THE ASTROPHYSICAL JOURNAL, 268:L33–L36, 1983 May 1

THE PULSATION PROPERTIES OF DB WHITE DWARFS: A PRELIMINARY ANALYSIS

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Received 1982 November 8; accepted 1983 January 19

ABSTRACT

We report preliminary results of a numerical investigation of the nonradial *g*-mode pulsation properties of evolutionary DB white dwarf models. We have solved the fully nonadiabatic equations for modes corresponding to spherical harmonic index $l = 1$ through 3. For each of the sequences of models we have examined ($M_\star = 0.6\ M_\odot$; and helium layer masses of $10^{-6}\ M_\star$ and $10^{-4}\ M_\star$), we find a nonradial *g*-mode instability strip about 3000 K wide. For models with standard ML1 convection, this strip lies in the effective temperature range 19,000 K $\gtrsim T_e \gtrsim$ 16,000 K. The boundaries of the instability strip are extremely sensitive to the assumed efficiency of convection, however, and for sequences with more efficient (ML3) convection, we find the instability strip to be in the range 29,000 K $\gtrsim T_e \gtrsim$ 26,000 K. Extrapolation of our calculations to 0.4 $M_\odot$ and 0.9 $M_\odot$ indicates that the instability strip boundaries are insensitive to uncertainties in the total stellar mass. The most unstable modes have *e*-folding times of the order of days.

Subject headings: stars: pulsation — stars: white dwarfs

I. INTRODUCTION

In the past few years, rapid progress has been made in the study of white dwarf pulsations on both the observational and theoretical fronts; this progress has been described in a recent review article by Winget and Fontaine (1982). In particular, recent studies of the *g*-mode pulsation properties of DA white dwarfs have suggested that hydrogen partial ionization is the destabilizing mechanism responsible for the ZZ Ceti variability (Winget 1981; Dolez and Vauclair 1981; Winget *et al.* 1982*b*; Starrfield *et al.* 1982). At the time of these studies, the only known examples of single pulsating white dwarfs were stars of spectral type DA, i.e., with spectra showing only hydrogen absorption lines. The theoretical analyses of Winget (1981) and Winget *et al.* (1982*b*), however, predicted that the DB white dwarfs should also exhibit pulsational instabilities in the nonradial *g*-modes. This prediction has been confirmed by the discovery of the first pulsating DB white dwarf, GD 358 (Winget *et al.* 1982*a*). In order to interpret this observational result and to enhance the prospects for finding additional pulsating helium white dwarfs, we have undertaken a detailed numerical survey of the pulsation properties of model DB white dwarfs. The preliminary results of this survey are reported here.

In § II we briefly discuss the equilibrium models. In § III we present a summary of the results of the pulsation analysis for sequences of 0.6 $M_\odot$ models, emphasizing the location of the blue and red edges of the theoretical instability strips. The implications of our results are discussed in § IV.

II. METHODS AND MODELS

Our equilibrium models are taken from evolutionary sequences of compositionally stratified DB white dwarf models constructed by Tassoul, Fontaine, and Winget (1983). A complete discussion of the models is presented in that paper, and some discussion is also given in Winget and Fontaine (1982). The compositional structure of the models consists of a layer of helium overlying a pure carbon core, with the composition transition zone computed using the diffusion equilibrium approach of Arcoragi and Fontaine (1980). The mass of the surface helium layer, M_{He}, is a free parameter, the range of which is restricted by the constraint that $\log M_{He}/M_\star < -2$; a more massive helium layer would be consumed by nuclear burning during the prior evolution of the white dwarf. For this preliminary analysis, we have examined sequences with surface helium layer masses given by $\log M_{He}/M_\star = -6$ and -4, and we have assumed a total stellar mass of 0.6 $M_\odot$ for our models. However, in view of the uncertainties in the mean mass of the DB white dwarfs (cf. Liebert 1980; Koester, Schulz, and Wegner 1981), we have extrapolated our results to models with masses of 0.4 $M_\odot$ and 0.9 $M_\odot$. The pulsation equations, and the numerical techniques used for their solution, have been described elsewhere (Winget *et al.* 1982*b*; Winget 1981; Tassoul, Fontaine, and Winget 1983).

The development of pulsational instabilities in our models is governed by the location and properties of the helium partial ionization zone. These properties are affected by uncertainties in the opacities, equation of state, and the description of convective energy transport. Our previous investigations of white dwarf pulsations have demonstrated the insensitivity to uncertainties in the opacities (Winget 1981; Winget *et al.* 1982*b*). Fontaine *et al.* (1974) have shown that, in the temperature range of the models we have examined, the effects of uncertainties in the convective energy transport are much more important than the uncertainties in the equation of state. In particular, the partial ionization zones are contained within the region where the free energy minimization techniques are both adequate and reliable (low-density regime). Thus uncertainties arising from interpolation and the treatment of pressure ionization are not important for these models. Therefore, the main uncertainty affecting our result is the assumed convective efficiency which calibrates the phenomenological treatment of convection we have employed, the mixing length theory. In order to investigate the uncertainties arising from the assumed convective efficiency, evolutionary sequences have been computed using both the standard Böhm-Vitense (1958) description (hereafter the "ML1" sequences), as well as the Böhm and Cassinelli (1971) description which assumes more efficient convection (hereafter the "ML3" sequences). In the latter description we also take the ratio of the mixing length to the pressure scale height to be 2.0 to enhance further the assumed convective efficiency.

We have computed the eigenvalues and eigenfunctions for nonradial *g*-mode oscillations with spherical harmonic indices $l = 1$, 2, and 3. For the purpose of these preliminary computations, we have ignored the Lagrangian perturbation of the divergence of the convective flux. The convective turnover time scales are significantly shorter than the relevant pulsation time scales for our models; we believe this implies that the convection pulsation interaction is weak, and we are currently investigating this question.

In order to identify the relative contributions of different regions in the models to the overall stability, or

TABLE 1

EQUILIBRIUM DB WHITE DWARF MODELS WITH PULSATION PROPERTIES

Sequence (1)	Model (2)	$T_e (10^3$ K) (3)	$\log L/L_\odot$ (4)	Short ${}^l\Pi_k$ (s) (5)	τ_e (yr) (6)	Long ${}^l\Pi_k$ (s) (7)	τ_e (yr) (8)	Max ${}^l\Pi_k$ (s) (9)	τ_e (yr) (10)
ML1:									
M60400.......	200	20.1	-1.62	...	...	...	...	...	...
	204	19.1	-1.71	...	...	...	...	...	...
	208	18.4	-1.78	${}^3108_3$	9×10^3	${}^1895_{18}$	8×10^{-1}	${}^3504_{24}$	7×10^{-2}
	224	16.9	-1.93	${}^3158_5$	5×10^3	${}^1652_{12}$	4×10^2	${}^2484_{15}$	7×10^1
	233	16.0	-2.03	${}^3163_5$	4×10^4	${}^1493_8$	1×10^4	${}^1493_8$	1×10^4
	236	15.6	-2.08	...	...	...	...	...	...
ML1:									
M60600.......	216	19.2	-1.70	...	...	...	...	...	...
	220	18.5	-1.77	${}^379_1$	7×10^5	${}^1727_{13}$	1×10^0	${}^3386_{17}$	2×10^{-2}
	228	17.6	-1.85	${}^382_1$	1×10^6	${}^11004_{18}$	2×10^0	${}^3405_{17}$	1×10^0
	236	17.0	-1.92	${}^3169_5$	2×10^3	${}^1610_{10}$	5×10^2	${}^1518_8$	1×10^2
	244	16.3	-2.00	...	...	...	...	...	...
ML3:									
M60600.......	152	30.0	-0.88	...	...	...	...	...	...
	156	28.6	-0.98	${}^378_2$	5×10^4	${}^3108_4$	3×10^3	${}^3108_4$	3×10^3
	160	27.1	-1.08	${}^2129_3$	1×10^4	${}^3303_{16}$	5×10^{-1}	${}^3291_{15}$	2×10^{-2}
	164	25.9	-1.16	...	...	...	...	...	...

instability, of a given mode, we have also computed the running integral giving the stability coefficient, κ, as described by Winget *et al.* (1982*b*).

We have examined the pulsation properties of models in the temperature range 30,000 K $\gtrsim T_e \gtrsim$ 16,000 K; the physical parameters of these models are listed in Table 1. We refer to these models using the notation originally developed for our ZZ Ceti calculations. Thus M60400 indicates a model from a sequence with a total stellar mass $M_\star = 0.6\ M_\odot$ (the first digit), and $\log M_{\rm He}/M_\star = -04$ (the second and third digits), and *zero* hydrogen content (the last two digits).

III. RESULTS OF THE PULSATION ANALYSIS

In addition to listing the physical parameters of the models, Table 1 also gives the periods and *e*-folding times for the shortest, the longest, and the most unstable modes. If there are no entries in columns (5)–(10), the model is stable. All of the unstable modes we have found are excited by driving from the surface He partial ionization zone. The most unstable modes have *e*-folding times of the order of 10^{-2} years—significantly shorter than the evolutionary time through the instability strip: $\tau \sim 2 \times 10^7$–1.4×10^8 years. In all cases the instabilities were always found to be confined to a narrow effective temperature range about 3000 K in width.

The locations of the blue and red edges of the instability strip are quite sensitive to the assumed efficiency of convection (see Table 1). For ML1 convection the instability strip is located in the effective temperature range 19,000 K $\gtrsim T_e \gtrsim$ 16,000 K, while for ML3 convection the strip is in the range 29,000 K $\gtrsim T_e \gtrsim$ 26,000 K. The location of these edges, as well as their sensitivity to the convective efficiency, can be understood by considering the local thermal time scale, $\tau_{\rm th}$, of the driving region (the helium partial ionization zone for these stars). As discussed in Winget *et al.* (1982*b*), in order for a given pulsation mode to be driven efficiently, the driving region must be able to respond thermally on the time scale of the pulsation. Thus $\tau_{\rm th} \sim \Pi$ for efficient driving. Consequently, the onset of *g*-mode instabilities occurs when $\tau_{\rm th}$ at the base of the helium partial ionization zone is comparable to the lowest *g*-mode periods, $\Pi \sim 10^2$ s. (Note that, for white dwarfs, as for most other stars, the partial ionization zone is coincident with the convection zone.) This condition is satisfied for 0.6 $M_\odot$, ML1 models when $T_e \sim$ 20,000 K, and for ML3 models when $T_e \sim$ 28,000 K; both of these values are completely consistent with the numerical results indicated in Table 1.

As the star cools below the blue edge, the helium partial ionization zone extends rapidly to greater depths, and its thermal time scale increases correspondingly. In addition, convection carries essentially all of the energy flux in the partial ionization zone for the cooler models. Because we allow for perturbations in the radiative flux only, the presence of efficient convection artificially removes a large part of the potential driving zone in our calculations. This results in longer growth times for the unstable modes in these models. This effect is not physical but reflects our inability to treat convection-pulsation interactions properly. As the star cools still further, the thermal time scale throughout the driving region (the inner half of the partial ionization zone; cf. Cox and Giuli 1968) becomes orders of magnitude larger than the periods of the low-order *g*-modes. Thus the driving zone is moved to the "adiabatic region," so that the models are stable, and a red edge is produced. This effect is not sensitive to the treatment of convection-pulsation interactions.

Since the mean mass of the DB white dwarfs is uncertain, we have computed a grid of He envelopes for models with total stellar masses of 0.406, 0.612, and 0.885 $M_\odot$. The temperature of the blue edge of the *g*-mode instability strip can be estimated for these envelopes by determining the effective temperature at which the thermal time scale at the base of the He partial ionization zone exceeds 10^2 s. For 0.406, 0.612, and 0.885 $M_\odot$ models with ML1 convection, we find respective blue edge temperatures of 19,000, 20,000, and 21,000 K. For the same masses and ML3 convection, we find blue edge temperatures of 27,000, 28,000, and 29,000 K. Note that the estimates obtained in this way for the 0.612 $M_\odot$ envelope models agree very well with the results of the detailed pulsation analyses of the 0.6 $M_\odot$ models, thus lending confidence in the estimates for the other stellar masses. These results show that the location of the instability strip is much less sensitive to the total stellar mass than to the assumed convective efficiency.

IV. DISCUSSION

The preliminary calculations reported here confirm and extend the earlier analyses of Winget (1981) and Winget *et al.* (1982*b*). The following conclusions can be drawn from our results:

1. Driving from the surface He partial ionization zone produces a nonradial *g*-mode pulsational instability strip in the theoretical DB evolutionary sequences. The unstable modes have periods in the range $10^2\ {\rm s} \lesssim \Pi \lesssim 10^3$ s.

2. The blue edge of the instability strip is between 29,000 K and 19,000 K, depending on the assumed convective efficiency.

3. The effective temperature of the blue edge is insensitive to the total mass of the model and to the mass of the surface helium layer, at least for the sequences examined thus far.

4. The width of the instability strip is approximately 3000 K.

5. Mode trapping, such as occurs in models of the ZZ Ceti stars (cf. Winget and Fontaine 1982), does not significantly affect the magnitudes of the growth rates

for the models with relatively thick surface helium layer masses we have considered here.

A comparison of our detailed numerical results with the periods, spectral type, and effective temperature of the pulsating variable star GD 358 (cf. Winget *et al.* 1982*a*) strongly supports the identification of that star with the nonradial *g*-mode DB instability strip discussed here. Further, the insensitivity of our results to both the surface helium layer mass and the total stellar mass indicates that most and possibly all DB white dwarfs with effective temperatures in the instability strip may pulsate. We can estimate the fraction of the total DB white dwarf population that may be expected to pulsate by taking the ratio of the time required for a DB model to evolve through the instability strip to the total evolutionary time over which the model could be identified as a DB. If we assume a mass of 0.6 $M_\odot$ and take the DB lifetime as the time to cool to about 13,000 K (below this temperature the star would be identified as a DC), we obtain the ratio of nonvariables to variables as about 4 to 1 for ML1 convection and 25 to 1 for ML3 convection.

The sensitivity of the blue edge temperature to the assumed convective efficiency suggests that an observational determination of the location of the instability strip may be extremely useful in calibrating the assumed convective efficiency appropriate for use with the mixing-length description of white dwarf convection. We note that a preliminary comparison that we have made of the available *International Ultraviolet Explorer* (*IUE*) data on GD 358[1] with the hot, high-gravity helium model atmospheres of Wesemael (1981) indicates an effective temperature near 30,000 K—suggesting efficient convection. This is in good agreement with similar results for the ZZ Ceti stars. This work demonstrates that, in order to discover additional pulsating DB white dwarfs in an efficient way, and also in order to interpret the one already discovered, it is imperative to establish a consistent and reliable temperature scale for the DB white dwarfs.

We are grateful to H. Saio for numerous useful discussions and for providing an opacity interpolation subroutine used in our computations. We would also like to thank E. L. Robinson, R. E. Nather, S. O. Kepler, and M. L. Aizenman for their strong encouragement and many valuable discussions. We would also like to thank the referee for suggesting important improvements in our discussion of the uncertainties in the properties of the equilibrium models. This research has been supported in part by the National Science Foundation under grants AST 8108691 through the University of Texas, grant AST 8111460 through the University of Rochester, and grant AST 8117486 through the University of Colorado, and in part by the Natural Sciences and Engineering Research Council of Canada.

[1] *IUE* spectra LWR 10668 and SWP 14015; observed by G. Vauclair.

REFERENCES

Arcoragi, J. P., and Fontaine, G. 1980, *Ap. J.*, **242**, 1208.
Böhm, K. H., and Cassinelli, J. 1971, *Astr. Ap.*, **12**, 21.
Böhm-Vitense, E. 1958, *Zs. f. Ap.*, **46**, 108.
Cox, J. P., and Giuli, R. T. 1968, *Principles of Stellar Structure* (New York: Gordon & Breach).
Dolez, N., and Vauclair, G. 1981, *Astr. Ap.*, **102**, 375.
Fontaine, G., Van Horn, H. M., Böhm, K.-H., and Grenfell, T. C. 1974, *Ap. J.*, **193**, 205.
Koester, D., Schulz, H., and Wegner, G. A. 1981, *Astr. Ap.*, **102**, 331.
Liebert, J. 1980, *Ann. Rev. Astr. Ap.*, **18**, 363.
Starrfield, S., Cox, A. N., Hodson, S., Pesnell, W. D. 1982, in *Pulsations in Classical and Cataclysmic Variable Stars* (Boulder: University of Colorado, Joint Institute for Laboratory Astrophysics), p. 78.
Tassoul, M., Fontaine, G., and Winget, D. E. 1983, in preparation.
Wesemael, F. 1981, *Ap. J. Suppl.*, **45**, 177.
Winget, D. E. 1981, Ph.D. thesis, University of Rochester.
Winget, D. E., and Fontaine, G. 1982, in *Pulsations in Classical and Cataclysmic Variable Stars* (Boulder: University of Colorado, Joint Institute for Laboratory Astrophysics), p. 46.
Winget, D. E., Robinson, E. L., Nather, R. N., and Fontaine, G. 1982*a*, *Ap. J.* (*Letters*), **262**, L11.
Winget, D. E., Van Horn, H. M., Tassoul, M., Hansen, C. J., Fontaine, G., and Carroll, B. W. 1982*b*, *Ap. J.* (*Letters*), **252**, L65.

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