$  line progressively weakens in absorption until, about  $M_v \approx -7$ , the core goes into emission. The H $\beta$  line follows a similar progression, but only reaches emission in the very highest luminosity stars. In retrospect, these properties are known and have been reasonably well studied. The earliest references that we find regarding the luminosity dependence of H $\alpha$  are by Butler and Seddon (1958) and Apt (Weymann 1963). Detailed studies of the H $\alpha$  line in Galactic supergiants have been conducted by Andrews (1968), Rosendhal (1973), and Ebbets (1982).

The present paper will describe in detail the relationships that we found involving H $\alpha$  and H $\beta$ . These relationships are of interest both with regard to the extragalactic distance scale problem and with regard to the problem of mass loss in supergiant stars. The analysis is preliminary in that more precise spectral classifications and reddening corrections will become available with the analysis of the spectrophotometry that we have already obtained at shorter wavelengths for a much larger sample of supergiants. In a later paper, we will present an analysis of the higher order Balmer lines, and we intend to extend our observations at H $\alpha$  and H $\beta$  to include all the objects already observed in the blue.

#### **II. OBSERVATIONS**

All the observations at H $\alpha$  and H $\beta$  were made with the R-C spectrograph and 40 mm ultraviolet SIT-vidicon detector on the 4 m telescope at Cerro Tololo Inter-American Observatory on 1982 December 8. Standard stars from the lists by Stone and Baldwin (1983) were observed in order to transform observed signals to relative fluxes. Atmospheric extinction corrections were applied with a standard CTIO program. A slit width of 1"5 projected on the sky was used. The spectrograph was rotated frequently to maintain a slit position perpendicular to the horizon to minimize uneven slit illumination as a function of wavelength as a consequence of atmospheric refraction. The data were smoothed to a spectral resolution of 6 Å. Integration times were varied from a few seconds to a few minutes depending on star brightness, so that the signal-tonoise ratio was always roughly the same.

Program stars were drawn from the list of LMC supergiants compiled by Rousseau *et al.* (1978) and the list of SMC supergiants compiled by Azzopardi and Vigneau (1975). Members of known multiple systems and stars in crowded regions were avoided. For our sample, we required that spectral classification and apparent magnitude information be available. In the case of the LMC, this information was taken from Rousseau *et al.* (1978). For the SMC, spectral types are from Azzopardi and Vigneau (1975) or the compilation by Humphreys (1983) and magnitudes are from Azzopardi and Vigneau (1975), Ardeberg and Maurice (1977), or Ardeberg (1980).

Stars were selected in a number of specific spectral bins, and uniformly in luminosity over an interval from  $M_v \sim -5$  to -9.5. From the SMC, 20 stars of type B8 and 23 stars of type A0 were selected for the program. In the LMC, 20 B5 stars, 22 A0 stars, and 12 A5 stars were observed. In addition, almost all remaining cataloged stars in the LMC brighter than  $M_v \sim -7$ in the range B5–A5 were observed. Some stars in our simple were known to have emission line characteristics, and others have been cited as examples of stars with anomalously broad Balmer absorption lines (Fehrenbach and Duflot 1972; Humphreys 1983). We neither favored nor avoided these kinds of stars, but tried to include them in rough proportion to their preponderance.

Equivalent widths in the H $\alpha$  and H $\beta$  lines for the stars in our samples are given in Table 1 (LMC) and Table 2 (SMC). The H $\alpha$  lines characteristically contain both a narrow core in emission or absorption and broad wings in emission. Consequently, we have tabulated both equivalent widths determined in a 20 Å window, which is a measure of the core line strength, and equivalent widths in a 75 Å window (-50 Å, +25 Å with respect to the central wavelength), which should give the total flux in the line.

Typical H $\alpha$  profiles at different luminosity intervals are illustrated in Figure 1. At high luminosities, a very asymmetric, broad component is inevitably apparent, stretching 40 Å or more to the blue and 15 Å to the red of the narrow component. The asymmetric broad component in emission is almost always present even for the lowest luminosity stars in our samples. Unfortunately, our signal-to-noise ratio is too poor to allow us to measure the equivalent widths in these broad but weak features with suitable accuracy. The relationships discussed in the following sections pertain to the *narrow* components only. Luminosity correlations also are found in the total profiles but the scatter is inevitably greater. For the moment, it cannot be decided whether this increased scatter is intrinsic or observational.

We are not aware of any discussion in the literature concerning the extremely broad and asymmetric component to the H $\alpha$ profiles. Such luminous stars have not received much quantitative attention at sufficiently low dispersions. To explain the widths of the lines in terms of a stellar wind, one would have to postulate that mass flows in A supergiants regularly attain



FIG. 1.—Representative H $\alpha$  spectra of A0 supergiants in the LMC. The stars are ordered by decreasing luminosity from the top star at  $M_v \approx -9$  to the bottom star at  $M_v \approx -5$ . Equivalent widths  $W_{\alpha}^{20}$  and  $W_{\alpha}^{75}$  were measured in the 20 Å and 75 Å windows indicated by the long dash and short dash lines, respectively.

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TABLE	1		
DOLANTS IN	тиг	Ŧ	N

		SUPER	RGIANTS IN '	THE LMC			
Name	V	B - V	V <sub>0</sub>	$W_{\alpha}^{75}$	$W_{\alpha}^{20}$	$W_{\beta}^{20}$	Туре
·······		20 $(B-V)$	B5 Stars $h_0 = -0.08$				
11.65	11.66	0.04	11.54	6.1	2.5	1 1	
50-66	10.63	+0.04	10.33	-6.4	-2.3 -3.3	-0.3	
69-66	13.17	+0.04	12.81	-1.7	0.9	2.4	
107-66	11.86	+0.01	11.59	- 5.2	-1.2	2.0	
125-66	12.74	0.08	12.74	-0.6	0.8	2.2	
58-67	11.34	0.00	11.10	-4.0	-0.9	0.4	
66-67	11.55	0.01	11.34	- 3.9	-0.3	1.2	
126-67	11.39	-0.01	11.18	- 5.4	- 2.0	1.3	
222-07	12.19	0.01	12.28	- 7.2	-1.0	2.7	
C 11	13.62	+0.01	13.26	-56	0.1	3.0	
82-68	9.88	-0.02	9.70	-11.2	- 5.3	5.0	
154-68	13.12	+0.03	12.79	0.7	1.5	2.3	
179-68	11.73	0.04	11.61	-6.8	-2.4	1.4	
92-69	10.68	+0.10	10.14	-19.7	-13.5	-2.7	
100-69	11.74	-0.02	11.56	- 3.2	-0.3	1.1	
143-69	10.81	+0.11	10.24	-10.5	- 5.9	-0.8	
278-69	12.70	+ 0.09	12.19	-4.4	-0.4	1.5	
102-70	12.36	0.00	12.12	-2.8	-0.5	2.3	
	11.59	-0.01	11.38	- 3.4	-1.2	0.5	
		(B-V)	$A_0 \text{ stars})_0 = +0.02$	* 40			
53-65	10.52	+0.13	10.19	-12.8	-6.8	-0.6	
67-65	11.44	+0.05	11.35	-3.6	-0.7	1.7	
G 431	13.40	+0.08	13.22	2.0	3.4	5.8	
12-66	10.81	+0.03	10.78	-6.5	-2.9	0.9	
29-66	11.59	+0.08	11.41	-2.3	-0.1	2.4	
154-66	10.81	+0.12	10.51	-4.1	-0.8	0.7	
11-67	12.08	+0.10	11.84	- 3.2	0.3	2.8	÷
6/-6/	11.34	+0.10	11.10	-4.3	- 1.9	1.4	
143-07	12 72	+0.03	12.66	- 2.8	-0.5	1.9	
201-67	9.88	+0.04	0.70	-10.1	-66	_09	
282-67	13 74	+0.03	13 71	59	6.0	6.0	
29-68	11.76	+0.07	11.61	-3.0	-0.6	2.3	
66-68ª	12.69	+0.11	12.42	-44.0	- 39.9	0.8	
84-69	12.13	+0.08	11.95	-3.5	-0.6	0.9	
170-69	10.34	+0.13	10.01	-8.2	-4.5	-0.3	
239-69	10.24	+0.35	9.25	14.0	-9.3	-0.9	
8-70	13.72	-0.02	13.72	-3.7	2.5	4.0	
52-70	13.00	+0.04	12.94	-2.9	1.0	2.9	
122-70	13.80	+0.01	13.80	-0.7	2.9	4.4	
G 117	14.0	+0.15	13.61	3.0	4.5	6.8	
14-/1	10.62	+ 0.09	10.41	- 9.0	- 3.3	0.5	
	*	(B-V)	$A_0 = +0.10$	)			
102-66	11.76	0.13	11.67	1.2	3.8	5.2	
G 140	13.23	0.04	13.23	1.1	4.3	6.3	
G 338	12.20	0.09	12.20	2.9	3.3	4.9	
208–67	10.85	0.16	10.67	3.1	2.9	3.9	
G 465	12.82	0.18	12.58	3.6	4.5	6.2	
G 8	13.92	0.14	13.80	1.8	4.9	6.8	
93-09	10.30	0.25	9.83	-0.2	- 1.3	2.0	
182-60	10.54	0.19	10.40	- 15.0	- 0.0 2 2	-0.1	
G 424	10.99	0.10	10.03	-121	_94	+.5 1 2	
262-69	11.42	0.36	10.64	-1.2	0.1	2.8	
G 441	12.30	0.14	12.18	2.1	4.6	5.9	
		16 Additiona	al Stars B5	-A5 with V	< 11		
58-66	10.24	+0.10	9.94	- 5.6	-2.6	-0.7	B9
17-67	9.69	+0.09	9.42	-12.3	-9.1	-1.2	<b>B9</b>
44-67	9.12	+0.19	8.79	-9.5	-4.9	0.5	A3
122-67	10.88	0.00	10.70	- 5.9	- 1.9	1.6	.B6
204-08	10.87	+ 0.04	10.75	-4.5	- 1.1	0.5	B0 B0
207-07	10.51	- v.vu	10.71	- 0.0	- 5.4	0.4	לע

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TABLE 1—Continued

Name	V	(B-V)	V <sub>0</sub>	$W_{\alpha}^{75}$	$W_{\alpha}^{20}$	$W_{\beta}^{20}$	Туре
93-68	10.74	+0.13	10.47	- 7.9	-1.5	1.9	A1
7-69	10.46	+0.17	9.95	- 7.0	-4.0	0.7	B9
16-69	10.72	+0.22	10.06	-27.5	-22.6	-4.5	B9
75-69	10.76	+0.08	10.37	-22.0	-17.0	-3.3	<b>B</b> 7
82-69	10.92	+0.04	10.72	- 3.4	-1.4	0.8	<b>B</b> 8
171-69	10.28	+0.24	9.68	-11.2	-7.5	-0.1	A1
211-69	10.36	+0.09	10.02	-10.3	- 5.7	-0.5	<b>B</b> 8
247-69	10.42	+0.17	9.73	-15.5	-9.8	-1.5	<b>B</b> 6
299-69	10.24	+0.23	9.67	- 7.9	- 4.8	0.5	A1

<sup>a</sup> Anomalous emission at Hα.

Names: Those stars numbered by declination zone are from Sanduleak (1969). The G series stars are from Fehrenbach and Duflot (1970). The single C series star is from Ardeberg *et al.* (1972).

Equivalent widths: Measured in either a 20 Å window ( $W^{20}$ ) or an asymmetric 75 Å window ( $W^{75}$ ), in angstroms.

velocities of 2000 km s<sup>-1</sup>. Such a high value is completely at variance with other evidence that indicates that winds around A supergiants are characterized by velocities of only a few hundred kilometers per second (Praderie, Talavera, and Lamers 1980; Kunasz and Morrison 1982). More study of the morphology of H $\alpha$  is clearly warranted.

The five stars with far-and-away the strongest emission at H $\alpha$  are *not* extremely luminous. Since these stars have characteristics that do not obey the relationships defined by the rest of our samples, we say these stars have "anomalous" H $\alpha$  emission. Curiously, four of the five have  $M_v = -6.0 \pm 0.2$ . We offer no explanation for why these stars are unusual.

#### III. HARD FACTS

#### a) The Large Magellanic Cloud

Our most important results are displayed graphically in Figures 2–5. The B5 star material is seen in Figure 2, the A0 in

Figure 3, and the A5 in Figure 4. The almost complete sample of stars B5–A5 with V < 11 is shown in Figure 5. In each case, the data at H $\alpha$  is displayed in the upper panel, and the data at H $\beta$  is seen in the lower panel. We plot equivalent width in a 20 Å window versus visual magnitude, corrected for reddening. The reddening corrections have been estimated by adopting intrinsic colors given by Flower (1977) with the assumption that  $A_v = 3.0E(B-V)$ . Negative equivalent widths correspond to lines in emission. The rough absolute magnitude scale assumes  $(m - M)_0 = 18.6$  and  $A_v = 0.4$  for the LMC.

Tight correlations are found in all the plots. The following properties are noted. (a) The H $\alpha$  relationships for the B5-A0 stars pass into emission around  $M_v \sim -7$ , while at H $\beta$  these stars pass into emission at  $M_v \sim -8.5$ . (b) At B5-A0, the rms scatter in the relationships is 0.3-0.4 mag. (c) There is only a modest temperature dependence in the H $\alpha$  relationships over the interval B5-A0. By A5 the emission effect is greatly reduced.



FIG. 2.—B5 stars in the LMC. The top panel illustrates the dependence of the equivalent width in the core of the H $\alpha$  line,  $W_{\alpha}^{20}$ , on visual magnitude. The H $\beta$  dependence is shown in the bottom panel.

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1984ApJ...281...67T

TABLE 2

Name	V	B-V	Vo	$W_{\alpha}^{75}$	$W_{\alpha}^{20}$	$W_{\beta}^{20}$
		20 B	8 Stars			
		$(B-V)_0$	= -0.02	5		
8	13.53	-0.06	13.53	-0.6	2.2	2.3
19	13.00	+0.00	12.92	1.2	2.8	3.5
20	12.14	+0.30	11.16	- 5.3	-0.8	1.5
31	12.52	+0.28	11.60	-3.0	0.4	2.1
45	14.15	+0.04	13.96	1.9	3.5	2.8
72	12.95	-0.07	12.95	-0.6	1.7	2.3
98	11.45	+0.06	11.20	-3.3	-0.2	1.3
122	12.79	+0.03	12.62	-4.2	-1.0	1.2
153	13.58	+0.00	13.50	-8.6	-1.2	3.0
172ª	13.37	+0.06	13.12	-82.4	-69.1	-3.5
184	14.18	-0.08	14.18	-0.5	1.2	3.1
185ª	13.28	+0.04	13.08	-63.5	-57.6	-3.4
200	12.10	+0.07	11.82	- 3.1	-0.7	1.2
2001 241ª	14 44	+0.07	14 16	- 58.8	-51.3	-1.2
263	12.85	+0.00	12.78	0.8	2.8	2.9
315	10.92	+0.09	10.58	-57	-21	11
387	11.40	$\pm 0.04$	11 20	-50	-11	1.2
<i>4</i> 15	10.58	+0.07	10.30	- 19.1	-123	-18
415	12.61	-0.06	12.61	-01	2.3	2.2
474	14 35	-0.09	14 35	24	3.6	3.6
+/+	14.55	0.07	14.55	2.1	5.0	510
		23 A	0 Stars			
		(B-V)	$_{0} = +0.02$	2		
49	13.92	(B-V) + 0.01	$\frac{0}{13.92} = +0.02$	0.6	3.0	3.0
49	13.92 12.96	(B-V) + 0.01 + 0.16	0 = +0.02 13.92 12.54	0.6 0.6	3.0 1.0	3.0 2.3
49 53 76	13.92 12.96 11.18	(B-V) + 0.01 + 0.16 + 0.08	$\begin{array}{r} 0 = +0.02 \\ \hline 13.92 \\ 12.54 \\ 11.00 \end{array}$	2 0.6 0.6 -8.6	3.0 1.0 -4.2	3.0 2.3 -0.1
49 53 76 105	13.92 12.96 11.18 12.22	(B-V) + 0.01 + 0.16 + 0.08 + 0.01	$\begin{array}{r} 0 = +0.02 \\ \hline 13.92 \\ 12.54 \\ 11.00 \\ 12.22 \end{array}$	$   \begin{array}{r}     0.6 \\     0.6 \\     -8.6 \\     -1.6   \end{array} $	3.0 1.0 -4.2 1.2	3.0 2.3 -0.1 2.7
49 53 76 105 136	13.92 12.96 11.18 12.22 10.96	(B-V) + 0.01 + 0.16 + 0.08 + 0.01 + 0.14	$\begin{array}{r} 0 = +0.02 \\ \hline 13.92 \\ 12.54 \\ 11.00 \\ 12.22 \\ 10.60 \end{array}$	$\begin{array}{r} 0.6 \\ 0.6 \\ -8.6 \\ -1.6 \\ -7.4 \end{array}$	3.0 1.0 -4.2 1.2 -5.3	$3.0 \\ 2.3 \\ -0.1 \\ 2.7 \\ 1.1$
49 53 76 105 136 156	13.92 12.96 11.18 12.22 10.96 14.17	(B-V) + 0.01 + 0.16 + 0.08 + 0.01 + 0.14 - 0.08	$\begin{array}{r} _{0} = +0.02 \\ \hline 13.92 \\ 12.54 \\ 11.00 \\ 12.22 \\ 10.60 \\ 14.17 \end{array}$	$\begin{array}{r} 0.6 \\ 0.6 \\ -8.6 \\ -1.6 \\ -7.4 \\ 1.9 \end{array}$	$3.0 \\ 1.0 \\ -4.2 \\ 1.2 \\ -5.3 \\ 4.2$	$3.0 \\ 2.3 \\ -0.1 \\ 2.7 \\ 1.1 \\ 4.9$
49 53 105 136 156 161	13.92 12.96 11.18 12.22 10.96 14.17 11.78	(B-V) + 0.01 + 0.16 + 0.08 + 0.01 + 0.14 - 0.08 + 0.02	$\begin{array}{r} 0 = +0.02 \\ \hline 13.92 \\ 12.54 \\ 11.00 \\ 12.22 \\ 10.60 \\ 14.17 \\ 11.78 \end{array}$	$\begin{array}{r} 2 \\ 0.6 \\ 0.6 \\ - 8.6 \\ - 1.6 \\ - 7.4 \\ 1.9 \\ - 0.2 \end{array}$	$3.0 \\ 1.0 \\ -4.2 \\ 1.2 \\ -5.3 \\ 4.2 \\ 2.1$	$   \begin{array}{r}     3.0 \\     2.3 \\     -0.1 \\     2.7 \\     1.1 \\     4.9 \\     1.8   \end{array} $
49         53         76         105         136         156         161         190	13.92 12.96 11.18 12.22 10.96 14.17 11.78 13.52	(B-V) + 0.01 + 0.16 + 0.08 + 0.01 + 0.14 - 0.08 + 0.02 - 0.01	$\begin{array}{c} 0 = +0.02 \\ \hline 13.92 \\ 12.54 \\ 11.00 \\ 12.22 \\ 10.60 \\ 14.17 \\ 11.78 \\ 13.52 \end{array}$	$\begin{array}{c} 0.6 \\ 0.6 \\ -8.6 \\ -1.6 \\ -7.4 \\ 1.9 \\ -0.2 \\ -2.7 \end{array}$	$3.0 \\ 1.0 \\ -4.2 \\ 1.2 \\ -5.3 \\ 4.2 \\ 2.1 \\ 1.1$	$   \begin{array}{r}     3.0 \\     2.3 \\     -0.1 \\     2.7 \\     1.1 \\     4.9 \\     1.8 \\     2.4   \end{array} $
49         53         76         105         136         156         161         190         205	13.92 12.96 11.18 12.22 10.96 14.17 11.78 13.52 12.30	(B-V) + 0.01 + 0.01 + 0.08 + 0.01 + 0.14 - 0.08 + 0.02 - 0.01 + 0.12	$\begin{array}{c} 0 = +0.02 \\ \hline 13.92 \\ 12.54 \\ 11.00 \\ 12.22 \\ 10.60 \\ 14.17 \\ 11.78 \\ 13.52 \\ 12.00 \end{array}$	$\begin{array}{c} 2 \\ \hline 0.6 \\ 0.6 \\ -8.6 \\ -1.6 \\ -7.4 \\ 1.9 \\ -0.2 \\ -2.7 \\ 0.0 \end{array}$	$3.0 \\ 1.0 \\ -4.2 \\ 1.2 \\ -5.3 \\ 4.2 \\ 2.1 \\ 1.1 \\ 1.6$	$ \begin{array}{r} 3.0\\ 2.3\\ -0.1\\ 2.7\\ 1.1\\ 4.9\\ 1.8\\ 2.4\\ 2.5 \end{array} $
49         53         76         105         136         156         161         190         205         211	13.92 12.96 11.18 12.22 10.96 14.17 11.78 13.52 12.30 11.50	(B-V) + 0.01 + 0.16 + 0.08 + 0.01 + 0.14 - 0.08 + 0.02 - 0.01 + 0.12 + 0.09	$\begin{array}{c} 0 = +0.02 \\ \hline 13.92 \\ 12.54 \\ 11.00 \\ 12.22 \\ 10.60 \\ 14.17 \\ 11.78 \\ 13.52 \\ 12.00 \\ 11.29 \end{array}$	$\begin{array}{c} 0.6\\ 0.6\\ -8.6\\ -1.6\\ -7.4\\ 1.9\\ -0.2\\ -2.7\\ 0.0\\ -3.3 \end{array}$	$3.0 \\ 1.0 \\ -4.2 \\ 1.2 \\ -5.3 \\ 4.2 \\ 2.1 \\ 1.1 \\ 1.6 \\ -1.1$	$   \begin{array}{r}     3.0 \\     2.3 \\     -0.1 \\     2.7 \\     1.1 \\     4.9 \\     1.8 \\     2.4 \\     2.5 \\     1.2 \\   \end{array} $
49           53           76           105           136           156           161           190           205           211           250	13.92 12.96 11.18 12.22 10.96 14.17 11.78 13.52 12.30 11.50 12.82	(B-V) + 0.01 + 0.16 + 0.08 + 0.01 + 0.14 - 0.08 + 0.02 - 0.01 + 0.12 + 0.09 - 0.02	$\begin{array}{c} 0 = +0.02 \\ \hline 13.92 \\ 12.54 \\ 11.00 \\ 12.22 \\ 10.60 \\ 14.17 \\ 11.78 \\ 13.52 \\ 12.00 \\ 11.29 \\ 12.82 \end{array}$	$\begin{array}{c} 2 \\ \hline 0.6 \\ 0.6 \\ -8.6 \\ -1.6 \\ -7.4 \\ 1.9 \\ -0.2 \\ -2.7 \\ 0.0 \\ -3.3 \\ 0.0 \end{array}$	$3.0 \\ 1.0 \\ -4.2 \\ 1.2 \\ -5.3 \\ 4.2 \\ 2.1 \\ 1.1 \\ 1.6 \\ -1.1 \\ 2.5 $	$ \begin{array}{r} 3.0\\2.3\\-0.1\\2.7\\1.1\\4.9\\1.8\\2.4\\2.5\\1.2\\3.3\end{array} $
49         53         76         105         136         156         161         190         205         211         250         270	13.92 12.96 11.18 12.22 10.96 14.17 11.78 13.52 12.30 11.50 12.82 11.42	(B-V) + 0.01 + 0.16 + 0.08 + 0.01 + 0.14 - 0.08 + 0.02 - 0.01 + 0.12 + 0.09 - 0.02 + 0.02 + 0.02	$\begin{array}{r} 0 = +0.02 \\ \hline 13.92 \\ 12.54 \\ 11.00 \\ 12.22 \\ 10.60 \\ 14.17 \\ 11.78 \\ 13.52 \\ 12.00 \\ 11.29 \\ 12.82 \\ 11.42 \end{array}$	$\begin{array}{c} 2 \\ \hline 0.6 \\ -8.6 \\ -1.6 \\ -7.4 \\ 1.9 \\ -0.2 \\ -2.7 \\ 0.0 \\ -3.3 \\ 0.0 \\ -3.2 \end{array}$	$3.0 \\ 1.0 \\ -4.2 \\ 1.2 \\ -5.3 \\ 4.2 \\ 2.1 \\ 1.1 \\ 1.6 \\ -1.1 \\ 2.5 \\ -0.7$	$\begin{array}{c} 3.0\\ 2.3\\ -0.1\\ 2.7\\ 1.1\\ 4.9\\ 1.8\\ 2.4\\ 2.5\\ 1.2\\ 3.3\\ 1.1\end{array}$
49         53         76         105         136         156         161         190         205         211         250         270         319 <sup>a</sup>	13.92 12.96 11.18 12.22 10.96 14.17 11.78 13.52 12.30 11.50 12.82 11.42 13.22	$\begin{array}{c} (B-V) \\ + 0.01 \\ + 0.16 \\ + 0.08 \\ + 0.01 \\ + 0.14 \\ - 0.08 \\ + 0.02 \\ - 0.01 \\ + 0.12 \\ + 0.09 \\ - 0.02 \\ + 0.02 \\ + 0.06 \end{array}$	$\begin{array}{c} 0 = +0.02 \\ \hline 13.92 \\ 12.54 \\ 11.00 \\ 12.22 \\ 10.60 \\ 14.17 \\ 11.78 \\ 13.52 \\ 12.00 \\ 11.29 \\ 12.82 \\ 11.42 \\ 13.10 \end{array}$	$\begin{array}{c} 2\\ \hline 0.6\\ 0.6\\ -8.6\\ -1.6\\ -7.4\\ 1.9\\ -0.2\\ -2.7\\ 0.0\\ -3.3\\ 0.0\\ -3.2\\ -24.9 \end{array}$	$3.0 \\ 1.0 \\ -4.2 \\ 1.2 \\ -5.3 \\ 4.2 \\ 2.1 \\ 1.1 \\ 1.6 \\ -1.1 \\ 2.5 \\ -0.7 \\ -17.5 \\ \end{array}$	3.0 2.3 -0.1 2.7 1.1 4.9 1.8 2.4 2.5 1.2 3.3 1.1 0.2
49         53         76         105         136         156         161         190         205         211         250         270         319 <sup>a</sup> 384	13.92 12.96 11.18 12.22 10.96 14.17 11.78 13.52 12.30 11.50 12.82 11.42 13.22 12.61	$\begin{array}{c} (B-V) \\ + 0.01 \\ + 0.16 \\ + 0.08 \\ + 0.01 \\ + 0.14 \\ - 0.08 \\ + 0.02 \\ - 0.01 \\ + 0.12 \\ + 0.09 \\ - 0.02 \\ + 0.02 \\ + 0.06 \\ - 0.04 \end{array}$	$\begin{array}{c} 0 = +0.02 \\ \hline 13.92 \\ 12.54 \\ 11.00 \\ 12.22 \\ 10.60 \\ 14.17 \\ 11.78 \\ 13.52 \\ 12.00 \\ 11.29 \\ 12.82 \\ 11.42 \\ 13.10 \\ 12.61 \end{array}$	$\begin{array}{c} 2\\ \hline 0.6\\ 0.6\\ -8.6\\ -1.6\\ -7.4\\ 1.9\\ -0.2\\ -2.7\\ 0.0\\ -3.3\\ 0.0\\ -3.2\\ -24.9\\ -4.6\end{array}$	$\begin{array}{r} 3.0\\ 1.0\\ -4.2\\ 1.2\\ -5.3\\ 4.2\\ 2.1\\ 1.1\\ 1.6\\ -1.1\\ 2.5\\ -0.7\\ -17.5\\ 1.2\end{array}$	3.0 2.3 -0.1 2.7 1.1 4.9 1.8 2.4 2.5 1.2 3.3 1.1 0.2 2.9
49         53         76         105         136         156         161         190         205         211         250         270         319 <sup>a</sup> 384         399	13.92 12.96 11.18 12.22 10.96 14.17 11.78 13.52 12.30 11.50 12.82 11.42 13.22 11.42 13.22 12.61 12.37	$\begin{array}{c} (B-V) \\ + 0.01 \\ + 0.16 \\ + 0.08 \\ + 0.01 \\ + 0.14 \\ - 0.08 \\ + 0.02 \\ - 0.01 \\ + 0.12 \\ + 0.09 \\ - 0.02 \\ + 0.02 \\ + 0.02 \\ + 0.06 \\ - 0.04 \\ - 0.01 \end{array}$	$\begin{array}{c} 0 = +0.02 \\ \hline 13.92 \\ 12.54 \\ 11.00 \\ 12.22 \\ 10.60 \\ 14.17 \\ 11.78 \\ 13.52 \\ 12.00 \\ 11.29 \\ 12.82 \\ 11.42 \\ 13.10 \\ 12.61 \\ 12.37 \end{array}$	$\begin{array}{c} 2\\ \hline 0.6\\ 0.6\\ -8.6\\ -1.6\\ -7.4\\ 1.9\\ -0.2\\ -2.7\\ 0.0\\ -3.3\\ 0.0\\ -3.2\\ -24.9\\ -4.6\\ 1.0\\ \end{array}$	$\begin{array}{r} 3.0\\ 1.0\\ -4.2\\ 1.2\\ -5.3\\ 4.2\\ 2.1\\ 1.1\\ 1.6\\ -1.1\\ 2.5\\ -0.7\\ -17.5\\ 1.2\\ 2.9\end{array}$	3.0 2.3 -0.1 2.7 1.1 4.9 1.8 2.4 2.5 1.2 3.3 1.1 0.2 2.9 2.8
49         53         76         105         136         156         161         190         205         211         250         270         319 <sup>a</sup> 384         399         430	13.92 12.96 11.18 12.22 10.96 14.17 11.78 13.52 12.30 11.50 12.82 11.42 13.22 12.61 12.37 14.20	$\begin{array}{c} (B-V) \\ + 0.01 \\ + 0.16 \\ + 0.08 \\ + 0.01 \\ + 0.14 \\ - 0.08 \\ + 0.02 \\ - 0.01 \\ + 0.02 \\ + 0.09 \\ - 0.02 \\ + 0.06 \\ - 0.04 \\ - 0.01 \\ + 0.03 \end{array}$	$\begin{array}{c} 0 = +0.02 \\ \hline 13.92 \\ 12.54 \\ 11.00 \\ 12.22 \\ 10.60 \\ 14.17 \\ 11.78 \\ 13.52 \\ 12.00 \\ 11.29 \\ 12.82 \\ 11.42 \\ 13.10 \\ 12.61 \\ 12.37 \\ 14.17 \end{array}$	$\begin{array}{c} 2\\ \hline 0.6\\ 0.6\\ -8.6\\ -1.6\\ -7.4\\ 1.9\\ -0.2\\ -2.7\\ 0.0\\ -3.3\\ 0.0\\ -3.2\\ -24.9\\ -4.6\\ 1.0\\ 2.7\\ \end{array}$	$\begin{array}{r} 3.0\\ 1.0\\ -4.2\\ 1.2\\ -5.3\\ 4.2\\ 2.1\\ 1.1\\ 1.6\\ -1.1\\ 2.5\\ -0.7\\ -17.5\\ 1.2\\ 2.9\\ 4.8\end{array}$	3.0 2.3 -0.1 2.7 1.1 4.9 1.8 2.4 2.5 1.2 3.3 1.1 0.2 2.9 2.8 5.7
49         53         76         105         136         156         161         190         205         211         250         270         319 <sup>a</sup> 384         399         430	13.92 12.96 11.18 12.22 10.96 14.17 11.78 13.52 12.30 11.50 12.82 11.42 13.22 12.61 12.37 14.20 12.93	$\begin{array}{c} (B-V) \\ + 0.01 \\ + 0.16 \\ + 0.08 \\ + 0.01 \\ + 0.14 \\ - 0.08 \\ + 0.02 \\ - 0.01 \\ + 0.12 \\ + 0.09 \\ - 0.02 \\ + 0.02 \\ + 0.02 \\ + 0.06 \\ - 0.04 \\ - 0.01 \\ + 0.03 \\ + 0.00 \end{array}$	$\begin{array}{c} 0 = +0.02 \\ \hline 13.92 \\ 12.54 \\ 11.00 \\ 12.22 \\ 10.60 \\ 14.17 \\ 11.78 \\ 13.52 \\ 12.00 \\ 11.29 \\ 12.82 \\ 11.42 \\ 13.10 \\ 12.61 \\ 12.37 \\ 14.17 \\ 12.93 \end{array}$	$\begin{array}{c} 2\\ \hline 0.6\\ 0.6\\ -8.6\\ -1.6\\ -7.4\\ 1.9\\ -0.2\\ -2.7\\ 0.0\\ -3.3\\ 0.0\\ -3.2\\ -24.9\\ -4.6\\ 1.0\\ 2.7\\ -1.2 \end{array}$	$\begin{array}{r} 3.0\\ 1.0\\ -4.2\\ 1.2\\ -5.3\\ 4.2\\ 2.1\\ 1.1\\ 1.6\\ -1.1\\ 2.5\\ -0.7\\ -17.5\\ 1.2\\ 2.9\\ 4.8\\ 1.3\end{array}$	3.0 2.3 -0.1 2.7 1.1 4.9 1.8 2.4 2.5 1.2 3.3 1.1 0.2 2.9 2.8 5.7 2.9
49         53         76         105         136         156         161         190         205         211         250         270         319 <sup>a</sup> 384         399         438         450	13.92 12.96 11.18 12.22 10.96 14.17 11.78 13.52 12.30 11.50 12.82 11.42 13.22 12.61 12.37 14.20 12.93 14.85	$\begin{array}{c} (B-V) \\ + 0.01 \\ + 0.16 \\ + 0.08 \\ + 0.01 \\ + 0.14 \\ - 0.08 \\ + 0.02 \\ - 0.01 \\ + 0.12 \\ + 0.09 \\ - 0.02 \\ + 0.02 \\ + 0.06 \\ - 0.04 \\ - 0.01 \\ + 0.03 \\ + 0.00 \\ - 0.02 \end{array}$	$\begin{array}{c} 0 = +0.02 \\ \hline 13.92 \\ 12.54 \\ 11.00 \\ 12.22 \\ 10.60 \\ 14.17 \\ 11.78 \\ 13.52 \\ 12.00 \\ 11.29 \\ 12.82 \\ 11.42 \\ 13.10 \\ 12.61 \\ 12.37 \\ 14.17 \\ 12.93 \\ 14.85 \end{array}$	$\begin{array}{c} 2\\ \hline 0.6\\ 0.6\\ -8.6\\ -1.6\\ -7.4\\ 1.9\\ -0.2\\ -2.7\\ 0.0\\ -3.3\\ 0.0\\ -3.2\\ -24.9\\ -4.6\\ 1.0\\ 2.7\\ -1.2\\ 0.5 \end{array}$	$\begin{array}{r} 3.0\\ 1.0\\ -4.2\\ 1.2\\ -5.3\\ 4.2\\ 2.1\\ 1.1\\ 1.6\\ -1.1\\ 2.5\\ -0.7\\ -17.5\\ 1.2\\ 2.9\\ 4.8\\ 1.3\\ 3.7\end{array}$	3.0 2.3 -0.1 2.7 1.1 4.9 1.8 2.4 2.5 1.2 3.3 1.1 0.2 2.9 2.8 5.7 2.9 4.9
49         53         76         105         136         156         161         190         205         211         250         270         319 <sup>a</sup> 384         399         430         438         450	13.92 12.96 11.18 12.22 10.96 14.17 11.78 13.52 12.30 11.50 12.82 11.42 13.22 12.61 12.37 14.20 12.93 14.85 13.14	$\begin{array}{c} (B-V) \\ + 0.01 \\ + 0.16 \\ + 0.08 \\ + 0.01 \\ + 0.14 \\ - 0.08 \\ + 0.02 \\ - 0.01 \\ + 0.12 \\ + 0.09 \\ - 0.02 \\ + 0.02 \\ + 0.06 \\ - 0.04 \\ - 0.01 \\ + 0.03 \\ + 0.00 \\ - 0.02 \\ + 0.01 \end{array}$	$\begin{array}{c} 0 = +0.02 \\ \hline 13.92 \\ 12.54 \\ 11.00 \\ 12.22 \\ 10.60 \\ 14.17 \\ 11.78 \\ 13.52 \\ 12.00 \\ 11.29 \\ 12.82 \\ 11.42 \\ 13.10 \\ 12.61 \\ 12.37 \\ 14.17 \\ 12.93 \\ 14.85 \\ 13.14 \end{array}$	$\begin{array}{c} 2\\ \hline 0.6\\ 0.6\\ -8.6\\ -1.6\\ -7.4\\ 1.9\\ -0.2\\ -2.7\\ 0.0\\ -3.3\\ 0.0\\ -3.2\\ -24.9\\ -4.6\\ 1.0\\ 2.7\\ -1.2\\ 0.5\\ 2.5\\ \end{array}$	$\begin{array}{r} 3.0\\ 1.0\\ -4.2\\ 1.2\\ -5.3\\ 4.2\\ 2.1\\ 1.1\\ 1.6\\ -1.1\\ 2.5\\ -0.7\\ -17.5\\ 1.2\\ 2.9\\ 4.8\\ 1.3\\ 3.7\\ 3.4\end{array}$	3.0 2.3 -0.1 2.7 1.1 4.9 1.8 2.4 2.5 1.2 3.3 1.1 0.2 2.9 2.8 5.7 2.9 4.0
49         53         76         105         136         156         161         190         205         211         250         270         319*         384         399         430         438         470	13.92 12.96 11.18 12.22 10.96 14.17 11.78 13.52 12.30 11.50 12.82 11.42 13.22 12.61 12.37 14.20 12.93 14.85 13.14 10.22	$\begin{array}{c} (B-V) \\ + 0.01 \\ + 0.16 \\ + 0.08 \\ + 0.01 \\ + 0.14 \\ - 0.08 \\ + 0.02 \\ - 0.01 \\ + 0.02 \\ + 0.09 \\ - 0.02 \\ + 0.02 \\ + 0.02 \\ + 0.00 \\ - 0.04 \\ - 0.01 \\ + 0.03 \\ + 0.00 \\ - 0.02 \\ + 0.01 \\ + 0.13 \end{array}$	$\begin{array}{c} 0 = +0.02 \\ \hline 13.92 \\ 12.54 \\ 11.00 \\ 12.22 \\ 10.60 \\ 14.17 \\ 11.78 \\ 13.52 \\ 12.00 \\ 11.29 \\ 12.82 \\ 11.42 \\ 13.10 \\ 12.61 \\ 12.37 \\ 14.17 \\ 12.93 \\ 14.85 \\ 13.14 \\ 9.89 \end{array}$	$\begin{array}{c} 2\\ \hline 0.6\\ 0.6\\ -8.6\\ -1.6\\ -7.4\\ 1.9\\ -0.2\\ -2.7\\ 0.0\\ -3.3\\ 0.0\\ -3.2\\ -24.9\\ -4.6\\ 1.0\\ 2.7\\ -1.2\\ 0.5\\ 2.5\\ -8.0\\ \end{array}$	$\begin{array}{r} 3.0\\ 1.0\\ -4.2\\ 1.2\\ -5.3\\ 4.2\\ 2.1\\ 1.1\\ 1.6\\ -1.1\\ 2.5\\ -0.7\\ -17.5\\ 1.2\\ 2.9\\ 4.8\\ 1.3\\ 3.7\\ 3.4\\ -3.3\end{array}$	3.0 2.3 -0.1 2.7 1.1 4.9 1.8 2.4 2.5 1.2 3.3 1.1 0.2 2.9 2.8 5.7 2.9 4.0 -0.6
49         53         76         105         136         156         161         190         205         211         250         270         319 <sup>a</sup> 384         399         430         438         450         475         478	13.92 12.96 11.18 12.22 10.96 14.17 11.78 13.52 12.30 11.50 12.82 11.42 13.22 12.61 12.37 14.20 12.93 14.85 13.14 10.22 11.56	$\begin{array}{c} (B-V) \\ + 0.01 \\ + 0.16 \\ + 0.08 \\ + 0.01 \\ + 0.14 \\ - 0.08 \\ + 0.02 \\ - 0.01 \\ + 0.02 \\ + 0.09 \\ - 0.02 \\ + 0.02 \\ + 0.02 \\ + 0.00 \\ - 0.04 \\ - 0.01 \\ + 0.03 \\ + 0.00 \\ - 0.02 \\ + 0.01 \\ + 0.13 \\ + 0.10 \end{array}$	$\begin{array}{c} 0 = +0.02 \\ \hline 13.92 \\ 12.54 \\ 11.00 \\ 12.22 \\ 10.60 \\ 14.17 \\ 11.78 \\ 13.52 \\ 12.00 \\ 11.29 \\ 12.82 \\ 11.42 \\ 13.10 \\ 12.61 \\ 12.37 \\ 14.17 \\ 12.93 \\ 14.85 \\ 13.14 \\ 9.89 \\ 11.32 \end{array}$	$\begin{array}{c} 2\\ \hline 0.6\\ 0.6\\ -8.6\\ -1.6\\ -7.4\\ 1.9\\ -0.2\\ -2.7\\ 0.0\\ -3.3\\ 0.0\\ -3.2\\ -24.9\\ -4.6\\ 1.0\\ 2.7\\ -1.2\\ 0.5\\ 2.5\\ -8.0\\ -2.7\end{array}$	$\begin{array}{r} 3.0\\ 1.0\\ -4.2\\ 1.2\\ -5.3\\ 4.2\\ 2.1\\ 1.1\\ 1.6\\ -1.1\\ 2.5\\ -0.7\\ -17.5\\ 1.2\\ 2.9\\ 4.8\\ 1.3\\ 3.7\\ 3.4\\ -3.3\\ 1.8\end{array}$	$\begin{array}{c} 3.0\\ 2.3\\ -0.1\\ 2.7\\ 1.1\\ 4.9\\ 1.8\\ 2.4\\ 2.5\\ 1.2\\ 3.3\\ 1.1\\ 0.2\\ 2.9\\ 2.8\\ 5.7\\ 2.9\\ 4.9\\ -0.6\\ 3.5\\ \end{array}$
49         53         76         105         136         156         161         190         205         211         250         270         319 <sup>a</sup> 384         399         430         438         450         470         475         478	$\begin{array}{c} 13.92\\ 12.96\\ 11.18\\ 12.22\\ 10.96\\ 14.17\\ 11.78\\ 13.52\\ 12.30\\ 11.50\\ 12.82\\ 11.42\\ 13.22\\ 12.61\\ 12.37\\ 14.20\\ 12.93\\ 14.85\\ 13.14\\ 10.22\\ 11.56\\ 13.89\\ \end{array}$	$\begin{array}{c} (B-V) \\ + 0.01 \\ + 0.16 \\ + 0.08 \\ + 0.01 \\ + 0.14 \\ - 0.08 \\ + 0.02 \\ - 0.01 \\ + 0.02 \\ + 0.02 \\ + 0.02 \\ + 0.02 \\ + 0.02 \\ + 0.03 \\ + 0.00 \\ - 0.04 \\ - 0.01 \\ + 0.03 \\ + 0.00 \\ - 0.02 \\ + 0.01 \\ + 0.13 \\ + 0.10 \\ - 0.02 \end{array}$	$_{0} = +0.02$ 13.92 12.54 11.00 12.22 10.60 14.17 11.78 13.52 12.00 11.29 12.82 11.42 13.10 12.61 12.37 14.17 12.93 14.85 13.14 9.89 11.32 13.89	$\begin{array}{c} 2\\ \hline 0.6\\ 0.6\\ -8.6\\ -1.6\\ -7.4\\ 1.9\\ -0.2\\ -2.7\\ 0.0\\ -3.3\\ 0.0\\ -3.2\\ -24.9\\ -4.6\\ 1.0\\ 2.7\\ -1.2\\ 0.5\\ 2.5\\ -8.0\\ -2.7\\ -3.5\end{array}$	$\begin{array}{r} 3.0\\ 1.0\\ -4.2\\ 1.2\\ -5.3\\ 4.2\\ 2.1\\ 1.1\\ 1.6\\ -1.1\\ 2.5\\ -0.7\\ -17.5\\ 1.2\\ 2.9\\ 4.8\\ 1.3\\ 3.7\\ 3.4\\ -3.3\\ 1.8\\ 1.2\end{array}$	$\begin{array}{c} 3.0\\ 2.3\\ -0.1\\ 2.7\\ 1.1\\ 4.9\\ 1.8\\ 2.4\\ 2.5\\ 1.2\\ 3.3\\ 1.1\\ 0.2\\ 2.9\\ 2.8\\ 5.7\\ 2.9\\ 4.9\\ 4.0\\ -0.6\\ 3.5\\ 3.6\end{array}$

<sup>a</sup> Anomalous emission at Hα.

Names: Catalog number in Azzopardi and Vigneau (1975).

Equivalent widths: Measured in either a 20 Å window ( $W^{20}$ ) or an asymmetric 75 Å window ( $W^{75}$ ), in ångstroms.

Ebbets (1982) emphasizes that the H $\alpha$  line strength is well correlated with mass loss. Hence in the LMC, mass loss in supergiants seems to be well correlated with luminosity. There is reason to be optimistic that these relationships can be used to determine distances to other star systems. Unfortunately, there is also The Case of The SMC.

### b) The Small Magellanic Cloud

The same information displayed for the LMC is shown for the SMC in Figure 6 (B8 stars) and Figure 7 (A0 stars). The rough absolute magnitude scale assumes  $(m - M)_0 = 19.0$  and  $A_v = 0.2$ . The same general trends are evident, but the rms scatter is now  $\pm 0.7$  mag, twice the amount found in the LMC data.

The increased scatter cannot be observational. All the material for both Clouds was obtained under photometric conditions, on the same night, and with the same instrument. The samples in the two galaxies range over the same interval of apparent magnitudes. Integration times, and hence the signalto-noise ratio, are comparable. Neither can the increased scatter be attributed to geometric factors. Even though Azzopardi and others have suggested the SMC is extended in the line-of-sight, it would take a prohibitive  $\pm 40\%$  effect to explain the scatter in the SMC diagrams. Azzopardi (1981) has noted that previous investigations involving Balmer line widths have also found greater scatter in SMC data sets compared with LMC material. In short, the greatly increased scatter in the SMC plots cannot be due to observational uncertainties, nor reddening, nor a range in distances, yet the samples are large enough that it must be real.

## IV. A PROBABLE METALLICITY EFFECT

There is strong evidence in Figure 8 that the scatter in the SMC data is *systematic*. For the A0 star samples in each of the two Clouds, the *differences* in equivalent widths between the H $\beta$  and H $\alpha$  lines are shown. These differences are clearly not the same in the LMC and SMC plots and, again, the reason cannot be attributed to observational, reddening, or distance effects. In the SMC, the equivalent width differentials are closer to zero.

We can postulate an explanation. The H $\alpha$  and H $\beta$  equivalent widths are dependent on both surface gravity conditions in the photosphere and the extended environment of the star where there is a stellar wind and mass loss. The effect of the extended envelope is manifestly important when the line is seen in emission and influences the H $\alpha$  line far more than the higher order Balmer lines. Suppose that in the LMC the lines tend to be more filled in by emission than they are in the SMC, with the emission being greatest at H $\alpha$ . The consequence would be a larger differential in the measured equivalent widths from H $\alpha$ to H $\beta$ , as is observed. Figure 8 tells us that mass loss is less important in the SMC stars than in the LMC stars.

If the stellar winds leading to mass loss are radiatively driven through opacity in the metal lines (Lucy and Solomon 1970; Castor, Abbott, and Klein 1975), then there may be a natural explanation for lower mass-loss rates in the SMC. Oxygen abundances deduced from studies of nebular emission regions are down from Galactic values by a factor of 2 in the LMC and a factor of 5 in the SMC (Dufour and Harlow 1977). Ultraviolet spectra of metal lines obtained with the *IUE* by Hutchings (1982) confirm our speculation that stellar winds are probably weaker in both Clouds than in the Galaxy. In the SMC, Hutchings argues that we are "on the threshold of where winds may or may not exist at all," and he finds that several SMC supergiants show no wind lines.

It remains to be explained why the scatter is so much greater in the SMC samples. Hutchings (1982) has pointed out that the radiation pressure induced by an optically thick metallic line is insensitive to metallicity effects, while if a line is optically thin then there is acute sensitivity. Perhaps the metal abundances of the SMC stars are low enough that an important fraction of the lines driving mass loss are optically thin, while the LMC stars have sufficient metal enrichment that the relevant lines are optically thick.

If this hypothesis is correct, then the scatter in Figure 7 (the SMC A0s) with respect to the tight relationship in Figure 3 (the



FIG. 3.-A0 stars in the LMC. The same format as Fig. 2. The open circle in both panels identifies a star with anomalous emission at Ha.

LMC A0s) would be to the left, to more positive equivalent widths (less emission) at a given luminosity. It is obviously important to be certain what is going on before the relationships are used to determine distances. To this end, we intend to return to the telescope to increase our sample and to obtain data on Galactic stars with yet higher metallicities.

#### V. SUMMARY

In the case of A and B supergiants in the LMC, there are good correlations between the equivalent widths of the  $H\alpha$  and

 $H\beta$  lines and intrinsic luminosity. The  $H\alpha$  line is seen in emission over a 2 mag interval at the highest luminosities, so in this regime equivalent widths can be easily measured at low dispersions. These properties make A–B supergiants excellent prospects as standard candles, since these kinds of stars are very bright, numerous, generally in fairly uncrowded fields, little reddened, and easily identified. It should be possible to calibrate out the modest temperature dependencies.

The correlations shown here have, in fact, been known for a long time. By looking in the Magellanic Clouds, however, we



FIG. 4.—A5 stars in the LMC. The same format as Fig. 2.

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FIG. 5.—An almost complete sample of B5–A5 stars with V < 11 in the LMC. The format is the same as in Fig. 2, except that different symbols identify different spectral types.

were able to accumulate a very large sample of the most luminous supergiants, free of distance and reddening problems. At low dispersion, it appears that almost all A and B supergiants brighter than -5 have broad, asymmetric emission components that extend blueward 2000 km s<sup>-1</sup>. Whatever their origin, we find that these broad features are weakly correlated with luminosity. More importantly, the narrow H $\alpha$  line cores are strongly correlated with luminosity. Evidently, mass loss is strongly correlated with luminosity and hence probably with stellar mass as well.

Problems become apparent when the SMC samples are considered. Scatter in the relationships is doubled. A comparison of equivalent widths in the H $\alpha$  and H $\beta$  lines and between the LMC and SMC suggests that there are metallicity complications. There is evidence that mass loss is less at a given luminosity in many SMC supergiants. If mass loss is driven by radiation pressure coupled through the opacity in metal lines, then there might be lower mass-loss rates in the SMC if the important metal lines tend to be optically thin. If these lines are systematically optically thick in the LMC, this fact might explain the much tighter relationship seen in this larger galaxy.

Our vested interest is to employ the relationships to obtain distances to giant galaxies in nearby groups. Such galaxies are usually high metallicity systems. Whatever problems there are with low abundance systems, the tight correlations seen within the LMC provide reason for optimism that A and B supergiants will provide good distances to large galaxies.

It is a tribute to the visitor facilities at Cerro Tololo Inter-American Observatory that we were able to observe 113 stars in a single night and then have reduced equivalent widths only a couple of months later. Special thanks go to Jack Baldwin for his aid with these facilities. This research was supported by NSF grants AST 79-27154 and AST 82-03971.



FIG. 6.—B8 stars in the SMC. Same format as Fig. 2. Open circles identify stars with anomalous emission at H $\alpha$ .





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FIG. 8.—Equivalent width differentials between H $\alpha$  and H $\beta$ . The top panel shows  $W_{\beta}^{20} - W_{\alpha}^{20}$  as a function of visual magnitude for stars in the LMC, and the bottom panel shows the same information for stars in the SMC. Distinct spectral types are distinguished by the symbols.

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Note added in proof—We had discounted a depth effect (Florsch, Marcout, and Fleck 1981, Astr. Ap., 96, 158) as the origin of the scatter in the SMC diagrams because, if the SMC is a single unit, the line-of-sight dimension would have to be implausibly larger than the dimensions of the SMC on the plane of the sky. However, the possibility raised by Mathewson and Ford (1983, IAU Symposium No. 108) that we are seeing two separate sub-systems in projection is a more tenable explanation of the greater SMC scatter. Unfortunately, very few of our stars have sufficiently well-determined radial velocities that they can be unambiguously identified with a specific Mathewson-Ford entity and, perhaps due to small number statistics, most of those cases that do exist happen to fall in their Mini-Magellanic Cloud (MMC: seven stars) rather their Small Magellanic Cloud Remnant (SMCR: two stars). We do obtain a 2  $\sigma$  result suggesting that the MMC is roughly at the LMC distance and the SMCR is ~0.5 mag farther away. This result contradicts the Mathewson-Ford conclusion based on the velocities of interstellar calcium lines that the SMCR is in front of the MMC.

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