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MASS LOSS FROM EVOLVED STARS. II. RADIO CONTINUUM EMISSION AND EVOLUTION TO PLANETARY NEBULAE

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ABSTRACT

The radio continuum properties of a sample of 31 evolved red giants with high mass loss rates, plus NGC 7027, were investigated by sensitive observations at 5 GHz (3 σ limit = 0.3–0.6 mJy) using the VLA. Emission was detected from four stars (CRL 618, R Aqr, and, tentatively, IRC +10216 and *o* Ceti).

R Aqr was found to be a double radio source, with a second continuum component about 6″ from the star. We suggest that the second source is a dense clump in the stellar wind. The detections of *o* Ceti and IRC +10216, and the upper limits for the continuum emission from other stars, are used to derive quite low limits for the fractional ionization [$n_e/n(\mathrm{H}) \lesssim 10^{-3}$] in the winds from cool evolved stars. The weak emission from the hotter stars α Ori and *o* Ceti may arise from optically thick chromospheres extended to several stellar radii, with $n_e/n(\mathrm{H}) \sim 0.1$, through which the wind accelerates.

The radio brightening of CRL 618 found by Kwok and Feldman has slowed. The combined data suggest that the ratio of the recent mass loss rate to outflow velocity is smaller than in the past and that the ionization front is slowing down while propagating through a circumstellar envelope of decreasing density. Models of the propagation of an ionization front into an envelope produced by copious mass loss ($\dot{M} \gtrsim 10^{-4}\ M_\odot\ \mathrm{yr}^{-1}$) suggest that the planetary nebulae produced by such stars remain radiation bounded. NGC 7027 probably belongs to this class of objects, CRL 618 is in the transition stage, and IRC +10216 may be a precursor.

Subject headings: nebulae: planetary — stars: late-type — stars: mass loss — stars: radio radiation

I. INTRODUCTION

Recent observations of molecular line emission from evolved red giant stars have shown that many of these objects have very high mass loss rates; values of $\dot{M}$ between about 10^{-7} and $10^{-4}\ M_\odot\ \mathrm{yr}^{-1}$ have been found (e.g., Zuckerman 1978; Knapp *et al.* 1982, hereafter Paper I). It has been proposed (e.g., Zuckerman 1978) that these objects are the precursors of planetary nebulae, with the nebulae being formed when the exposed stellar core of the hot white dwarf ionizes the surrounding envelope. The mechanism causing the mass loss is not, however, well understood. One proposed mechanism involves the deposition of plasma (Alfvén) wave momentum into the lower chromosphere, resulting in the driving of a wind (e.g., Hartmann and McGregor 1980). This mechanism also results in an extended, partially ionized chromosphere; that around α Ori may have been detected at radio frequencies (Altenhoff, Oster, and Wendker 1979).

Both of the considerations discussed above suggest that some mass-losing stars may be detectable in the radio continuum. This paper therefore presents the results of a sensitive 6 cm continuum survey of 31 evolved stars known to be undergoing copious mass loss. Only four of the objects were detected (two tentatively), with the detection limit being $\sim$0.3 mJy.

The observations are described in § II. The symbiotic star R Aqr, known to be a radio source, was observed and found to be double. We interpret the second component as due to a density fluctuation in the stellar wind. The interpretation of these observations is discussed in Appendix A.

In § III, we discuss the limits set by our observations on the presence of extended chromospheres. In § IV, we examine the data for evidence of planetary nebula formation. This section includes estimates of the time scales for ionization of the circumstellar material using simple models of the propagation of an ionization front into the circumstellar cloud, with evolution of the central star included in the calculations. The properties of CRL 618 and NGC 7027 are discussed in the context of these models. The discussion in both § III and § IV requires a revision of the Wright-Barlow (1975) formulation of the continuum emission from a steady stellar wind for the cases of finite envelope size and variable mass loss rate. This discussion is presented in Appendix B. The conclusions are given in § V.

II. CONTINUUM OBSERVATIONS OF EVOLVED STARS

Radio continuum observations of 31 evolved stars known to be undergoing mass loss, and of NGC 7027, were made with the National Radio Astronomy

Observatory's Very Large Array (VLA) on 1981 October 29 and 30, with a bandwidth of 50 MHz centered at 4885 MHz (6 cm). The C configuration was chosen for these observations, since the continuum sources may be extended on scales of seconds of arc (see the discussion by White and Becker 1982). A total of 30 minutes of integration time was spent on each source. Nearby standard phase calibrators were observed between each source for 4 minutes, and the flux densities were measured relative to an adopted flux of 5.36 Jy for 3C 48 (Baars *et al.* 1977). Due to excellent weather conditions, phase closure within 3° was obtained after the data were edited. The visibility data were transformed to maps with a synthesized beamwidth of 3".5 × 5".7 using standard VLA data reduction programs. The maps were cleaned (Högbom 1974) and restored with a 4".5 circular beam.

Four of the 31 stars were detected, with strong emission from CRL 618 and R Aqr and tentative (3 σ) emission from IRC +10216 and *o* Ceti, and all were unresolved by the 5" beam of the VLA. R Aqr was found to be a double source, with a second component about 6" from the primary source; this observation, and its implications for mass loss processes, is analyzed in Appendix A. The integrated flux densities in mJy per beam for the detected stars were found from the uncleaned maps by fitting an elliptical Gaussian. The detection limit for each star was taken to be 3 times the rms noise in the same beam area determined off source. The results are summarized in Table 1, which first gives the star's name, its distance D in pc, the terminal outflow velocity of the wind, V_0, and the mass-loss rate $\dot{M}$ in $M_\odot$ yr^{-1}. The distances are in most cases very uncertain (see the discussion by Morris *et al.* 1979 and in Paper I). The terminal outflow velocities are found from observations of thermal millimeter-wavelength emission lines, mostly CO. The mass loss rates are likewise mostly from CO observations. References for these quantities are given in column (5) of Table 1.

Next in Table 1 we list the 6 cm continuum flux from the object, or its 3 σ upper limit. The ionized fraction of the wind (or its upper limit) was calculated from

$$\dot{M}(\mathrm{H}^+) = \frac{0.095\mu V_0 S_\nu{}^{3/4} D^{3/2}}{Z\gamma^{1/2} g^{1/2} \nu^{1/2}} M_\odot \ \mathrm{yr}^{-1} \tag{1}$$

(Wright and Barlow 1975), where $\dot{M}(\mathrm{H}^+)$ is the mass loss rate for ionized material, S_ν is the flux of the

TABLE 1

5 GHz Continuum Emission from Evolved Stars

Star (1)	D (pc) (2)	V_0 (km s^{-1}) (3)	$\dot{M}$ ($M_\odot$ yr^{-1}) (4)	Reference (5)	S (6 cm) (mJy) (6)	n_e/n_H (7)
IRC +10011	510	24	2.6×10^{-5}	1	<0.45	$<9 \times 10^{-4}$
IRC +40004	1140	22	3.6×10^{-5}	2	<0.34	$<2 \times 10^{-3}$
o Ceti	77	5	2.0×10^{-7}	1	0.74 ± 0.25	$<2 \times 10^{-3}$
IRC +50096	680	18	2.2×10^{-5}	1	<0.30	$<9 \times 10^{-4}$
NML Tau	270	28	4.2×10^{-6}	1	<0.33	$<2 \times 10^{-3}$
CRL 618	1700	18	2.7×10^{-4}	1	16.2 ± 0.1	see text
S CMi	580	18	(6.3×10^{-6})	1	<0.48	$<3 \times 10^{-3}$
IRC +60144	960	25	3.3×10^{-5}	2	<0.39	$<2 \times 10^{-3}$
IRC +60150	300	18	3.7×10^{-6}	2	<0.30	$<2 \times 10^{-3}$
IRC +50137	1790	23	1.2×10^{-4}	2	<0.30	$<1 \times 10^{-3}$
R Aur	366	11	1.3×10^{-6}	2	<0.33	$<4 \times 10^{-3}$
IRC + 70066	736	21	1.3×10^{-5}	2	<0.30	$<2 \times 10^{-3}$
CRL 865	1580	14	1.5×10^{-4}	2	<0.77	$<9 \times 10^{-4}$
IRC +60169	1040	19	2.3×10^{-5}	2	<0.36	$<2 \times 10^{-3}$
VY CMa	1500	37	3.0×10^{-4}	3	<0.30	$<4 \times 10^{-4}$
OH 231.8+4.2	2000	23	3.0×10^{-4}	2, 4	<0.39	$<5 \times 10^{-4}$
IRC +10216	290	17	1.5×10^{-4}	1	0.42 ± 0.10	$<4 \times 10^{-5}$
RS Cnc	410	11	3.4×10^{-6}	1	<0.39	$<2 \times 10^{-3}$
R LMi	395	6	1.0×10^{-6}	1	<0.42	$<4 \times 10^{-3}$
R Leo	304	7	8.5×10^{-7}	1	<0.56	$<4 \times 10^{-3}$
CIT 6	190	17	3.2×10^{-6}	1	<0.56	$<1 \times 10^{-3}$
V Hya	400	18	9.2×10^{-6}	1	<0.33	$<1 \times 10^{-3}$
RT Vir	1000	11	2.0×10^{-5}	2	<0.33	$<1 \times 10^{-3}$
RX Boo	225	8	1.8×10^{-6}	1	<0.49	$<1 \times 10^{-3}$
IRC +20326	1140	19	1.0×10^{-4}	1	<0.36	$<2 \times 10^{-3}$
CRL 2135	1900	20	1.5×10^{-4}	1	<0.32	$<7 \times 10^{-4}$
CRL 2155	1330	20	7.4×10^{-5}	1	<0.36	$<9 \times 10^{-4}$
CRL 2199	2000	15	1.2×10^{-4}	1	<0.38	$<8 \times 10^{-4}$
χ Cyg	97	8	1.8×10^{-7}	1	<0.30	$<4 \times 10^{-3}$
CRL 2688	1000	19	1.3×10^{-4}	1	<0.32	$<3 \times 10^{-4}$
R Aqr	260	30	1.0×10^{-7}	5	7.9 ± 0.2	see text

References.—(1) Paper I. (2) Knapp 1982. (3) Bowers, Johnston, and Spencer 1981. (4) Morris, Bowers, and Turner 1982. (5) Gregory and Seaquist 1974; radio continuum emission estimate for $\dot{M}$.

source in Jy, ν is the observing frequency in Hz, V_0 the outflow velocity in km s^{-1}, and D the distance in kpc. Assuming $\mu = 1.4$, $\gamma = n_e/n_i = 1$, $Z = 1$, and using Mezger and Henderson's (1967) formula for the Gaunt factor, equation (1) gives

$$\dot{M}(\mathrm{H}^+) = 1.7 \times 10^{-6} V_0 T_e^{-0.075} \times S_\nu^{3/4}(5\ \mathrm{GHz}) D^{3/2}\ M_\odot\ \mathrm{yr}^{-1}\ . \quad (2)$$

Values or upper limits for $\dot{M}(\mathrm{H}^+)$ are listed for each observed star in Table 1. The implications of these results for the proposed mass loss mechanisms are discussed in the following section.

III. IRC +10216, *o* CETI, AND THE IONIZATION OF STELLAR WINDS

Apart from CRL 618 and R Aqr, weak continuum emission was tentatively detected from two other objects, with sources of 0.42 ± 0.1 mJy and 0.74 ± 0.25 mJy being found at the positions of IRC +10216 and *o* Ceti, respectively. None of the other stars were detected, and the observations set low limits on the fractional ionization of the stellar winds (Table 1).

IRC +10216 is a late-type star at a distance of 290 pc (Herbig and Zappala 1970), which is losing mass at an extremely rapid rate ($\sim 10^{-4}\ M_\odot\ \mathrm{yr}^{-1}$; Paper I, Kwan and Linke 1982) and has been suggested as a possible planetary nebula precursor (Zuckerman 1978; Terzian 1980). The model by Crabtree and Martin (1979) predicts that the 6 cm flux from the stellar surface and the hot dust is ~ 0.1 mJy. Assuming that the remaining observed flux is due to ionization in the wind, equation (2) gives $\dot{M}(\mathrm{H}^+) = 5.4 \times 10^{-9}\ M_\odot\ \mathrm{yr}^{-1}$ under conditions to be discussed below, so that the fraction of ionized gas is $\sim 3.6 \times 10^{-5}$.

For *o* Ceti, a continuum source of strength 0.74 ± 0.25 mJy was found at $\alpha = 02^{\mathrm{h}}16^{\mathrm{m}}49^{\mathrm{s}}.83$, $\delta = -3°12'19''.0$ (1950), which agrees well with the position of the star when proper motion is taken into account. This is too strong to be emission from the surface of *o* Ceti ($S_\nu < 10^{-6}$ mJy) or from the accretion disk of the companion (cf. the model of Yamashita and Maehara 1978). The ionized fraction of the stellar wind is $\sim 2.0 \times 10^{-3}$ using equation (2). Although Mira has a compact companion, the model of Yamashita and Maehara (1978) predicts $S_\nu(5\ \mathrm{GHz}) = 3 \times 10^{-5}$ mJy.

Together with α Ori, which has an ionized fraction of ~ 0.02 (Altenhoff, Oster, and Wendker 1979), these stars represent the only late-type (nonbinary) stars from which continuum emission has been detected. The remaining stars in Table 1 have very low fractions of ionized gas. The present observations are of interest in the context of proposed mass loss mechanisms which involve the deposition of acoustic (Renzini *et al.* 1977) or Alfvén (Hartmann and MacGregor 1980) wave momentum into the atmosphere. In these models, the associated deposition of wave energy gives rise to an ionized chromospheric region extended to several stellar radii with a temperature of 5000–10,000 K.

The values and upper limits for $n_e/n(\mathrm{H})$ in Table 1 are only weakly dependent on distance [$\dot{M}(\mathrm{H}^+) \propto D^{3/2}$, while $\dot{M}(\mathrm{H}) \propto D^2$] and on electron temperature ($T_e^{0.075}$, through the Gaunt factor). However, equation (1), taken from the discussion by Wright and Barlow (1975), contains several additional assumptions: first, that the ionized fraction x and the wind velocity V_0 are constant, so that $n_e = Ar^{-2}$; second, that the outer radius of the ionized region R_2 is much greater than the characteristic length scale $L = [\kappa(\nu, T_e)A^2]^{-1/3}$, where κ is the free-free absorption coefficient; and third, that the inner radius R_1 is much less than R_2. These conditions may not hold for the case of an extended (several R_*) ionized chromosphere, outside which the wind becomes neutral, such as that predicted by the models of Renzini *et al.* (1977) and Hartmann and MacGregor (1980). Some features of this general case are discussed in Appendix B, where it is shown that the resulting continuum spectrum no longer has a constant spectral index but is characterized by a critical frequency ν_c such that for $\nu \leq \nu_c/2$ the emission is optically thick, while for $\nu \gtrsim \nu_c$ the spectrum turns over and becomes optically thin. If $\dot{M}$ is constant, but through the chromosphere x/V varies as $(x_2/V_0)(R_2/r)^{\beta-2}$, where R_2 is the radius beyond which $V = V_0$ and x drops to zero, then $n_e(r) = Ar^{-\beta}$, where $A = x_2 \dot{M} R_2^{\beta-2}/4\pi\mu m_{\mathrm{H}} V_0$, and the critical frequency from equation (B6) is

$$\nu_c = \left[\frac{0.212}{T_e^{1.35}}\left(\frac{x_2 \dot{M}}{4\pi\mu m_{\mathrm{H}} V_0}\right)^2 \frac{1}{R_2^3}\right]^{1/2.1} . \quad (3)$$

For the detected stars, the radii of the extended chromospheres R_2 can be determined by assuming that the observed flux S_ν is optically thick, as long as the condition $\nu \leq \nu_c/2$ is satisfied. We assume a typical temperature for the chromosphere of $T_e = 7000$ K and use the values for $\dot{M}$, V_0, and D from Table 1, and, for α Ori, from Paper I.

For α Ori, the stellar radius is $0''.037$ (Lynds, Worden, and Harvey 1976; Altenhoff, Oster, and Wendker 1979), which gives $\nu_c \sim 300 x_2^{0.095}(R_*/R_2)^{1.43}$ GHz. From the observed spectrum of Altenhoff *et al.*, $\nu_c \leq 5$ GHz. For $R_2 = 3\ R_*$, we find a consistent solution if $x_2 \lesssim 0.07$. For *o* Ceti, the stellar radius is $0''.014$ (Bonneau *et al.* 1982), giving $\nu_c \sim 460 x_2^{0.095}(R_*/R_2)^{1.43}$ GHz. If the emitting region found in the present observations is optically thick at 5 GHz, the chromospheric radius is 3 R_*, which requires $x_2 \gtrsim 0.09$.

For IRC +10216, the bolometric flux of 3.2×10^{-5} ergs cm^{-2} s^{-1} gives, assuming $T_* = 2000$ K, $R_* = 0''.039$, and hence $\nu_c \sim 2.2 \times 10^4 x_2^{0.095}(R_*/R_2)^{1.43}$ GHz. If the emitting region is optically thick, then its radius is only $0''.033$ and thus IRC +10216 may not have an extended optically thick chromosphere. If the outflow is optically thin, we find the previously estimated ionized fraction of $\sim 4 \times 10^{-5}$. A similar discussion to the above examples can be made for the upper limits of Table 1.

Thus the present observations set quite low limits for the ionized fraction near the base of the stellar wind or for the sizes of extended chromospheres around the stars; *IUE* observations (Linsky 1981) also demonstrate the absence of chromospheres in late-type giants. The detection of radio emission from α Ori and o Ceti suggests that these hotter stars may have extended chromospheres such as that predicted by Hartmann and MacGregor 1980); however, the cooler stars do not seem to have these extended ionized regions, and may be losing mass by some other mechanism.

IV. CRL 618, NGC 7027, AND THE LIFETIMES OF PROTO–PLANETARY NEBULAE

It has been suggested by several authors (e.g., Zuckerman 1978) that carbon stars with high mass loss rates are the precursors of planetary nebulae, and this suggestion is strongly supported by the detection of continuum emission from CRL 618 (Kwok and Feldman 1981) and of a molecular cloud associated with NGC 7027 (Mufson, Lyon, and Marionni 1975). Here, we investigate the evolution of CRL 618 and the lifetime of the proto–planetary nebula stage for objects produced by stars with high mass loss rates.

a) *CRL 618*

CRL 618 is a strong source of thermal molecular line emission (Zuckerman *et al.* 1976), with line shapes similar to those produced by mass loss from evolved stars. The mean mass loss rate inferred from the CO observations is $\sim 2.7 \times 10^{-4}$ $M_\odot$ yr^{-1} at $D = 1.7$ kpc (Paper I). Wynn-Williams (1977) observed radio continuum emission, which was shown by Kwok and Feldman (1981) to be thermal, optically thick at long wavelengths, and increasing in intensity with time. From these observations, Kwok and Feldman inferred that the emission is produced by a compact central H II region whose diameter is growing with time (0".15 to 0".22 in about 3 years) due to the advance of an ionization front at 95–140(D/1.7) km s^{-1} into the surrounding envelope of assumed constant density. Our observations (Table 1) show that CRL 618 has further brightened to 16.2 ± 0.2 mJy at 5 GHz (1981 Oct. 29), giving an angular size of 0".239 and showing that the velocity of the ionization front has slowed to 50(D/1.7) km s^{-1}.

In the following, we investigate what these observations can tell us about the run of density in the surrounding envelope and hence the history of mass loss from the star; since the expansion velocity of the H II region is much greater than the sound speed in the H II region, the density distribution is unchanged by the passage of the front on short time scales and reflects that produced by the mass loss. Assume that the density can be written as $n_e = Ar^{-\beta}$. Then it can be shown (see Appendix B) that the H II region is optically thick ($S_\nu \propto \nu^2$) for $\nu \leq \nu_c/2$ and optically thin above this frequency, where $\nu_c^{2.1} = 0.212 A^2 T_e^{-1.35} R_2$ (cgs units) and R_2 is the radius of the ionization front. The data of Kwok and Feldman (1981) show that ν_c has decreased with time, with $\nu_c/2 > 15$ GHz in 1977 and $\nu_c/2 \sim 10$ GHz in 1980. Since during the same time interval the flux in the optically thick regime increased, R_2 increased and therefore $\beta > 0.5$. Hence the ionization front is advancing through a region of decreasing density as would be expected if the central star were ionizing the neutral circumstellar envelope produced by a wind. Furthermore, since the flux at $\nu_c/2$ has decreased with time, we find $\beta > 1.55$. For ease of computation, we assume $\beta = 2$ (constant mass loss rate). Using the optically thick flux of 14.2 mJy at 5 GHz (Kwok and Feldman 1981) to obtain $R_2 \sim 2.8 \times 10^{15}$ (D/1.7 kpc) cm and equation (B6) with $\nu_c \sim 20$ GHz and $T_e = 2 \times 10^4$ K, we find $4\pi A \mu m_{\rm H} = \dot{M}/V_0 \sim 4.9 \times 10^{14} (D/1.7 \text{ kpc})^{3/2}$ g cm^{-1} for the 1980 epoch. The CO observations give $\dot{M}/V_0 \sim 9.5 \times 10^{15} (D/1.7 \text{ kpc})^2$ for the larger and hence older envelope. The mass loss rate has therefore decreased by a factor of up to $19(D/1.7 \text{ kpc})^{1/2}$ close to the end of the mass-losing phase (or the outflow velocity has increased by a like amount). The density at the outer edge of the ionizing front from the 1980 observations is $n_e \sim 2 \times 10^6 (D/1.7 \text{ kpc})^{-1/2}$ cm^{-3}, consistent with the emission measure of 10^9 cm^6 pc observed by Kwok and Feldman (1981), but much larger than the densities of 10^3–10^5 derived from optical forbidden-line measures by Westbrook *et al.* 1975). This, the detection of H_2 emission (Beckwith, Persson, and Gatley 1978; Thronson 1981) and the bipolar appearance of CRL 618 could be due to the presence of large density inhomogeneities and/or nonspherical geometry, which complicate the interpretation.

Ignoring these difficulties, we now use the above values of the H II region parameters and the flux data of CRL 618 in Table 1 and from Kwok and Feldman to examine the change in the ionizing flux of the central star. We take the gas-to-dust number density to be $\Sigma_d/\pi a^2$, where a is the radius of a dust grain. Then the velocity of the ionization front is

$$\dot{R}_2 = V_0 + \frac{[N_*(t) + N_c]}{4\pi A} \exp\left[\Sigma_d A\left(\frac{1}{R_2} - \frac{1}{R_1}\right)\right] - \frac{\alpha^{(2)}}{\Sigma_d}, \tag{4}$$

where $N_*(t)$ is the photon luminosity of the star, V_0 is the constant gas outflow velocity, $N_c = 4\pi A \alpha^{(2)}/\Sigma_d$, $\alpha^{(2)}$ is the hydrogen recombination coefficient, and R_1 is the inner radius of the H II region (see below for derivation). With the present observations, equation (4) can be evaluated at two epochs: the midpoint of the 1977–1980 period where $R_{2a} \sim 2.4 \times 10^{15}$ cm and $\dot{R}_{2a} \sim 100$ km s^{-1}, and the midpoint of the 1980–1981 period where $R_{2b} \sim 2.9 \times 10^{15}$ cm and $\dot{R}_{2b} \sim 50$ km s^{-1}. We assume that during the period of the observations the inner edge of the H II region (radius R_1) has been coasting outward at V_0, and hence we need to know R_1 at only a single instant. From the optically thick flux in 1977, $R_2(1977) \sim 1.9 \times 10^{15}$ cm and, for the same value of A determined above, $\nu_c(1977) \sim 35$ GHz. With these values and the optically thin flux at 90 GHz, we can use equation (B9) to find $R_1(1977) \sim 1.5 \times 10^{15}$ cm. Then for $\Sigma_d = 10^{-21}$ cm^2 (Spitzer 1978) and $\alpha^{(2)}$

for $T_e = 2 \times 10^4$ K, the change in the ionizing flux can be written as

$$\frac{N_*(t_b) + N_c}{N_*(t_a) + N_c} \sim 4.2 \exp\left(-\frac{V_0 \Delta t}{R_1{}^2} 1.7 \times 10^{16} \text{ cm}\right),$$

where Δt is the interval between the two epochs (2.3 years). Since $\dot{R}_{2a}$, $\dot{R}_{2b}$, and V_0 are much smaller than $\alpha^{(2)}/\Sigma_d$, the variation of $N_*(t)$ depends mainly on the change of the inner radius. For $V_0 = 18$ km s^{-1} (Table 1), N_* has increased by a factor of about 1.6 over the value of $\sim 10^{48}$ photons s^{-1} found for the 1977–1980 epoch from equation (4). For $V_0 > 26$ km s^{-1}, N_* has decreased, which seems more reasonable in light of the decrease in the velocity of the ionization front and the negative density gradient. The large luminosity is characteristic of hot planetary nebula nuclei, and the rapid radio brightening suggests that the star is evolving rapidly. This is in agreement with Paczyński's (1971) models for massive (1.2 $M_\odot$) nuclei. These observations thus suggest that toward the end of the mass loss phase of its precursor, the mass loss rate decreased and/or the outflow velocity increased, and that the object is now evolving toward a planetary nebula by ionizing the precursor's envelope.

An object in a similar stage of evolution to CRL 618 may be CRL 2688, for which Beckwith, Persson, and Gatley (1978) also detect H_2 emission. In the present observations (Table 1) we do not detect radio continuum from the object, so that it may be in a slightly earlier stage than CRL 618. Since the models discussed herein suggest that the radio continuum flux for such objects increases rapidly with time, continued searches for continuum emission from CRL 618 should be made.

b) The Lifetimes of Proto-Planetary Nebulae and NGC 7027

We now examine the continued evolution of an object such as CRL 618, and assume for the sake of simplicity that the precursor asymptotic branch star loses mass copiously in the form of a steady stellar wind over several times 10^4 years, producing an extensive circumstellar envelope, until the hot core of the star is exposed and ionizes the surrounding envelope, producing a planetary nebula. We ignore any previous phase of mass loss which may have taken place while the star was on the red giant branch (cf. Reimers 1981; the mass loss rates found by Reimers are generally very much lower than those found in Paper I and are therefore relatively unimportant). We also assume that the H is in atomic form in calculating the motion of the ionization front. The state of the hydrogen in these envelopes is not known, though both observations and theory suggest that it is molecular (Vardya 1966; Beckwith, Persson, and Gatley 1978; Zuckerman, Terzian, and Silverglate 1980). In any case, the more important source of UV opacity is dust, and the dissociation of H_2 will only have the effect of slowing the ionization front still further from the rates calculated below.

As an example, we examine the ionization of the envelope produced by a star with constant mass loss $\dot{M}$ for 2.5×10^4 years at a constant outflow velocity $V_0 = 20$ km s^{-1} (cf. Paper I). At $t = 0$, the mass loss stops and the remnant core is "turned on" as a planetary nebula nucleus, while the envelope continues to coast outward at V_0. We neglect the effects of a fast wind from the central star, described by Kwok (1982), which may give rise to a compressed shell structure such as that seen in NGC 7027 (Bignell 1983). The ionizing flux from the core N_* as a function of time is displayed in Figure 2 of Giuliani (1981) and is based on the core star models of Paczyński (1971) (masses of 1.2 $M_\odot$, 0.8 $M_\odot$, and 0.6 $M_\odot$), and of Härm and Schwarzschild (1975) (mass of 0.625 $M_\odot$). Tabular values of $N_*(t)$ were used in the numerical calculations discussed below. Similar values of $N_*(t)$ are found from the more recent models of Schönberner (1979).

The number of ionizing photons per unit area at a given radius can be found by integrating the Strömgren relation (eq. [A1]) with $n_e = n_i = A/r^2$, assuming spherical symmetry. Thus

$$N(r) = N_*(t) - 4\pi A^2 \alpha^{(2)}\left(\frac{1}{R_1} - \frac{1}{r}\right), \tag{5}$$

where $R_1 = V_0 t$. The velocity of the ionization front is

$$\dot{R}_2 = V_0 + \frac{N(R_2)}{4\pi n_e R_2{}^2} \tag{6}$$

$$= V_0 + \frac{N_*(t)}{4\pi A} - A\alpha^{(2)}\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \tag{7}$$

(Spitzer 1978). Since, roughly, $N_*(t) \propto t^{-1}$ (Giuliani 1981) and $R_1 \propto t$, the ionization front will propagate very slowly for sufficiently high mass loss rates. This behavior is very different from that produced when the ionizing flux is constant. In such a case, the ionization front propagates very rapidly once the inner radius has expanded sufficiently that the opacity is small.

Dust in the circumstellar envelope will further slow the advance of the ionization front. The effect of dust modifies equation (A1) to:

$$\frac{dN(r)}{dr} = -4\pi\alpha^{(2)} n_e{}^2 r^2 - \pi a^2 n_d N(r) \tag{8}$$

(cf. Kwok and Purton 1979), where $n_d = Cr^{-2}$ is the density of dust and a the mean particle size. Equation (8) integrates to:

$$N(r) = [N_*(t) + N_c] \exp\left[\pi a^2 C\left(\frac{1}{r} - \frac{1}{R_1}\right)\right] - N_c, \tag{9}$$

where $N_c = 4\pi A^2 \alpha^{(2)}/a^2 C$. Here, we assume

$$\Sigma_d = \pi a^2 C/A = 10^{-21} \text{ cm}^2 \tag{10}$$

(Spitzer 1978), which leads to

$$\dot{R}_2 = V_0 + \frac{[N_*(t) + N_c]}{4\pi A} \exp\left[\Sigma_d A\left(\frac{1}{R_2} - \frac{1}{R_1}\right)\right] - \frac{\alpha^{(2)}}{\Sigma_d}. \tag{11}$$

Equations (7) and (11) can be solved by a modified Newton-Raphson scheme, for various values of the input parameters (the core models and the mass loss rates). Since these equations include the assumption that the front is always R-type and models the recombination after core flashes as the starward motion of the ionization front, these models give the approximate large scale propagation of the front rather than a detailed description of its evolution. Sample results are shown in Figure 1, in which the distance from the star of the inner and outer radii of the envelope, and of the ionization front, are shown as a function of time for two different sets of input parameters. The effects of dust are included in the calculations. The low-mass cores, which have a large flux at late times, are always able to ionize all of the surrounding gas. The high-mass cores, on the other hand, which cool rapidly, are not able to ionize all of the gas in the more massive envelopes, which are always radiation bound. It is likely, though not yet proven, that high-mass cores occur in stars with high initial masses, and therefore high mass loss rates.

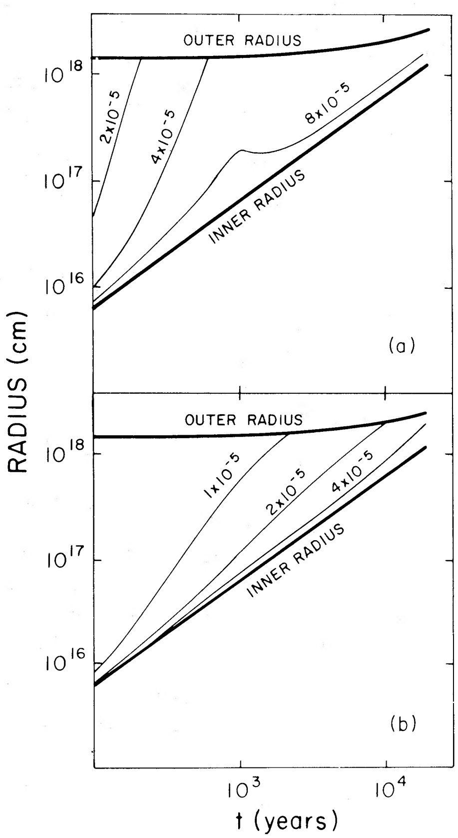

FIG. 1.—Models for the growth with time of an ionization front in a spherically symmetric dusty proto-planetary nebula. The nebula is assumed produced by constant mass loss rate $\dot{M}$ at constant velocity; the sizes of the inner and outer radii of the envelope are shown as a function of time. The position of the ionization front as a function of time is shown for several values of $\dot{M}$: (a) for a core model of $m_c = 0.8\ M_\odot$ and (b) $m_c = 1.2\ M_\odot$ (Paczyński 1971).

In Table 2, we list the time to ionize the entire envelope for various model stars.

As an example, NGC 7027 is classified as a planetary nebula because the central ionized region is now visible. The extinction toward the ionized region due to the circumstellar cloud is calculated from

$$A_v = 1.5 \times 10^{22} \left(\frac{\dot{M}}{M_\odot\ \mathrm{yr}^{-1}}\right)\left(\frac{V_0}{\mathrm{km\ s}^{-1}}\right)\left(\frac{R_2}{\mathrm{cm}}\right)^{-1} \mathrm{mag}\ , \tag{12}$$

if the gas-to-dust ratio in these objects is the same as that in the general interstellar medium. For NGC 7027, our VLA observations give a diameter for the ionized region of $10'' \pm 1''$, in good agreement with the diameter of the optical nebula. Taking the distance to be 1100 pc, the mass loss rate to be a $4 \times 10^{-4}\ M_\odot\ \mathrm{yr}^{-1}$, and the outflow velocity to be 22 km s^{-1} (Paper I), we find $R_2 = 8 \times 10^{16}$ cm and $A_v = 3$ mag, in good agreement with the measured color excess of 1 mag (Pottasch *et al.* 1982).

CRL 618 is identified as a proto-planetary nebula, since the ionized region is not yet visible. As the ionization front expands, the extinction drops and the nebula will become visible. Taking the extinction at which this happens to be 5 mag, we use equation (12) to calculate the relative lifetimes of the model objects with $\dot{M} \sim 10^{-4}\ M_\odot\ \mathrm{yr}^{-1}$ in Table 2 as proto-planetary nebulae.

Thus the lifetimes of the objects in these various evolutionary stages are: copious mass loss $\sim 2.5 \times 10^4$ years; proto-planetary nebula ~ 500 years; ionization-bound planetary nebula $\sim 2 \times 10^4$ years. From Table 1, there are 10 objects with $\dot{M} \sim 10^{-4}\ M_\odot\ \mathrm{yr}^{-1}$ which have been observed in the continuum; since all of these are strongly detected in the CO line, it is likely that the mass loss has been under way for more than 10^4 years. Of these, CRL 618 and perhaps IRC +10216 are proto-planetary nebulae. Two, possibly three, planetary nebulae with extensive surrounding molecular clouds are known—NGC 7027, Vy2-2 (Kwok 1981), and IC 418 (Mufson, Lyon, and Marionni 1975)—and more may exist (cf. the list of compact planetaries given by Kwok (1981) and the H_2 observations of Beckwith, Persson, and Gatley 1978). There are not, at present, sufficient data to test the predictions of Table 2, but the agreement so far is suggestive.

The models discussed herein give a second important result. For stars with high mass loss rates, the ionization fronts propagate very slowly and do not reach the edge of the envelope before the cores cool. Thus, intermediate-mass stars with high mass loss rates and large core masses produce envelopes in which much of the material is never ionized. IRC +10216, for example, with its high $\dot{M}$ and high core mass (inferred from the m_c–L_c relationship of Paczyński 1971) may return most of its mass to the interstellar medium in the form of cold neutral gas and dust. Since stars of initial mass 2–6 $M_\odot$ such as IRC +10216 are responsible for much of the

TABLE 2

$\dot{M}$ ($M_\odot$ yr^{-1})	1.0×10^{-5}	2.0×10^{-5}	4.0×10^{-5}	8.0×10^{-5}	1.6×10^{-4}
Envelope Mass ($M_\odot$)	0.24	0.48	0.96	1.92	3.84
A. Time Scales in Years for Ionization of Circumstellar Envelopes					
Core Mass:					
0.6 $M_\odot$ (no dust)	1.0×10^3	1.0×10^3	1.5×10^3	2.0×10^3	5.0×10^3
0.6 $M_\odot$ (dust)	1.0×10^3	1.0×10^3	1.5×10^3	3.0×10^3	7.0×10^3
0.6254 $M_\odot$ (no dust)	3.0×10^2	5.0×10^2	1.5×10^3	2.0×10^3	7.0×10^3
0.6254 $M_\odot$ (dust)	4.0×10^2	1.0×10^3	2.0×10^3	4.0×10^3	8.0×10^3
0.8 $M_\odot$ (no dust)	$< 10^2$	$< 10^2$	$< 10^2$	8.0×10^2	2.0×10^4
0.8 $M_\odot$ (dust)	$< 10^2$	2.0×10^2	6.0×10^2	$> 2.0 \times 10^4$	$> 2.0 \times 10^4$
1.2 $M_\odot$ (no dust)	2.0×10^3	1.5×10^4	$> 2.0 \times 10^4$	$> 2.0 \times 10^4$	$> 2.0 \times 10^4$
1.2 $M_\odot$ (dust)	2.5×10^3	1.7×10^4	$> 2.0 \times 10^4$	$> 2.0 \times 10^4$	$> 2.0 \times 10^4$
B. Lifetime in Proto-Planetary Nebula Stage for Stars with $\dot{M} > 10^{-4}$ $M_\odot$ yr^{-1}					
1.2 $M_\odot$ (dust)	...	...	200	400	800

mass returned to the interstellar medium (ISM), these results show that this mass is returned with the dust unharmed by UV radiation from the central star. This result is particularly important in light of the discussion of Natta and Panagia (1981), who suggest that the dust-to-gas ratio in planetary nebulae decreases with time, presumably due to destruction of the dust, and that, as a result, the dust-to-gas ratio in the Galaxy is decreasing, or there is another source of dust. Our results modify this conclusion: in the most massive envelopes, the bulk of the circumstellar material is never ionized and the dust is injected, unharmed, into the ISM.

V. CONCLUSIONS

In this paper, we have explored the radio continuum properties of cool evolved stars from the point of view of the mass loss mechanisms from such stars and of their subsequent evolution through the planetary nebula stage. Thirty-two objects with high mass loss rates were searched for 6 cm continuum emission to a 3 σ sensitivity of 0.3–0.6 mJy with the VLA; five were detected. These are the known sources NGC 7027, CRL 618, and R Aqr, and new detections of two objects, IRC +10216 and o Ceti. The evolution of such objects to the planetary nebula stage was studied by calculating the propagation of an ionization front into the remnant neutral envelope produced by continuous mass loss in the red giant stage. We find:

1. None of the 30 stars observed has a hot, compact companion except for the known cases of R Aqr and Mira.

2. The fractional ionization in the envelopes is small, $\lesssim 10^{-3}$ in most cases. The hotter stars α Ori and o Ceti may have optically thick chromospheres extending to several stellar radii, as predicted by models of the mass loss process which use the momentum in acoustic or Alfvén waves. The cooler stars, however, do not appear to have such regions.

3. The continuum flux of CRL 618 continues to increase, as found by Kwok and Feldman (1981), but the expansion velocity of the ionization front has decreased. The density close to the star falls off with radius, and $\dot{M}/V_0 \sim 1/20$ of that found from CO observations of the larger envelope, suggesting that the mass loss rate for the star decreased toward the end of the mass loss phase.

4. The calculations of the propagation of an ionization front in these objects show that, for red giants with low mass loss rates ($\dot{M} < 10^{-4}$ $M_\odot$ yr^{-1}), the envelope is rapidly ionized in $\lesssim 10^3$ years after the core star is exposed, producing a planetary nebula. For stars with high mass loss rates ($\dot{M} \gtrsim 10^{-4}$ $M_\odot$ yr^{-1}), however, the ionization front propagates very slowly ($\sim$20 km s^{-1}), and much of the envelope is not ionized before the core star cools. Thus, intermediate-mass stars ($M \sim$ 2–6 $M_\odot$) return most of their material to the ISM in the form of cold, neutral gas; in particular, the dust formed in the red giant envelope is injected into the ISM unaffected by the ultraviolet radiation from the central star.

5. R Aqr was found to be a double radio source. A model of the source shows that the mass outflow from the primary red giant is ionized by a hot compact companion. Because of the rapid density increase toward the primary, the gas within one or two stellar radii remains neutral; the SiO maser emission seen from this star presumably arises in this gas. This ionization of the stellar wind causes the stronger radio source, which is coincident with the star. The second source, 6″ from the primary, may be due to a dense clump of ejected gas ($M \sim 10^{-7}$ $M_\odot$) recently ionized by the companion. These observations suggest that mass loss from this star, and perhaps others, takes place both as a steady wind and in massive individual clumps.

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APPENDIX A

THE DOUBLE SOURCE R AQUARII

R Aqr is a symbiotic long-period late-type (M7) Mira variable, at a distance of 260 pc (Gregory and Seaquist 1974). The Mira is known to be losing mass, since it is an SiO maser source (Lépine, LeSqueren, and Scalise 1978; Zuckerman 1979; Spencer *et al.* 1981) but remains undetected in the thermal millimeter-wavelength lines of CO and SiO, and is not an OH maser.

As well as the M7 star, the R Aqr system must contain a hot component, which has never been seen directly. In 1922 to 1933, the optical spectrum was dominated by blue light (Merrill 1935). The present UV spectrum shows the presence of a hot component (Michalitsianos, Kafatos, and Hobbs 1980). Also, in contrast to other late-type stars (see Table 1) R Aqr is a strong radio continuum source ($\sim$8 mJy, Gregory and Seaquist 1974; Bowers and Kundu 1979; Ghigo and Cohen 1981). The presence of the SiO maser means that, close to the red giant, the wind is neutral, and thus the hot component is likely to be a binary companion to the Mira, rather than the hot central core of the Mira almost evolved to a planetary nebula nucleus.

The 6 cm radio continuum map made in 1981 October is shown in Figure 2. The observation is remarkable in showing that the radio source associated with R Aqr is double, consisting of a primary source (7.8 ± 0.1 mJy) coincident with the star, as found in previous observations, and, detected for the first time, a second source of strength 1.6 ± 0.1 mJy at a distance of $6''.0 \pm 0''.5$ from the primary, at a position angle of 22°. Both radio sources are unresolved in our observations. This double structure was also found by Sopka *et al.* (1982) in observations made at almost the same time as ours.

The inner structure of the R Aqr nebulosity contains several knots and clumps. In plates of the star taken in 1960 and 1970 (copies of which were kindly given to us by G. H. Herbig) there are two knots of gas to the northeast

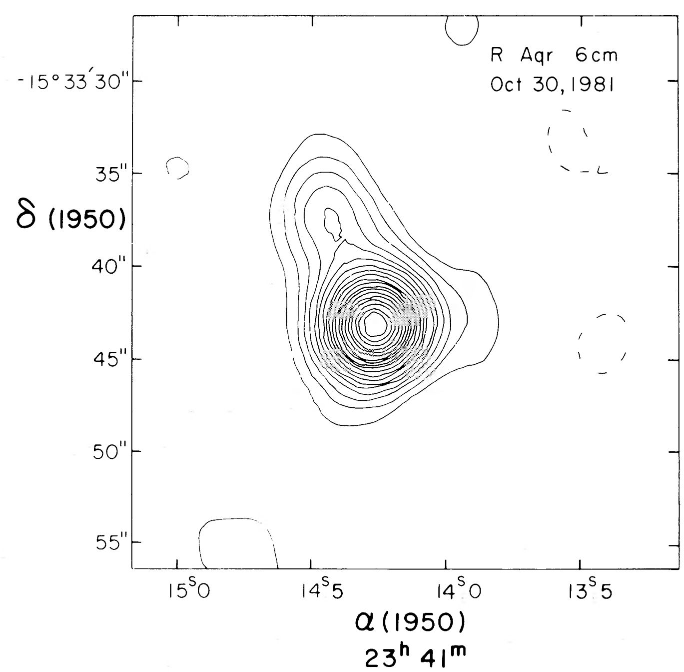

FIG. 2.—5 GHz VLA observation of R Aqr. The map has been cleaned and restored with a circular Gaussian beam of diameter 4″.5. The peak flux is 7.8 mJy; the contours are drawn every 5% of the peak flux. Negative contours are dashed.

of the star. The inner clump, coincident with the second radio source, has undergone a sudden brightening in its optical emission in plates taken in 1980 (Wallerstein and Greenstein 1980; Herbig 1981). The available evidence therefore suggests that this clump has brightened sometime in the past several years. It is unlikely that the radio source was ejected from R Aqr, for several reasons: (1) it is coincident with an optical feature whose position has remained stationary over the past several years, (2) the two sources do not appear to be part of a continuous distribution (Fig. 2), and (3) the observed radial velocities of the clump (~ -70 km s^{-1}, Sopka *et al.* 1982) are not large.

As a model for the R Aqr system, we suggest that the circumstellar material lost from the Mira is ionized by the hot binary companion, with the stronger radio source arising from ionization of the steady wind component while the weaker source is due to the recent ionization of an individual clump in the wind. First, we consider the ionization of the steady wind. In a spherical coordinate system centered on the companion with the polar axis along the line joining the two stars, the extent of the H II region $R_0(\theta)$ is given by the Strömgren relation

$$N_* = 4\pi \int_0^{R_0(\theta)} R^2 \alpha^{(2)} n_e n_i \, dR \,, \tag{A1}$$

where N_* is the stellar ionizing luminosity, $\alpha^{(2)}$ is the total H recombination coefficient to the second level, and n_e and n_i are the electron and proton densities. The orbital parameters of the binary system are unknown. We assume a circular orbit of radius a; Wallerstein and Greenstein (1980) suggest a mean separation of 2.4×10^{14} cm, corresponding to an orbital period of about 50 years. The H II region is steadily fed by the neutral wind from the Mira, and we assume that once the wind is ionized it has an expansion velocity V_0 of 30 km s^{-1} (Wallerstein and Greenstein 1980). Since this is about 3 times the sound speed, we can set $n_e = n_i = Ar^{-2}$, where r is the distance from the Mira, $A = \dot{M}/4\pi\mu m_{\mathrm{H}} V_0$, and μ is the molecular weight. Since $r^2 = R^2 + a^2 - 2aR \cos\theta$, equation (A1) gives

$$N_* = 4\pi A^2 \alpha^{(2)} \left(-\frac{1}{3a} + \frac{1}{p} - \frac{a}{p^2} + \frac{a^2}{3p^3} \right), \tag{A2}$$

for $\theta = 0$, where $p = a - R_0(0)$ is the "inner" Strömgren radius measured from the Mira. In the opposite direction ($\theta = \pi$), the "outer" Strömgren radius $q = a + R_0(\pi)$ is given by

$$N_* = 4\pi A^2 \alpha^{(2)} \left(\frac{1}{3a} - \frac{1}{q} + \frac{a}{q^2} - \frac{a^2}{3q^3} \right). \tag{A3}$$

We see that q approaches infinity when $N_* > 4\pi A^2 \alpha^{(2)}/3a$, and for this flux, $p = a/2$. As the companion moves through the envelope, it will form an irregular toroidal H II region whose maximum extent in the orbital plane is sketched in Figure 3*a*. In the regime where the opacity $\kappa(\nu, T_e)n^2(q)q$ is much smaller than 1 and $p \ll q$, it can be shown (Appendix B) that Wright and Barlow's (1975) calculations for the flux from a star with a constant loss of ionized gas become valid (eq. [2]).

For R Aqr, with $V_0 = 30$ km s^{-1}, $T_e = 10^4$ K, and $D = 260$ pc, equation (2) gives $\dot{M}(\mathrm{H}^+) = 9 \times 10^{-8}$ $M_\odot$ yr^{-1}. With $\alpha^{(2)} = 1.5 \times 10^{-13}$ cm^3 s^{-1}, this gives $A = 6.7 \times 10^{34}$, $q = \infty$, and $p = 6 \times 10^{13}$ cm for $N_* = 10^{44}$ photons s^{-1}. This suggestion, that the primary radio emission from R Aqr is due to ionization of an $n \propto r^{-2}$ envelope, is supported by other evidence. The spectral index of the continuum emission from R Aqr is 0.56 (Sopka *et al.* 1982; Kafatos, Hollis, and Michalitsianos 1983, and other references quoted above), which is similar to that expected from a star with a constant mass loss rate (Panagia and Felli 1975; Wright and Barlow 1975). Also, Johnson (1982) finds that the electron density obtained from *IUE* data is more consistent with an r^{-2} density gradient than with a constant density envelope.

The geometry sketched in Figure 3*a* now finds direct application in a discussion of the radio brightening of the second radio source. Recombination, occurring in the shadow of the neutral wind zone near the primary, will distort the ionized region so that its appearance is like that sketched in Figure 3*b*. The gas and dust close to the star will shield any condensation on the opposite side of the primary from the ionizing radiation of the secondary until the latter's orbit brings it near syzygy with the condensation (Fig. 3*c*). Once the companion moves out of the shadow of the Mira variable, it begins to ionize the condensation. The sudden brightening of the radio knot may be the result of this rapid ionization.

From the radio flux, and time scale arguments, we can obtain approximate values for the density, size, and shape of the gas clump, which ionizes and recombines on the observed time scale, $\lesssim 10$ years, which, with $t(\mathrm{recomb}) = 1/n_e \alpha^{(2)}$, gives $n_e > 2 \times 10^4$ cm^{-3}.

The clump can be simply modeled as a pillbox of width b and radius x with its axis in the plane of the sky. The requirement that a white dwarf producing 10^{44} ionizing photons per second (Kafatos, Michalitsianos, and Hobbs 1980) at a distance of 6″ (2×10^{16} cm) from the condensation can ionize it in less than 10 years then gives $b \lesssim 2 \times 10^{14}$ cm.

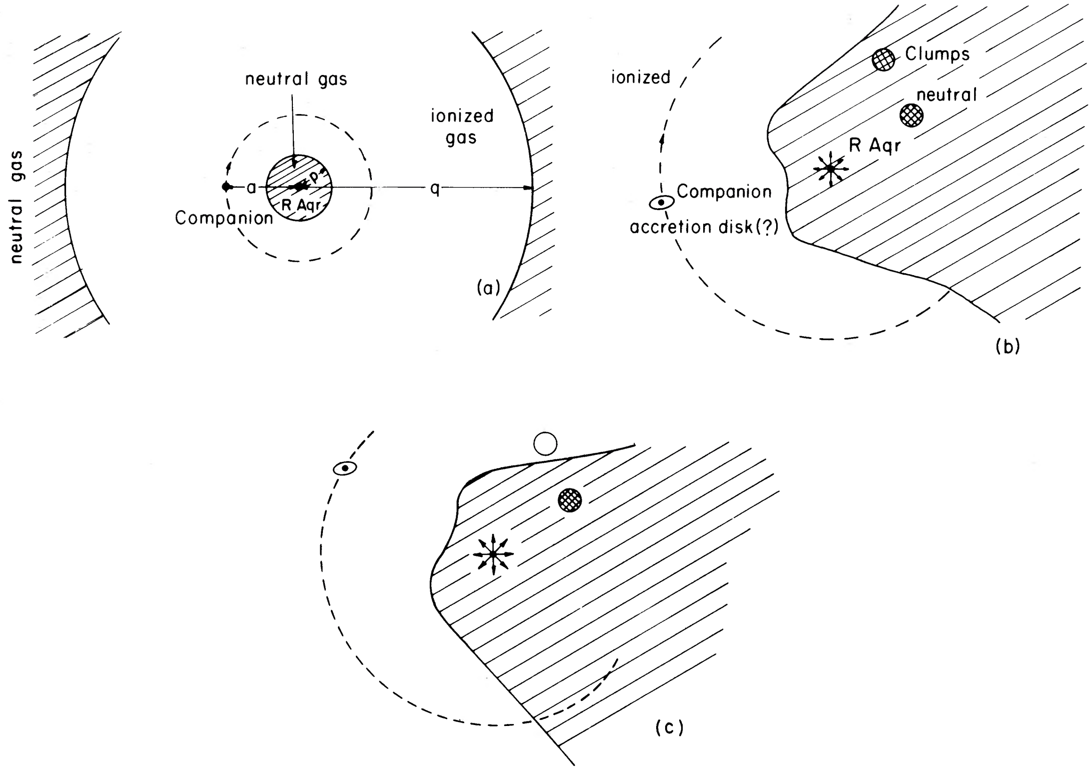

FIG. 3.—Geometry of the R Aqr system. (*a*) Large scale geometry; the orbit of the blue companion is assumed circular. The neutral wind from R Aqr is ionized by the companion; the inner portion of the wind (within radius *p*) remains neutral. The ionized region of the nebula is radiation bounded, and outside radius *q* the wind is again neutral. (*b*) The inner region of the system. The recombination time for the wind is short enough that the dense region near R Aqr shelters the gas behind it, which remains neutral. (*c*) As the companion moves in its orbit, the embedded clumps become ionized (see text).

The measured 6 cm flux of 1.6 mJy also provides a lower limit to the density and volume. Assuming that the clump is optically thin,

$$S_\nu = \frac{2\nu^2 kT}{c^2} \frac{1}{D^2} n_e^2 \pi x^2 b \kappa_\nu \,, \tag{A4}$$

where κ_ν is the free-free absorption coefficient (see eq. [B3]). With the above estimates for n_e and b, we find $x > 10^{15}$ cm, with corresponding angular size 0."4. Assuming $\mu = 1.4$, the mass is $\sim 1.6 \times 10^{-7}\, M_\odot$. The mass and size estimates are consistent with values suggested by Schwarzschild (1975) and Elitzur (1981) for large convection cells.

Ultraviolet observations suggest that the companion is hotter and more luminous than typical white dwarfs, with a luminosity close to 1 $L_\odot$, and this could be due to accretion by the condensed object of mass lost by the primary. The accretion rate is (Warner 1972):

$$\dot{\mathcal{M}} = \frac{G^2 m^2 \dot{M}}{2(V_0{}^2 + V_{\rm orb}{}^2 + c^2)^{3/2} V_0 a^2} \,, \tag{A5}$$

where c is the sound speed, m the mass of the compact star, and $V_{\rm orb}$ the orbital speed. Assuming that the mass of the companion is 1 $M_\odot$ and that $\dot{M} = 9 \times 10^{-8}\, M_\odot\ {\rm yr}^{-1}$, the luminosity of the companion is:

$$L = \frac{Gm\dot{\mathcal{M}}}{r_{\rm WD}} = 1.2 \times 10^{33} \left(\frac{10^9 \text{ cm}}{r_{\rm WD}}\right) \text{ergs s}^{-1} \,, \tag{A6}$$

where $r_{\rm WD}$ is the radius of the companion. This predicts a luminosity for the companion in the neighborhood of 1 $L_\odot$. The accretion of a dense clump of material could have produced the brightening of the system seen between 1922 and 1933.

APPENDIX B

THE RADIO SPECTRUM OF AN IONIZED SHELL

In this section, we develop expressions for the flux emitted by ionized gas outflowing from a central source in the case where (1) the ionized gas has a finite outer radius, (2) the inner and outer radii are of comparable extent, and (3) the ionized gas density has an arbitrary power-law dependence on radius, $n_e(r) = n_i(r) = Ar^{-\beta}$. Assume that the gas has a uniform electron temperature T_e and is confined to a spherical region with inner radius R_1 and outer radius R_2. Then the observed flux at frequency ν from an unresolved source at distance D is

$$S_\nu = B(\nu, T_e)D^{-2} \int_0^{R_2} (1 - e^{-\tau})2\pi q dq \,, \tag{B1}$$

where q is the impact parameter of the line of sight (cf. Fig. 1 of Wright and Barlow 1975). The optical depth is

$$\tau(q) = 2\kappa(\nu, T_e)A^2 \int r^{-2\beta} dl \,, \tag{B2}$$

where $l = (r^2 - q^2)^{1/2}$ and the integration is from l' to $(R_2{}^2 - q^2)^{1/2}$. For $q > R_1$, $l' = 0$, and for $q < R_1$, $l' = (R_1{}^2 - q^2)^{1/2}$.

The absorption coefficient may be approximated by

$$\kappa(\nu, T_e) = 0.212 T_e^{-1.35} \nu^{-2.1} \text{ cm}^5 \tag{B3}$$

(Mezger and Henderson 1967), where ν is in Hz.

Now, in equation (B2) set

$$t = \frac{R_2{}^2 - (q^2 + l^2)}{R_2{}^2 - q^2} \,,$$

$y = q/R_2$, and $\delta = R_1/R_2$, to give, for $y > \delta$,

$$\tau_1(y) = 2\left(\frac{\nu_c}{\nu}\right)^{2.1} (1 - y^2)^{1/2} \tilde{F}(1 - y^2) \,, \tag{B4}$$

and for $y < \delta$,

$$\tau_2(y) = 2(\nu_c/\nu)^{2.1}[(1 - y^2)^{1/2}\tilde{F}(1 - y^2) - \delta^{1-2\beta}(1 - y^2/\delta^2)^{1/2}\tilde{F}(1 - y^2/\delta^2)] \,, \tag{B5}$$

where the critical frequency ν_c in cgs units is

$$\nu_c = \left(\frac{0.212A^2}{T_e^{1.35} R_2{}^{2\beta-1}}\right)^{1/2.1} \text{ Hz} \,, \tag{B6}$$

and $\tilde{F}(x)$ is the hypergeometric function $F(\beta, 1; 3/2; x)$. For $\beta = 2$ (constant mass loss at constant velocity) $\tau_1(y)$ and $\tau_2(y)$ reduce to the forms given by Kwok (1977). For other values of β,

$$S_\nu = B(\nu, T_e)\pi\left(\frac{R_2}{D}\right)^2 \left(1 - 2\int_0^\delta e^{-\tau_2(y)} y dy - 2\int_\delta^1 e^{-\tau_1(y)} y dy\right). \tag{B7}$$

Consider first the low-frequency optically thick limit $\nu \ll \nu_c$. Here $\tau_2(y)$ is large, and the first integral in (B7) can be neglected. The second integral has a significant contribution only near the limb of the source ($y = q/R_2 \sim 1$) where the source is becoming optically thin (eq. [B4]). Under these conditions the second integral in equation (B7) can be evaluated by Laplace's method (cf. Bender and Orszag 1978, p. 261). The asymptotic expression for the flux in the limit $\nu \ll \nu_c$ is

$$S_\nu \approx \frac{2\pi k T_e}{c^2} \nu^2 \left(\frac{R_2}{D}\right)^2 \left[1 - \frac{1}{2}\left(\frac{\nu}{\nu_c}\right)^{4.2} + 2\beta\left(\frac{\nu}{\nu_c}\right)^{8.4} + \cdots\right]. \tag{B8}$$

The expansion breaks down rapidly as $\nu \to \nu_c$, but equation (B8) allows a fairly good estimate of ν_c to be obtained from the observed spectrum; for $\nu \lesssim \nu_c/2$, the spectral index is $+2$, while for larger ν the emission becomes optically thin and $d \ln S_\nu/d\nu$ decreases. In the limit $\nu \gg \nu_c$, direct expansion of the exponentials in equation (B7) leads to the optically thin expression

$$S_\nu \approx \frac{2\pi k T_e}{c^2} \nu^2 \left(\frac{R_2}{D}\right)^2 \left(\frac{\nu_c}{\nu}\right)^{2.1} \frac{4}{2\beta - 3} (\delta^{3-2\beta} - 1) \,. \tag{B9}$$

This formula is valid as long as $\delta = R_1/R_2 \lesssim 1$. If the emitting region is sufficiently thick (small δ) and $\beta > \frac{1}{2}$, there will be a range of frequencies, $\nu_c \ll \nu \ll \nu_c \delta^{(1-2\beta)/2.1}$, over which the Wright and Barlow formulation holds.

REFERENCES

Altenhoff, W. J., Oster, L., and Wendker, H. J. 1979, *Astr. Ap.*, **73**, L21.
Baars, J. W. M., Genzel, R., Pauliny-Toth, I. I. K., and Witzel, A. 1977, *Astr. Ap.*, **61**, 99.
Beckwith, S., Persson, S. E., and Gatley, I. 1978, *Ap. J.*, **219**, L33.
Bender, C., and Orszag, S. 1978, *Advanced Mathematical Methods* (New York: McGraw-Hill).
Bignell, C. 1983, in *IAU Symposium 103, Planetary Nebulae*, ed. D. R. Flower (Dordrecht: Reidel), in press.
Bonneau, D., Foy, R., Blazit, A., and Labeyrie, A. 1982, *Astr. Ap.*, **106**, 235.
Bowers, P. F., Johnston, K. J., and Spencer, J. H. 1981, *Nature*, **291**, 382.
Bowers, P. F., and Kundu, M. R. 1979, *A.J.*, **84**, 791.
Crabtree, T., and Martin, P. G. 1979, *Ap. J.*, **227**, 900.
Elitzur, M. 1981, in *Physical Processes in Red Giants*, ed. I. Iben and A. Renzini (Dordrecht: Reidel).
Ghigo, F. D., and Cohen, N. L. 1981, *Ap. J.*, **245**, 988.
Giuliani, J. L., Jr. 1981, *Ap. J.*, **245**, 903.
Gregory, P. C., and Seaquist, E. R. 1974, *Nature*, **247**, 532.
Härm, R., and Schwarzschild, M. 1975, *Ap. J.*, **200**, 324.
Hartmann, L., and MacGregor, K. B. 1980, *Ap. J.*, **242**, 260.
Herbig, G. H. 1981, *IAU Circ.*, No. 3535.
Herbig, G. H., and Zappala, R. R. 1970, *Ap. J.*, **162**, L15.
Högbom, J. A. 1974, *Astr. Ap. Suppl.*, **15**, 417.
Johnson, H. M. 1982, *Ap. J.*, **253**, 224.
Kafatos, M., Hollis, J. M., and Michalitsianos, A. G. 1983, *Ap. J. (Letters)*, **267**, L103.
Knapp, G. R. 1982, unpublished.
Knapp, G. R., Phillips, T. G., Leighton, R. B., Lo, K.-Y., Wannier, P. G., Wootten, H. A., and Huggins, P. J. 1982, *Ap. J.*, **252**, 616 (Paper I).
Kwan, J., and Linke, R. A. 1982, *Ap. J.*, **254**, 587.
Kwok, S. 1977, *Ap. J.*, **214**, 437.
———. 1981, in *Physical Processes in Red Giants*, ed. I. Iben and A. Renzini (Dordrecht: Reidel), p. 421.
———. 1982, *Ap. J.*, **258**, 280.
Kwok, S., and Feldman, P. A. 1981, *Ap. J. (Letters)*, **247**, L67.
Kwok, S., and Purton, C. R. 1979, *Ap. J.*, **229**, 187.
Lépine, J. R. D., Le Squeren, A. M., and Scalise, E., Jr. 1978, *Ap. J.*, **225**, 869.
Linsky, J. L. 1981, in *Physical Processes in Red Giants*, ed. I. Iben and A. Renzini (Dordrecht: Reidel), p. 247.
Lynds, C. R., Worden, S. P., and Harvey, J. W. 1976, *Ap. J.*, **207**, 174.
Merrill, P. W. 1935, *Ap. J.*, **81**, 312.
Mezger, P. G., and Henderson, A. P. 1967, *Ap. J.*, **147**, 471.
Michalitsianos, A. G., Kafatos, M., and Hobbs, R. W. 1980, *Ap. J.*, **237**, 506.
Morris, M., Redman, R. O., Reid, M. J., and Dickinson, D. 1979, *Ap. J.*, **229**, 257.
Morris, M., Bowers, P. F., and Turner, B. E. 1982, *Ap. J.*, **259**, 625.
Mufson, S. L., Lyon, J., and Marionni, P. A. 1975, *Ap. J. (Letters)*, **201**, L85.
Natta, A., and Panagia, N. 1981, in *Physical Processes in Red Giants*, ed. I. Iben and A. Renzini (Dordrecht: Reidel), p. 427.
Paczyński, B. E. 1971, *Acta Astr.*, **21**, 47.
Panagia, N., and Felli, M. 1975, *Astr. Ap.*, **39**, 1.
Pottasch, S. R., Goss, W. M., Arnal, M., and Gathier, R. 1982, *Astr. Ap.*, **106**, 229.
Reimers, D. 1981, in *Physical Processes in Red Giants*, ed. I. Iben and A. Renzini (Dordrecht: Reidel), p. 269.
Renzini, A., Cacciani, C., Ulmschneider, P., and Schmitz, F. 1977, *Astr. Ap.*, **61**, 39.
Schönberner, D. 1979, *Astr. Ap.*, **79**, 108.
Schwarzschild, M. 1975, *Ap. J.*, **195**, 137.
Sopka, R. J., Herbig, G. H., Kafatos, M., and Michalitsianos, A. G. 1982, *Ap. J. (Letters)*, **258**, L35.
Spencer, J. H., Winnberg, A., Olnon, F. M., Schwartz, P. R., Mathews, H. E., and Downes, D. 1981, *A.J.*, **86**, 392.
Spitzer, L. 1978, *Physical Processes in the Interstellar Medium* (New York: Wiley).
Terzian, Y. 1980, *Quart. J.R.A.S.*, **21**, 82.
Thronson, H., Jr. 1981, *Ap. J.*, **248**, 984.
Vardya, M. S. 1966, *M.N.R.A.S.*, **134**, 347.
Wallerstein, G., and Greenstein, J. L. 1980, *Pub. A.S.P.*, **92**, 275.
Warner, B. 1972, *M.N.R.A.S.*, **159**, 95.
Westbrook, W. E., Becklin, E. E., Merrill, K. M., Neugebauer, G., Schmidt, M., Willner, S. P., and Wynn-Williams, C. G. 1975, *Ap. J.*, **202**, 407.
White, R. L., and Becker, R. H. 1982, *Ap. J.*, **262**, 657.
Wright, A. E., and Barlow, M. J. 1975, *M.N.R.A.S.*, **170**, 41.
Wynn-Williams, C. G. 1977, *M.N.R.A.S.*, **181**, 61p.
Yamashita, Y., and Maehara, H. 1978, *Pub. Astr. Soc. Japan*, **30**, 409.
Zuckerman, B. 1978, in *IAU Symposium 76, Planetary Nebulae: Observations and Theory*, ed. Y. Terzian (Dordrecht: Reidel).
———. 1979, *Ap. J.*, **230**, 442.
Zuckerman, B., Gilra, D. P., Turner, B. E., Morris, M., and Palmer, P. 1976, *Ap. J. (Letters)*, **205**, L15.
Zuckerman, B., Terzian, Y., and Silverglate, P. 1980, *Ap. J.*, **241**, 1014.

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