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SECULAR VARIATION AND SHORT-TERM FLUCTUATIONS OF THE PULSE PERIOD OF VELA X-1

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ABSTRACT

The pulse period of Vela X-1 was further measured between 1981 December and 1982 December with the *Hakucho* satellite. Combining the newly obtained pulse measurements with the results reported earlier, we have by joint fitting derived the orbital elements of the binary system, which are essentially the same as those given in a previous paper. There is no evidence for the change in orbital period, nor for the advance of periastron. For 14 separate data sets from 1979 March to 1982 December, we have obtained the pulse period, P, and the rate of period change, $\dot{P}$, for each set. The pulse period is found to increase at an average rate of $\dot{P}/P \approx 2.7 \times 10^{-4}$ yr^{-1} between 1979 March and 1981 December and to stay at a nearly constant value of $P = 282.93$ s between 1981 December and 1982 December. Fluctuations in the pulse period change are found within the time scale of an orbital period superposed on the secular trend of pulse period change. The value of $\dot{P}$ fluctuates between $\dot{P}/P \approx (4 \times 10^{-3})$ and (-3×10^{-3}) yr^{-1}. A correlation is found between the long-term period change and the orbital phase dependence of the X-ray spectrum, but no conspicuous correlation with X-ray luminosity is observed.

Subject headings: stars: neutron — stars: pulsation — X-rays: binaries

I. INTRODUCTION

Since the discovery of the binary X-ray source Vela X-1 (4U 0900−40) by Forman *et al.* (1973) and of its 283 s pulsation by McClintock *et al.* (1976), the binary system consisting of this X-ray pulsar and an early-type supergiant HD 77581 has been intensively investigated both in the X-ray and optical ranges (e.g., van Paradijs *et al.* 1977; Conti 1978; Rappaport and Joss 1982). Various X-ray properties have also been observed on the X-ray light curve, the energy spectrum, and the pulse profile (e.g., Nagase *et al.* 1983; White, Swank, and Holt 1983 and references therein).

The pulse period of Vela X-1 has been monitored over 5 years from 1975 to 1979 by *Copernicus* and *Ariel 5* (Charles *et al.* 1978), by *COS B* (Ögelman *et al.* 1977; Molteni *et al.* 1982), by *OSO 8* (Becker *et al.* 1978), by *SAS 3* (Rappaport, Joss, and McClintock 1976; Rappaport and Joss 1977; Rappaport, Joss, and Stothers 1980), and by *HEAO 1* (Bautz *et al.* 1983). The long-term history of the pulse period displayed an average spin-up rate of $\dot{P}/P = -1.5 \times 10^{-4}$ yr^{-1} except for a temporal spin-down episode in 1975 November.

In a previous paper (Nagase *et al.* 1981) we reported on the spin-down of Vela X-1 between 1979 March and 1980 March as well as on the spin-down during 1980 March at a rate of $\dot{P}/P \approx 2.6 \times 10^{-3}$ yr^{-1}. The observation of Vela X-1 with the X-ray astronomy satellite *Hakucho* between 1980 December and 1981 March revealed that the increase in the pulse period continued until 1981 March at a secular rate of $\dot{P}/P \approx 3 \times 10^{-4}$ yr^{-1}, while the rate of period change fluctuated considerably from orbit to orbit (Nagase 1981).

We have further observed Vela X-1 with *Hakucho* between 1981 December and 1982 March and also in 1982 December. Combining the new observations with those previously obtained, we have determined the pulse arrival times through more than 14 cycles of the binary orbit during eight separate time intervals between 1979 March and 1982 December.

The joint-fitting of all the pulse arrival-time data consisting of 14 separate data sets over the time span longer than 3 years gives the orbital elements of Vela X-1 with good accuracy, which are consistent with those derived previously from *SAS 3* 1975/1978 data (Rappaport, Joss, and Stothers 1980). Hence, the orbital elements of Vela X-1 seems to be stable from 1975 to 1982 within the accuracy of determination. A slightly improved upper limit for the rate of apsidal motion is obtained.

The joint-fitting has also confirmed the new features of pulse period change of Vela X-1 reported in previous papers

TABLE 1

VELA X-1 OBSERVATION WITH *HAKUCHO* (1979.3–1982.12)

DATA SET No.	OBSERVATION DATE (UT) (y/m/d)	SPAN (days)	NO. OF EPOCHS[a]	FLUX (counts s^{-1}) Total[b]	S.D.[c]	$L_x(\times 10^{36})$[d] (ergs s^{-1})
1	1979/3/8.1–21.8	13.7	35	...	...	...
2	1980/3/5.2–15.0	9.8	56	15.5	1.1	1.1
3	1980/3/15.1–24.0	8.9	59	19.6	1.0	1.1
4	1980/12/16.5–26.2	9.7	49	16.7	1.9	1.2
5	1981/1/7.8–18.9	11.1	54	27.1	1.5	1.7
6	1981/1/19.0–30.7	11.7	55	14.7	0.8	1.2
7	1981/3/2.7–15.0	12.3	68	17.9	1.2	1.0
8	1981/12/14.6–25.6	11.0	26	24.9	3.2	2.0
9	1982/1/18.8–29.8	11.0	29	21.3	3.7	1.8
10	1982/1/30.6–2/8.6	9.0	20	18.0	2.6	1.3
11	1982/2/8.7–18.4	9.7	22	10.9	2.7	0.9
12	1982/2/19.4–26.6	7.2	22	15.6	9.2	1.4
13	1982/2/27.0–3/6.5	8.5	43	15.5	5.7	1.1
14	1982/12/9.9–24.6	14.7	47	13.3	4.1	0.9

[a] Number of pulse arrival times used in the present fitting.

[b] Flux observed in the energy range 1–22 keV, averaged over each observation span.

[c] Standard deviation of the observed flux averaged over each one pulse. Statistical uncertainty of each one-pulse averaged flux is typically ± 0.5 counts s^{-1}.

[d] The distance to Vela X-1 is assumed to be 1.2 kpc. The luminosity is estimated from the average flux in the H band (9–22 keV).

(Nagase *et al.* 1981; Nagase 1981). The observations between 1981 December and 1982 December indicate that the secular spin-down of Vela X-1 has ceased and that the pulse period stays nearly constant during this time interval. A change in the pulse period on the time scale of a few days, as partly reported earlier (Hayakawa and Nagase 1982), is also examined by a more extensive analysis including the $\ddot{P}$ term.

The time variations of pulse period provide a means to diagnose physical properties of the accreting neutron star (Rappaport and Joss 1977; Lamb, Pines, and Shaham 1978*a*, *b*; Ghosh and Lamb 1978, 1979; Wang 1981). We investigate correlations of the pulse period change with the X-ray luminosity and with the orbital phase dependence of the X-ray spectrum, the latter being related to the pattern of stellar wind fed to the neutron star (Nagase *et al.* 1983). No significant correlation is found with the former, but the correlation with the latter supports an earlier suggestion (Hayakawa and Nagase 1982) that the wind matter is more concentrated behind the X-ray source when the spin-down is prominent.

II. OBSERVATIONS

Vela X-1 was observed with the Japanese astronomical satellite *Hakucho* between 1979 March and 1982 December as summarized in Table 1. All the observed data were divided into 14 data sets in which each data set covered the time interval of nearly one orbital period of Vela X-1. The data sets 2 and 3, 5 and 6, and 9–13 were obtained during continuous observations. The data from the counter with a honeycomb field-of-view (5°.8 FWHM) and an effective area of 82.5 cm^2 (FMC-2) were used in the present analyses of timing and flux determination, while the counters with coarse (CMC-1 and CMC-2) and fine (FMC-1) modulation collimaters were used to determine the aspect of the satellite.

In Figure 1 the long-term light curves of Vela X-1 observed from 1980 March to 1982 December are shown for the energy ranges 1–9 keV (L band) and 9–22 keV (H band), in which each data point represents the X-ray flux averaged over one pulse period ($\sim$283 s) after subtracting the background and correcting for the satellite aspect. The light curve during 1979 March is discarded from this figure because during this observation period, which was just after the launch of the *Hakucho* satellite, both the attitude and the observation mode were frequently changed for initial functional test.

In the fifth column of Table 1 is given the X-ray flux in the range 1–22 keV averaged for all available pulse-averaged X-ray count rates in each observation interval. As Vela X-1 often exhibits an erratic variation of intensity by a factor of 3 or 4 times the average on the time scale of hours to days, the standard deviation of the observed pulse-averaged fluxes is also listed for each observation interval to indicate the variability measure of flux. This standard deviation is compared with the statistical uncertainty typically of ± 0.5 counts s^{-1} for the pulse-averaged flux.

An approximate X-ray luminosity corresponding to each set of observation interval is listed in the last column of Table 1, in which the distance of 1.2 kpc is assumed for Vela X-1 (Hiltner, Werner, and Osmer 1972). Because the flux in the L band is affected by the absorption by circumstellar matter, the total X-ray luminosity is derived from the X-ray flux observed in the H band, assuming a power-law spectrum with an index of $\alpha = 1.2$ (Nagase *et al.* 1983) together with the spectral steepening above 20 keV (Becker *et al.* 1978; Staubert *et al.* 1980; Bautz *et al.* 1983).

III. TIMING ANALYSIS

Because of a large variability of X-ray intensity on the time scale of minutes to days, the pulse profile of Vela X-1 changes from pulse to pulse and from one satellite orbit to another. This gives rise to a restriction to the determination of pulse arrival times when we compute the cross-correlation

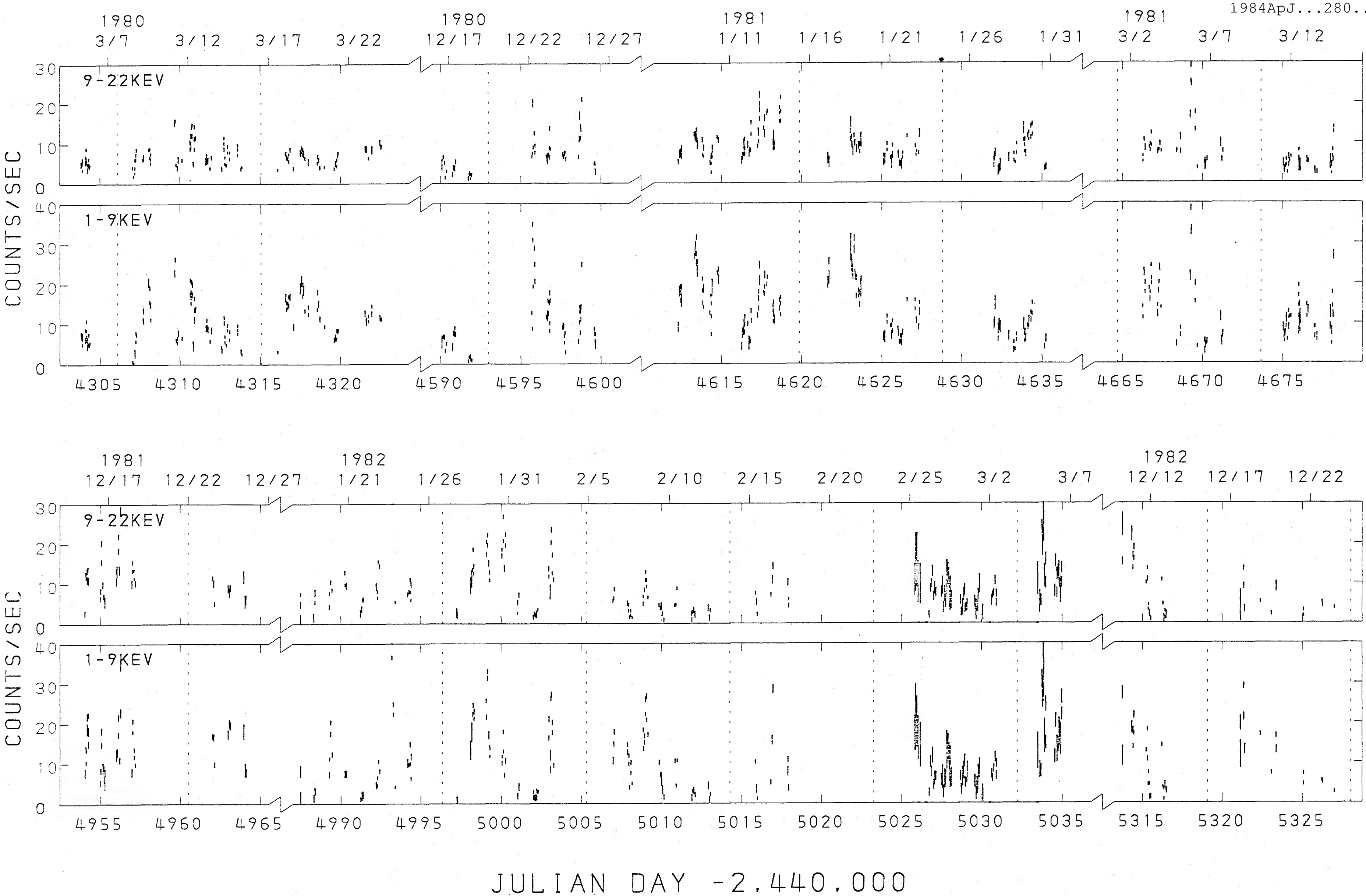

FIG. 1.—Light curves of Vela X-1 in the energy ranges 1–9 keV and 9–22 keV observed with *Hakucho* satellite between 1979 March and 1982 December. Each data point represents the X-ray flux averaged over one pulse after subtracting the background and correcting for the satellite aspect. The dashed lines in the figure represent the eclipse centers.

of each observed pulse profile with a template pulse profile, in which the template is constructed from the pulse profile superposed over a long time interval folding modulo a temporal pulse period. Since we have found that a sharp dip in the pulse profiles is characteristic to Vela X-1 and relatively stable, we have used this dip as a fiducial point of pulse arrival time and determined the center of the dip utilizing the parabolic fitting to the dip profile, in which each pulse profile is obtained by folding several successive pulses modulo a temporal pulse period. Pulse arrival times derived with the above method jitter less than those with the cross-correlation method, though consistent with each other in the overall behavior.

During the fourteen intervals of observations in Table 1, a total of 585 pulse arrival times were obtained with errors ranging from 2 s to 10 s due to the variation of the X-ray intensity and the drift of the satellite axis. The number of pulse arrival times available for each set of observation interval is listed in the fourth column of Table 1. All epochs of pulse arrival times are converted to the heliocentric pulse arrival times by correcting for the orbital motion of the earth.

We have further carried out a simultaneous fit to all of the pulse arrival times for all data sets listed in Table 1. For each data set $k (k = 1, 2, \ldots, 14)$ the arrival time t_{nk} of the nth pulse $(n = 0, 1, 2, \ldots)$ is fitted to the expression

$$t_{nk} = t_{0k} + P_{0k} n + \tfrac{1}{2} P_{0k} \dot{P}_k n^2 + a_x \sin i F(\theta, \omega, e, \tau, \dot{\omega}), \quad (1)$$

where P_{0k} is the pulse period at the initial epoch t_{0k}, and $\dot{P}_k$ is the average rate of period change for the kth set. The last term represents the effect of an eccentric orbit with projected semimajor axis $a_x \sin i$, longitude of periastron ω, eccentricity e, time of periastron passage τ, and mean anomaly θ, where $\theta = 2\pi(t - \tau)/P_{\rm orb}$, and $P_{\rm orb}$ is the orbital period. The orbital elements except for the longitude of periastron were assumed to be constant during the observation intervals of 3.8 years from 1979 March to 1982 December. The rate of change in the longitude of periastron $\dot{\omega}$ was included in the function F of equation (1) to examine the rate of apsidal motion.

The 585 pulse arrival times were first fitted to equation (1) with $\dot{\omega}$ fixed to be zero (i.e., under the assumption of no apsidal motion), a total of 47 free parameters including five orbital parameters, $P_{\rm orb}$, $a_x \sin i$, ω, e, and τ, and t_{0k}, P_{0k}, and $\dot{P}_k$ in 14 data sets. The result gives a good fit with the orbital parameters consistent with the previous work (Rappaport, Joss, and Stothers 1980). The joint-fitting also gives the pulse periods consistent with those derived from the Doppler fitting for individual data sets. The rates of pulse period change are found significant for some data sets. The fitting by equation (1) with a total of 48 free parameters including $\dot{\omega}$ scarcely improves the fit. The best fit value of $\dot{\omega} = 0\overset{\circ}{.}3 \pm 1\overset{\circ}{.}1$ yr^{-1} is obtained for the rate of apsidal motion, together with the orbital parameters and pulse periods consistent with those derived above. The orbital elements obtained from this fit are given in Table 2.

The orbital elements derived by Rappaport, Joss, and Stothers (1980) from the joint-fitting of *SAS 3* 1975 and 1978 data are also listed in the same table. Both sets of orbital elements are consistent with each other at the 2 σ level or better. The binary orbital period of 8.9642 ± 0.0006 days derived from the data between 1979 and 1982 is consistent with that of 8.9649 ± 0.0002 days derived from the data between 1975 and 1978 by Rappaport, Joss, and Stothers (1980). These two sets of data give the upper limit of $|\dot{P}_{\rm orb}/P_{\rm orb}| \lesssim 6.7 \times 10^{-5}$ yr^{-1} (2 σ level) for the rate of change in the binary period.

The pulse periods and the rates of change in the pulse period for the 14 data sets obtained from the joint-fitting to equation (1) are summarized in Table 3 together with the initial epochs of individual data sets. The values of pulse period P_{0k} thus derived are shown in Figure 2 by filled circles together with those obtained from the earlier observations (*open circles*). The changes of pulse period during each observation interval, $(\Delta P/\Delta t)$, are indicated by the arrows attached to the open and filled circles for the data for which significant rates of change of pulse period $\dot{P}$ are obtained.

In contrast to the overall tendency of spin-up until 1978, a secular spin-down at an average rate of $\dot{P}/P \simeq 2.7 \times 10^{-4}$ yr^{-1} is apparent for the 3 years between 1979 and 1981, whereas the pulse period stayed at about a constant value of $P \simeq 282.93$ s thereafter until 1982 December.

Superposed on such a secular trend of period change, the joint-fit reveals significant rates of pulse period change during several orbital cycles as shown in Table 3. Their absolute values in respective data sets are often as large as $|\dot{P}/P| \simeq 3 \times 10^{-3}$, which is larger by an order of magnitude than the secular spin-down rate. Moreover, the value of $\dot{P}/P$ changes erratically and sometimes reverses its sign. Such a short-term period change has been recently reported also by Molteni *et al.* (1982) and by Bautz *et al.* (1983).

We have examined the residuals of the pulse arrival times with respect to the fitted ones to see if there is any random

TABLE 2

ORBITAL ELEMENTS OF VELA X-1 DERIVED FROM THE COMBINED FITTING

Element	*SAS 3*[a] 1975/1978	*Hakucho* 1979–1982
Projected semimajor axis; $a_x \sin i (lt - s)$	113.0 ± 0.8[b]	114.1 ± 0.5[b]
Mass function; $f(M)$ $(M_\odot)$	19.3 ± 0.4	19.9 ± 0.5
Eccentricity; e	0.092 ± 0.005	0.080 ± 0.006
Periastron longitude; ω	$154° \pm 5°$	$157\overset{\circ}{.}3 \pm 2\overset{\circ}{.}1$
Periastron passage time; τ (JD $-2,440,000$)	3823.40 ± 0.13	5329.55 ± 0.06
Orbital period; $P_{\rm orb}$ (days)	8.9649 ± 0.0002	8.9642 ± 0.0006
Apsidal motion; $\dot{\omega}$(yr^{-1})	$0\overset{\circ}{.}4 \pm 1\overset{\circ}{.}7$	$0\overset{\circ}{.}3 \pm 1\overset{\circ}{.}1$

[a] From Table 1 of Rappaport, Joss, and Stothers 1980.
[b] Quoted uncertainties are all single parameter 1 σ confidence limits.

TABLE 3

PULSE PERIOD OF VELA X-1 DERIVED FROM THE COMBINED ORBITAL FIT WITH P AND $\dot{P}$

Data Set No.	Temp. Origin of Epoch T_{0k}(JD −2,440,000)	Pulse Period P_{0k}(s)	Period Change Rate $\dot{P}_k(\times 10^{-9})$
1.	3941.24214 ± 0(4)	282.7462 ± 0.0035	−21.1 ± 6.2
2.	4303.82974 ± 0(2)	282.7931 ± 0.0025	10.8 ± 6.1
3.	4313.67855 ± 0(3)	282.8063 ± 0.0029	29.0 ± 6.9
4.	4590.02166 ± 0(2)	282.8814 ± 0.0023	−26.1 ± 6.1
5.	4612.30311 ± 0(2)	282.8437 ± 0.0019	35.7 ± 3.5
6.	4623.47671 ± 0(3)	282.8850 ± 0.0024	4.3 ± 4.2
7.	4666.27380 ± 0(2)	282.9030 ± 0.0017	10.4 ± 3.2
8.	4953.06897 ± 0(3)	282.9454 ± 0.0039	19.2 ± 8.0
9.	4988.32990 ± 0(3)	282.9497 ± 0.0029	−9.6 ± 5.3
10.	5000.07671 ± 0(3)	282.9321 ± 0.0039	−1.5 ± 9.0
11.	5009.16068 ± 0(3)	282.9252 ± 0.0051	23.3 ± 12.5
12.	5019.85251 ± 0(5)	282.9367 ± 0.0088	6.4 ± 26.3
13.	5027.49546 ± 0(3)	282.9264 ± 0.0049	19.8 ± 14.0
14.	5313.43500 ± 0(2)	282.9265 ± 0.0018	4.5 ± 3.3

NOTE.—All quoted uncertainties are single parameter 1 σ confidence limits.

fluctuation of pulse period on the time scale shorter than an orbital period derived from the present analysis. The number distribution of the residuals of pulse arrival times, that is the difference between the observed pulse arrival time and those estimated from the best fit parameters of equation (1), is shown in Figure 3. This distribution is reasonably fitted by the Gaussian distribution function with the standard deviation of $\sigma = 3.5 \pm 0.2$ s as shown by a dashed line in Figure 3. This value is comparable to the typical uncertainty in the determination of pulse arrival times.

The fluctuation of pulse period as revealed by the variation of $\dot{P}$ from one set to another indicates that the pulse period variation can be better expressed by a quadratic form than a linear form. We therefore carried out fitting by adding a term $(\frac{1}{6})P_{0k}{}^2\ddot{P}_k n^3$ to equation (1). Significant contributions of the $\ddot{P}$ term were obtained for data sets 2–7, in which a

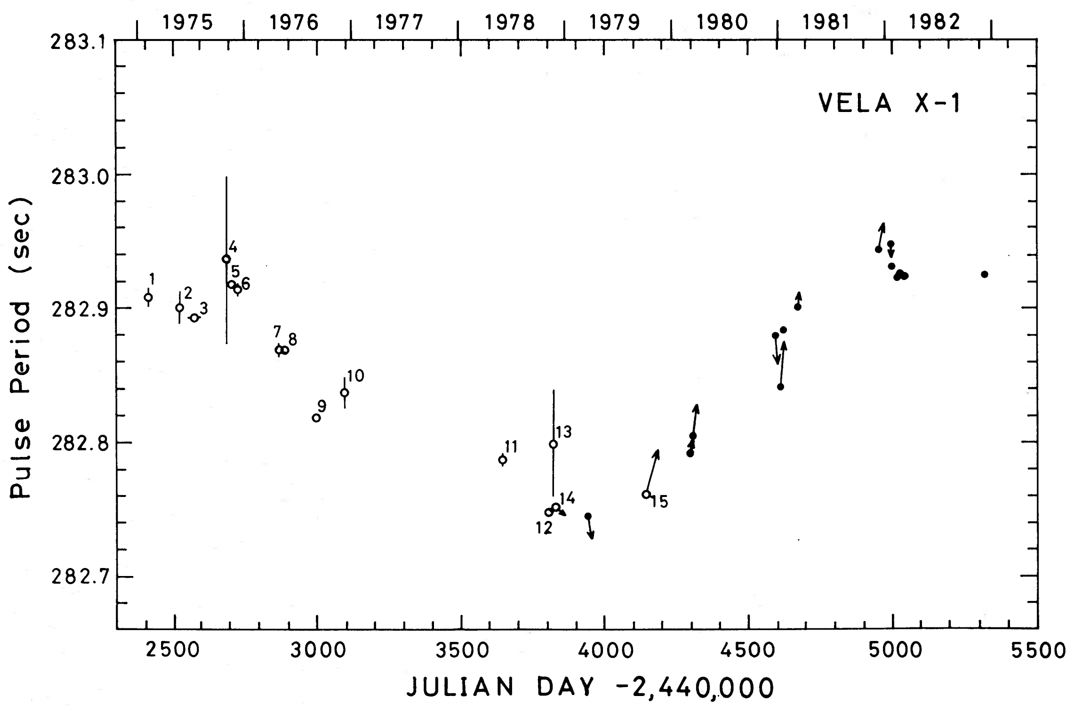

FIG. 2.—The long-term history of the pulse period change of Vela X-1. Open circles with the numbers 1–15 attached show the pulse periods measured previously (1, 2, and 4 from Charles *et al.* 1978; 3 from Rappaport, Joss, and McClintock 1976; 5 from Ögelman *et al.* 1977; 6, 8, and 10 from Becker *et al.* 1978; 7 from Rappaport and Joss 1977; 9 and 15 from Molteni *et al.* 1982; 11 and 14 from Bautz *et al.* 1983; 12 from Rappaport, Joss, and Stothers 1980; and 13 from Staubert *et al.* 1980). Filled circles are derived from the present combined fitting to all the data observed with *Hakucho*. The arrows represent the changes in pulse period during the observation intervals. No arrow is drawn if the rate of period change obtained is insignificant.

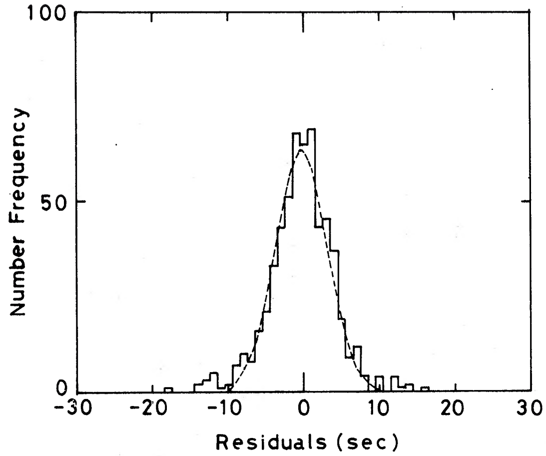

FIG. 3.—Number frequency distribution of residual delays of pulse arrival times from the best fit estimate with eq. (1) in the text. The dashed line represents the Gaussian distribution function with $\sigma = 3.5$ s.

total of 341 pulse arrival times are available with relatively good accuracy. The result of quadratic fitting is compared with that of linear fitting in Figure 4. This indicates that the pulse period sometimes changes on a time scale of about one-half of the orbital period. Pulse period variations of still shorter time scales are hardly obtainable in the present accuracy of observation, as we found no improvement of fitting by the addition of $\dddot{P}$ and higher order terms.

IV. DISCUSSION

a) *Orbital Parameters*

The orbital parameters derived from the present analysis show good agreement with those derived from the joint-fit of *SAS 3* 1975 and 1978 data by Rappaport, Joss, and Stothers (1980) within an accuracy of 2 σ limit as compared in Table 2. The influence of short-term fluctuations of pulse period in the determination of orbital parameters can be eliminated by combining all the sets of pulse arrival-time data for fitting.

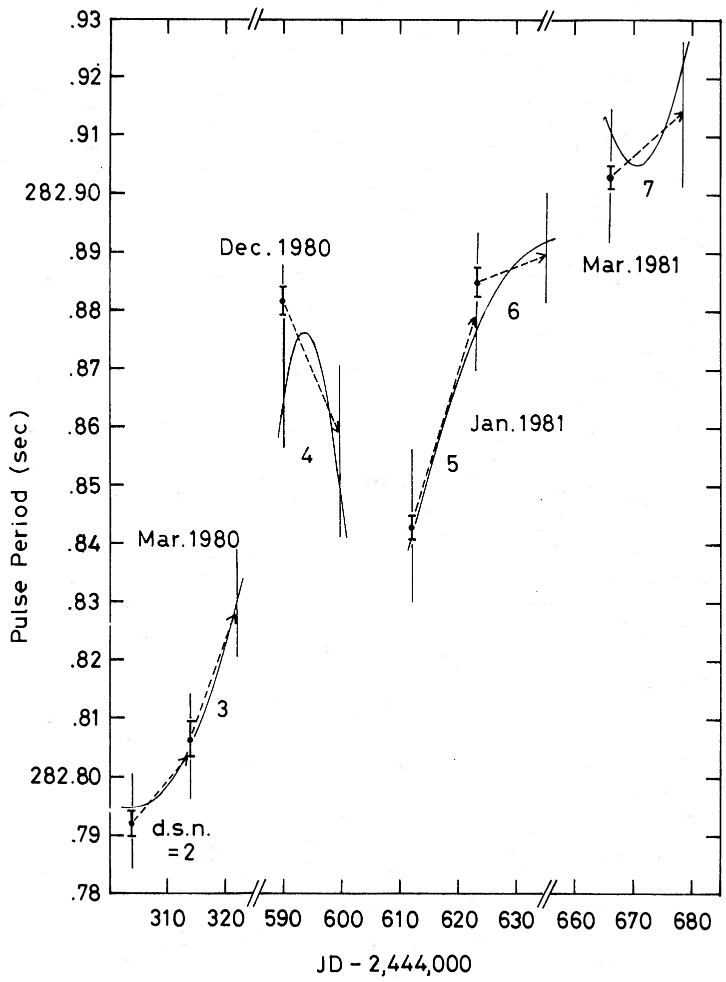

FIG. 4.—Intrinsic pulse period variations of Vela X-1 between 1980 March and 1981 March (data sets 2–7) reconstructed from the derived best fit P and $\dot{P}$ (*dashed line*) and P, $\dot{P}$, and $\ddot{P}$ (*solid line*) with fixed orbital elements of Table 2. In the latter fitting, data sets 2 and 3, and 5 and 6 are combined together and considered as single data sets, respectively.

We note that there is as yet no positive evidence for apsidal motion. The best fit value of $\dot{\omega} = 0\overset{\circ}{.}3 \pm 1\overset{\circ}{.}1$ yr^{-1} gives an upper limit of $\dot{\omega} \lesssim 2\overset{\circ}{.}5$ yr^{-1} (97% confidence) for the rate of apsidal motion. This value of upper limit slightly improves the results of $\omega \lesssim 3\overset{\circ}{.}8$ yr^{-1} derived from *SAS 3* observations (Rappaport, Joss, and Stothers 1980). The joint-fit by combining our data sets with *SAS 3* 1975 data (Rappaport, Joss, and McClintock 1976) gives the best fit value of $\dot{\omega} = -0\overset{\circ}{.}74 \pm 0\overset{\circ}{.}72$ yr^{-1}, or the upper limit of apsidal motion $\dot{\omega} \lesssim 2\overset{\circ}{.}2$ yr^{-1} (97% confidence). This limit yields a restriction to the apsidal motion constant k which reflects the structure of the companion star HD 77581. Using the same orbital and stellar parameters as Rappaport, Joss, and Stothers (1980), we obtain a limit of $\log k \lesssim -2.7$ (97% confidence).

b) Spin Reversal in Disk-fed Accretion

The observed secular spin-down makes it difficult to interpret this X-ray pulsar by the disk-fed accretion model developed by Rappaport and Joss (1977) and by Ghosh and Lamb (1978, 1979), unless the magnetic momentum of Vela X-1 is unusually greater than that estimated for other known X-ray pulsars. The corotation radius for Vela X-1 with the long pulse period of 283 s would require the surface magnetic field of $\gtrsim 10^{13}$ gauss for the reversal of accretion torque to occur by the interchange of the radii of corotation and magnetopause.

c) X-Ray Luminosity and Spin Period Change

The X-ray luminosity averaged over each data set ranges from 1×10^{36} ergs s^{-1} to 2×10^{36} ergs s^{-1}, assuming the distance to Vela X-1 of 1.2 kpc (Hiltner, Werner, and Osmer 1972), as seen in Table 1. If the distance to Vela X-1 is assumed to 2.2 kpc (Conti 1978), the above value increases by a factor of 3.3. There is no clear evidence for the change in X-ray luminosity between the period of secular spin-down from 1979 March to 1981 December and the period of constant spin rate thereafter to 1982 December.

The X-ray luminosities observed during the spin-down phase are also comparable to those observed during the spin-up phase between 1975 and 1978 by McClintock *et al.* (1976), by Watson and Griffiths (1977), and by Becker *et al.* (1978). Evidence for the correlation of spin period change with a drastic change of X-ray luminosity is not obtained. The deviation of X-ray flux from the average as given in Table 1 arises from the intensity variability on the time scale of minutes to days and is sometimes associated with flares of an intensity increase by an order of magnitude (Watson and Griffiths 1977; Charles *et al.* 1978; Tsunemi *et al.* 1981). Such variations seem to occur at random without correlation with spin period change.

d) Ineffectiveness of Internal Torque

In a previous paper (Nagase *et al.* 1981) we suspected a possibility of spin-down between 1979 March and 1980 March as due to the internal torque acting between crust and core of the neutron star. However, such a long-term tendency of spin-down continuing over 3 years as observed with *Hakucho* can hardly be explained by this mechanism, because such a long coupling time between crust and core seems to be unlikely. It is also difficult to attribute the short-term pulse period fluctuation to an internal origin as explained below.

The accumulation of a noiselike variation in the internal torque on the crust of a neutron star has been considered as a cause of pulse period fluctuations by Lamb, Pines, and Shaham (1978*a*, *b*). The crust pinning and unpinning of vortex lines in the rotating neutron star superfluid and the fracture of crust lattice by the Magnus force are considered as a possible explanation of glitch phenomena in radio pulsars (Ruderman 1976). The condition to unpin the vortex lines is given by equating the Magnus force and the pinning force for the angular velocity difference $\delta\Omega$ as

$$\frac{h}{2m_n}\rho R\delta\Omega \approx f_p\frac{\pi\xi^2}{b^3}\,, \tag{2}$$

in which h, m_n, ρ, R, ξ, and b are the Planck constant, the neutron mass, the density of inner crust, the radius of the neutron star, the size of vortex core, and the crust lattice spacing between nuclei, respectively. The pinning force f_p is estimated as

$$f_p \approx \frac{\Delta^2}{E_F}k_F{}^3\xi^2 \approx 10^8 \text{ dyn}\,,$$

by using the BCS energy gap of $\Delta \sim 2$ MeV, the Fermi energy of $E_F \sim 10$ MeV, the corresponding wave number of $k_F \sim 8 \times 10^{12}$ cm^{-1}, and the size of vortex core of $\xi \sim E_F/(k_F\Delta) \sim 10^{-12}$ cm (Ruderman 1976). Equations (2) and (3) yield the critical value for the difference of angular velocity between the crust and the core;

$$(\delta\Omega)_{\text{crit}} \approx 30\left(\frac{f_p}{10^8\text{ dyn}}\right)\left(\frac{\xi}{10^{-12}\text{ cm}}\right)\left(\frac{b}{5\times10^{-12}\text{ cm}}\right)^{-3} \times\left(\frac{\rho}{3\times10^{-13}\text{ g cm}^{-3}}\right)^{-1}\left(\frac{R}{10^6\text{ cm}}\right)^{-1}\text{ s}^{-1}\,. \tag{4}$$

On the other hand, the difference of angular velocity arising from the observed fluctuation of pulse period $\Delta P/P \approx 1 \times 10^{-4}$ is estimated to be

$$(\delta\Omega)_{\text{obs}} \approx \frac{I}{I_{\text{crust}}}\left(\frac{\Delta P}{P}\right)\Omega \approx (2\times10^{-6})\text{–}(2\times10^{-4})\text{ s}^{-1}\,, \tag{5}$$

assuming the ratio of the moment of inertia of the whole neutron star I to that of crust I_{crust} to be 1–10^2. The observed value estimated in equation (5) is smaller by several orders of magnitude than that required in equation (4). Therefore, it is difficult to explain the observed period fluctuation by vortex pinning and unpinning in the neutron star interior. The fracture of crust lattice by the Magnus force seems more difficult because this requires an angular velocity difference about an order of magnitude greater than that for crust unpinning.

e) Stellar Wind Origin of Pulse Period Change

In the case of accretion from a stellar wind, torque variations could be produced by inhomogeneities in the flow of stellar wind (Henrichs 1982 and references therein). The radial density gradient in the spherically expanding stellar wind gives the net angular momentum per unit mass (Shapiro and Lightman 1976)

$$l = \frac{1}{2}v_x a\left(\frac{r_a}{a}\right)^2\eta\,, \tag{6}$$

where v_x, a, and r_a are the orbital velocity of the X-ray star, the stellar separation of the system, and the accretion radius, respectively. A numerical factor η is unity in the above case where the wind velocity is assumed constant. In the presence of radial velocity gradient, the value of η is (Wang 1981)

$$\eta = 1 + \frac{7}{2}\frac{a}{v_w}\left(\frac{\partial v_w}{\partial r}\right)_{r=a} \approx 1 + \frac{7}{4(a/R_* - 1)}, \qquad (7)$$

where v_w is the wind velocity, $(\partial v_w/\partial r)_{r=a}$ is its gradient at the center of a cylinder in which the accreting gas is captured, and R_* is the radius of the companion star. With the values of $a = 54\ R_\odot$, $R_* = 31\ R_\odot$ (Rappaport and Joss 1982), we have $\eta = 3.4$.

The accretion radius for the X-ray star with mass of M_x is estimated to be

$$r_a = \frac{\zeta 2GM_x}{{v_{\rm rel}}^2} \approx 6 \times 10^{10}\zeta\left(\frac{M_x}{1.9\ M_\odot}\right)\left(\frac{v_{\rm rel}}{910\ {\rm km\ s^{-1}}}\right)^{-2}\ {\rm cm}, \qquad (8)$$

where $v_{\rm rel}$ is the wind velocity relative to the X-ray star, $v_{\rm rel} = {v_x}^2 + {v_x}^2$, and ζ is a factor near unity. For $M_x \approx 1.9\ M_\odot$, $v_x \approx 300$ km s^{-1} (Rappaport and Joss 1982), and $v_w \approx 860$ km s^{-1} (Dupree *et al.* 1980); hence, $v_{\rm rel} \approx 910$ km s^{-1}, and equation (8) gives the ratio of $r_a/a \simeq 0.016$ for Vela X-1. Equation (6) together with equation (8) gives a maximum rate of period change assuming that all the captured angular momentum is transferred to the neutron star,

$$-\frac{\dot P}{P} = \frac{P}{2\pi}\frac{l\dot M}{I} \approx 4.2 \times 10^{-4}\zeta^2\eta\left(\frac{\dot M}{10^{16}\ {\rm g\ s^{-1}}}\right)\left(\frac{I}{10^{45}\ {\rm g\ cm^2}}\right)^{-1} \times \left(\frac{v_{\rm rel}}{910\ {\rm km\ s^{-1}}}\right)^{-3}\ {\rm yr^{-1}}, \qquad (9)$$

in which the accretion rate of $\dot M \approx 10^{16}$ g s^{-1} corresponds to an X-ray luminosity of $\sim 2 \times 10^{36}$ ergs s^{-1}. The moment of inertia of the neutron star is normalized to 10^{45} g cm^2 (Arnett and Bowers 1977).

The value of $\dot P/P \approx -1.5 \times 10^{-3}$ yr^{-1} is obtained from equation (9) adopting $\zeta = 1$, $\eta = 3.4$. The absolute value is comparable with the measured rate of short-term period change, as shown in Table 3. However, equation (9) holds for a wind with radial density and/or velocity gradients expected for a homogeneous flow, and $\dot P$ is always negative for such a case; i.e., the neutron star accreting a stellar wind is spun up in the sense of its orbital motion around the companion star. Reversal in the direction of accretion torque may occur (*i*) if there are significant azimuthal anisotropies in the stellar wind density and/or velocity, or (*ii*) if the neutron star has an accreting surface tilted at certain angles with respect to the direction of the orbital motion (Wang 1981). In case (*i*), the spin-down takes place for

$$\frac{\partial v_w}{\partial \phi} \lesssim -\frac{1}{3}v_x, \quad \text{or} \quad \frac{\partial \rho}{\partial \phi} \gtrsim 2\rho\frac{v_x}{v_w}, \qquad (10)$$

where ρ is the density of the wind at the orbit of the neutron star.

For a neutron star with a tilted accreting surface the wind velocity relative to the neutron star is expressed as ${v_{\rm rel}}^2 = {v_r}^2 + (v_x - v_\phi)^2$, where v_r and v_ϕ are the radial and azimuthal components of the wind velocity; hence $v_x - v_\phi$ represents the azimuthal velocity of the neutron star relative to the material coming from the primary. The numerical expression of angular momentum of captured matter is also derived by Wang (1981) for this case. It should be noted that the neutron star is spun up if $v_\phi < v_x$ and is spun down if $v_\phi > v_x$, provided that the wind density does not depend on the azimuthal angle.

In the previous paper (Nagase *et al.* 1983), we suggested the orbital phase dependence of the X-ray absorption measure which was derived from the energy spectrum and the softness ratio of X-rays observed in 1980 March. This feature of X-ray absorption seems to be maintained during the course of secular spin-down of Vela X-1. In Figure 5*a* we show the orbital phase dependence of the ratio of X-ray flux in the L band to that in the H band together with that of the H band flux and L band flux for the data observed between 1980 March and 1981 March, folded modulo the orbital period of 8.9642 days. The corresponding absorption measure is marked on the right side ordinate of the softness ratio.

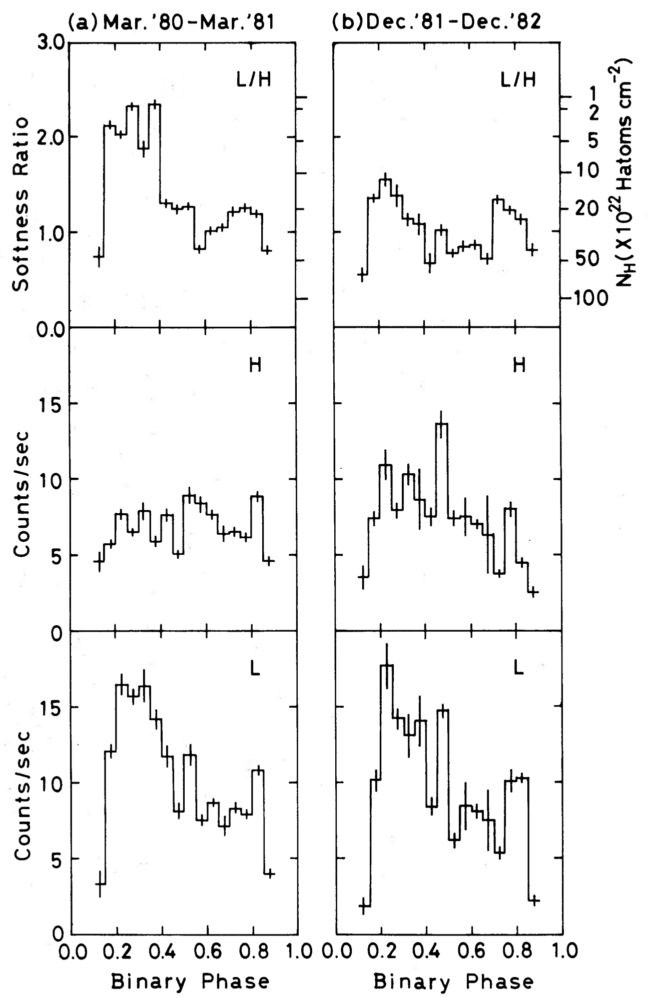

FIG. 5.—The softness ratio (L band/H band) and the X-ray fluxes of the H band and L band are shown (*a*) for the observation period between 1980 March and 1981 March (data sets 2–7), and (*b*) for the observation period between 1981 December and 1982 December (data sets 8–14), folded modulo the binary period 8.9642 days.

TABLE 4

AVERAGE FLUX AND SOFTNESS RATIO IN THE ORBITAL PHASES 0.2–0.4 AND 0.6–0.8

Period	Φ_{orb}	L band Flux (counts s^{-1})	H band Flux (counts s^{-1})	Softness Ratio (L/H)	$\langle N_H \rangle$ ($\times 10^{23}$ cm^{-2})
1980/3–1981/3	0.2–0.4	15.6 ± 0.5	6.7 ± 0.3	2.15 ± 0.04	0.3
	0.6–0.8	8.2 ± 0.3	6.7 ± 0.2	1.4 ± 0.02	2.4
1981/12–1982/12	0.2–0.4	14.3 ± 0.8	9.6 ± 0.5	1.23 ± 0.05	2.0
	0.6–0.8	7.9 ± 0.7	6.4 ± 0.8	0.86 ± 0.05	4.0

As the total interstellar column density in the direction toward Vela X-1 is 6×10^{21} H atoms cm^{-2} (Daltabuit and Meyer 1972), most of the variation in the absorption measure of $(0.1–5) \times 10^{23}$ H atoms cm^{-2} should be caused by the inhomogeneities of circumstellar absorbing matter. The absorption measure in the rear side of the orbiting neutron star ($\Phi_{orb} \approx 0.6–0.8$) is by an order of magnitude greater than that in the front side ($\Phi_{orb} \approx 0.2–0.4$). This feature of anisotropic absorption may indicate the existence of an azimuthal dependence in the flow of stellar wind surrounding the neutron star, in agreement with the result of an Hα observation by Bessel, Vidal, and Wickramasinghe (1975).

The orbital phase dependence of the softness ratio (L band/H band) and that of the H band flux and L band flux are shown in Figure 5*b* for the constant pulse period phase between 1981 December and 1982 December. The orbital phase dependence of the softness ratio in this period is different from that in the spin-down phase, which seems to be due to the enhancement of the H band flux in the earlier phase of orbit as seen Figure 5*b*. The values of L band flux, H band flux, and the softness ratio averaged over the binary phases $\Phi_{orb} = 0.2–0.4$ and $\Phi_{orb} = 0.6–0.8$ are summarized in Table 4 together with the approximate absorption measure. The softness ratio in the earlier phase is 1.89 ± 0.04 times larger than that in the later phase during the spin-down phase from 1980 March to 1981 March. On the contrary, during the constant pulse-period phase from 1981 December to 1982 December, the softness ratio in the earlier phase is only 1.42 ± 0.08 times larger than that in the later phase.

The difference between the softness ratios in the spin-down and constant period cases is due to that between the H band light curves, as seen from Figure 5. In the constant period phase the H band flux in $\Phi_{orb} = 0.2–0.4$ is appreciably higher than that in $\Phi_{orb} = 0.6–0.8$. The orbital phase dependences of the X-ray intensities in the low- and high-energy range during the spin-up period (Watson and Griffiths 1977; Holt *et al.* 1979) are different from those in the spin-down period, as noted by Hayakawa and Nagase (1982).

Summarizing the above we remark that the secular variation of the spin period is related to the orbital phase dependence of the X-ray spectrum, but not to other quantities, as far as the observed data thus far available are concerned. We shall not go into discussions how these two are related to one another, since this is beyond the scope of the present paper which describes the pulse period variations of Vela X-1 observed by *Hakucho*.

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