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APPROXIMATIONS TO THE RADII OF ROCHE LOBES

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ABSTRACT

Effective radii of Roche lobes were computed and are compared with the results of Kopal and of Pacyński. A convenient approximation is given, whose derivative is smooth and fairly accurate.
Subject heading: stars: binaries

When the effective radius r_L of a Roche lobe is required Pacyński's approximation (Pacyński 1971) is commonly used:

$$r_L = \max\left[0.46224\left(\frac{q}{1+q}\right)^{1/3},\quad 0.38 + 0.2\log_{10} q\right],\qquad 0 < q < 20\,, \quad (1)$$

where q is the mass ratio and the separation of the two stellar centers is unity. This formula agrees with the tabulation of Kopal (1959) to within $\sim 2\%$. However, its derivative is discontinuous by $\sim 20\%$ at $q \approx 0.523$, where the two arguments in equation (1) are equal, and this can lead to awkward nonsmoothness when following numerically the evolution of a semidetached or contact system near this mass ratio.

Using a rather straightforward but accurate integration procedure, I evaluated Roche lobe volumes and hence effective radii to five or more significant figures. These are approximated, to better than 1% over the whole range, by

$$r_L = \frac{0.49q^{2/3}}{0.6q^{2/3} + \ln(1+q^{1/3})}\,, \quad 0 < q < \infty\,. \quad (2)$$

This function has a continuous derivative, and because the maximum errors are not concentrated to a small range of q, as they are in equation (1), the derivative can also be expected to be accurate to $\sim 1\%$. Table 1 compares r_L obtained by my integrations, by Kopal (1959), and by approximations (1) and (2).

If a star just fills its Roche lobe, its mean density ρ (g cm^{-3}) is related to the orbital period P (days) by

$$P(\rho)^{1/2} = 0.1375\left(\frac{q}{1+q}\right)^{1/2} r_L^{-3/2}\,, \quad (3)$$

where r_L is still in units of the orbital separation. Hence, $P(\rho)^{1/2}$ is a function of q only, which we can approximate via equations (1) or (2); this is also given in Table 1. Over the "useful" range $0.01 \lesssim q \lesssim 10$, $P(\rho)^{1/2}$ varies by less than 20% from the value 0.37, and over the more

TABLE 1

VOLUME RADII OF ROCHE LOBES

q	r_L[a]	r_L[b]	r_L[c]	$P(\rho)^{1/2}$ [c]	r_L[d]	$P(\rho)^{1/2}$ [d]
∞	0.8149	0.8149	...	...	0.8167	0.1863
1000	0.7817	0.764	...	...	0.7853	0.1975
100	0.7182	0.7047	...	...	0.7203	0.2238
10	0.5803	0.5776	0.580	0.297	0.5782	0.2982
2.5	0.4621	0.4606	0.460	0.373	0.4599	0.3726
1	0.3799	0.3785	0.380	0.415	0.3789	0.4168
0.4	0.3026	0.3012	0.304	0.438	0.3031	0.4405
0.1	0.2054	0.2041	0.208	0.438	0.2068	0.4409
0.01	0.1012	0.0988	0.099	0.438	0.1020	0.4199
0.001	0.0482	0.0461	0.046	0.438	0.0484	0.4086
0	0	0	0	0.438	0	0.4009

[a] Integration; this paper.
[b] Kopal 1959.
[c] Pacyński 1971, i.e., eqs. (1) and (3).
[d] This paper, eqs. (2) and (3).

restricted range of $0.01 \lesssim q \lesssim 1$, of particular interest in cataclysmic binaries, $P(\rho)^{1/2}$ varies by less than 3% from the value 0.43.

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