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THE ANGULAR MOMENTUM CONTENT OF GALAXIES

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ABSTRACT

A schema of galaxy formation is developed in which the environmental influence of large-scale structure plays a dominant role. This schema was motivated by the observation that the fraction of E and S0 galaxies is much higher in clusters than in low-density regions and by an inference that those spirals that are found in clusters probably have fallen in relatively recently from the low-density regions. It is proposed that the tidal field of the Local Supercluster acts to determine the morphology of galaxies through two complementary mechanisms. In the first place, the supercluster can apply torques to protogalaxies. Galaxies which collapsed while expanding away from the central cluster decoupled from the external tidal field and conserved the angular momentum that they acquired before collapse. Galaxies which formed in the cluster while the cluster collapsed continued to feel the tidal field. In the latter case, the spin of outer collapsing layers can be halted and reversed, and tends to cancel the spin of inner layers. The result is a reduction of the total angular momentum content of the galaxy. In addition, the supercluster tidal field can regulate accretion of fresh material onto the galaxies since the field creates a Roche limit about galaxies and material beyond this limit is lost. Any material that has not collapsed onto a galaxy by the time the galaxy falls into a cluster will be tidally stripped.

The angular momentum content of that part of the protogalactic cloud which has not yet collapsed continues to grow linearly with time due to the continued torquing by the supercluster and neighbors. Galaxies at large distances from the cluster core can continue to accrete this high angular momentum material until the present, but galaxies that enter the cluster are cut off from replenishing material. Consequently, low angular momentum systems formed in the environment that produced the initial cluster because of incipient tidal locking and were subsequently deprived of fresh infall material because of a restrictive Roche limit. High angular momentum galaxies formed where the tidal spin-up mechanism was uninhibited and where accretion continued for a long time, perhaps until today.

The distinctive morphological components, disks and spheriods, are determined by the specific angular momentum of a system. If the angular momentum content is high, then the collapse factors required to achieve centrifugal equilibrium are not large and disks can be formed. When the angular momentum content is low, collapse factors are larger, central densities are expected to be higher, and, if star formation occurs before the equilibrium condition is achieved, a spheroid is formed.

Subject headings: cosmology — galaxies: clustering — galaxies: evolution — galaxies: formation — galaxies: structure

I. INTRODUCTION

The Hubble morphological sequence contains three principal subunits: ellipticals, lenticulars (or S0), and spirals plus irregulars. Elliptical galaxies are distinguished from lenticulars and spirals by three major characteristics. They have no extremely flattened component, the radial distribution of the logarithmic surface luminosity falls off with distance to the $\frac{1}{4}$ power rather than in a linear fashion, and the ratio of rotational to random motions is quite reduced. It is deduced that the stars in elliptical galaxies (at least giant ellipticals) are predominantly *not* in circular orbits and, consequently, that the angular momentum per unit mass in these systems is reduced compared with that in disk systems (by a factor of 6 according to Fall 1982). The difference between elliptical and lenticular galaxies, on the one hand, and spiral galaxies, on the other, is the presence or absence of an extreme Population I component. Spiral galaxies contain a significant amount of interstellar gas and the traits of recent star formation, while the visible material in elliptical and S0 galaxies is overwhelmingly composed of old stars.

Dressler (1980) demonstrated in a quantitative fashion that the likelihood of the occurrence of each of these types of galaxies is a strong function of environment. The fraction of ellipticals to lenticulars to spirals increases dramatically with an increase in the local density of galaxies. Among galaxies brighter than -19 in the Virgo Cluster, 55% are E or S0, while among luminous galaxies in the general field of the Local Supercluster (excluding condensed groups), only 17% are these gas-poor types (Tully 1982).

In the preceding paper (Tully and Shaya 1984), it was demonstrated that the flux of spiral and irregular galaxies from the field into the Virgo Cluster is very high at the present epoch. Plausibly *all* the gas-rich galaxies that are observed in the cluster today have arrived within the last one-third to one-half of a Hubble time. By contrast, very few gas-poor systems are falling into the cluster. It follows that either the spirals and irregulars are being transformed into ellipticals and lenticulars (Spitzer and Baade 1951; Toomre and Toomre 1972; Silk and Norman 1981; Schweizer 1982; Farouki and Shapiro 1982; Gunn and Gott 1972; Cowie and Songaila 1977) or the morphology of galaxies is sensitively dependent on environmental factors at the epoch of galaxy formation (Dressler 1980; Larson, Tinsley, and Caldwell 1980; Gunn 1982; Bothun 1982). In the former picture, there probably would be a trend toward an increased fraction of gas-poor systems with time. In the latter, there could actually be a dilution of the early/late ratio toward gas-rich systems as galaxies fall into a cluster.

We cannot make a decisive choice between these two possibilities if, indeed, only one of these general schemata is operative. However, the concept that spirals are converted into ellipticals has problems (Ostriker 1980; van den Bergh 1982). Instead, we have a model to propose which presumes the other possibility: that the environment plays a fundamental role in the determination of the angular momentum content of galaxies and, hence, in the determination of morphology. The one fundamental assumption we must make is that galaxies were/are continuing to form during the time of cluster formation.

Our contention is that the tidal field of the emerging cluster or supercluster can have major effects on protogalaxies. The cluster tidal field supplies a torque on nonspherical protogalaxies which supplies increasing amounts of angular momentum to succeeding shells of infalling matter. However, in some environments this mechanism fails. If a galaxy is forming in a region which is collapsing to form a cluster, the strength of the tidal field grows with time and can result in spin reversal of the later layers of material falling into the separate protogalaxies. Compared with galaxies at large in the supercluster, galaxies which form in a cluster have less time to acquire angular momentum, and in addition the last material that they receive is a massive shell of low or negative angular momentum material which arrives just as the cluster as a whole collapses. Any shells that have not reached the galaxy by the time it falls into the cluster are tidally stripped, and after this epoch the only fresh material a galaxy can receive would be that due to random motions and collisions.

II. THE TIDAL FIELD OF THE LOCAL SUPERCLUSTER

The effect of the tidal field of clusters on galaxies could easily compete with the tidal field between neighboring galaxies if galaxies formed at the same time as the large-scale structure, especially if the mass-to-light ratio is considerably higher in clusters than in galaxies. For instance, if the Local Supercluster contains a mass of $10^{15}\ M_\odot$ above the mean density within the radius of our Galaxy, this supercluster inhomogeneity has an associated tidal field at our position equal to that of a $10^{12}\ M_\odot$ galaxy at a distance of 1.7 Mpc. Perhaps it is this tidal field which is responsible for the spin-up of the larger and more isolated spiral galaxies.

The tidal interaction mechanism (Hoyle 1949; Peebles 1969; Thuan and Gott 1977; Efstathiou and Jones 1979) can provide angular momentum to galaxies moving apart from their neighbors in the following way. The torque on a protogalaxy due to the summed tidal field of neighboring galaxies acting upon the quadrupole moment of the protogalaxy is nearly constant before the protocloud has begun to collapse (see Thuan and Gott 1977). During this time, the moment of inertia is growing sufficiently that the protogalaxy rotates by only a few degrees. As the galaxy collapses, the quadrupole moment falls off, and, because galaxies are moving apart, so does the strength of the tidal field. Thus the torque drops precipitously, permitting the galaxy to spin up freely while contracting.

The tidal acceleration at a distance r from the center of a galaxy which is itself a distance R from the center of a spherically symmetric density distribution is to first order (Olson 1980)

$$g^{\rm tidal} = D(R)\left[\frac{3\boldsymbol{R}(\boldsymbol{r}\cdot\boldsymbol{R})}{R^2} - \boldsymbol{r}\right], \qquad (1)$$

where the tidal field strength D is defined by

$$D(R) = \frac{GM(<R)}{R^3} - \frac{4}{3}\pi G\rho(R); \qquad (2)$$

$M(<R)$ is the mass internal to the sphere of radius R, and $\rho(R)$ is the mass density at R.

The time evolution of D was calculated for two spherically symmetric multishelled models of the Local Supercluster using the standard parameterized solutions to the Freidmann equation in a bound universe (eqs. [5] and [6] in Tully and Shaya 1984). In the first case, presented in Figure 1*a*, the initial conditions were a point mass placed in a homogeneous universe at 3×10^7 yr. A similar model of the formation of a supercluster from the gravitation of a protocluster was investigated in detail by Shectman (1982). Parameters were varied to arrive at the lowest point mass which would result, at $t_0 = 12$ Gyr, in the accretion of at least $5 \times 10^{14}\ M_\odot$ within 3 Mpc and of no more than $1 \times 10^{15}\ M_\odot$ within 17 Mpc. The minimum seed mass required was $2.5 \times 10^{13}\ M_\odot$. In the second case, illustrated in Figure 1*b*, conditions which might characterize the present epoch were specified; $t_0 = 12$ Gyr, a central point mass of $4 \times 10^{14}\ M_\odot$ representing the Virgo Cluster today, and a density falloff proportional to $1/R^2$ normalized to provide a total of $1 \times 10^{15}\ M_\odot$ within $R = 17$ Mpc. The solid curves in these two diagrams demonstrate the dependence of the tidal field strength as a function of time at several present epoch distances from the central mass. For early times, $D \sim a^{-3}[\delta(<R) - \delta(R)] \sim t^{-4/3}$. The tidal field curves break upward in collapsing regions, and break downward near the present epoch in regions below critical density as expansion ceases to be approximated by an Einstein–de Sitter model universe. During the expansion phase the tidal field varies by only a modest amount between these two quite different models and with radius from the central cluster over distances comparable to the scale of the present supercluster. If one is impressed with the seeming universality of the angular momentum content of disk galaxies as might be implied by the tight correlations found between the luminosities, dimensions, and rotation of galaxies, the explanation may lie in the nearly constant tidal field supplying torque to all nearby galaxies.

The angular momentum imparted to galaxies by the tidal field of the supercluster can be estimated. The torque N exerted upon the quadrupole moment Q of an ellipsoidal protogalaxy with a major axis a and minor axis b is taken to be (see Thuan and Gott 1977)

$$N = \tfrac{3}{4}DQ\sin 2\theta, \qquad (3)$$

$$Q = \tfrac{2}{5}M(a^2 - b^2), \qquad (4)$$

where θ is the position angle between the source of the tidal field and the major axis of the protogalaxy. We want to calculate the angular momentum per unit mass, or specific angular momentum, J/M:

$$J/M = 1.2\int_{t=0}^{t=t_m} N/M\,dt, \qquad (5)$$

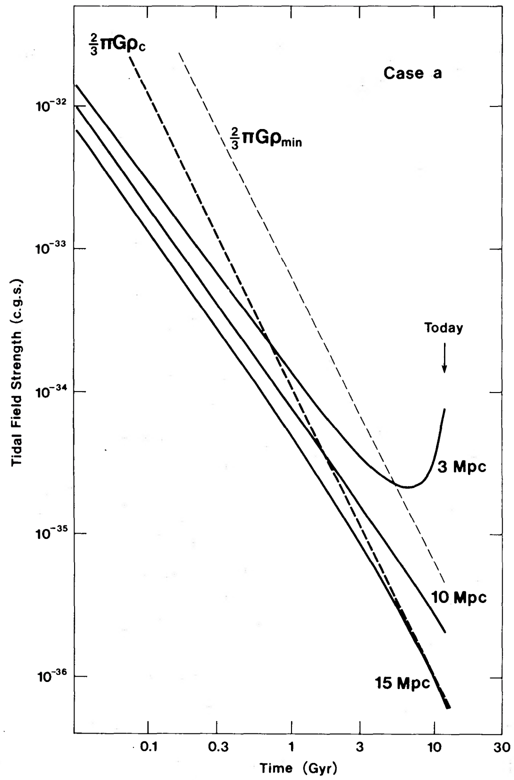

FIG. 1*a*

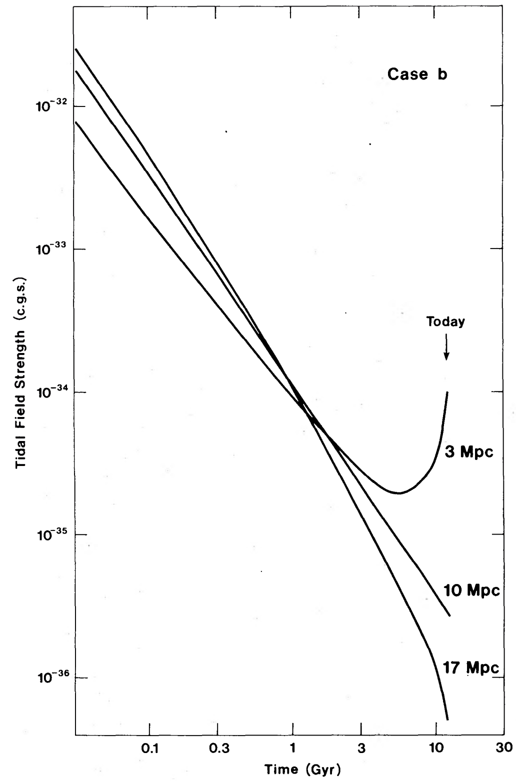

FIG. 1*b*

FIG. 1.—(*a*) The tidal field strength associated with a model of the supercluster which grew about a seed point mass. The solid curves indicate the evolution of the tidal field strength at various present epoch positions. A bound system expands with a density which places it just above the heavy dashed critical density curve until, at some point, it evolves to the right, reaches minimum density at the intersection of the thin dashed line, and collapses. However, if a bound shell about a galaxy following such a trajectory intersects the tidal field curve associated with the position of the galaxy in the supercluster then the shell will be tidally disrupted. (*b*) The tidal field strength associated with a model which was allowed to develop such that today there is $4 \times 10^{14}\ M_\odot$ within the central cluster and $1 \times 10^{15}\ M_\odot$ within the radius of the Local Group.

where the integral is calculated up to the moment of maximum expansion at t_m, and augmented 20% to account for the angular momentum acquired during the collapse phase. The augmentation factor was determined through the numerical modeling discussed further on.

Given $D = D_1(t/t_1)^{-4/3}$ and $a = a_m(t/t_m)^{2/3}$ in a closed part of the universe in expansion, where t_1 is any arbitrary initial reference epoch at an early time, $\theta_1 = 45°$, and assuming the shape of the protocloud does not change:

$$J/M = 0.36\, D_1 t_1^{4/3}(a_m^2 - b_m^2)t_m^{-1/3}\ . \qquad (6)$$

The moment of maximum expansion for a spherical system is:

$$t_m = \pi(a_m^3/8GM)^{1/2}\ , \qquad (7)$$

so substituting for a_m in equation (6):

$$J/M = 0.36\, D_1 t_1^{4/3}(8GM/\pi^2)^{2/3}(1 - b_m^2/a_m^2)t_m\ . \qquad (8)$$

It will be seen with the numerical models that $(b/a)^2 \ll 1$ most of the time since the minor axis collapses much earlier than the major axis provided the initial $b/a < 0.9$. A characteristic supercluster value for the combination $D_1 t_1^{4/3}$ can be obtained from Figure 1 to be 9×10^{-13} in cgs units. Hence:

$$J/M \approx 1.1 \times 10^{28} M_{10}^{2/3}\, t_m\ \mathrm{cm^2\ s^{-1}}\ , \qquad (9)$$

where M_{10} is the mass of the galaxy in units of $10^{10}\ M_\odot$ and the age is in Gyr. The specific angular momentum grows linearly with the collapse time scale. If galaxies continue to form over an extended period of time, it is the late-arriving material which contributes most of the angular momentum.

III. A COMPARISON WITH OBSERVATIONS

The dependence of the specific angular momentum on mass to the $\frac{2}{3}$ power has been demonstrated in several data sets. Heidmann (1968) found this dependence in a sample of 48 galaxies when she compared the indicative parameters $J/M \sim aW$ and $M \sim aW^2$, where a is an isophotal dimension of the galaxy and W is the H I line profile width. Nordsieck (1973) computed the angular momentum and mass of 16 galaxies from models based on rotation curve information and found the same dependence. Nordsieck expressed concern that observational errors might have produced a spurious power law correlation which would render the slope meaningless (since one is essentially plotting aW against aW^2). However, Vettolani *et al.* (1980) found a relationship between Heidmann's indicative angular momentum and *luminosity* in two independent samples:

$$J/M \sim L^{0.73 \pm 0.05}\ ,$$

which would be consistent with the other results if the ratio of mass to light is constant.

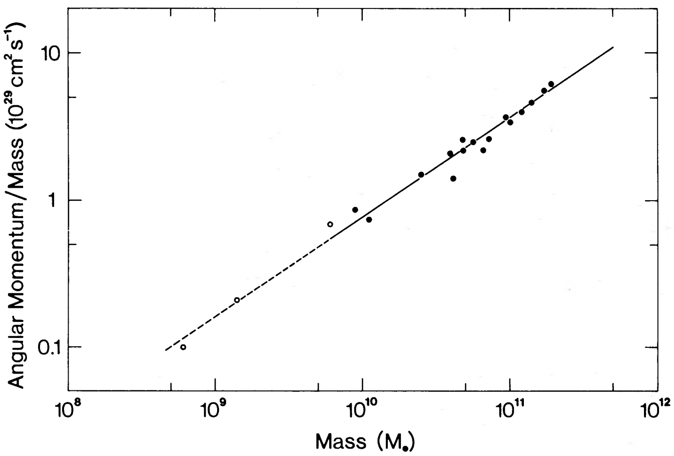

FIG. 2.—Observed specific angular momentum as a function of mass. The filled circles correspond to data from Nordsieck. The three additional galaxies indicated by the open circles are, from right to left, NGC 3109, the SMC, and DDO 125. The straight line illustrates the fit determined by Nordsieck. These data are consistent with a distance scale in accordance with a global value of the Hubble constant of 75 km s^{-1} Mpc^{-1}.

We present Nordsieck's material, with his least-squares fit, in Figure 2. His data base has been extended a decade to lower masses with the inclusion of the three irregular galaxies NGC 3109, the SMC and DDO 125. The required information for these systems was extracted from models by Tully *et al.* (1978), which are only subtly different from the models by Nordsieck. The $\frac{2}{3}$ power law is now found to hold over two and a half decades in mass. The strong correlation over this large interval cannot be due to measurement error.

It should be noted that the dimensions which enter the indicative parameters are isophotal, while the dimensions entering the mass modeling come from characteristics of the rotation curves. We found it amusing that our equation (3) in Tully and Shaya (1984), which is a correlation between isophotal dimensions and rotation, could be directly reformulated in the form $J/M \sim M^{0.63}$, though in retrospect we are merely confirming Heidmann's results.

Unfortunately, the observations do not serve as proof of the validity of a tidal torquing theory because, as Thompson (1974) has discussed, there are a variety of theories that predict dependencies of specific angular momentum on mass with powers close to $\frac{2}{3}$. Within the framework of the tidal spin-up mechanism, it might be possible to distinguish between the relative importance of the supercluster and near neighbors. If the large-scale influence of the supercluster is most important, then there should be a tendency for spin axes to align perpendicular to the supercluster potential gradient. On the observational side the situation is confused, with reports of significant alignment in some environments and not in others (Thompson 1976; see review by Helou and Salpeter 1982).

In any event, we can resolve if the angular momentum acquired from torquing by the supercluster is sufficient to explain the observations illustrated in Figure 2. Neglecting for the moment the possible existence of invisible matter, the observed specific angular momentum of a galaxy of mass $1 \times 10^{10}\ M_\odot$ is 8×10^{28} cm^2 s^{-1}. Equation (9) tells us that our mechanism produces this amount after $t_m = 7$ Gyr at representative locations in the supercluster. Since the collapse time is $2t_m$, *we must continue to accrete material into galaxies essentially until the present epoch* to explain the angular momentum content by torquing from the supercluster tidal field. According to Huchtmeier and Richter (1982), roughly one-third of galaxies mapped in the 21 cm line have H I envelopes larger than 2 Holmberg radii, and some extend up to 8 Holmberg radii. It is possible that many of these galaxies are accreting material out of their original protoclouds.

If we continue to ignore the possibility that massive halos exist, then the problem posed in the preceding paragraph can be turned around to ask where collapsing shells are falling from today. If $t_m = \frac{1}{2}t_0 \approx 6$ Gyr, then equation (7) can be rewritten:

$$a_m \approx 100\, M_{10}^{1/3}\ \mathrm{kpc}\ , \qquad (10)$$

where the mass of the galaxy M_{10} is in units of $10^{10}\ M_\odot$. Equations (1) and (3) from Tully and Shaya (1984) can be combined to give an empirical relationship between luminosity, L_B, and the Holmberg radius, a_{Ho}. If we assume a mass-to-light ratio of $M/L_B = 6$ (Faber and Gallagher 1979, with a distance scale adjustment), then we can derive a relationship between the Holmberg radius and total mass:

$$a_{\mathrm{Ho}} \approx 8\, M_{10}^{1/3}\ \mathrm{kpc}\ . \qquad (11)$$

It follows that gas falling into the Holmberg radius of a galaxy today is collapsing by a factor $a_m/a_{\mathrm{Ho}} \approx 13$, independent of the mass of the galaxy.

The problem turns out to be more complicated, however. If there is no massive halo, material collapsing only a factor of 13 will not be in centrifugal equilibrium if it has only the specific angular momentum acquired by supercluster torques at a random location. In this case, material collapsing today will fall to smaller radii, and material that would fall to the equilibrium condition at the Holmberg radius would require longer than the age of the universe to do so. It will be pointed out in § VII that this dilemma can be resolved if either the specific angular momentum is greater than we calculate or the collapse is aided by the existence of a halo of dissipationless matter.

IV. THE FORMATION OF LOW ANGULAR MOMENTUM SYSTEMS THROUGH INCIPIENT TIDAL LOCKING

According to linear perturbation theory, in a region with closure density, enhancements of arbitrarily small amplitude develop into density excesses of at least unity by the time of complete collapse. Cluster collapse may assist galaxy accretion, provided material is available. In fact, a significant number of galaxies may form which would not have formed in noncollapsing regions.

Given the possibility that galaxies evolve significantly during the collapse phase in clusters, the differences in populations of galaxy types can be understood in terms of the tidal interaction mechanism for the spin-up of galaxies. This mechanism might work differently in regions collapsing while the individual protogalaxies are contracting as compared with regions expanding during protogalaxy contraction.

In an expanding region, the torque on the protogalaxy falls off once the galaxy begins to collapse. In a collapsing region, however, the torque will not drop off since the tidal field is increasing with time. If a galaxy accretes material while collapsing to join a cluster, then the torque on the infalling material might remain high even after the material has rotated enough so that the torque is opposed to the spin. Rotation can *reverse sign*. Late-arriving material that is so affected will have angular momentum content which partially cancels that of material that had fallen in earlier. Undoubtedly tremendous dissipation would occur over the entire galaxy, and partial tidal locking could be expected. The natural outcome of this process would be that clusters should predominantly contain galaxies of low specific angular momentum at the epoch of the initial collapse.

The effects of an external field on collapsing ellipsoids were calculated by numerical integration. The acceleration due to self-gravitation within a uniform ellipsoid is (see White and Silk 1979, and references therein):

$$\frac{d^2 a_i}{dt^2} = -2\pi G \sum_{i=1}^{3} \left[\beta_i \rho_e + \left(\frac{2}{3} - \beta_i\right)\rho_b\right] a_i \,, \tag{12}$$

where ρ_e is the density of the ellipsoid, ρ_b is the density of the region outside the ellipsoid, and a_i are the principal axes. The coefficients β_i are given by

$$\beta_i = a_1 a_2 a_3 \int_0^\infty (a_i^2 + \lambda)^{-1} \prod_{j=1}^{3} (a_j^2 + \lambda)^{-1/2} d\lambda \,. \tag{13}$$

These coefficients depend only on the two axial ratios of the ellipsoid. They were calculated for 100 different combinations of axial ratios, and an interpolation scheme was devised to find the coefficients for any shape of the ellipsoid as it evolved.

For the case of a galaxy forming in a cluster, the cluster was represented by a two-density-component model; a sphere of density ρ_s and radius R_s, and a surrounding region of arbitrary shape with density ρ_b. The initial density of the sphere was selected such that collapse occurs at $t = 2$ Gyr, and the initial density of the surrounding region was chosen to be at the critical density of the beginning epoch, $t_i = 0.003$ Gyr, $\rho_c = (6\pi G t_i^2)^{-1}$. The tidal strength on a galaxy embedded in the surrounding region at a distance R from the cluster center is (see eq. [2])

$$D = \frac{4}{3}\pi G \left[\frac{\rho_s R_s^3 + (R^3 - R_s^3)\rho_b}{R^3} - \rho_b\right] . \tag{14}$$

The tidal tensor, $\boldsymbol{E}$, for the case of one principal axis perpendicular to the direction toward the cluster center is

$$\boldsymbol{E} = \frac{D}{2}\begin{pmatrix} -2 & 0 & 0 \\ 0 & 6\sin^2\theta - 1 & 3\sin(2\theta) \\ 0 & 3\sin(2\theta) & 6\cos^2\theta - 1 \end{pmatrix} , \tag{15}$$

where θ is the angle between the third axis and the direction of the cluster center. The diagonal components of the tidal tensor added to equation (12), plus a centrifugal term $\Omega^2 a_i$, give the acceleration of the principal axes. The off-diagonal components give rise to the torque expressed by equation (3). This model constrains the shape to ellipsoids, although in reality the forces resulted in more complicated configurations. Nevertheless, this procedure should give the correct order of magnitude for the angular momentum that would be generated in real galaxies.

Figure 3*b* illustrates the tidally induced reversal described above. An ellipsoid, at $R/R_s = 1.001$, with the initial axial ratios $(b/a)_i = (c/a)_i = 0.6$, reaches maximum expansion at $t = 0.75$ Gyr, rotates past $\theta = 0$, and has its rotation halted and reversed before the galaxy and the cluster finish collapsing at $t = 2$ Gyr. The quantity $J/M^{5/3}$, the axial ratios, and the position angle are all independent of the size of the galaxy. The shell that we follow is only slightly inside the Roche limit for the galaxy. The collapse of the major axis is retarded due to the tidal field. It expands to a radius far in excess of the radius anticipated by equation (7) for spherical systems because of nonsphericity and tidal distention.

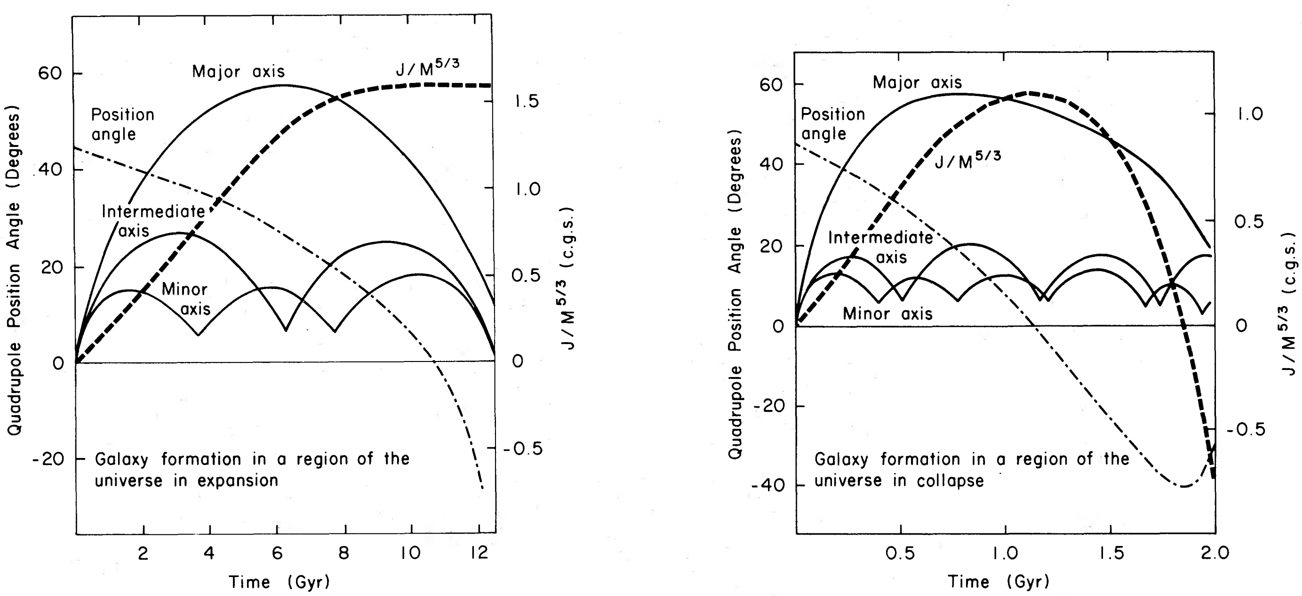

FIG. 3*a* FIG. 3*b*

FIG. 3.—(*a*) A model of shell collapse in a region of the universe in expansion. The three solid curves trace the expansion and eventual collapse along the major and two minor axes. The dot-dashed line records the rotation in the alignment of the major axis (see left scale). The heavy dashed curve follows the development of the angular momentum in scale-free units $J/M^{5/3}$ (see right scale). (*b*) A model of shell collapse in a region of the universe in collapse. The curves provide the same information as in Fig. 3*a*. The reversal in the direction of change of the position angle corresponds to a change in sign of the spin vector.

The nonsphericity and the tidal field cause the minor axes to collapse earlier. As a consequence, the quadrupole moment becomes large at an early time [i.e., $(b/a)^2 \ll 1$ most of the time].

For comparison, an example of the collapse of a protogalaxy in an expanding region is shown in Figure 3*a*. Here, the tidal field strength D was changed with time according to the early time solution $D \propto t^{-4/3}$. In this case the system has rotated through only a small angle before collapse, and the angular momentum content of the galaxy grows monotonically. The shell expands to a radius in excess of the radius expected if the system were spherical. As a result, the shell acquires more angular momentum than predicted by equation (8).

We feel we have demonstrated with our modeling that torque reversal *can occur* with plausible supercluster tidal fields. However, it takes some tuning of the tidal field and collapse time scales. The formation of real protogalaxies must be highly chaotic, with torques changing as successive layers of material arrive. The final specific angular momentum content of a galaxy will depend in a complicated way on the rate of mass accretion in the environment of the growing cluster tidal field.

V. ROCHE LIMITS IN THE SUPERCLUSTER TIDAL FIELD

Intergalactic material will fall into a galaxy only if the overall density is great enough to overcome the distending force of the tidal field. This restriction requires that

$$D < 2\pi G\langle\rho\rangle/3 \,, \tag{16}$$

where $\langle\rho\rangle$ is the density of the galaxy as seen by the infalling material. In Figure 1*a* there is a line which shows the evolution of $2\pi G/3$ times the critical density for closure and another which gives $2\pi G/3$ times the density at turnaround for objects reaching maximum expansion at any given time. Any bound shell about a galaxy will follow a locus on Figure 1 just above the critical density line until it is about to collapse, then break off to the right and reach minimum density at the turnaround line. If the shell has not collapsed by the time its $2\pi G\rho/3$ line has intersected the tidal field line at the appropriate radius from the cluster, then the shell will be tidally disrupted and will be lost to the galaxy.

There are two separate points to be made. In the first place, it can be seen from Figure 1*a* that most galaxies in the Local Supercluster must be close to being tidally limited today. Consequently, it is probably the case that most galaxies have received substantial amounts of material falling from *nearly* their Roche limits, as defined by the summed tidal field of the supercluster and neighboring galaxies. This material is rich in specific angular momentum. Equating the tidal acceleration directed away from the center of the protogalaxy to the gravitational acceleration at maximum expansion and using equation (7) for the time of maximum expansion, one finds:

$$D_{\rm Roche} = \pi^2/16t_m^2 \,. \tag{17}$$

The specific angular momentum at the time of maximum expansion generated with this strong a tidal field is found by substituting equation (17) into equation (8) (minus the augmentation factor):

$$J/M = 6.0 \times 10^{28} M_{10}^{2/3} t_m^{1/3} \text{ cm}^2 \text{ s}^{-1} \,. \tag{18}$$

This formula provides an upper limit to the specific angular momentum that can be provided by the tidal torque mechanism, except that it does not take into account nonsphericity and the distortions caused by the tidal field. Both of these factors can cause further increases in the specific angular momentum.

The second point is that close to the Virgo Cluster accretion of gas onto galaxies is strongly retarded by the cluster tidal field at the present epoch. *The instant a galaxy enters a cluster, it can no longer accrete gas out of its primordial cloud.* Only material already infalling is not tidally restricted. Galaxies in the cluster have been deprived of intergalactic gas for up to 85% of the age of the universe (roughly how long ago the cluster formed), while in field galaxies the accretion of material can be an ongoing process which replenishes the gas lost to star formation.

It has been estimated by Larson, Tinsley, and Caldwell (1980) that the time scale for the depletion of gas in galaxies is 1 to 3 Gyr, and a claim is made that if a galaxy is deprived of infalling gas, then star formation ceases and an S0 is created. It is observed that the relative percentage of S0's is strongly dependent on local density (Dressler 1980). The restrictive Roche limit about cluster galaxies due to the strong local tidal fields may account for these observations.

It was pointed out by Tully and Shaya (1984) that if spirals in the Virgo Cluster are deficient in H I, then so are galaxies falling toward the cluster. Material in uncollapsed shells about these systems would be hung up by the cluster tidal field, and the present accretion rate of gas into these infalling galaxies would already be reduced from the levels experienced by isolated galaxies.

VI. ACCRETION COLLAPSE FACTORS: DID GALAXIES FORM RECENTLY?

Until now in our discussion, we have simply accepted that galaxies formed recently enough to experience the effects of the supercluster tidal field. In retrospect, there is an argument that supports this assumption.

Gas falling into a rotationally supported disk will settle into centrifugal equilibrium. *Upper limits* to the rotational motion that this gas could have had as it just began to fall in are given by the tidal field experienced at the Roche limit. The requirement that angular momentum be conserved defines a collapse factor, and the Roche limit case provides a *lower limit* to this parameter. A lower limit for the turnaround radius implies a *lower* limit to the age of the universe at the time of shell collapse.

The argument will be repeated in a more quantitative fashion. A shell of material in centrifugal and gravitational equilibrium will rotate with angular velocity ω_c at a radius a_c:

$$\omega_c^2 = g(e)G\alpha_c M_m/a_c^3 \,. \tag{19}$$

The parameter $g(e)$ depends on the geometry of the system, ranging from $g(e = 0) = 1$ in the spherical case to $g(e = 1) = 3\pi/4$ for a flattened disk. The parameter α_c allows us to account for the possibility that there are both dissipative and nondissipative components to the mass that constitute the galaxy. The dissipative component could have collapsed much further than the nondissipative component (Fall and Efstathiou 1980). We define α_c to be the ratio of the mass within the radius of the dissipative collapse to the total mass enclosed by the pertinent shell at maximum expansion:

$$\alpha_c \equiv M_c/M_m \,. \tag{20}$$

With a collapse factor

$$f \equiv a_m/a_c \tag{21}$$

and using equation (7):

$$\omega_c = \frac{\pi}{2^{3/2}} g(e)^{1/2} \alpha_c^{1/2} f^{3/2} t_m^{-1} \tag{22}$$

is the angular velocity after collapse.

The angular velocity at maximum expansion, ω_m, is

$$\omega_m = J/I , \tag{23}$$

where the moment of inertia is $I = \frac{1}{5}M(a^2 + b^2)$ and equation (6) gives the angular momentum (minus 20%). If the tidal field at maximum expansion for a shell is D_m, then:

$$\omega_m = \frac{3}{2}\left(\frac{a^2 - b^2}{a^2 + b^2}\right) D_m t_m . \tag{24}$$

Now, with conservation of angular momentum:

$$\omega_m a_m^2 = 2\omega_c a_c^2 , \tag{25}$$

where the factor of 2 comes about because the moment of inertia is modified by the evolution from $a \gg b$ before collapse to $a = b$ after collapse. Rewritten in terms of the collapse factor:

$$f = (2\omega_c/\omega_m)^{1/2} , \tag{26}$$

and substituting for the angular velocities using equations (22) and (24):

$$f = \frac{2\pi^2}{9} g(e)\alpha_c \left(\frac{a^2 + b^2}{a^2 - b^2}\right)^2 D_m^{-2} t_m^{-4} . \tag{27}$$

There is a very strong dependence on the tidal field strength in the sense that if the tidal torquing is *weak*, then the angular momentum acquired is low and the collapse factor is *large*. A limiting case is provided by assuming that the tidal field is set by the Roche limit. After substitution for the tidal field strength using equation (17), one finds that the collapse factor is dependent *only* on *geometric factors* and not on time:

$$f_{\rm lim} = \frac{512}{9\pi^2} g(e)\alpha_c \left(\frac{a^2 + b^2}{a^2 - b^2}\right)^2 . \tag{28}$$

In the case of a flattened disk, $(b/a)^2 \approx 0$ and $g(e) = 3\pi/4$, so

$$f_{\rm lim} = 13.6\alpha_c \tag{29}$$

and can be much *greater* if $D_m < D_{\rm Roche}$.

We now ask: How old was the universe when matter collapsed to a specified radius? Equation (7), the expression for the collapse time, $t_c = 2t_m$, can be rewritten incorporating the definitions of α_c and f:

$$t_c = (\pi^2/2G)^{1/2} f^{3/2} a_c^{3/2} \alpha_c^{1/2} M_c^{-1/2} . \tag{30}$$

Flat rotation curves in disk galaxies imply $M_c = M_{\rm Ho}(a_c/a_{\rm Ho})$, and equation (11) in cgs units is $M_{\rm Ho} = 1.4 \times 10^{-24} a_{\rm Ho}^3$, so that

$$t_c = 0.24 f^{3/2} \alpha_c^{1/2} (a_c/a_{\rm Ho}) \tag{31}$$

with units of time in Gyr. The substitution of the limiting value of the collapse factor (eq. [29]) gives:

$$t_c \geq 12\alpha_c^2 (a_c/a_{\rm Ho}) \text{ Gyr} . \tag{32}$$

These formulae provide some rather interesting restrictions in the (f, α)-plane, as we show in Figure 4. Equation (29) provides one restriction. Shells that collapse with maximum allowable angular momentum will be described by a point lying on this locus, while shells that acquire less angular momentum can only fall to the right. The dotted lines map out collapse isochrones in accordance with equation (31) for the case $a_c = a_{\rm Ho}$. Values of t_c greater than the age of the universe are excluded, creating the forbidden zone in the upper right of the diagram.

A third restriction can be imposed if some additional lore about the mass distribution in galaxies is incorporated. Assume that $M(a) \sim a$, consistent with the flat rotation curves that are observed, and assume, as a limiting case, that the fraction of dissipative mass is *negligible* compared to the nondissipative material. Then

$$\alpha_c \equiv M_c/M_m = a_c/(a_m/2) = 2f^{-1} , \tag{33}$$

since the dissipationless material will collapse by a factor of 2 from the radius of maximum expansion. Combined with equation (29), this limit implies $f \geq 5.2$.

A more general assumption would be that the fraction of dissipative mass, M_c^b, to nondissipative mass within the same radius, M_c^v, is $\gamma_c \equiv M_c^b/M_c^v$. If we still hold that $M^v(a) \sim a$, then

$$\alpha_c \equiv \frac{M_c^v + M_c^b}{M_m^v + M_c^b} = \frac{2\gamma_c + 2}{2\gamma_c + f} \tag{34}$$

since $M_c^v/M_m^v = 2f^{-1}$. Curves corresponding to a few representative values of γ have been plotted on Figure 4, with the limiting case $\gamma = 0$ creating yet another forbidden zone at small values of α.

Finally, a fourth restriction can be imposed through equation (27) and the interdependence of D and t illustrated in Figure 1. In expanding parts of the supercluster, $D \sim t^{-4/3}$ over most of time up to the present. With a choice of a "typical" supercluster tidal field strength at a given time and our usual values for $g(e)$ and b/a, we have

$$f \approx 100(10/t_c)^{4/3}\alpha_c , \tag{35}$$

with time in Gyr. The dependence on time can be eliminated in the case of collapse to a specific fraction of the Holmberg radius by combining the above equation with equation (31):

$$f = 24\alpha_c^{1/9}(a_{\rm Ho}/a_c)^{4/9} . \tag{36}$$

In general, galaxies in the expanding region of the supercluster will acquire sufficient angular momentum that they will collapse by no more than this amount to the specified radius. This limit, applied in the case of collapse to the Holmberg radius and illustrated in the lower right of Figure 4, completes the enclosure of a "permitted" zone in the collapse factor–dissipative/nondissipative mass fraction plane. The collapse factor is smaller than the right-hand limit if additional angular momentum has been acquired from the tidal field of neighbors. Collapse to radii smaller than the Holmberg radius occurred earlier on, when the shells had less angular momentum, and for these shells the collapse factor would have been larger.

It is seen in Figure 4 that the *lower limit* for the epoch of collapse to the Holmberg radius is 2 Gyr. Since we estimate (Tully and Shaya 1984) that the core of the Virgo Cluster collapsed at about this time, it follows that at least the outer parts of galaxies were still being accreted when the supercluster was already a well-developed entity.

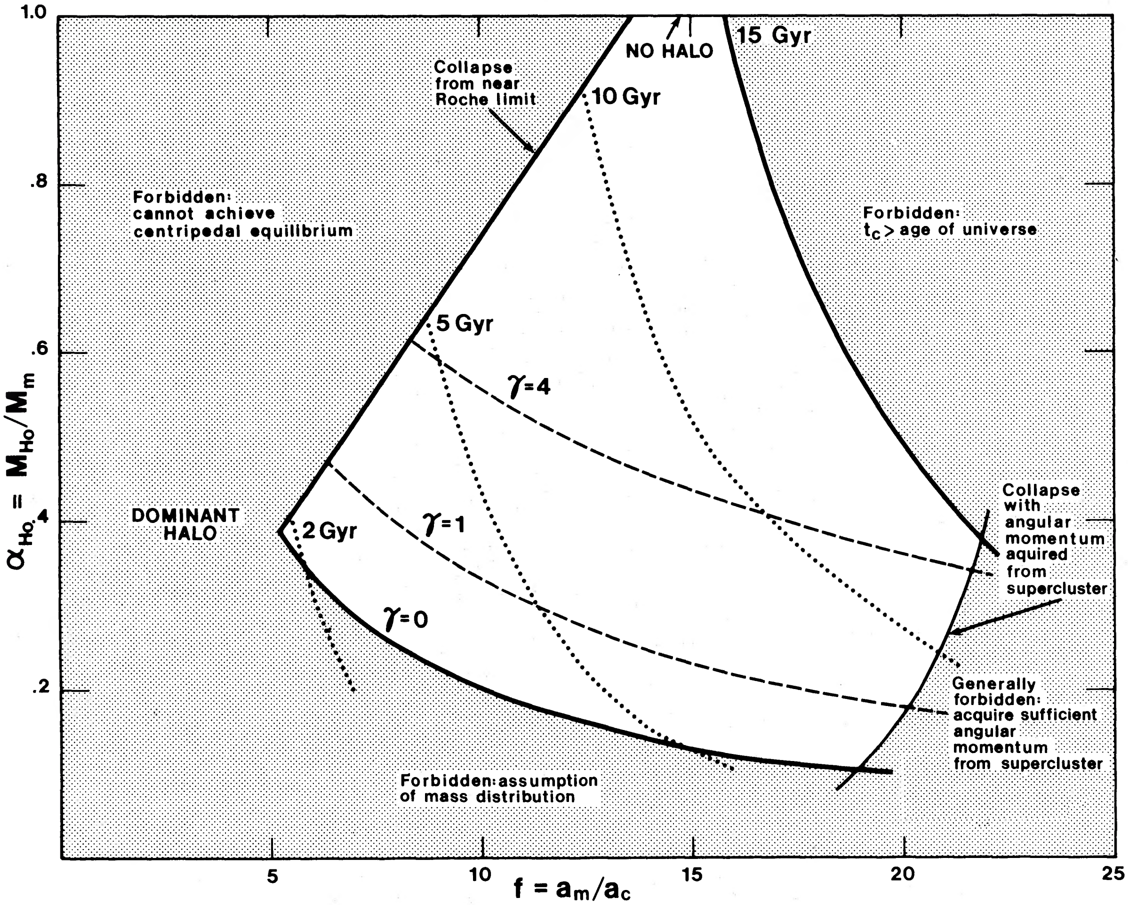

FIG. 4.—Collapse factor f, versus α, the fraction of the total mass within the radius of the dissipational collapse. If there is no dissipationless halo, then $\alpha = 1$; and if there is such a halo, then α is small. A permitted region in the (f, α)-plane is bounded by limits described by equations (29), (31), (33), and (36). This diagram illustrates the case of collapse to the Holmberg radius.

VII. ACCRETION COLLAPSE FACTORS: DIFFERENCES BETWEEN SPHEROIDS AND DISKS

There has been discussion of a mechanism by which the large-scale environment can regulate the angular momentum content of the outer layers of galaxies-in-formation. We will now consider how differences in angular momentum content might give rise to differences in morphological type. We suppose that all kinds of galaxies began as undifferentiated pertubations to the background density, with masses within the restricted range that could cool in less than a Hubble time (Rees and Ostriker 1977), and that the principal differences between the galaxy types arose from differences in the collapse factors required for the outer layers to reach centrifugal equilibrium. The accretion rate during the latest epoch plays a secondary role governing galaxy cosmetics.

An important observed difference between ellipticals and spirals is that the luminosity distribution is distinctly more centrally concentrated in ellipticals (Okamura, Kodaira, and Watanabe 1984; de Vaucouleurs 1977). Faber (1982) has argued that the chief cause for the higher central densities in ellipticals compared with spirals was not earlier collapse but larger collapse factors. Large collapse factors in the luminous material of ellipticals are indicated by the relatively low ratio of nonluminous to luminous mass that are inferred in these galaxies (Tinsley 1981). Collapse of the disks in spiral galaxies was halted by rotational support because the specific angular momentum is high (Fall and Efstathiou 1980). Observations of rotational to dispersive velocities in galaxies (Binney 1976; Illingworth 1977) indicate that specific angular momentum is lower in ellipticals than in spirals (Fall 1982) and is particularly low in giant ellipticals (Davies *et al.* 1983). The lower specific angular momentum in ellipticals would certainly result in larger collapse factors. It remains to be answered why ellipticals did not, even so, reach a disk configuration, only tighter and with more rapid rotation.

The spheroidal shapes of ellipticals and the bulges of spirals indicate that these structures fragmented into stars and clusters before centrifugal equilibrium could be achieved. Dissipation was halted (White and Rees 1978), and binding energy was subsequently deposited into stellar orbits, limiting further collapse to a factor of 2 in accordance with the virial theorem. Mathews (1972) showed that star formation occurs efficiently only if the ratio of densities in dissipative to nondissipative material exceeds several hundred (see Faber 1982). Evidently, this critical ratio in densities is achieved at collapse factors between those needed by the angular momentum–rich material to achieve equilibrium in the disks of spirals and those the angular momentum–poor material would have required to reach equilibrium.

Kashlinsky (1982) has argued that there is competition between fragmentation and coalescence, such that if the fragmentation time scale is the shorter, then star formation occurs and stops dissipative collapse; whereas if the coalescence time scale is the shorter, then collapse continues to a disk. Fragmentation dominates if angular momentum is low (the mean free path between collisions is long) whereas coalescence counteracts fragmentation if angular momentum is high. As with the proposal by Mathews, the juncture between these cases depends on the ratio of dissipative to nondissipative densities. In the thick disks observed in some spirals and lenticulars, star formation may have occurred before dissipative collapse was completed (Jones and Wyse 1983).

We can calculate the collapse factors that are expected as a function of radius if galaxies build up over time as a consequence of shell collapse to centrifugal equilibrium configurations with no angular momentum transfer. Suppose that galaxies acquire only the "typical" amount of angular momentum that leads to collapse factors defined by equation (36). Substitution into equation (31) gives the corresponding collapse time scale:

$$t_c = 28\alpha_c^{2/3}(a_c/a_{\rm Ho})^{1/3}\,. \qquad (37)$$

The parameter α_c depends on the ratio of dissipational to dissipationless matter as described by equation (34). In Table 1, there is a summary of the expected collapse factors and time scales at representative radii under several assumptions about the relative importance of dissipational and dissipationless material. The parameters associated with collapse to the Holmberg radius in a Roche limiting tidal field are also tabulated.

One point can be made that was already apparent in Figure 4. If galaxies do not contain any dark halo and have only the angular momentum content acquired due to supercluster torques at a "typical" location, then collapse time scales are much greater than the age of the universe. *Either the amount of dissipationless mass must be comparable to or greater than the amount of dissipational mass, or most galaxies are almost tidally limited.* The former possibility was recognized as a solution to the collapse time scale problem by Efstathiou and Jones (1979) and Fall and Efstathiou (1980).

A second remark stimulated by Table 1 is that collapse factors to small fractions of the Holmberg radius are inevitably rather large. They become large enough that it would not be surprising if our calculations become inapplicable because fragmentation occurs and collapse is halted before the centrifugal equilibrium condition is achieved. We might associate material that does not dissipatively collapse all the way to centrifugal equilibrium with a bulge.

Most recently, Davies *et al.* (1983) have noted that there are systematic differences between giant and small elliptical galaxies in the sense that, while giant ellipticals manifestly are not rotating fast enough to be rotationally supported, it is possible that small ellipticals *are* so supported. The bulges of spiral galaxies have photometric and kinematic properties similar to the small ellipticals. Kormendy (1977) pointed out another observational property of interest: the surface brightness within the effective diameter of small ellipticals tends to be greater than that of giant ellipticals. Both the kinematic and photometric properties can be explained *if small ellipticals* (and bulges) *collapse farther than giant ellipticals*. By collapsing farther, densities go up and the systems come closer to satisfying the centrifugal equilibrium condition.

We can speculate why small ellipticals might collapse more than giant ellipticals. Rees and Ostriker (1977) have demonstrated that above a limit of about $10^{12}\ M_\odot$ cooling time scales exceed collapse time scales. In the case of small ellipticals and bulges, cooling times should be sufficiently short that collapse is free-fall. Perhaps in giant ellipticals cooling times become significant and collapse is held up. There would be more time for stars to be formed at lower densities in the systems that are to become giant ellipticals.

VIII. CONCLUSIONS

1. At the present epoch, the tidal field of central clusters and superclusters can compete with the tidal field of neighboring galaxies, especially if M/L associated with the central cluster is much higher than that associated with individual galaxies.

2. At least the outer disks of spiral galaxies must have been forming later than when the universe was 2 Gyr old ($z \sim 2.4$) if disks are in centrifugal equilibrium and angular momentum was acquired by tidal torquing, whether from the supercluster or nearest neighbors. This limit is set by the observed dimensions and masses of galaxies and a minimum collapse factor associated with collapse from the Roche limit. Since the initial collapse of the Virgo Cluster occurred at $t \sim 2$ Gyr, the cluster tidal field must have been significant during the period of galaxy formation.

3. For collapse to a centrifugal equilibrium condition at the

TABLE 1

COLLAPSE FACTORS

		No Halo: $M_c^b/M_c^v = \infty$ ($\gamma_c = \infty$)			Equal Dissipational and Dissipationless Components: $M_c^b/M_c^v = 1$ ($\gamma_c = f/2$)			Dominant Halo: $M_c^b/M_c^v = 0.1$ ($\gamma_c = f/20$)		
Tidal Field	$a_c/a_{\rm Ho}$	α_c	t_c (Gyr)	f	α_c	t_c (Gyr)	f	α_c	t_c (Gyr)	f
"Typical" tidal field	0.1	1.0	13	67	0.52	8	62	0.12	3	53
	0.25	1.0	18	44	0.53	11	41	0.14	5	36
	0.5	1.0	22	33	0.53	14	30	0.16	7	27
	1.0	1.0	28	24	0.54	18	22	0.18	9	20
Roche limiting field	1.0	1.0	12	14	0.78	7	11	0.47	3	6

Holmberg radius in less than the age of the universe, either there must exist a dissipationless halo of comparable mass to the mass of the dissipational material or galaxies were formed out of protogalactic clouds that were almost tidally limited.

4. Low angular momentum systems, or components of systems, collapse further than high angular momentum systems and achieve higher central densities. Equilibrium collapse factors can become so large if the angular momentum content is low that a system is probably held up above equilibrium by the formation of stars in noncircular orbits. Small ellipticals and the bulges of spirals apparently collapse further than giant ellipticals because densities are higher and they are closer to the centrifugal equilibrium condition.

5. Galaxies which form while expanding away from the central cluster (in at least one dimension) can receive a torque from the supercluster which is not strongly dependent on radial position and which ensures that most galaxies in the Local Supercluster will have similar values of $J/M^{5/3}$. The angular momentum per unit mass in a protogalactic cloud increases linearly with time. If all the angular momentum in spirals comes from torquing from the supercluster, galaxies must be accreting significant amounts of material up to the present epoch to account for the observed angular momentum levels.

6. Galaxies which form while collapsing toward the central cluster will not decouple from the cluster tidal field as they collapse. The last layer to be accreted will reverse in spin during infall and lead to a reduction of the angular momentum of the total system. Dissipative processes would probably abet a tendency for the systems to tidally lock toward the cluster core.

7. The tidal field of the cluster eventually cuts off accretion of fresh material by galaxies because they cannot pull in material from outside their Roche limit. This limit requires that the time scale for a shell to collapse into a galaxy be less than the time scale for collapse into a cluster. Consequently, a galaxy ceases to accrete material out of its primordial protocloud upon entering a cluster. If star formation can deplete the gas content of a galaxy in less than a Hubble time, then it would be anticipated that clusters would contain many galaxies devoid of gas and young stars.

To summarize: An elliptical galaxy would be a system that formed while collapsing toward the central cluster and that gave up angular momentum to the cluster through incipient tidal locking. Because of low angular momentum content, collapse factors required to attain rotational equilibrium were large, and densities became high enough before the equilibrium condition could be achieved that star formation occurred, dissipative collapse was halted, and a spheroid was formed. The system was deprived of fresh material early on, due to a restrictive Roche limit in the strong tidal field of the cluster. The small number of isolated elliptical galaxies might have collapsed either from spherical protosystems, or from ones initially aligned in the direction of the tidal field so that they received no torques, or they collapsed in their entirety relatively early on when collapse factors were large.

A lenticular galaxy might have initially formed while expanding away from the central cluster but fell back into the cluster early on. It has subsequently been cut off from fresh, high angular momentum material, and the consequences are a stunted disk and no recent star formation. Lenticulars in the field may have exhausted their supply of intergalactic material through competition with neighbors.

A spiral galaxy would have formed while expanding away from the cluster. On collapsing, it decoupled from the outside tidal field and conserves the angular momentum it received as a protocloud. Either the supercluster or neighbors could have provided the external tidal field. Material that collapsed earlier had lower specific angular momentum and would have led to the formation of a bulge. Higher angular momentum material, arriving later, would collapse by smaller factors and be able to achieve the centrifugal equilibrium condition in a disk. Fresh, high angular momentum material has continued to build up the disk until the present day.

Although our ideas have evolved, E. S. began working on the problem of tides with Jim Peebles, and his initial influence is appreciated. We enjoyed stimulating conversations with George Efstathiou and Mike Fall on the topics discussed in this article. Financial support was provided by the Honolulu Chapter of the Achievement Rewards for College Scientists to E. S. and by NSF grants AST 79-26040 and AST 82-03971.

REFERENCES

Binney, J. 1967, *M.N.R.A.S.*, **177**, 19.
Bothun, G. D. 1982, *Ap. J. Suppl.*, **50**, 39.
Cowie, L. L., and Songaila, A. 1977, *Nature*, **266**, 501.
Davies, R. L., Efstathiou, G., Fall, S. M., Illingworth, G., and Schechter, P. L. 1983, *Ap. J.*, **266**, 41,
de Vaucouleurs, G. 1977, in *The Evolution of Galaxies and Stellar Populations*, ed. B. M. Tinsley and R. B. Larson (New Haven: Yale Univ. Obs.), p. 43.
Dressler, A. 1980, *Ap. J.*, **236**, 351.
Efstathiou, G., and Jones, B. J. T. 1979, *M.N.R.A.S.*, **186**, 133.
Faber, S. M. 1982, in *Astrophysical Cosmology*, ed. H. A. Bruck, G. V. Coyne, and M. S. Longair (Vatican: Pontificiae Academiae Scientarum), pp. 191, 219.
Faber, S. M., and Gallagher, J. S. 1979, *Ann. Rev. Astr. Ap.*, **17**, 135.
Fall, S. M. 1982, in *IAU Symposium 100, Internal Kinematics and Dynamics of Galaxies*, ed. F. Athanassoula (Dordrecht: Reidel), p. 391.
Fall, S. M., and Efstathiou, G. 1980, *M.N.R.A.S.*, **193**, 189.
Farouki, R. T., and Shapiro, S. L. 1982, *Ap. J.*, **259**, 103.
Gunn, J. E. 1982, in *Astrophysical Cosmology*, ed. H. A. Bruck, G. V. Coyne, and M. S. Longair (Vatican: Pontificiae Academiae Scientarum), p. 233.
Gunn, J. E., and Gott, J. R. 1972, *Ap. J.*, **176**, 1.
Heidmann, N. 1968, *Ap. Letters*, **3**, 153.
Helou, G., and Salpeter, E. E. 1982, *Ap. J.*, **252**, 75.
Hoyle, F. 1949, in *Problems of Cosmical Aerodynamics* (International Union of Theoretical and Applied Mathematics and International Astronomical Union), p. 195.
Huchtmeier, W. K., and Richter, O. G. 1982, *Astr. Ap.*, **109**, 331.
Illingworth, G. 1977, *Ap. J.* (*Letters*), **218**, L43.
Jones, B. J. T., and Wyse, R. F. G. 1983, *Astr. Ap.*, **120**, 165.
Kashlinsky, A. 1982, *M.N.R.A.S.*, **200**, 585.
Kormendy, J. 1977, *Ap. J.*, **218**, 333.
Larson, R. B., Tinsley, B. M., and Caldwell, C. N. 1980, *Ap. J.*, **237**, 692.
Mathews, W. G. 1972, *Ap. J.*, **174**, 101.
Nordsieck, K. H. 1973, *Ap. J.*, **184**, 735.
Okamura, S., Kodaira, K., and Watanabe, M. 1984, *Ap. J.*, **280**, in press.
Olson, D. W. 1980, *Ap. J.*, **236**, 335.
Ostriker, J. P. 1980, *Comments Ap.*, **8**, 177.
Peebles, P. J. E. 1969, *Ap. J.*, **155**, 353.
Rees, M. J., and Ostriker, J. P. 1977, *M.N.R.A.S.*, **179**, 541.
Shectman, S. 1982, *Ap. J.*, **262**, 9.
Schweizer, F. 1982, *Ap. J.*, **252**, 455.
Silk, J., and Norman, C. 1981, *Ap. J.*, **247**, 59.
Spitzer, L., and Baade, W. 1951, *Ap. J.*, **113**, 413.
Thompson, L. A. 1974, Ph.D. thesis, University of Arizona.
———. 1976, *Ap. J.*, **209**, 22.
Thuan, T. X., and Gott, J. R., III. 1977, *Ap. J.*, **216**, 194.
Tinsley, B. M. 1981, *M.N.R.A.S.*, **194**, 63.
Toomre, A., and Toomre, J. 1972, *Ap. J.*, **178**, 623.
Tully, R. B. 1982, *Ap. J.*, **257**, 389.
Tully, R. B., Bottinelli, L., Fisher, J. R., Gouguenheim, L., Sancisi, R., and van Woerden, H. 1978, *Astr. Ap.*, **63**, 37.

Tully, R. B., and Shaya, E. J. 1984, *Ap. J.*, **281**, 31.
van den Bergh, S. 1982, *Pub. A.S.P.*, **94**, 459.
Vettolani, G., Marano, B., Zamorani, G., and Bergamini, R. 1980, *M.N.R.A.S.*, **193**, 269.
White, S. D. M., and Rees, M. J. 1978, *M.N.R.A.S.*, **183**, 341.
White, S. D. M., and Silk, J. 1979, *Ap. J.*, **231**, 1.

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