THE ASTROPHYSICAL JOURNAL, **268**:246–249, 1983 May 1

DIFFERENTIAL ROTATION IN F STARS: A COMPARISON BETWEEN THEORY AND OBSERVATION

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Received 1982 May 24; accepted 1982 October 26

ABSTRACT

Gray recently observed seven stars on or slightly above the main sequence in the spectral region between F2 and F8, looking at the Fourier transforms of their line profiles. His observations show no or very small differential rotation in the examined stars, contradicting the theoretical predictions of Belvedere *et al.* However, in the spectral range examined by Gray, only an F5 V model was computed, adopting for it the standard equatorial velocity of 30 km s^{-1}, thus guessing the differential rotation behavior of main-sequence stars between F5 and G2 (Sun) by simply connecting these two models with a convenient curve.

Here we compute models for each star Gray observed, using the same differential rotation computer code as in Belvedere *et al.* and the observed rotational velocities given by Gray. The results indicate that the computed theoretical models are compatible with Gray's observations, but at the same time they sensibly differ from the guessed curve given by Belvedere *et al.* for main-sequence stars in the range F5–G2. The conclusions seem now in favor of a possible differential rotation minimum around F7 for main-sequence stars in the range F5–M0, contrary to the previous deductions of a minimum in G5.

These new results are explained and discussed on the basis of interaction of rotation with convection and also in the framework of the very recent observations by Wöhl.

Subject headings: convection — stars: interiors — stars: rotation

I. INTRODUCTION

Recently, seven F stars on or just above the main sequence have been investigated for differential rotation by Gray (1982) by looking at the Fourier transforms of their line profiles. The method for detecting differential rotation, the discussion on the effects of the macroturbulence, the criteria followed for the selection of the observed stars, and the details of the instrumentation and data reduction can be found in Gray's paper.

What we want to discuss here deals with the implications of Gray's analysis on the models of stellar differential rotation by Belvedere, Paternò, and Stix (1980, hereafter BPS). In the introduction to his paper, Gray states:

> There is also good theoretical reason to look in the F star region of the main sequence. Models calculated by Belvedere, Paternò, and Stix . . . predict a maximum differential rotation for F stars and a minimum at G5. Therefore we look to the F stars for our candidates of study.
>
> Unfortunately, the results are negative, and no differential rotation can be seen in the F stars.

We point out that the results summarized in Figure 5 of BPS can barely be compared with Gray's observations because the BPS curve, describing the differential rotation in the spectral region F5–G2 (Sun), was obtained simply connecting in a convenient way the points corresponding to the only computed model (F5 V) and to the Sun's model, which was used as calibration point (see BPS, § 2).

Moreover, BPS is only concerned with main-sequence stars, while among the seven stars studied by Gray, only three are on the main sequence, two of which (F7) in the spectral region considered in BPS.

Therefore, to adequately compare the theory on which the BPS models are based with observations, we compute models of differential rotation concerning the spectral type and luminosity class stars observed by Gray using the same BPS computer code and the rotational velocities given by Gray.

The results indicate that the computed strength of differential rotation for Gray's stars is compatible with the observations of no or very small differential rotation within the various uncertainties, concerned in particular with macroturbulence, which Gray carefully points out in his paper. However, the newly calculated points differ sensibly from the hypothesized curve given in BPS, indicating that a differential rotation minimum around F7, for main-sequence stars in the spectral region F5–M0, may be plausible.

In the following we shall discuss these new results in the framework of the same theory on which the BPS models are based, namely the interaction of rotation with convection, and show that there is no contradiction between the results of the computed BPS models and the present results of Gray's star models.

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II. THEORETICAL MODELS AND RESULTS

Basic assumptions and methods of computation are the same as those adopted and described in BPS. We recall that we assume for stellar models the same rotation-convection interaction coupling constant ϵ which reproduces the Sun's observed differential rotation. This interaction is described by the factor $f(r) = \omega l^2/\nu$ that determines the strength of the differential rotation in the models, where r is the radial distance from the star's center, ω the angular velocity of the star, l the scale height of convective motions and ν the turbulent kinematic viscosity. The factor $f(r)$ represents essentially the ratio of Coriolis frequency to the growth rate of convective motions, and it is especially important at the bottom of the convection zone. In the BPS parameterization, $f(r)$ appears in the perturbed convective transport coefficient $k(r, \theta)$, where θ is the polar angle, which represents the driving force of the large scale meridional circulation which in turn generates the differential rotation. However, this latter depends on $f(r)$ in a no simple way, because of the combined effect of the driving terms in the equations, which include $k(r, \theta)$ and its first derivative (for details see Belvedere and Paternò 1977). Nevertheless, we do expect that the larger $f(r)$ at the bottom of the convection zone, the stronger is the influence of Coriolis force on convective motions and therefore the stronger is the differential rotation. Since at the bottom of the convection zone $l \approx d$, where d is the depth of the convection zone, we do expect that stars with stronger differential rotation have also larger ratios $F = \omega d^2/\nu$. The dimensionless factor F can then be considered as a measure of differential rotation, even if we do not expect they are linearly dependent.

All computations reported here have been carried out with a turbulent Prandtl number $\sigma = \nu/\nu_k = 0.01$, where ν_k is the turbulent diffusion coefficient for heat. As explained in § 3 of BPS, ν_k is determined from the structure parameters of the stellar models and thus is fixed for each spectral type. The free parameter is σ which therefore determines ν. The choice of $\sigma = 0.01$ has been explained in § 4 of BPS, and, on the other hand, its value affects only slightly the differential rotation especially in the vicinity of G0.

Table 1 shows the values of the structure and rotation parameters adopted for modeling the observed stars, the corresponding theoretical differential rotation rates expressed by the parameter $\alpha = \omega_2/(\omega_0 + \omega_2)$, where ω_0 represents the polar angular velocity and $\omega_0 + \omega_2$ the equatorial angular velocity as in Gray's formulation, and the dimensionless ratio F. For comparison, two stellar models computed in the BPS paper are also shown, namely F5 V and G5 V, which show, respectively, the maximum and the minimum value of α in the BPS model sequence.

Note that the parameter $\alpha = (\Omega_e - \Omega_p)/\Omega_e$, where Ω_e and Ω_p are, respectively, the surface equatorial and polar angular velocities, is related to the parameter $(\Omega_e - \Omega_p)/\Omega_0$ expressing the strength of the differential rotation in BPS models, where Ω_0 is a reference angular velocity. This latter can be expressed in terms of Ω_e through the relationship $\Omega_0 = \Omega_e/[1 - \epsilon\omega_2(R_s)/2]$, where the quantity $\omega_2(R_s)$, with R_s the star's radius, is the result of the computations, and Ω_e is assumed to be the same as the observed angular velocity of the star ω (see BPS, § 3, for further details). Therefore we obtain $(\Omega_e - \Omega_p)/\Omega_0 = \alpha[1 - \epsilon\omega_2(R_s)/2]$, which means that α is smaller than $(\Omega_e - \Omega_p)/\Omega_0$ since $\epsilon\omega_2(R_s)$ is a negative quantity.

Of course, the structure parameters listed in Table 1 are all suitable values adopted according to the present state of knowledge (we refer to BPS for the bibliography), except the ω values, computed directly from the $V \sin i$ values given by Gray, including the statistical factor $4/\pi$.

As one can see from Table 1, it seems that the theoretically calculated α-values show a clear evidence of very low or no differential rotation in the observed stars, in reasonable agreement with Gray's results. We have to recall that some Gray's α-values substantially different from zero are to be considered as very uncertain upper limits derived under unrealistic assumptions (see Gray 1982, §§ V*b* and V*c*). The conclusions

TABLE 1

STRUCTURE AND ROTATION PARAMETERS FOR EACH STELLAR MODEL AND CORRESPONDING THEORETICAL α VALUES

Star	Spectrum	R_s	$V \sin i\ (4/\pi)$	ω	g_s	P_s	T_s	$(\Delta\nabla T)_s$	ν_k	D	α	F
ξ Oph	F2 V	9.0(+8)	26.2	2.9(−5)	2.5(+2)	3.9(+3)	7.4(+3)	8.0(−3)	3.9(+11)	0.04	0.000	9.6(+0)
α CMi	F5 IV–V	1.2(+9)	3.6	3.0(−6)	1.3(+2)	5.1(+3)	7.1(+3)	5.8(−3)	2.2(+11)	0.09	0.015	1.6(+1)
θ UMa	F6 IV	1.5(+9)	8.1	5.4(−6)	9.6(+1)	4.5(+3)	7.0(+3)	1.5(−3)	9.5(+11)	0.09	0.001	1.0(+1)
γ Ser	F6 IV–V	1.1(+9)	13.7	1.2(−5)	1.6(+2)	5.5(+3)	7.0(+3)	5.7(−3)	2.1(+11)	0.11	0.024	8.3(+1)
θ Boo	F7 V	7.7(+8)	36.3	4.7(−5)	2.6(+2)	1.2(+4)	6.6(+3)	1.0(−3)	3.8(+10)	0.12	0.082	1.1(+3)
τ Boo	F7 V	7.7(+8)	18.8	2.4(−5)	2.6(+2)	1.2(+4)	6.6(+3)	1.0(−3)	3.8(+10)	0.12	0.042	5.4(+2)
θ Dra	F8 IV–V	1.7(+9)	35.4	2.1(−5)	7.4(+1)	2.7(+3)	6.4(+3)	7.0(−4)	1.2(+12)	0.12	0.021	7.5(+1)
BPS	F5 V	8.4(+8)	38.0	3.6(−5)	2.4(+2)	9.0(+3)	7.1(+3)	5.9(−3)	1.2(+10)	0.10	0.849	2.1(+3)
BPS	G5 V	6.5(+8)	1.9	2.3(−6)	2.9(+2)	1.2(+4)	6.0(+3)	3.4(−3)	6.9(+9)	0.36	0.169	1.8(+3)

NOTE.—R_s is the stellar radius; $V \sin i$ is taken from Gray 1982, and ω is the corresponding angular velocity; g_s, P_s, T_s, and $(\Delta\nabla T)_s$ are, respectively, the surface gravity, pressure, temperature, and superadiabatic temperature gradient; ν_k is the turbulent diffusivity for heat (note that the turbulent kinematic viscosity is $\nu = \sigma\nu_k$, where the adopted Prandtl number is $\sigma = 0.01$); D is the depth of the convection zone in units of the stellar radius; $F = \omega d^2/\nu$ is the factor that parameterizes the strength of differential rotation, with d the depth of the convection zone. Units are MKS, except the values of $V \sin i$ which are given in km s^{-1}. For comparison, the BPS standard F5 V and G5 V, which have, respectively, a maximum and a minimum of α in the BPS spectral type sequence, are also shown.

of Gray's analysis are clearly in favor of rigid rotation, so that we do not report here Gray's α-values, referring the reader to Gray's Table 3.

In Figure 1 we compare the computed values of the parameter α for Gray's stars with BPS models. Gray's stars are indicated by circles (main-sequence stars) and squares (above–main-sequence stars), while dotted circles indicate BPS models computed in the spectral region F5–K0. Note that in the present work the differential rotation of BPS models is expressed in terms of the parameter α, different from Figure 5 of BPS, as explained above. The dashed curve, reproducing the BPS continuous curve in the spectral range F5–K0, represents a convenient curve which estimates the behavior of differential rotation in the region where intermediate models were not computed.

As is evident from Figure 1, there is a noticeable discrepancy between the BPS predictions in the spectral range F5–G0 and the present results. This is especially evident for the F7 V stars τ Boo and θ Boo which have rotational velocities and depth of the convection zone comparable to those adopted for modeling the BPS F5 V standard star.

III. DISCUSSION

The discrepancy of the present results when compared with those obtained in BPS paper can be explained looking at the combined effect of the depth of the convection zone, the angular velocity of rotation, and the turbulent viscosity, which are the basic ingredients of the BPS parameterization of the interaction of rotation with convection described by the dimensionless factor F.

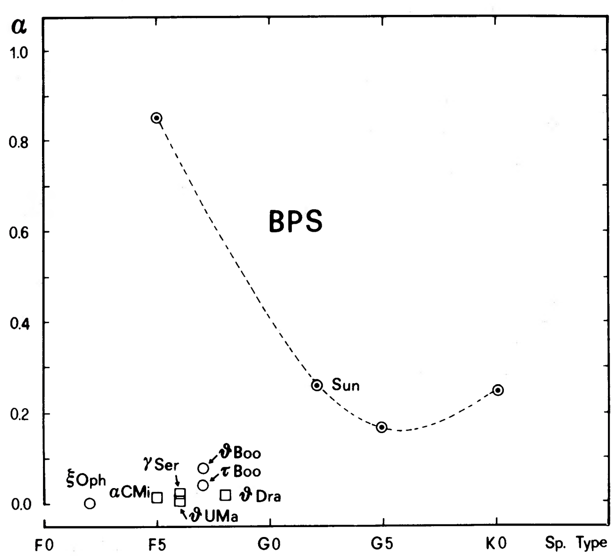

FIG. 1.—The computed strength of differential rotation α is plotted vs. spectral type for the seven stars observed by Gray. Circles and squares indicate main-sequence and above–main-sequence stars, respectively. The differential rotation of the main-sequence standard star BPS models, in the spectral region F5–K0, is also shown by dotted circles, together with the hypothesized curve describing the behavior of differential rotation in the regions where intermediate models were not computed.

For instance, ξ Oph has a very shallow convection zone and thus cannot develop differential rotation. θ UMa and α CMi have rotational velocities remarkably lower than the average and radii larger than those characteristic of the main sequence, so that the angular velocities which result are low, and consequently also their differential rotation is small. θ Dra and γ Ser have angular velocities high and convection zones deep enough to develop an appreciable differential rotation, but the competitive role of v is sufficient for reducing it to some 2%. This role is more evident in the case of τ Boo and θ Boo which have angular velocities and depths of the convection zone comparable to those that in a F5 spectral type produce large differential rotation (see BPS, Table 3 and Fig. 5). For main-sequence stars, the change of the surface structure parameters from F5 to F7 produces a significant change in the surface super-adiabatic gradient, thus causing an increase of v_k, through relationship (4) of BPS, which in turn determines an increase of v. This limits differential rotation to values between 4% and 8%. These values are still acceptable considering the uncertainties of Gray's measurements. For τ Boo and θ Boo, F values are, respectively, about 5×10^2 and 10^3, smaller than the F value of the BPS F5 V star which is 2.1×10^3, and even smaller than the BPS G5 V star which has $F = 1.8 \times 10^3$ and $\alpha = 0.17$. All other stars here considered, which have Fs ranging from 10 to 10^2, have much smaller α's ranging from zero to 0.024. From Table 1 it appears indeed that α is rather low and slightly sensible to F for relatively low F-values, while it increases rapidly for relatively high F-values ($F \gtrsim 1.5 \times 10^3$).

If one accepts that the factor F is a measure of the strength of differential rotation (and the detailed calculations shown in Table 1 seem to indicate so), thus the discrepancy of the results shown in Figure 1 can easily be understood and accounted for in the framework of the same BPS theory. The very reason of this discrepancy is that in BPS we did not compute any single model in the spectral region F5–G2 (Sun), thus inferring a differential rotation profile as a function of spectral type which turned out to be wrong in the light of the present results. These results indicate that a relative minimum of differential rotation for main sequence stars lies in the F7–F8 region, while the differential rotation drops to zero in the region F5–F2 due to the decrease of the thickness of the convection zone, and it has a maximum at F5 for fast rotating stars.

As far as the stars above the main sequence are concerned, we can have some indication about their differential rotation profile as a function of spectral type if we combine the present results with those recently obtained by Belvedere, Chiuderi, and Paternò (1982), concerning the luminosity class III stars of spectral type later than G0. These combined results indicate that there is a differential rotation maximum around G0, and that differential rotation drops to zero for earlier and later spectral types. In the first case this is due to the combined effect of the depth of the convection zone and turbulent kinematic viscosity; in the second case this essentially depends on the increase of the star's radius which reflects on the angular velocity decrease.

A word of caution about the meaning of differential rotation profiles as function of spectral type is needed. In fact the factor F, whose value is a convenient measure of the strength of differential rotation, is the product of the two quantities d^2/v and ω. The first depends only on the structure parameters of the star; therefore, it would give rise to fixed differential rotation profiles as function of spectral type for each luminosity class, if all the stars were to rotate at the same ω. This factor is essentially responsible for the relative minimum in F7 region for main-sequence stars, even if one considers rotators as fast as the standard F5 of BPS. The second is a peculiar characteristic of the single stars; therefore it is meaningless to speak about differential rotation profiles as functions of spectral type unless one specifies some average ω for each spectral type. Observations show in fact a large spread of ω's within the same spectral type.

At this point it would be extremely interesting to test the BPS theory looking at F5 main-sequence stars having rotational velocities close to 40 km s^{-1}, which would develop very high rates of differential rotation. As already explained, F7 V stars have a smaller d^2/v, even if their spectral type is close to F5 V and their rotation rates are about the same as to F5 V type. Thus they cannot develop a significant differential rotation. The same argument applies to F5 stars above the main sequency whose angular velocities are comparable with those of a F5 V spectral type. This happens because the theory is extremely sensitive to the factor F especially for large Fs. To stress this point, it is sufficient to compare the F5 V with the G5 V BPS models as given in Table 1. The F ratio of F5 to G5 is in fact only 1.16, while the α ratio is about 5.

Last but not certainly least for the importance they can have for the present discussion, we should quote the very recent observations of differential rotation carried out by Wöhl (1982). Among the stars observed by Wöhl, there is in fact a fast rotating F5 V star. The very preliminary results of Wöhl seem to indicate an upper limit of $\alpha = 0.4$ for this star. Should this preliminary result be confirmed by a deeper analysis of the data and possibly by further observations of fast rotating F5 V stars, we have to reconsider the BPS theory at least for the effects produced by fast rotation. As mentioned in § 4 of BPS, the assumption that ϵ is a constant could be no longer valid at high rotation rates. This could reflect into a weakening of the rotation–convection interaction, thus causing a decrease of differential rotation.

IV. CONCLUSIONS

We have shown that the discrepancy between Gray's observations and BPS predictions of differential rotation, for main-sequence stars in the spectral region F5–G2 (Sun), is only apparent. When we compute, using the same BPS theory, appropriate models which describe the observed stars, we find out that the differential rotation around F7 V spectral type is very low, and it is compatible with Gray's measurements. These new results indicate that a minimum of differential rotation in the range F5 V to M0 V is located around F7 spectral type instead of G5 as previously stated in BPS.

However, the preliminary results of very recent observations of differential rotation carried out by Wöhl indicate an upper limit of 40% differential rotation for a F5 V star rotating at the same angular velocity as the standard F5 V star considered in the BPS model sequence which has about 85% differential rotation. The preliminary results of this observation could really contradict the conclusions of BPS theory concerning with fast rotating stars, and a possible explanation for that is given in § III.

However, until further observations will confirm the preliminary results of Wöhl, we think that the present differential rotation observations can still be explained in the framework of the BPS theory of the interaction of rotation with convection.

We are most grateful to Professor David F. Gray for letting us see his paper prior to publication and for useful comments.

REFERENCES

Belvedere, G., Chiuderi, C., and Paternò, L. 1982, *Astr. Ap.*, **105**, 133.
Belvedere, G., and Paternò, L. 1977, *Solar Phys.*, **54**, 289.
Belvedere, G., Paternò, L., and Stix, M. 1980, *Astr. Ap.*, **88**, 240 (BPS).
Gray, D. F. 1982, *Ap. J.*, **258**, 201.
Wöhl, H. 1983, in *IAU Symposium 102, Solar and Stellar Magnetic Fields*, ed. J. O. Stenflo (Dordrecht: Reidel), in press.

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