THE ASTROPHYSICAL JOURNAL, 272:48–53, 1983 September 1

FOCUSING OF HIGH-MACH NUMBER JETS BY AN AMBIENT MEDIUM

DAVID EICHLER[1]
Institute for Theoretical Physics, University of California, Santa Barbara; and Astronomy Program, University of Maryland
Received 1982 July 12; accepted 1983 February 2

ABSTRACT

It is argued that a precessing jet will, at some sufficiently large scale, be focused by an ambient plasma of finite pressure such that its angular spread on larger scales is much smaller than that of the precession cone at small scales. More generally, the collimation of a moderately wide jet can be sharpened by the ambient medium, and a smeared-out beam can be recollimated. Shapes of channels formed by such recollimated beams are calculated. The focal length along the precession axis is found to be nearly independent of the original opening angle if the other jet parameters are fixed. The results are compared with real sources where there is some observational evidence for jet focusing, such as SS 433.
Subject headings: galaxies: jets — hydromagnetics — plasmas

I. INTRODUCTION

The lobes of very extended radio galaxies often contain hot spots that subtend opening angles as small as $\sim 10^{-2}$ radians as seen from the central source. One obvious interpretation is that the lobes are being fed by jets that are extremely well collimated, their opening angles presumably being comparable to the solid angle subtended by the hot spot. Further evidence of very narrow collimation is provided by direct observations of radio and optical jets on scales of 3–100 kpc. The jet in NGC 6251, a good and not atypical example, has an opening angle of 3° sin ϕ (Readhead, Cohen, and Blandford 1978), where ϕ is the angle between the jet and our line of sight. From such observations, it might reasonably be inferred (1) that the central source of the jets can aim material with an accuracy of a degree or so, and (2) that the central source remains pointed at the hot spot over long time scales ($\gtrsim 10^6$ yr) as jitter or precession in the beam direction over shorter time scales would essentially smear it.

In a previous paper (Eichler 1982, hereafter Paper I) it was suggested that the first conclusion could be avoided if the beam is overcollimated in an axisymmetric way, so that the fluid elements are directed toward the axis and eventually crash into each other there. Also see Cantó (1980) and Cantó *et al.* (1981), who independently proposed a similar model in the context of Herbig-Haro objects, though with different assumptions regarding the ambient pressure profile. Depending on how much transverse motion is dissipated following impact at the axis, the resulting opening angle of the jet (at least the part of the jet that is overcollimated), after such dissipation has occurred, can be much smaller than any characteristic angle in the source itself. On the other hand, the jet direction is well defined only to the extent that the axis of symmetry is stable.

The discovery of SS 433 and its being identified as a precessing beam may be an important counterexample to the second conclusion. For this source there is little doubt that the inner jets are precessing at an angle of 20° to the precession axis. As has been noted (e.g., Linfield 1981*b*), the opening angle indicated by the radio "ears" at the periphery of the associated supernova remnant (SNR) W50 (Geldzahler, Pauls, and Salter 1980) is considerably less than 20° and is suggestive of recollimation at some intermediate scale (see Fig. 2). In addition, there is an increasing list of jets that seem well collimated at large scales while displaying some misalignment at smaller scales. Published examples include observations of 3C 390.3 and Cyg A, where there is misalignment of about 5° between the VLBI jets and the hot spots (Linfield 1981*a*), NGC 6251, where there is 5° misalignment between the nuclear jet and the outer jet (Cohen and Readhead 1980), and 3C 273, where there is strong misalignment, probably enhanced by projection effects, between the nuclear jet and outer jet (Linfield 1981*b*). The apparent misalignment angle is in some cases greater than the opening angle of the jet at larger scales. For these extragalactic sources there is no direct evidence that the misalignment is due to precession. However, in some highly asymmetric sources such as 3C 273, the nuclear jet shows curvature, and a strong case can be made (Begelman, Blandford, and Rees 1980; Linfield 1981*b*) that the curved pattern results from the precession of a relativistic jet.

In the particular model suggested in Paper I, the ejected material is collimated into a jet while it is highly supersonic. (This assumption is made so that the jet is stable to centrifugal instabilities, which typically require that sound can propagate across the jet over the hydrodynamic time scale.) As such, the model requires that the pressure fall off fairly rapidly as a function of distance from the source. If, for example, the ejected material were to expand into a homogeneous

[1] Alfred P. Sloan Fellow.

external medium, it would merely create a spherical cavity bounded by a termination shock. In order to be overcollimated without being stopped, on the other hand, the fluid must slip along a curved surface.

In this paper it is noted that if the ejected material is already collimated to some degree, overcollimation is possible over a much broader range of density profiles than assumed in Paper I. In particular, a homogeneous medium, such as the interstellar medium of the parent galaxy or the surrounding intergalactic medium, could overcollimate a jet that was essentially free on smaller scales.

It is thus argued that the compact source from which the jets originate need not point steadily at the hot spot. The effects of small-angle precession, or fast jitter, which effectively broaden the opening angle of the jet, can be overcome by recollimation in the large-scale environment into which it expands, provided that the distribution of jet directions is axisymmetric and that the long-term average of the jet direction points at the hot spot.

In § II, the model is presented for recollimation of a precessing jet which is assumed to form a hollow cone of opening semiangle θ_0. The case of the hollow cone is particularly easy to calculate. The same model would also apply to a filled-in cone, where because the average angular spread is somewhat less than θ_0, the jet would be even easier to collimate. In § III the condition is derived for keeping the cone interior free of ambient material, so that the ambient pressure creates a net force toward the axis. In § IV the model is compared with some of the sources mentioned above. In § V, the stability of the flow is discussed.

The basic picture proposed here has very recently been discussed independently by Norman *et al.* (1982, 1983), who have performed axisymmetric hydrodynamic simulations that confirm it. A similar picture has been discussed recently by Königl (1982) in the context of Herbig-Haro objects. These papers are referred to in more detail in later sections.

II. BASIC CALCULATIONS

Below, we consider the recollimation of a precessing jet. We assume that the precession period is small compared with the large-scale hydrodynamic time so that the material in the precession cone blends together over the larger time scale to form a smooth surface. Although the precession "cone" (which ultimately develops a more complicated shape) is not completely hollow (because of a finite intrinsic opening angle of the jet and possibly other effects), we assume, for the purpose of calculating the shape of the channel evacuated by the jet, that all the material is emitted in a cone of opening semiangle θ_0. It is shown below that for small θ_0, only a small fraction of the flow is needed in the interior of the channel to keep it evacuated. Hence, we also neglect any pressure in the interior of the precession cone. The shape of the channel is then determined in a straightforward manner by equating the ambient pressure P with the centrifugal force per unit area (C.P., for centrifugal pressure) exerted by the jet material as it shoots along the channel wall.

$$P = \text{C.P.} \equiv \frac{\sigma v^2}{R_C}\,, \tag{1}$$

where σ is the surface density, v is the velocity of the beam material, and R_C is the radius of curvature of the fluid trajectories.

The power output, L, of the beam is

$$L = \tfrac{1}{2}\sigma v^3(2\pi x)\,, \tag{2}$$

where x is the cross-sectional channel radius. The jet axis is taken to be the y-axis. We have assumed a nonrelativistic jet for convenience, but what follows is trivially extended to relativistic jets.

The quantity R_C^{-1}, for the present purposes, is expressed most conveniently as

$$R_C^{-1} = \frac{d}{ds}\chi = \frac{d}{ds}\arctan\left(\frac{dy}{dx}\right), \tag{3}$$

where χ is the angle made by the tangent of the flow trajectory with the x-axis, and ds is the length element $(dx^2 + dy^2)^{1/2}$.

Combining the above equations, we obtain

$$P = \frac{L}{v\pi x}\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{-3/2}\frac{d}{dx}\frac{dy}{dx}\,. \tag{4}$$

Normalizing distance scales to $(L/\pi vP)^{1/2}$, we can rewrite equation (4) as

$$x = (1 + y'^2)^{-3/2}y''\,, \tag{5}$$

where y' is dy/dx.

A first integral is obtained by rewriting equation (5) as

$$x = \frac{d}{dx}\frac{y'}{(1 + y'^2)^{1/2}}$$

or

$$\frac{y'}{(1 + y'^2)^{1/2}} = \frac{x^2}{2} + a\,, \tag{6}$$

where a is a constant of integration that is easily shown to be $\sin\theta_0$.

Finally,

$$y' = \left(\frac{x^2}{2} + a\right)\bigg/\left\{\left[1 - \left(\frac{x^2}{2} + a\right)^2\right]^{1/2}\right\}. \tag{7}$$

It is clear that the maximum channel width, where $y' = \infty$, is $(2 - 2\sin\theta_0)^{1/2}$.

Equation (7) is easily integrated numerically, and the solutions of channel shapes for $\theta_0 = 10°$ and $20°$ are shown in Figure 1. The length of the channel y_c before it closes up is about $y_c = 3(L/\pi vP)^{1/2}$. The quantity y_c is weakly dependent on θ_0 for a given L because the curvature of the channel wall produced by a given pressure is inversely proportional to its cross-sectional radius, which is the scaling relation for a given y_c.

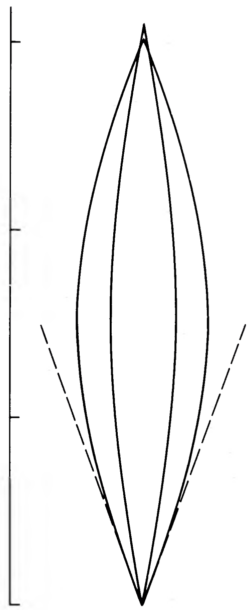

FIG. 1.—The channel shapes resulting from precession cones with semiangles of 10° and 20° are displayed. The dashed lines show the original 20° cone. The length units marked on the vertical axis are $(L/\pi v P)^{1/2}$.

The flow continues beyond the focal length y_c, perhaps after some dissipation there, with a net opening angle that is clearly much less than θ_0. The focusing mechanism continues to operate, assuming the ambient medium is homogeneous, for as long as stability considerations permit.

III. EVACUATION OF THE CHANNEL INTERIOR

It remains to be shown that the presence of ambient material in the interior of the channel can be justifiably neglected under reasonable assumptions. Suppose that the ram pressure at the axis of the precession cone $\rho_a v^2$ is some small fraction ε of $\bar{\rho} v^2$, the ram pressure averaged over the solid angle of the precession cone, where

$$\frac{1}{2} \bar{\rho} v^2 = \frac{L}{\pi v y^2 \theta_0^2} . \tag{8}$$

Such a finite ram pressure could arise because the beam in any instantaneous orientation has a finite spread that is small but nonvanishing at an angle θ_0 from its direction of maximum intensity. Moreover, the ratio of density at the precession axis to that at the precession cone is enhanced by a significant factor when the average over the precession cycle is considered (as opposed to merely considering the angular spread of the inner jet at any instant), for the precession axis receives a steady supply, while any given direction on the precession cone is supplied only over a relatively small duty cycle. Ambient fluid could remain inside the channel only if it can withstand the head-on ram pressure due to the flow inside the channel; i.e., only if $P = \rho_a v^2$ for some $y < y_c$. Moreover, the physical collimation is completely effective if the channel is evacuated out to $y_c/2$, for beyond that point, the material is overcollimated and flows toward the axis. However, according to the above calculations, $P = 9L/\pi v y_c^2$, whereas $\rho_a v^2 = (2\epsilon L)/(\pi v y^2 \theta_0^2)$. It follows that if $\rho_a/\bar{\rho} \geq (9/8)\theta_0^2$, then the interior of the channel is evacuated by the beam at $y < y_c/2$.

If the inner jet has an intrinsic opening angle $\delta\theta \ll \theta_0$ through which most of its power flows, then $\bar{\rho} = (\delta\theta/\theta_0)^2 \rho_{\max}$, and the condition for evacuating the channel can be expressed as $\rho_a/\rho_{\max} \geq (9/8)(\delta\theta)^2$.

IV. APPLICATION TO SOME ASTROPHYSICAL SOURCES

a) *SS 433*

For SS 433 and its associated supernova remnant, the narrow opening angle suggested by the X-ray emission (Seward *et al.* 1980) and radio "ears" implies a value for y_c of $y_{20} \times 10^{20}$ cm, $y_{20} < 3$ (see Fig. 2). Given a luminosity of $L_{39} \times 10^{39}$ ergs s^{-1} (on each side of the compact source) and a jet velocity of 0.26c, the required pressure inside the remnant is about $3.7 L_{39} y_{20}^{-2} \times 10^{-11}$ ergs cm^{-3} = $12 L_{39} y_{20}^{-2} P_{\rm IS}$, where $P_{\rm IS}$ is the total interstellar pressure, 2 eV cm^{-3}. It is not unreasonable that a supernova remnant roughly 50 pc in radius, still expanding, would have a pressure an order of magnitude or so above the interstellar pressure. If the jet power makes its way, ultimately, into the rest of the remnant, the pressure in the remnant is roughly $1.5 L_{39} \tau_4 \times 10^{-11}$ ergs cm^{-3}, where $\tau_4 \times 10^4$ yr is the time over which the jets have been turned on.

It should be noted that the emitting material of the X-ray jet, which has a density of ~ 0.1 cm^{-3} (Seward *et al.* 1980), is too dense to be the ejected material itself. We may suppose, however, that at and beyond the focal point y_c, the convergent flow is very violent and heats the surrounding material. Unsteadiness in the flow could have the same effect by causing the channel walls to fluctuate. Königl (1982) suggests a different model for the X-ray jet that does not require focusing of the ejected material at the scale of 30 kpc.

An electron density of 0.1 cm^{-3} (Seward *et al.* 1980) and a global temperature of 10^6 K (Königl 1982) yield an ambient electron pressure of 1.3×10^{-11} ergs cm^{-3}, which is probably accompanied by a comparable ion pressure. This is in rough agreement with the estimated pressure required for the model if L_{39}, and $y_{20} \sim 1$.

b) *Extragalactic Jets*

Typical numbers for extragalactic radio sources appear, on the whole, to admit jet focusing by an extragalactic medium on the scale of giant radio lobes. Writing the relevant numbers as $L = L_{46} \times 10^{46}$ ergs s^{-1}, $y_c = y_{300} \times 300$ kpc, and $v = \beta c$, an extragalactic pressure of order $(L_{46} 10^{-12})/(y_{300}^2 \beta)$ ergs cm^{-3} is required. This seems consistent with present knowledge, particularly given that giant radio galaxies are strongly correlated with clusters of galaxies, where pressures of 10^{-11} ergs cm^{-3} or more are not uncommon. It is, of

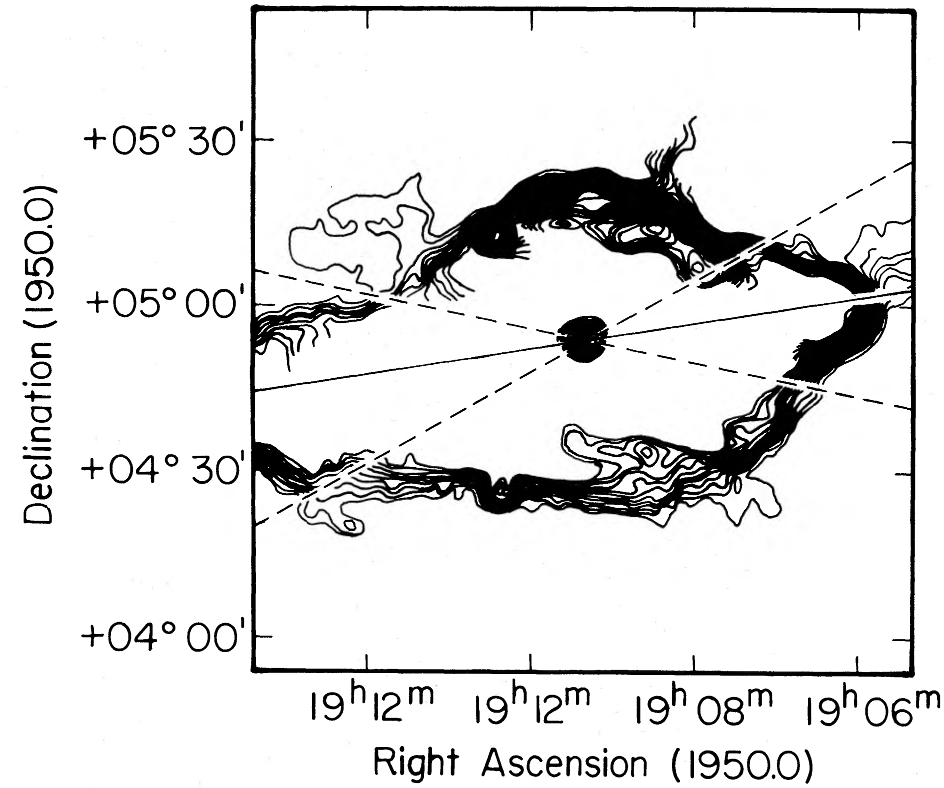

FIG. 2.—The outer radio contours of the supernova remnant taken from Geldzahler, Pauls, and Salter (1980) are shown. The dashed lines represent the extension of the 20° precession cone of SS 433. Comparing with Fig. 1, the focal length cannot be more than twice the radius of the remnant, some 50 pc.

course, advisable to check the numbers on a case-by-case basis.

The referee has noted that the collimation of supersonic flow might produce limb brightening, which, because it is not observed, would create difficulties for the model. Before this conclusion can be established, however, one must be sure that the thickness of the radio profile is established by the flow pattern and not by the (possibly larger) scale of diffusion of the energetic electrons. Moreover, unsteadiness in the flow would create shocks in the beam. The relative contributions in energetic particle production of the interior shocks and those at the wall of the channel could depend on the respective magnetic field geometries of the shocks (e.g., Edmiston, Kennel, and Eichler 1982), which in turn depend on that of the beam. The nature of this dependence, in turn, depends on the Alfvén Mach number of the flow. As far as the author is aware, theoretical predictions of jets' radio profiles have not yet been done with all these factors in mind. They would seem to depend on numerous details that are not yet fully understood.

In any case, it is clear that, at least in some sources, very narrow collimation occurs at smaller scales. The jet in NGC 6251 has an opening angle of $\sim 3° \sin \phi$ (Readhead, Cohen, and Blandford 1978) (where ϕ is the angle between the jet and the line of sight) at a scale of order kiloparsecs. Letting $y_c = y_{22} \times 10^{22}$ cm and $L/v = 10^{35}$ ergs cm^{-1} (Readhead, Cohen, and Blandford 1978) requires a pressure P of $3 \times 10^{-9}/y_{22}^2$ ergs cm^{-3}. NGC 6251 has been observed by the imaging proportional counter on the *Einstein Observatory* to be a point source with a luminosity of $\sim 3 \times 10^{42}$ ergs s^{-1} (W. H. Ku, private communication). This is consistent with the existence of hot confining gas at a temperature of 10^7 K and a pressure of 10^{-9} ergs cm^{-3} over a region ~ 3 kpc in size; significantly higher or lower temperatures allow higher pressures.

The same considerations imply that if the knots in the jet of M87 are focal points in the flow, then a pressure of $10^{-9}(L_{43}/\beta)(10^{21}\ \mathrm{cm}/y_c)^2$ ergs cm^{-3} is required to focus it, where $L_{43} \times 10^{43}$ ergs s^{-1} is the beam power, and βc is the velocity. If y_c is taken to be the distance between knot A and the nucleus, ~ 1 kpc/sin ϕ, or the distance between adjacent knots, ~ 200 pc/sin ϕ, the model is consistent with the pressure inferred by Schreier, Gorenstein, and Feigelson (1982) of $\sim 3 \times 10^{-10}$ ergs cm^{-3}, given the remaining uncertainties. Schreier *et al.* also note that the minimum pressure in knot A is apparently greater than the ambient pressure. It is easily shown that this is always expected at a focal point, assuming that transverse motion is converted to heat.

However, a detailed quantitative model of M87 is not attempted here.

V. STABILITY AND CONFRONTATION WITH OBSERVATIONS

Rigorous demonstration of the stability of hydrodynamic flows comes with great difficulty because of the large number of instabilities that could be present, and because global stability could depend on the saturation of various instabilities in the nonlinear regime. We therefore confine this discussion to be qualitative on the theoretical side and include empirical evidence for the stability of high–Mach number, collimated flows.

A vortex sheet is known to be unstable (Miles 1958) to compressible Kelvin-Helmholtz instabilities whose wave vector points at an angle θ to the relative velocity vector u in such a way as to satisfy

$$u \cos \theta < (c_{1s}^{2/3} + c_{2s}^{2/3})^{3/2} ,$$

where c_{1s} and c_{2s} are the sound velocities on either side of the vortex sheet. Roughly speaking, perturbations do not grow if their phase velocity is supersonic in the frame of the fluid. A possible physical interpretation is that if the fluid elements are to conspire to create an instability, they must be able to communicate across the spatial scale of the instability before being convected across a comparable distance. It is therefore widely suspected (e.g., Blandford and Pringle 1976) that Kelvin-Helmholtz instabilities do not disrupt a beam if it is sufficiently supersonic that sound cannot propagate across its thickness in the time required for the fluid to travel 1 scale height.

Axisymmetric numerical simulations (Norman *et al.* 1982) show that for Mach numbers greater than 5, beams can propagate indefinitely without being disrupted by axisymmetric instabilities. While these simulations do not include non-axisymmetric instabilities, which may under some circumstances be stronger than axisymmetric ones (Hardee 1978), they are consistent with the qualitative picture described above.

The most convincing evidence that high–Mach number jets are stable over a distance that is long compared with y_c is supplied by the observations of laboratory jets (Love *et al.* 1959). These jets emerge from solid nozzles. As the material emerges from the nozzle, it is greatly underexpanded and diverges from the axis of symmetry at an angle that is typically 45° or more. *Thus, the jet is not very well collimated by the nozzle.* Some distance from the nozzle, while still diverging, its Mach number is much larger than the value at the nozzle opening. The material is observed to crash into the wall of the channel, forming a sheath of shocked material that shoots along the wall, similar to the picture outlined in Paper I and here. The walls of the channel are bent inward by the pressure of the ambient material so that the jet is focused. The focal length is typically 2–3 times $(L/\pi v P_a)$, where P_a is the ambient pressure. As the laboratory shocks are not radiative, the material bounces at the focal length (in some cases forming a Mach disk), and the pattern of divergence and focusing is repeated. When the focusing is repeated over many cycles, the opening angle of the jet taken over many focal lengths from the nozzle is extremely small.

When Mach disks form in laboratory jets, the pattern does not repeat many times because there is a vortex sheet between the subsonic material that has just passed through the Mach disk shock and the slipstream of material that remains supersonic by going around it. The vortex surface is stretched and scrambled within two cycles because it is in contact with subsonic material, at which point the jet becomes turbulent and diverges at a much larger opening angle.

If, on the other hand, the shocks are radiative, or for some other reason dissipative, shocked material would tend to remain supersonic, and the flow would probably be more stable. In experiments where the original opening angle is not very large, Mach disks do not form, and nowhere does the flow become subsonic. Under such circumstances, the jet propagates over many more cycles before becoming turbulent.

The jet in M87 does not extend into distant lobes characteristic of classical double radio sources. It is also knotty, and detailed maps (Biretta, Owen, and Hardee 1983) of the knots are suggestive of shocks whose normal is coincident with the jet axis. A simple interpretation is that they are Mach disks. We speculate that the modest extent of the M87 jet is connected to the existence of Mach disks in the flow, which are disruptive. More generally, this speculation predicts an inverse correlation between the penetrating power of jets and their degree of knottiness.

VI. DISCUSSION

We have argued that a jet that is directed with a finite opening angle into a homogeneous medium having a finite pressure is never totally free, because at some sufficiently large scale the ambient pressure will force the flow lines back toward the axis of the jet. This presents the possibility of focusing jets that are not particularly well collimated by their source—in particular, jets that are emitted by precessing sources such as SS 433. The focal length is nearly independent of the original opening angle for a given luminosity, so if all the other parameters of the system are measurable, it is relatively straightforward to establish whether this effect is important on any given scale.

In regard to the reported misalignments between VLBI structure and the jets they feed into at larger scales, we have noted that if the misalignments are due to precession that preserves an overall axis of symmetry, the effect discussed here could account for them.

As a final point, we note that the focal point y_c is always smaller than the length at which the head of the jet would come into ram pressure equilibrium with the ambient pressure of the external medium, which is on the order of $(L/Pv\delta\theta^2)^{1/2}$. Physically, the point is that it is much easier to guide a flow than to stop it. Thus, a beam that is in complete steady state with an ambient medium is generally focused.

This research was supported in part through the National Science Foundation, grant PHY77-27084 and grant AST80-02673. I am grateful to M. Milgrom, A. Königl, R. Blandford, and M. Smith for helpful discussions.

REFERENCES

Begelman, M. C., Blandford, R. D., and Rees, M. J. 1980, *Nature*, **287**, 307.

Biretta, J. A., Owen, F. N., and Hardee, P. E. 1983, preprint.

Blandford, R. D., and Pringle, J. E. 1976, *M.N.R.A.S.*, **176**, 433.

Cantó, J. 1980, *Astr. Ap.*, **86**, 327.

Cantó, J., Rodríguez, L. F., Barral, J. F., and Carral P. 1981, *Ap. J.*, **244**, 102.

Cohen, M. H., and Readhead, A. C. S. 1980, *Ap. J.* (*Letters*), **233**, L101.

Edmiston, P., Kennel, C. F., Eichler, D. 1982, *Geophys. Res. Letters*, **9**, 531.

Eichler, D. 1982. *Ap. J.*, **263**, 571 (Paper I).

Geldzahler, B. J., Pauls, T., and Salter, C. J. 1980, *Astr. Ap.*, **84**, 237.

Hardee, P. E. 1979, *Ap. J.*, **234**, 47.

Königl, A. 1982, *Ap. J.*, **261**, 115.

Linfield, R. 1981*a*, *Ap. J.*, **244**, 436.

———. 1981*b*, *Ap. J.*, **250**, 464.

Love, E. S., Grigsby, C. E., Lee, L. P., Woodling, M. J. 1959, NASA Technical Report TR-R-6.

Miles, J. W. 1958, *J. Fluid Mech.*, **4**, 538.

Norman, M. L., Smarr, L., Winkler, R. H. A., and Smith, M. D. 1982, *Astr. Ap.*, **113**, 285.

———. 1983, preprint.

Readhead, A. C. S., Cohen, M. H., and Blandford, R. D. 1978, *Nature*, **272**, 131.

Schreier, E. J., Gorenstein, P., and Feigelson, E. D. 1982, *Ap. J.*, **261**, 42.

Seward, F., Grindlay, J., Seaquist, E., and Gilmore, W. 1980, *Nature*, **287**, 806.

DAVID EICHLER: Astronomy Program, University of Maryland, College Park, MD 20742