THE ASTROPHYSICAL JOURNAL, 274:755–762, 1983 November 15

PERIOD AND AMPLITUDE VARIATIONS OF POLARIS

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Received 1983 March 28; accepted 1983 April 21

ABSTRACT

A reinterpretation of the pulsational $O-C$ residuals for Polaris (α Ursae Minoris), combining photometric and radial velocity observations, shows that the pulsational period has been steadily increasing for the last 100 years, the ephemeris being

$$\mathrm{JD(max.\ light)} = 2{,}431{,}495.813 + 3.9696E + 1.9877\times10^{-7}E^2.$$

This corresponds to a period increase rate of 316 s per century. The $O-C$ residuals from this ephemeris are significant, and cannot be explained by light-time effects caused by the orbital motion of Polaris around its unseen companion, or by gravitational effects of the companion at closest approach. Alternatively they may be produced by an unknown phenomenon intrinsic to the star.

The photometric amplitude is found to have decreased since about the 1940s. Consistently, the pulsational radial velocity amplitude has also decreased. The connection of this phenomenon to the star's leaving the instability strip is considered.

Subject headings: stars: Cepheids — stars: individual — stars: pulsation

I. INTRODUCTION

Polaris (α Ursae Minoris), the nearest bright star to the north celestial pole, has been an outstandingly interesting star for the past 130 years. Its peculiar position in the sky contributed to the early discovery of the star as a variable (Seidel 1852; Schmidt 1856), and at the beginning of this century the Cepheid nature of the star was suspected (Pannekoek 1913). Polaris is at present the Cepheid with the smallest light amplitude ($\Delta V \sim 0.05$ mag) and is one of 14 small-amplitude ($\Delta V < 0.3$ mag) galactic Cepheids presently known (Arellano Ferro 1983). Polaris is a single-line spectroscopic binary; its unseen companion is probably a late A or early F main-sequence star (Fernie 1966). The orbital period is about 30 years (Roemer 1965). A third member of the system is an 8th mag F star 19″ away with which the Cepheid forms a visual binary. The system membership of 13th and 12th mag stars 43″ and 83″ away is not clear.

Polaris was observed as part of a more extensive study of the pulsational properties of small-amplitude yellow supergiants, in which the period plays an important role. The secular increase of the period of Polaris has long been a well known fact (Petrov 1949; Roemer 1965; Fernie 1966), a circumstance that makes it difficult to combine the enormous number of existing observations to determine the period very accurately. Fernie (1966) analyzed the times of maximum light listed by Petrov (1949), while Roemer (1965) analyzed the times of minimum radial velocity from data obtained at the Lick Observatory. Both sets of data span the whole of this century.

Several circumstances moved me to do the analysis again. Fernie (1966) computed the ephemeris for times of maximum light using only those times of maximum since 1927; i.e., more than half the available data in Petrov's (1949) table were left out. The reason for this was that, as Fernie comments, for the 50 years prior to 1927 Petrov's figures show the period had been decreasing. This conclusion is not supported by the times of minimum radial velocity, which show that the period was increasing, as can be seen in Figure 46 of Roemer (1965). In § III it is shown that a reinterpretation of the photometric $O-C$ residuals, as listed by Petrov (1949), can bring the photometric and the radial velocity observations into agreement, and suggests that the period has been steadily increasing for the past 100 years. Secular light amplitude changes in Polaris have also been found; they are discussed in § IV.

II. OBSERVATIONS

New photoelectric observations were made through Johnson *UBVRI* filters. From 1980 September to 1981 October the 0.6 m telescope at the David Dunlap Ob-

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TABLE 1

PHOTOMETRIC VALUES OF COMPARISON STARS

Star	V	σ_V	$U-V$	σ_{UV}	$B-V$	σ_{BV}	$V-R$	σ_{VR}	$V-I$	σ_{VI}
HR 285 ...	4.220	0.021	3.013:	0.023	1.213	0.008	0.899	0.014	1.441	0.018
HR 286 ...	6.467	0.026	...	...	0.094	0.022	0.129	0.020	0.090	0.024

NOTE.—A colon indicates uncertain due to neutral density filter.

servatory (DDO) was used. This telescope is equipped with a two-channel chopping photometer, an EMI9658R photomultiplier, and a pulse counting system. A diaphragm of 30″ was chosen. From 1981 September 11 to September 26, the Number 4, 0.4 m telescope at Kitt Peak National Observatory (KPNO) was used. On this occasion an S-20 FW130 photomultiplier and a pulse counting system were used. The diaphragm was 20″.

Accurate photometry is not an easy task when Polaris is the object. Besides the difficulties involved in observing near the pole, no good comparison star exists nearby; also the star is so bright ($V \sim 2$ mag) that it requires a neutral density filter, and special care has to be taken in isolating the star from its visual companion. The unseen companion is not bright enough to contaminate the observations (Fernie 1966).

All the observations were made differentially using the star HR 285 as a comparison. Its constancy was tested against HR 286 and had been previously tested by Henden (1980). Observations of standard stars during six nights of the Kitt Peak run allowed the observations to be tied into the standard Johnson system. In Table 1 the photometric values and the standard deviations of the mean of HR 285 and HR 286 are given. In Table 2 the photometric values of Polaris are given. Light and color curves are shown in Figure 1.

TABLE 2

UBVRI PHOTOMETRY OF POLARIS

V	$U-V$	$B-V$	$V-R$	$V-I$	HJD (2,440,000.+)
1.919	...	...	...	...	4521.8965
1.951	...	0.554	...	...	4522.8281
1.997	...	0.575	...	...	4535.8403
1.965	...	0.546	...	...	4536.8174
2.013	...	0.564	0.539	0.761	4715.6011
1.989	...	0.561	0.514	0.771	4741.6450
1.985	1.137	0.588	0.510	0.778	4861.9790
1.964	1.143	0.590	0.510	0.782	4862.9331
1.946	1.160	0.584	0.488	0.758	4863.9443
1.986	1.164	0.606	0.493	0.783	4864.9775
1.960	1.139	0.595	0.488	0.766	4867.9312
2.008	1.187	0.617	0.510	0.787	4868.9185
1.946	1.133	0.586	0.476	0.744	4874.9365
1.971	...	0.532	0.503	0.757	4886.8667

The radial velocities were measured from photographic spectra at reciprocal dispersions of 8, 12, 15 and 16 Å mm^{-1}, obtained from 1980 June to 1981 August with the Cassegrain spectrograph on the 1.88 m telescope of the David Dunlap Observatory and from 1981 August 10 to August 17 with the Cassegrain spectrograph on the 1.83 m telescope of the Dominion Astrophysical Observatory (DAO) at Victoria. The emulsions were vacuum-sensitized IIa-O and 098, which cover the wavelength ranges 4000–5000 Å and 5300–6700 Å, respectively. The projected slit was 24 μm. The reduction techniques were as described by Lane and Percy (1979). About 30 lines were identified and measured on all plates. The radial velocities are reported in Table 3. Comparison with standard stars observed on the same nights leads to an estimate of the accuracy of the radial velocities between 1 and 1.5 km s^{-1}. The radial velocity curve is shown in Figure 2. The scatter around the mean variation is of the order of 1 km s^{-1}, which is consistent with the internal accuracy of the radial velocities.

III. PERIOD CHANGE

If the ephemeris for the time of maximum light given by Petrov (1949) ($JD_{max} = 2{,}418{,}985.86 + 3.968333E$) and the cycle numbers listed in his table are used, the resulting $O-C$ diagram suggests that the period was decreasing before 1927 and increasing thereafter. It is proposed here that this is a misinterpretation produced by erroneous cycle numbers assigned in Petrov's table to the four oldest times of maximum. These are key data for the extension of the $O-C$ diagram to the beginning of the history of the star. In the present interpretation, two cycles should be added to the first and second entries and one cycle should be added to the third and fourth entries. This makes the $O-C$ diagram consistent with an increasing period over the whole century.

According to this interpretation of the $O-C$ diagram, the information contained in both the photometry and the radial velocities can be combined to derive a new ephemeris which is valid for all the existing observations and which leads to the period and the period change rate at the present time.

The ephemeris computed by Stebbins (1946) to calculate the time of maximum light,

$$C = 2{,}431{,}495.99 + 3.96961E, \qquad (1)$$

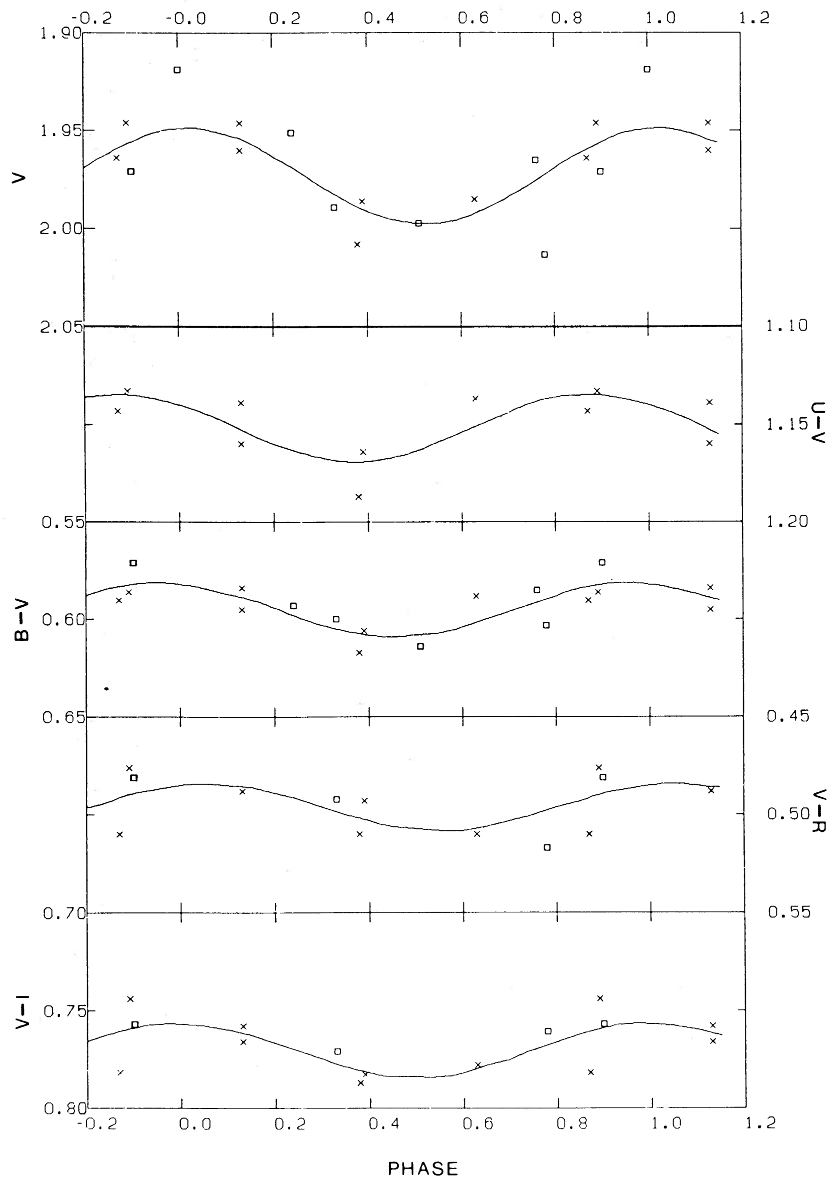

FIG. 1.—Polaris light and color curves. Symbols are: *crosses*, Kitt Peak; *squares*, DDO. The points at phases 0.0 and 0.78 were not included in the sinusoidal fit.

was first used to compute the $O-C$ residuals. For small-amplitude short-period yellow supergiants it has been observed that the time of maximum light does not coincide with the time of minimum radial velocity, but the latter is shifted between 0.03 and 0.13 to later phases (Arellano Ferro 1983). This was taken into account by shifting the $O-C$ residuals of the radial velocities by the proper amount. The shift for Polaris was estimated from Figures 1 and 2. It was found that $\phi^V_{\rm max}-\phi^{\rm RV}_{\rm min}=-0.053$ or $3.969(\phi^V_{\rm max}-\phi^{\rm RV}_{\rm min})=-0.21$ days. The $O-C$ residuals for the time of maximum light and for the time of minimum radial velocity, the latter already shifted -0.21 days, are listed in Tables 4 and 5, respectively. Figure 3 shows the combined $O-C$ diagram. The agreement between the two sets of observations is excellent. This diagram clearly shows the increasing nature of the period. The solid line is a least-squares fit parabola which can be parameterized by

$$O-C=-0.177-(5.271\times10^{-5})E+(1.9877\times10^{-7})E^2. \qquad (2)$$

Equations (1) and (2) lead to the ephemeris for the observed maxima of light (or minima of radial velocities

TABLE 3

RADIAL VELOCITIES OF POLARIS

Plate Number	Dispersion (Å mm^{-1})	V_r ($km\ s^{-1}$)	p.e. ($km\ s^{-1}$)	HJD (2,440,000.+)[a]
44803	12	−18.34	0.26	4432.626 +
44863	12	−20.45	0.24	4453.673 +
44982	12	−21.72	0.26	4489.776 +
44496	12	−19.31	0.21	4491.793 +
44997	12	−18.77	0.22	4491.800 +
45066	16	−19.70	0.42	4509.773 ×
45120	16	−20.40	0.31	4521.790 ×
45192	16	−18.96	0.39	4548.852 ×
45200	16	−19.66	0.42	4549.498 ×
45278	16	−20.24	0.45	4569.876 ×
45544	16	−19.84	0.45	4667.477 ×
45557	16	−18.24	0.40	4672.691 ×
45599	16	−19.94	0.37	4684.646 ×
45632	12	−20.34	0.18	4689.807 +
45706	16	−21.26	0.49	4710.582 ×
45745	16	−20.50	0.45	4724.660 ×
45763	16	−18.39	0.41	4726.646 ×
45798	16	−22.53	0.40	4732.552 ×
45808	12	−20.72	0.26	4737.640 +
45809	12	−21.27	0.23	4737.643 +
45810	12	−21.50	0.26	4737.646 +
45850	16	−19.58	0.33	4746.686 ×
45881	16	−19.69	0.39	4757.572 ×
45914	16	−21.63	0.44	4768.781 ×
45981	8	−18.66	0.18	4774.632 +
46001	8	−18.14	0.17	4802.581 +
46072	12	−20.40	0.25	4822.847 ×
88799	15	−19.90	0.32	4829.822 *
88813	15	−20.86	0.36	4830.853 *
88828	15	−21.57	0.39	4831.881 *
88840	15	−19.80	0.35	4832.947 *
88852	15	−19.19	0.36	4833.876 *
88867	15	−21.25	0.40	4834.876 *
46141	12	−18.15	0.30	4837.567 +
46586	12	−17.63	0.17	4988.603 +

[a] ×, DDO plates. *, DAO plates. +, DDO plates courtesy of N. R. Evans and K. Kamper.

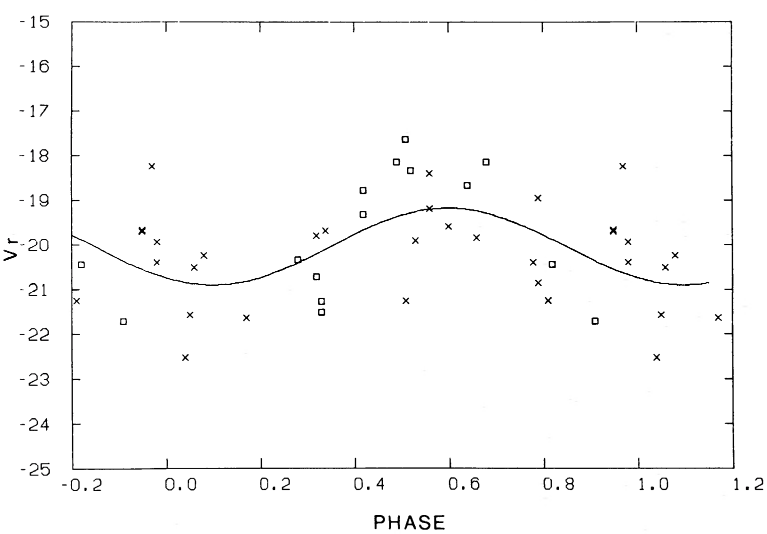

FIG. 2.—Polaris radial velocity curve. The symbols are: *crosses*, this work; *squares*, courtesy of N. R. Evans and K. Kamper.

TABLE 4

$O-C$ RESIDUALS OF THE MAXIMUM LIGHT FOR POLARIS

JD_{max} (2,400,000.+)	E	$O-C$ (days)
7696.57	−5997	6.33
8228.45	−5863	6.28
13045.81	−4649	4.53
13843.40	−4448	4.24
17791.79	−3453	2.86
18985.86	−3152	2.08
19299.15	−3073	1.77
22954.08	−2152	0.69
23620.77	−1984	0.49
23851.23	−1926	0.71
24406.28	−1786	0.01
24755.57	−1698	−0.02
24894.82	−1663	0.29
25120.73	−1606	−0.07
25454.46	−1522	0.22
27069.75	−1115	−0.12
27089.48	−1110	−0.24
27494.45	−1008	−0.17
27661.22	−966	−0.13
27689.13	−959	0.00
27784.28	−935	−0.12
28335.90	−796	−0.28
28431.25	−772	−0.20
29050.91	−616	0.20
30610.66	−223	−0.11
31495.99	0	0.00
32437.05	237	0.26
39253.23	1954	0.62
43779.41	3094	1.45
44517.92	3280	1.61

appropriately shifted),

$$O = \mathrm{JD(max.\ light)} = 2{,}431{,}495.813 + 3.9696E + (1.9877\times10^{-7})E^2. \tag{3}$$

Now, given an epoch E, the period at that epoch, $P(E)$, can be predicted by differentiating equation (3),

$$P(E) = \frac{dO}{dE} = 3.9696 + 2(1.9877\times10^{-7})E. \tag{4}$$

For example, let us compute $P(E)$ for $E = 3676$. This epoch is the mean epoch between time of maximum as indicated by the V-magnitudes in Table 2 (JD 2,444,517.91) and Henden's (1980) time of maximum, which is the second latest set of observations. Equation (4) leads to the period $P(3676) = 3.9710$ days. The Fourier analysis of the V magnitudes in Table 2 combined with those of Henden gives the period $P_f = 3.9715 \pm 0.0009$, in reasonable agreement with the predicted period $P(3676)$. From equation (4) it can be concluded that the period is increasing at a rate of about 316 s per century.

TABLE 5

$O-C$ RESIDUALS OF THE MINIMUM RADIAL VELOCITIES FOR POLARIS[a]

JD_{min} (2,400,000.+)	E	$O-C$ (days)
13879.39...........	−4439	4.29
14899.10...........	−4182	3.81
15097.71...........	−4132	3.94
15541.89...........	−3620	3.52
15982.51...........	−3909	3.52
16639.55...........	−3819	3.29
16652.67...........	−3740	2.81
17030.01...........	−3645	3.04
17799.86...........	−3451	2.78
18156.80...........	−3361	2.46
18589.13...........	−3242	2.10
18882.43...........	−3178	1.65
19271.93...........	−3080	2.13
19593.34...........	−2999	2.00
19803.51...........	−2940	1.78
20200.31...........	−2846	1.62
20648.65...........	−2733	1.39
20906.67...........	−2668	1.39
21561.50...........	−2503	1.23
21779.82...........	−2448	1.23
22307.30...........	−2315	0.75
22513.91...........	−2263	0.94
22902.70...........	−2165	0.71
23176.78...........	−2096	0.88
23692.37...........	−1966	0.42
23974.28...........	−1895	0.49
24339.22...........	−1803	0.23
24736.34...........	−1703	0.39
25125.40...........	−1605	0.42
25950.58...........	−1523	0.10
25867.34...........	−1418	0.05
26133.43...........	−1351	0.17
26466.87...........	−1267	0.17
26840.02...........	−1173	0.17
27220.87...........	−1077	−0.06
27550.51...........	−994	0.10
28062.33...........	−865	−0.16
29563.10...........	−487	0.10
29821.12...........	−422	0.10
30265.56...........	−310	−0.06
34533.21...........	765	0.26
35283.41...........	954	0.20
35589.15...........	1031	0.28
36442.81...........	1246	0.48

[a] Residuals are shifted −0.21 days; see text.

IV. AMPLITUDE VARIATION

The most striking feature of the light, color, and radial velocity curves is the smaller than expected amplitude exhibited by the star ($\Delta V \sim 0.05$ mag and $2K \sim 2.0$ km s^{-1}). This circumstance makes one wonder what has been known of the amplitude of this star in the past, other than that it is small. The best source of information for this is again a table given by Petrov (1949), where he lists light amplitudes and their uncertainties

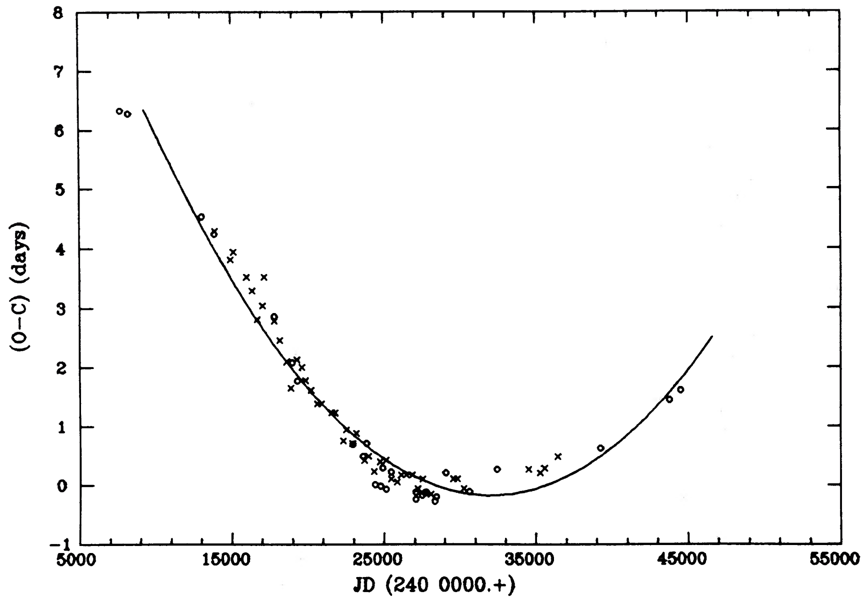

FIG. 3.—Period change of Polaris. All the $O-C$ residuals from photometric (*open circles*) and radial velocity observations (*crosses*) since 1879 have been combined. The radial velocities have been shifted $-0^{d}.21$ due to the observed phase lag between light and radial velocity curves (see text). The solid curve is a parabolic fit to the data. All $O-C$ residuals were computed with the ephemeris $C = 2{,}431{,}495.99 + 3.96961E$ (Stebbins 1946).

from 1879 to 1947. The only problem in using these amplitudes is that they are not homogeneous, in the sense that they have been measured by different techniques and at different wavelengths, and, as is well known for Cepheids and Cepheid-like stars, the amplitude in blue light is larger than in yellow light. Therefore, a straightforward comparison of amplitudes is inadequate. In Figure 4 the amplitudes, as listed by Petrov, have been plotted as a function of time. I have searched the original sources and have distinguished, whenever possible, between photoelectric, photographic and visual measurements as described in the legend of the figure. All the photoelectric measurements in Figure 4 were made through a blue filter. An F beside the plus signs means that the amplitude is listed by Florja (1935), but the original source is not available and there is no information about the nature of the observations. Similarly, the nonlabeled plus signs are listed by Petrov (1949), but no information on the nature of the observations could be found. The error bars are the uncertainties listed by Petrov and were drawn only for the photoelectric and photographic amplitudes. The amplitudes in the Johnson B filter for the three latest sets of observations were computed by the author. Error bars are a measurement of the scatter of the observations about the mean variation.

The first three observations on Figure 4 are visual; hence the amplitudes should be increased by about 0.025 mag, which is the average difference between yellow and blue amplitudes shown by the three latest photoelectric light curves. For the lowest three points between JD 2,423,000 and JD 2,428,000 the original papers are not available to the author and so the nature of these amplitudes is not known, but judging by their large uncertainties (0.09, 0.034, and 0.06 mag, respectively) the quality of these observations may not be very high and their positions may not be inconsistent with the cluster of points at amplitudes larger than 0.12 mag, especially if they are photoelectric yellow or visual observations.

Figure 4 strongly suggests that the amplitude of the variation started to decline approximately in 1945 (~ JD 2,431,000) and has been decreasing until the present. This trend is significant and must be real, especially if one considers that before 1945 there are a number of carefully made photoelectric observations obtained through blue filters very similar to the Johnson B filter used in the three latest observations. The amplitude appears constant (within the errors) before 1945. Due to the scarcity of observations and to the lower quality of earlier observations, it is not possible to say if the amplitude was increasing before 1905 (~ JD 2,417,000).

Consistent with the light amplitude variation is the behavior of the radial velocity amplitude. From 1900 to 1950 the radial velocity amplitude remained constant between 5 and 6 km s^{-1} (Roemer 1965). The 1980–1981 radial velocity curve in Figure 2 shows an amplitude of about 2 km s^{-1} with a dispersion of ± 1 km s^{-1}, i.e., both the light and the radial velocity curves show that the amplitude has decreased.

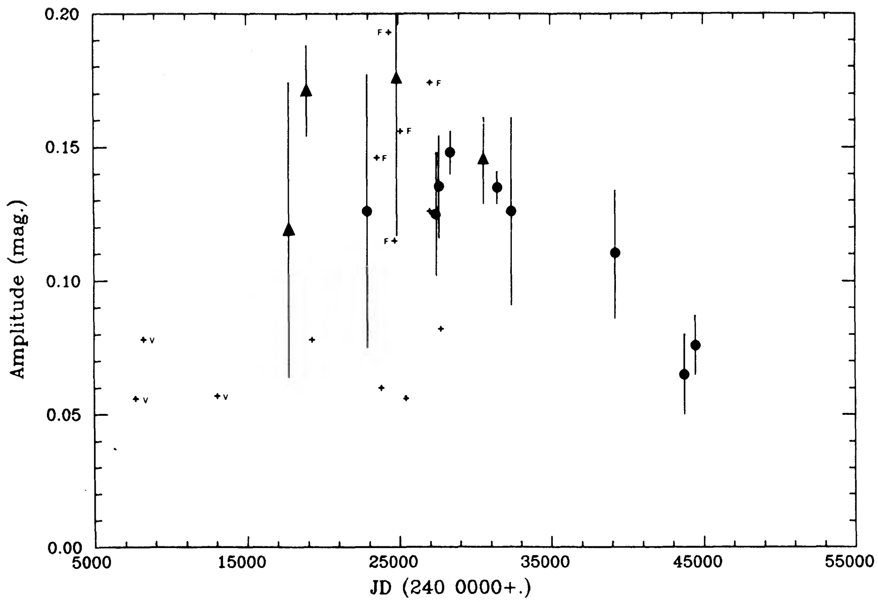

FIG. 4.—Long term amplitude change of Polaris. Error bars are plotted only for photoelectric (*dots*) and photographic (*triangles*) observations. All photoelectric observations were made through a blue filter. Visual observations are labeled *V*. Plus signs labeled F represent observations listed in Florja (1935), and nonlabeled plus signs represent observations listed in Petrov (1949), but there is no certainty on the nature of these observations.

V. DISCUSSION

As presented in § III, the photometric observations available since about 100 years ago indicate the increasing nature of the period. This is supported by the more extensive radial velocity observations. The parabolic interpretation of the $O - C$ diagram of Figure 3 does not produce a perfect fit to the data; instead, there are residuals of 0.3–0.4 days about the mean parabolic variation. This interpretation is not the only plausible one. An equally valid interpretation could be made by representing the $O - C$ residuals by three straight lines and two abrupt period changes at JD 2,426,000 and JD 2,437,000. Still, under such an interpretation the fit is poor, especially before JD 2,426,000 which is the best covered portion of the $O - C$ diagram. Of the two above interpretations, the parabolic case is preferred since it is simpler. The differences between the $O - C$ residuals and the parabolic fit in Figure 3 have been calculated, and they are plotted in Figure 5. It does not seem possible that these differences are produced by light-time effects caused by the orbital motion of Polaris around its unseen companion. The maxima of the variation shown in Figure 5 are separated by about 50 years. Since only one "cycle" of such differences is available and the scatter is considerable, it is difficult to estimate the cycle-length, but it is hard to reconcile it with the 30.5 year orbital period found from the variations of the systemic velocity (Roemer 1965). From the orbital elements given by Roemer (1965) one can calculate that the maximum $O - C$ value produced by light-time effects is about 0.5 hr, whereas the semiamplitude shown in Figure 5 is at least 7 hr ($0\overset{d}{.}3$), i.e., too large to be explained by light-time effects.

If the differences from the parabola are not due to light-time effects, the question of what causes them then arises. The orbit of Polaris around its unseen companion is considerably eccentric, $e = 0.639$ (Roemer 1965), and then one may wonder whether the proximity of the companion can alter the period—and probably the amplitude—of the variation of the Cepheid. Taking $\mathscr{M} = 6\mathscr{M}_\odot$ for Polaris (Arellano Ferro 1983), $\mathscr{M} = 2\mathscr{M}_\odot$ for the companion, the orbital elements (Roemer 1965), and assuming $i = 90°$, it can be found that the nearest approach of the two stars is about 3.5 AU. There are binary Cepheids that make significantly closer approaches to their companion than 3.5 AU, e.g., SU Cyg (N. R. Evans, private communication) and FF Aql (Abt 1959), but their $O - C$ deviations are much smaller than those in Figure 5 (Szabados 1977, 1980). Then alterations of the pulsational period due to gravitational interactions are unlikely.

There is also the possibility that the period has been steadily increasing (parabolic fit) but fluctuations around the mean variation are produced by an unknown phenomenon intrinsic to the star. In a star in which the pulsation amplitude has suffered the variations noted in Polaris (§ IV), fluctuations in the period change rate should probably not be surprising.

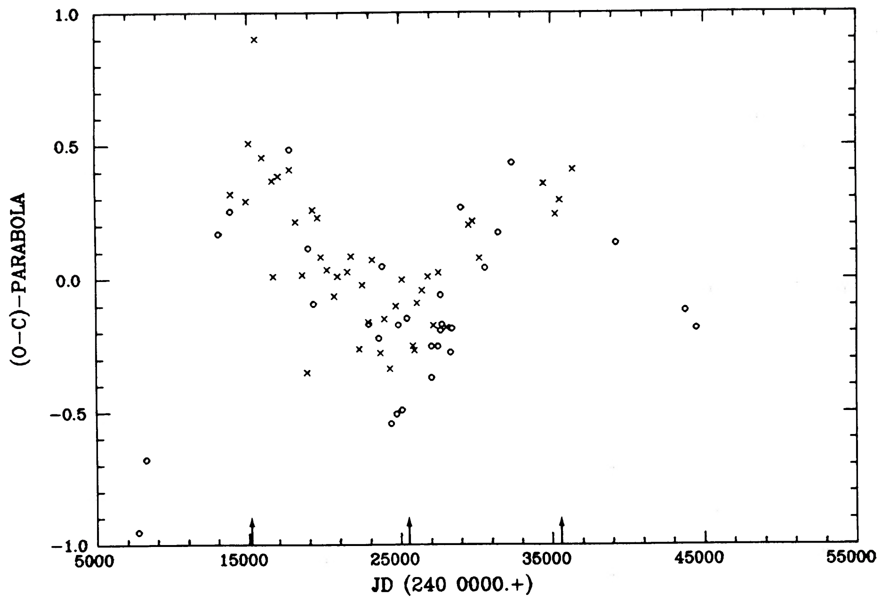

FIG. 5.—The differences of the residuals $O - C$ minus the parabolic fit in Fig. 3, show the imperfectness of the parabolic fit. Three consecutive periastron passages are indicated by arrows on the horizontal axis. These differences do not seem to be the consequence of orbital light-time effects or gravitational interaction with the companion (see text).

The amplitude variations found in Polaris are not unknown in small-amplitude Cepheids; e.g., HR 7308 presents a beating effect (Burki and Mayor 1980) which has been shown to have a period of about 1210 days (Breger 1981), although this star shows no sign of a period change. Of course, at present nothing can be predicted for the future of the amplitude of Polaris. It is possible that the amplitude will start increasing again and even that this is a periodic phenomenon with an enormous "beating" period ($\sim$100 years!). It is also possible that the amplitude will continue decreasing and the star will eventually stop pulsating. If the decreasing rate continues as it seems to have been during the last 40 years, it would take only another 40 years to become a nonvariable star! Is it possible that we are witnessing the evolution of the star as it evolves out of the instability strip? The position of Polaris and other small-amplitude yellow supergiants on the H-R diagram has recently been determined (Arellano Ferro 1983). The star is found to be very near the red edge of the instability strip, which is not unexpected if the star is evolving out of the instability region. If such is the case, the star is evolving to the red, which is consistent with the increasing nature of the period. However, finding an answer to these questions will require further observations over the coming decades.

I am grateful to Drs. J. R. Percy and N. R. Evans for multiple discussions and enlightening ideas, and to Drs. N. R. Evans and K. Kamper for letting me use and publish their radial velocities. This work was supported by Consejo Nacional de Ciencia y Tecnología de México (CONACYT), by the Natural Sciences and Engineering Research Council of Canada (research grant to Dr. J. R. Percy), and by the Carl Reinhart Travel Fund of the University of Toronto.

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