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THE EFFECTS OF MASS AND METALLICITY UPON PLANETARY NEBULA FORMATION

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ABSTRACT

We construct a parameterized function which describes the possible dependence of planetary nebula formation upon metal abundance and stellar mass. Data on galaxies in the Local Group compared with predictions made from the parameterized function indicate that heavy element abundance is the principal agent influencing the formation of planetary nebulae; stars which are rich in heavy elements are the progenitors of planetary nebulae. Our analysis, when compared with the observations, argues for a modest degree of pre-enrichment in a few of the sample galaxies. The heavy element dependence of planetary nebula formation also accounts for the deficit of planetary nebulae in the nuclei of NGC 221 and NGC 224, and in the bulge of our Galaxy.

Subject headings: nebulae: abundances — nebulae: planetary

I. INTRODUCTION

Our understanding of the formation of planetary nebulae would be considerably enhanced if it was possible to determine those stellar properties which affect nebula formation and evolution. A great deal of effort has recently gone into measuring elemental abundances in planetary nebulae (PN) (Kaler 1980*a*, *b*, *c*; Dinerstein 1980; Barker 1978*a*, *b*, *c*, 1980; Peimbert 1973; Shields 1978) with the results sometimes supporting metallicity effects in PN formation and other times not. On the theoretic side, PN formation models (Barkat and Tuchman 1980; Tuchman, Sack, and Barkat 1979; Kwok, Purton, and FitzGerald 1978) are ad hoc with reference to stellar masses only to provide the right sort of stars for PN formation at the end of stellar evolution. In this paper, we make a first step toward determining the connection between stellar properties and PN formation.

The approach we take is to consider PN as statistical phenomena within a galaxy. To select isolated stars and anticipate their evolution into PN is beyond our capabilities. But, within a given group of stars, we might expect a certain frequency of PN. Relating the mean properties of the group to frequency of formation then tells us which stars produce PN. The prescription we adopt to determine the expected frequency of PN, within a given group of stars, depends upon three parameters of the group: its age, its rate of star formation, and its average metallicity. While our description does have some dependence upon the model of galaxy evolution adopted, it is independent of an explicit model of PN formation. The results obtained by our approach should provide useful information on the mechanism of PN formation and may restrict the class of viable models.

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Having obtained an expression for the relative number of PN within a given group of stars, we next turn to the observed frequencies of PN in order to get an estimate of how effective stellar properties are in influencing PN formation. The strong distinction of abundances, kinematics, and location between Population I and Population II objects makes our Galaxy an ideal target for investigation. However, the plethora of incidental problems—selection effects, survey incompleteness, confusion in distance determinations, and uncertainties in abundance measurements, to name a few—makes it impossible to utilize our Galaxy alone. A reliable measure of what affects PN formation may be obtained by looking at other galaxies. It is possible to determine with some confidence the numbers of PN in nearby galaxies. These counts, along with the global properties of each external stellar system, provide the necessary data.

The test of our approach is in the results. We are able to distinguish mass from metallicity effects by the unique behavior of each as a function of the properties of stellar systems. Even in the face of simplified models of galaxy evolution and uncertainties in the observed parameters, the signatures of mass or metal dependence or both together are sufficiently distinct to allow a determination of which effect is the more pronounced.

We start by constructing a simple description of PN frequency which incorporates the effects of mass and metallicity. This is done in § II. The first part of § II emphasizes the mass dependency of the description, while the second part is concerned with the construction of the metallicity dependence. In § III we present the observational data and compare these data with the prescription defined in § II. We also point out a few peculiar and unanticipated results which are described by our prescription and are born out by the observations.

Our results and their implications are discussed in § IV, and we make our concluding remarks in § V.

II. RELATIVE NUMBERS OF PLANETARY NEBULAE

One would anticipate from the Vogt-Russel theorem that only a star's mass and composition determine whether the star will form a PN. Since PN are observed in both Population I and Population II distributions, it seems unlikely that mass has a pronounced effect on PN formation except as a general effect: very massive stars from supernovae; very low mass stars have not yet evolved. However, one cannot entirely discount a mass effect in PN formation without probing the issue. Similarly, the role of metal abundance does not appear to be obvious at first glance. Once again, Population I and Population II stars form PN, and the mean abundances of the two stellar populations are quite different.

We will take advantage of the Vogt-Russel theorem and hypothesize that only mass and element abundance (metallicity) can play a role in PN formation. We will also assume that the two effects are separable, i.e., there is a function $\mathscr{N}$ which describes the mass effect and which is, at least to first order, independent of element abundance; likewise, there is a function ϕ which describes the effect of element abundance and which to first order is independent of stellar mass. By mass effect we mean the possible variations of the lower and upper mass limits for stars able to form PN. We will check both these assumptions in § IV. If the number of PN per unit stellar mass at a given epoch is κ, then $\kappa \propto \mathscr{N}\phi$.

The mass effect is best introduced by counting stars which are able to form PN. This we proceed to do.

a) Star Counts

To simplify the calculations and streamline our arguments, we consider an idealized galaxy comprised of a homogeneous assembly of stars and gas. A good measure of the numbers of PN one expects to find in a galaxy is simply the number of stars within the galaxy which could form PN. As such, it is necessary to determine the number $\mathscr{N}$ of stars which, at a time T after galaxy formation, have completed their main-sequence life and have evolved to red giants. We are, therefore, assuming for the present that the number of PN is proportional to the number of red giants. Four factors enter the star count function $\mathscr{N}$: the initial mass function (IMF) $\psi(m)$; the birth rate function (BRF) $F(t)$; and the lower and upper mass limits for stars able to form PN, M_l and M_u, respectively. The last two parameters, especially, incorporate the mass dependence of PN formation.

Instead of determining the total number of stars formed over a galaxy's lifetime, as was the case with Salpeter (1955), we need determine only the number of stars which are now red giants. The implicit assumption in determining the star count function is that the lifetime of a red giant or PN is short compared to the main-sequence lifetime.

The best determination of the IMF in the solar neighborhood (Miller and Scalo 1979) yields an IMF which is not a simple power law over the entire range of stellar masses. We will be concerned with stars with masses ranging between approximately 1 $M_\odot$ to possibly 5 $M_\odot$. The limits of $\psi(m)$ found by Miller and Scalo (1979) over the range 1 $M_\odot$ to 5 $M_\odot$ are consistent with Salpeter's (1955) IMF. We shall therefore adopt Salpeter's (1955) IMF; $\psi(m) \propto m^{-2.35}$.

Variations in the IMF on the scale size of star clusters have been reported (Da Costa 1977; Freeman 1977; Scalo 1978). In addition, deviations from the local IMF have been reported for M104 (van den Bergh 1976; Schweizer 1978) and NGC 2841 (Kormendy 1977). In each of the latter cases, it is the number of high mass O-type stars which was found to be deficient. There appears to be no significant deviation from the locally determined IMF on large scales—averaging over an entire galaxy (Butcher 1977; Hardy 1977; Lequeux 1979). We can, therefore, use the Salpeter IMF as a fair representation of the global IMF in a galaxy. The possible effects of variations in the IMF will be considered later.

We will assume that the BRF is monotonically decreasing with time. The BRF is also parameterized to account for the differences among three major classes of galaxies which we consider subsequently: irregulars, spirals, and ellipticals. The birth rates of stars in these galaxy types are at present very different. Irregulars have high rates of star formation, while spirals have a lower rate of star formation. Ellipticals show little or no star formation. *A priori* we choose a BRF of the form

$$F(t) \propto \exp(-t/\lambda)\,, \tag{1}$$

where t is the time of the observation after galaxy formation and λ is a constant which depends upon galaxy type. The advantages in choosing a BRF like equation (1) are that (1) it can fairly represent the differences in BRF as a function of galaxy type; (2) it is monotonically decreasing in time; (3) it is supportive of our current views of galaxy formation wherein stars form very rapidly during the initial stages of galaxy formation and subsequently the birth rate decreases; and (4) it is a simple expression. Equation (1) suffers from the fact that it no doubt oversimplifies the actual state of affairs in a galaxy. However, we shall argue in § IV that the results support the use of expression (1) as a good approximation to the BRF for spiral and elliptical galaxies. While expression (1) may describe the mean trend of star formation in irregulars, its general application is suspect; in fact, a significant burst of star formation during the SMC's recent past is required to account for its current, large number of PN.

The lifetime of a star of mass m on the main sequence, $\tau(m)$, is approximately

$$\tau(m) = 10^{10}\ \mathrm{yr}\ (m/M_\odot)^{-3}\ . \tag{2}$$

If T denotes the present epoch, then at the time stars of mass m were formed, stars which are now red giants, the BRF had a value

$$F[T - \tau(m)] = \exp\{-[T - \tau(m)]/\lambda\} . \quad (3)$$

Since the IMF indicates the relative number of stars formed at a given mass m, then the total number of stars with masses m to $m + dm$ which are now red giants is

$$d\mathcal{N}(m) \propto \psi(m)F[T - \tau(m)]dm$$
$$\propto m^{-2.35} \exp\left\{-\frac{[T - \tau(m)]}{\lambda}\right\} . \quad (4)$$

Naively, one would expect the total number of PN to be proportional to $\mathcal{N}$, the number of red giants.

Integrating over the allowed masses, M_l to M_u, then gives the total number of stars which are potential PN, i.e., stars which are now red giants:

$$\mathcal{N} \propto \int_{M_l}^{M_u} d\mathcal{N}(m) \propto \int_{M_l}^{M_u} m^{-2.35} \exp\left\{-\frac{[T - \tau(m)]}{\lambda}\right\} dm . \quad (5)$$

Equation (5) also yields a reasonable estimate of the possible number of PN, provided the PN lifetime is short relative to the red giant stage and provided PN lifetime is constant regardless of other parameters—mass and metallicity.

Values of λ will differ according to galaxy type. Taking the age of the universe to be 10–15 Gyr, then λ may be found for each galaxy type by considering the presently observed rates of star formation and the fraction of mass in the form of gas. Irregulars are observed to have a relatively high rate of star formation (Searle and Sargent 1972; Searle, Sargent, and Bagnuolo 1973) and a large fraction of their mass in gaseous form (Allen 1965; Roberts 1975). These observations are consistent with irregulars having gone through approximately one e-folding of their evolution. Consequently we set $\lambda_{\rm irr} \sim 10$ Gyr. Spirals have only a few percent of their mass in gaseous form (Roberts 1975) and are not nearly as active as irregulars in forming stars; therefore, we expect spirals to have undergone two or three e-folding times; we set $\lambda_{\rm spiral} \sim 5$ Gyr. Ellipticals represent the extreme case with little or no star formation. We set $\lambda_{\rm ellip} \sim 1$ Gyr. The precise values of λ are, as we discuss in § IV, not essential to the argument providing we are not orders of magnitude out of line.

Values for $\mathcal{N}$ are presented in Table 1. The constant of proportionality has been determined so that the largest values of $\mathcal{N}$ are approximately unity. We have determined $\mathcal{N}$ for three different galaxy ages T, and values of the time constant λ ranging from 10^9 yr to T. For each T and λ, upper and lower mass limits were set (M_u, M_l) in columns (2) through (7). The last column, column (8), gives values of $\mathcal{N}$ where M_l, the lower mass limit, is set equal to the main-sequence turnoff mass corresponding to a star with an age T; the upper mass limit is 5 $M_\odot$. By comparing values of $\mathcal{N}$ in columns (2) with (3), (4) with (5), and (6) with (7), we see that the upper mass limit has little effect on the value of $\mathcal{N}$, as one would expect. Since the majority of stars are of low mass, extending the upper mass limit produces very few extra stars. The only way by which $\mathcal{N}$ can be induced to vary more than 25% for a change in λ of a factor of 10 or more, is to constrain M_l, that is, to fix M_l above the main sequence turnoff mass

TABLE 1

THE STAR COUNT FUNCTION

λ (Gyr) (1)	$M_u = 5$, $M_l = 0.95$ (2)	$M_u = 10$, $M_l = 0.95$ (3)	$M_u = 5$, $M_l = 1.0$ (4)	$M_u = 10$, $M_l = 1.0$ (5)	$M_u = 5$, $M_l = 1.1$ (6)	$M_u = 10$, $M_l = 1.1$ (7)	M_u M_l (8)
			$T = 10$ Gyr				1 5
1	...	...	3.56	3.56	3.56	3.57	3.56
5	...	...	4.30	4.44	3.90	3.04	4.30
10	...	...	3.58	3.77	2.79	2.98	3.58
			$T = 15$ Gyr				0.8735 5
1	0.115	0.115	0.0240	0.0240	2.40×10^{-3}	2.40×10^{-3}	2.79
5	2.05	2.10	1.58	1.63	1.07	1.12	3.43
10	2.52	2.64	2.17	2.29	1.69	1.81	3.33
15	2.34	2.47	2.07	2.20	1.68	1.81	2.91
			$T = 20$ Gyr				0.793 5
1	7.70×10^{-4}	6.10×10^{-4}	1.60×10^{-4}	1.60×10^{-4}	1.60×10^{-4}	1.60×10^{-4}	2.47
5	0.750	0.770	0.580	0.600	0.390	0.410	2.82
10	1.53	1.60	1.31	1.39	1.03	1.10	2.99
20	1.62	1.72	1.45	1.55	1.20	1.29	2.51

corresponding to a given galaxy age. From the data in column (8), it is easy to see that $\mathcal{N}$ will not vary by more than $\sim 20\%$ even if λ changes by a factor of 20 provided that all the stars between the main sequence turnoff at an epoch T and the upper mass limit of 5 $M_\odot$ are potential PN. Changing the upper limit to 3 $M_\odot$ would not change our results significantly. The occurrence of white dwarfs in several open clusters (Romanishin and Angel 1980; Tinsley 1975) suggests that PN may form from stars with masses as large as 4 or 5 $M_\odot$.

b) *The Metallicity Function*

If metals do play a role in the formation of PN, then it is necessary to consider the distribution of heavy elements among the stars as well as the overall metal abundance. The distribution function we adopt is for a closed, instant recycling, initially enriched system (Tinsley 1976). The distribution function $f(s)$ describes the fraction of stars with heavy element abundance of relative abundance s compared to the interstellar medium:

$$f(s) = \frac{1 - g^{bs+a}}{1 - g} , \tag{6}$$

where g is the fraction of the total mass in gaseous form, and a and b describe the abundance of the gas prior to star formation. The possible effects of metallicity are incorporated into our description by presuming that element abundance affects PN formation as a power law with exponent n. Weighting the relative number of stars in an interval s to $s + ds$, $[df(s)/ds]ds$, by the power law dependence gives the differential of our metallicity function, viz. $d\phi/ds$. Thus using equation (6), we obtain

$$\phi_n(z, g) = \int_0^1 \frac{df}{ds}(sz)^n ds$$

$$= z^n \frac{g^a b \ln g}{g - 1} \int_0^1 g^{bs} s^n ds . \tag{7}$$

The value of z in equation (7) is a measure of the actual metal abundance of the interstellar medium for the system under consideration. The subscript n has been introduced to denote possible variations in the power law exponent.

Using equation (6) introduces another free parameter into the analysis; the amount of pre-enrichment of the initial gas. For a system starting with metal-free gas, $a = 0$, $b = 1$ where as for the solar neighborhood, $b = 1.205$, $a = -0.205$ (Tinsley 1976) indicating that some pre-enrichment occurred; the gas from which the stars in the solar neighborhood formed was enriched to ~ 0.17 of solar abundance. We will include two cases in an analysis: $a = 0$, $b = 1$ corresponding to no pre-enrichment, and variable amounts of pre-enrichment (corresponding to modest pre-enrichment) which may vary from galaxy to galaxy. We expect $\phi_n(z, g)$ to illustrate the effect metal abundance has in PN formation, even for galaxies of different Hubble type. Both z and g are set by direct observations. The value of n can be determined by comparing predicted and observed numbers of PN within galaxies of different types.

Presented in Table 2 are values of the metallicity function $\phi_n(z, g)$ for different values of metal abundance z, gas fraction g, and power law exponent n for the two cases: no pre-enrichment and modest pre-enrichment. For fixed n and z and no pre-enrichment, ϕ_n will vary by a factor of 10 or more as g varies from 0.5 to 10^{-6} with ϕ_n decreasing as g decreases. The behavior of ϕ_n with pre-enrichment is somewhat more complicated in that ϕ_n has a minimum at $g \sim 10^{-6}$ and varies only by a factor of 2 for g ranging from 0.9 to 10^{-6}. In each case, the signature of metal dependence is marked. It is quite evident that the metallicity effect is far more pronounced than the mass effect.

III. OBSERVATIONAL DATA

For each of several galaxies, four quantities are needed to test the results of § II: the total galaxy mass M, the fraction of mass in gaseous form g, the total number of PN N, and the mean metallicity of the interstellar medium z.

The gas content at the last epoch of stellar formation is what one requires to determine the appropriate value of g. For spirals and irregulars, one need only make a direct measurement. However, of the ellipticals in our sample, only NGC 185 and NGC 205 have appreciable quantities of interstellar matter which may reflect the true value of g for these galaxies.

To obtain values for N we use the data provided by Jacoby (1980). Jacoby (1980) has obtained a luminosity function for PN in the Magellanic Clouds. This luminosity function is assumed to be universal so that it could be applied to a number of galaxies in the Local Group. Using the luminosity function, one can convert the observed number of PN within a galaxy to the total number. We have corrected a numerical error in a coefficient used to calculate the number of PN beyond the first six magnitudes of the luminosity function (c.f., Jacoby § IV*d*, who derives a value of 2.10 which is to multiply the number of PN in the first six magnitudes of the luminosity function; the correct value is 1.84). Changing the constant merely scales the number of PN proportionately. The principal difficulty associated with Jacoby's analysis is that the luminosity function is used without regard to a low luminosity cutoff. It may be that the shape of the luminosity function is universal. But mass or metallicity effects may lead to a shift in magnitude: the brightest PN in a galaxy which is metal-poor, for example, may be fainter than the brightest in another galaxy which is metal-rich. Thus, we would obtain a false estimate of the total number of PN since one implicitly assumed different limiting magnitudes for PN in each galaxy. The prospect of low mass stars producing low mass PN further compounds the problem since the faint end of the luminosity function would grow more quickly than that found by Jacoby. However, since reliable photometry is unavailable for the brightest PN in most of the galaxies in the sample, one can do

TABLE 2

THE METALLICITY FUNCTION $\phi_n(z, g)$

z	g = 0.5	g = 0.1	g = 10^{-3}	g = 10^{-6}
		$n = 1$		
10^{-3}	4.43×10^{-4}	3.23×10^{-4}	1.44×10^{-4}	7.24×10^{-5}
	(4.467×10^{-4})	(4.07×10^{-4})	(4.10×10^{-4})	(3.47×10^{-4})
0.1...........	0.0443	0.0323	0.0144	0.00724
	(0.0467)	(0.0407)	(0.0410)	(0.0847)
0.25	0.111	0.081	0.036	0.018
	(0.127)	(0.101)	(0.102)	(0.211)
0.5...........	0.221	0.161	0.072	0.0362)
	(0.223)	(0.203)	(0.205)	(0.423)
1	0.443	0.323	0.144	0.0724
	(0.467)	(0.407)	(0.410)	(0.847)
		$n = 2$		
10^{-3}	2.77×10^{-7}	1.70×10^{-7}	4.06×10^{-8}	1.05×10^{-8}
	(2.89×10^{-7})	(2.01×10^{-7})	(9.78×10^{-8})	(1.02×10^{-7})
0.1...........	2.77×10^{-3}	1.70×10^{-3}	4.06×10^{-4}	1.05×10^{-4}
	(2.89×10^{-3})	(2.01×10^{-3})	(9.78×10^{-4})	(1.02×10^{-3})
0.25	0.017	0.011	0.0025	6.6×10^{-4}
	(0.0181)	(0.0126)	(0.0061)	(0.0064)
0.5...........	0.0693	0.0425	0.0102	0.0026
	(0.0723)	(0.0503)	(0.0245)	(0.0255)
1	0.277	0.170	0.0406	0.0105
	(0.289)	(0.201)	(0.0978)	(0.102)
		$n = 3$		
10^{-3}	2.00×10^{-10}	1.10×10^{-10}	1.66×10^{-11}	2.27×10^{-12}
	(2.07×10^{-10})	(1.26×10^{-10})	(3.44×10^{-11})	(1.83×10^{-11})
0.1...........	2.00×10^{-4}	1.10×10^{-4}	1.66×10^{-5}	2.27×10^{-6}
	(2.07×10^{-4})	(1.26×10^{-4})	(3.44×10^{-5})	(1.83×10^{-5})
0.25	3.13×10^{-3}	1.72×10^{-3}	2.59×10^{-4}	3.55×10^{-5}
	(3.24×10^{-3})	(1.98×10^{-3})	(5.38×10^{-4})	(2.86×10^{-4})
0.5...........	0.025	0.0138	2.08×10^{-3}	2.84×10^{-4}
	(0.0259)	(0.0158)	(4.30×10^{-3})	(2.29×10^{-3})
1	0.200	0.110	0.0166	2.27×10^{-3}
	(0.207)	(0.126)	(0.0344)	(0.0183)

NOTE.—Values of $\phi_n(z, g)$ determined for the modest pre-enrichment case are enclosed in parentheses.

no better at present than to take Jacoby's data at face value.

Where direct measurement of abundance is not possible, we use a photometric measure of metallicity (van den Bergh 1967): the reddening free parameter Q [$Q = (U-B)-0.72(B-V)$]; Q is roughly correlated with metal abundance.

We now consider each galaxy in the sample.

LMC.—Feast (1964) has estimated the total mass of the LMC to be 1.0×10^{10} $M_\odot$. McGee and Milton (1966) find the total mass of neutral hydrogen to be 6×10^8 $M_\odot$ so that g is 0.06. The correct number of PN is 873 ± 253 (Jacoby 1980); the relative number of PN per unit stellar mass, κ, is therefore $(9.7 \pm 3) \times 10^{-8}$. Heavy element abundance in Population I stars is about 0.85 solar (van den Bergh 1975).

SMC.—Hindman (1967) has estimated the mass to be 1.5×10^9 $M_\odot$, while the H I mass is 4.8×10^8 $M_\odot$. Therefore, $g = 0.30$. Jacoby (1980) estimates that there are 250 ± 78 PN which gives $\kappa = (25 \pm 8) \times 10^{-8}$. Butler (1978), Osmer (1976), and van den Bergh (1968, 1975) suggest that the SMC is underabundant by a factor of 4 relative to the Sun.

Fornax.—Assuming that the M/L value is the same as that for NGC 205 (Tonry and Davis 1980), then the total mass of Fornax is 1.1×10^9 $M_\odot$. There is no detectable gas in this galaxy (Hodge 1971): we set $g = 10^{-6}$. One PN has been found by Danzinger *et al.* (1978) in a search which is complete over 2.3 mag; Jacoby (1980) concludes that there are 8 ± 3 PN. Thus, $\kappa = (0.7 \pm 0.25) \times 10^{-8}$. Abundance measurements of globular clusters indicates a wide range of metal abundances within Fornax (van den Bergh 1969). The most metal rich globular cluster in Fornax has an abundance between that of M92 and M5 in our galaxy. We therefore place the abundance in Fornax to be 0.1–0.2 solar.

NGC 147.—Taking the M/L of NGC 147 to be the same as that of NGC 205 (Tonry and Davis 1980), then the mass of NGC 147 is 1.7×10^9 $M_\odot$. Since there is no detectable gas in this galaxy (Hodge 1971) we put $g = 10^{-6}$. Jacoby (1980) determines that there are 32 ± 10 PN so that $\kappa = (1.8 \pm 0.6) \times 10^{-8}$. No direct measurements have been made to determine the metallicity of NGC 147. However, the photometric parameter Q has a value $Q_{\rm NGC\,147} = -0.33$ (de Vaucouleurs, de Vaucouleurs, and Corwin 1976) which is similar to that for the SMC, $Q_{\rm SMC} = -0.35$. Therefore, we presume similar metal abundances for the two; NGC 147 has an abundance approximately 0.2–0.25 solar.

NGC 185.—If the M/L for NGC 185 is the same as that for NGC 205 (Tonry and Davis 1980), then the total mass for NGC 185 is 2.2×10^{-9} $M_\odot$. There is approximately 4×10^4 $M_\odot$ in the form of dust and gas in NGC 185 (Hodge 1963) so that $g = 2 \times 10^{-5}$. The number of PN is 32 ± 10 (Jacoby 1980) which yields $\kappa = (1.4 \pm 0.5) \times 10^{-8}$. The value of Q is $Q_{\rm NGC\,185} = -0.285$ (de Vaucouleurs, de Vaucouleurs, and Corwin 1976) which is close to that for the LMC, $Q_{\rm LMC} = -0.27$. Therefore we assume the abundance of NGC 185 to be approximately one-half solar.

NGC 205.—Velocity dispersion measurements in NGC 205 suggest a total mass for this galaxy of 1.7×10^{10} $M_\odot$ (Tonry and Davis 1980). Hodge (1973) puts the gas content at $\sim 5 \times 10^4$ $M_\odot$ so that $g \sim 3 \times 10^{-6}$. The estimated number of PN is 144^{+45}_{-52} which gives $\kappa = (0.8^{+0.25}_{-0.29}) \times 10^{-8}$. Line strengths in NGC 205 indicate that this galaxy is metal poor (McClure and van den Bergh 1968; Spinrad and Peimbert 1975), a conclusion supported by a comparison of stellar populations between NGC 205 and our galaxy (Baum and Schwarzschild 1954) and the extreme value of Q, $Q_{\rm NGC\,205} = -0.42$ (de Vaucouleurs, de Vaucouleurs, and Corwin 1976). A reasonable estimate for the metal abundance in NGC 205 is 0.1–0.2 solar (Spinrad and Peimbert 1975).

NGC 221.—The most recent measurement of the mass of NGC 221 gives a value 2.2×10^9 $M_\odot$ (Tonry and Davis 1980). There is little or no detectable gas in NGC 221 (Hodge 1971); therefore, we set $g = 10^{-6}$. Jacoby estimates that there are 223^{+75}_{-110} PN which makes $\kappa = (10^{+3.5}_{-5.1}) \times 10^{-8}$. Faber (1973) finds the abundance in NGC 221 to be near 0.75 solar; this agrees with the results of Spinrad (1966) who argued for abundance near solar, and of the photometric parameter, $Q_{\rm NGC\,221} = -0.21$ (de Vaucouleurs, de Vaucouleurs, and Corwin 1976), which is close to the value for M31 ($Q_{\rm M31} = -0.16$).

NGC 224.—Rubin and Ford (1970) give the mass to be 2.1×10^{11} $M_\odot$ with neutral hydrogen making up 7.6×10^9 $M_\odot$; g has a value 0.036. Jacoby (1980) lists the number of PN as $17{,}734 \pm 5911$; hence, $\kappa = (8.4 \pm 2.8) \times 10^{-8}$. The nuclear region of NGC 224 appears to have a greater than solar abundance of metals based upon TiO band strengths (Spinrad and Taylor 1971). To match stellar models with the spectra of NGC 224 requires the model stars to have abundances of 10%–300% greater than solar for the heavy elements (N, Na, Mg, Ti) (Spinrad and Peimbert 1975). Given that there is a correlation between galaxy mass and galaxy metallicity (Spinrad and Peimbert 1975), and since NGC 224 is roughly twice as massive as our Galaxy, we set the abundance of NGC 224 to be 1.2 solar.

NGC 6822.—Rogstad, Rougoor, and Whiteoak (1967) have determined the total mass to be 2.4×10^9 $M_\odot$ and the H I mass to be 1.5×10^8 $M_\odot$; g is 0.062. There are 32 ± 11 PN in this galaxy (Jacoby 1980) which means $\kappa = (1.3 \pm 0.4) \times 10^{-8}$. The metal abundance is approximately one-sixth solar (Spinrad and Peimbert 1975), based upon emission line strengths for H II regions.

The Halo.—While the actual mass of the Halo is difficult to determine, van den Bergh (1973) has found an upper limit for $\kappa_{\rm Halo}$ based upon the mass per RR Lyrae star in globular clusters, the total number of RR Lyrae stars in the Halo, and the observed number of Halo PN; he finds $\kappa < 0.5$. The gas content of the Halo is unknown; we assume $g < 10^{-3}$. To obtain an estimate of the Halo metallicity, we have averaged values of [m/H] for globular clusters in Alcaino's (1977) list; an abundance of approximately one-fourth solar is found.

The Galaxy.—Innanen (1966) determines the mass of the Galaxy to be 1.3×10^{11} $M_\odot$. Since the total gaseous mass is poorly known, we will presume that the Galaxy has a value of g comparable to the average for Sc galaxies: $g = 0.08$ (Roberts 1975). Based upon comparisons with NGC 224, Jacoby (1980) estimates the number of PN in the Galaxy to be $10{,}000 \pm 4000$; therefore, $\kappa = (7.7 \pm 3.0) \times 10^{-8}$. The metallicity for the Galaxy is assumed to be Solar.

In Table 3 are the observed values of κ and the predicted κ for various ϕ_n, n from 0 to 3. All values of κ have been normalized to NGC 224. We also give κ for the cases with no metallicity dependence but having low mass cutoff values above the main-sequence turnoff. In Table 3, column (1) contains galaxy names; column (2) contains values of heavy element abundance relative to solar; column (3) lists fraction of total mass in gaseous form; column (4) records observed values of κ. The following columns contain predicted values: column (5) for $n = 0$, M_l = main-sequence turnoff mass, no mass nor metallicity effects; column (6) for linear dependence on metallicity; column (7) for quadratic dependence on metallicity; column (8) for cubic dependence on metallicity; column (9) no metallicity dependence, low mass cutoff $M_l = 1.1$ $M_\odot$; column (10) no metallicity dependence, low mass cutoff $M_l = 1.0$ $M_\odot$; column (11) no metallicity dependence, low mass cutoff $M_l = 0.95$ $M_\odot$. Bracketed values in columns (6), (7), and (8) are for fixed values of pre-enrichment. Quantities in columns (9), (10), and (11) are for a galaxy age of 15 Gyr. Values of χ^2 are given in Table 4 for each ϕ_n

TABLE 3

COMPARISON OF OBSERVED AND PREDICTED VALUES OF κ

GALAXY (1)	$z/z_\odot$ (2)	g (3)	κ(observed) (10^{-8} PN/$M_\odot$) (4)	κ(predicted)[a] (10^{-8} PN/$M_\odot$) $n=0$ (5)	$n=1$ (6)[b]	$n=2$ (7)[b]	$n=3$ (8)[b]	$M_l = 1.1\ M_\odot$ (9)[c]	$M_l = 1.0\ M_\odot$ (10)[c]	$M_l = 0.95\ M_\odot$ (11)[c]
LMC	0.85	0.06	9.7 ± 3	8.5	7.0	5.2	3.9	8.5	8.5	8.5
					(6.1)	(4.3)	(2.9)			
SMC	0.25	0.30	25 ± 8	8.5	2.8	0.8	0.2	13.7	11.7	9.4
					(2.0)	(0.5)	(0.1)			
Fornax	0.15	10^{-6}	0.7 ± 0.25	8.5	0.3	0.01	0.0005	0.02	0.1	0.4
					(2.3)	(0.07)	(0.003)			
NGC 147	0.20	10^{-6}	1.8 ± 0.6	8.5	0.4	0.2	10^{-4}	0.02	0.1	0.4
					(3.1)	(0.2)	(0.006)			
NGC 185	0.5	2×10^{-5}	1.4 ± 0.5	8.5	1.3	0.2	0.04	0.02	0.1	0.4
					(4.7)	(0.7)	(0.1)			
NGC 205	0.15	3×10^{-6}	0.8 ± 0.25	8.5	0.3	0.02	0.0007	0.02	0.1	0.4
					(2.4)	(0.07)	(0.003)			
NGC 221	0.75	10^{-6}	$10^{+3.5}_{-5.1}$	8.5	1.5	0.3	0.07	0.02	0.1	0.4
					(11.6)	(1.8)	(0.4)			
NGC 224	1.2	0.036	8.4 ± 2.8	8.5	8.5	8.5	8.5	8.5	8.5	8.5
					(8.5)	(8.5)	(8.5)			
NGC 6822	0.167	0.06	1.3 ± 0.4	8.5	1.4	0.2	0.03	8.5	8.5	8.5
					(1.2)	(0.2)	(0.02)			
Halo	0.25	10^{-6}	<0.5	8.5	0.05	0.004	0.0001	0.02	0.1	0.4
					(3.9)	(0.2)	(0.01)			
The Galaxy	1.0	0.08	7.7 ± 3.0	8.5	8.6	8.0	6.8	8.5	8.5	8.5
					(7.3)	(6.2)	(5.4)			

[a] Values of κ are normalized to NGC 224 with $\kappa = 8.5 \times 10^{-8}$ PN $M_\odot^{-1}$.

[b] Values calculated for pre-enrichment ($b = 1.205$, $a = -0.205$) appear in parentheses below corresponding values calculated without pre-enrichment ($b = 1$, $a = 0$).

[c] Values calculated assuming no metallicity dependence for various lower mass cutoff values; normalized to NGC 224 with $\kappa = 8.5 \times 10^{-8}$ PN $M_\odot^{-1}$; $T = 15$ Gyr.

and lower mass cutoff. Also in Table 4 are the number of degrees of freedom for each model and the 99.9%, one-sided, level of confidence for χ^2.

Two other possible models have been introduced in Table 4. Model h corresponds to ϕ_1 with variable pre-enrichment and a regular prescription for the SFR described in § II*a*, assumed to hold universally; model i uses ϕ_1, variable pre-enrichment, and a SFR for the SMC which is slightly greater than that implied by § II*a*. As shown in the Appendix, if the SMC has maintained its current, apparent high rate of star formation for the past 2–3 $\times 10^9$ yr, then the correct, predicted number of PN in the SMC is a factor of 10 more than that given in Table 3. The results of model i are shown in Figure 1.

Model a, corresponding to no mass or metallicity dependence for PN formation, is ruled out by the exceeding large value of χ^2. Likewise, models with no

TABLE 4

χ^2 FOR MODELS OF PN FORMATION

Model	Parameters	Degrees of Freedom	$\chi^2_{0.999}/\nu$	χ^2/ν
a. $n = 0$ (no mass or metallicity effects)	$\kappa_{\rm NGC\ 224} \equiv \kappa$	10	3.0	258
b1. $n = 1$	κ, z, g	7	3.3	3.5
b2. ($n = 1$)	κ, z, g, fixed pre-enrichment	7	3.3	26
c1. $n = 2$	κ, z, g	7	3.3	7.8
c2. ($n = 2$)	κ, z, g, fixed pre-enrichment	7	3.3	7.0
d1. $n = 3$	κ, z, g	7	3.3	9.1
d2. ($n = 3$)	κ, z, g, fixed pre-enrichment	7	3.3	9.0
e. $M_l = 1.1$	κ, M_l	9	3.1	982
f. $M_l = 1.0$	κ, M_l	9	3.1	361
g. $M_l = 0.95$	κ, M_l	9	3.1	343
h. $n = 1$, variable pre-enrichment	κ, z, g, pre-enrichment	7	3.3	2.50
i. $n = 1$, variable pre-enrichment, accelerated SFR in SMC	κ, z, g, pre-enrichment SFR in SMC	6	3.5	0.23

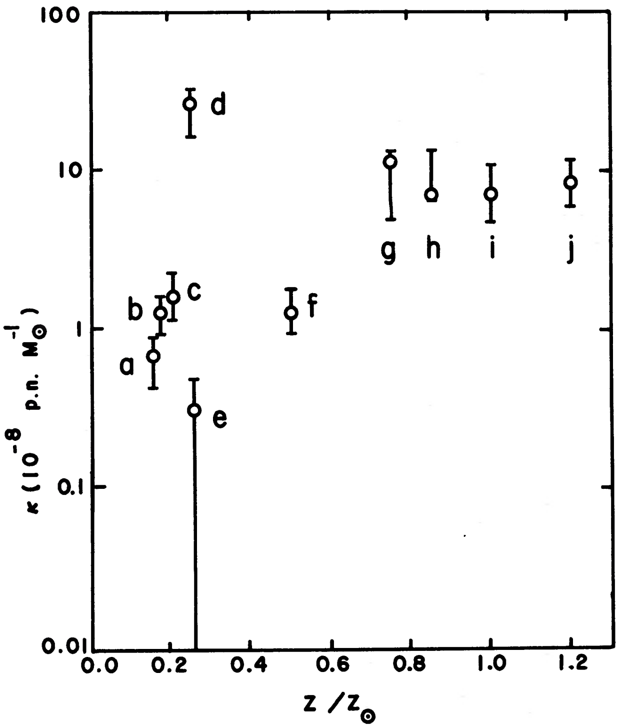

FIG. 1.—Dependence of the number of PN per unit mass, κ, upon heavy element abundance $z/z_{\odot}$. Error bars indicate the estimated ranges of observed values of κ. Circles denote predicted values for κ normalized to NGC 224 [$(\kappa_{\text{NGC 224}}) = 8.5 \times 10^{-8}$ PN $M_{\odot}^{-1}$] assuming linear dependence on metallicity, pre-enrichment, and a prolonged burst of star formation in the SMC. Quantities in square brackets denote the amount of pre-enrichment as a percentage of present heavy element abundance: (*a*) Fornax and NGC 205 [0.10]; (*b*) NGC 6822 [0.0]; (*c*) NGC 147 [0.12]; (*d*) SMC [0.0]; (*e*) Halo [0.0]; (*f*) NGC 185 [0.0]; (*g*) NGC 221 [0.17]; (*h*) LMC [0.0]; (*i*) Galaxy [0.17]; (*j*) NGC 224 [0.17].

metallicity effects, invoking low mass cutoff, models e, f, and g, are excluded because of their χ^2 values. Model b1, linear dependence on metal abundance without pre-enrichment, is marginally unacceptable at the 99.9% level of confidence. Only two models are acceptable at the 99.9% level: models h and i. The difference between the two is the SFR in the SMC. Model h assumes a regular, monotonic SFR, while model i indicates the result of an increased rate of SFR over the past 4–5 $\times 10^9$ yr.

It is quite clear that metallicity is the principal factor influencing PN formation.

IV. DISCUSSION

Before proceeding to a more general discussion, there are three points to consider: the possible effects of varying the IMF, the possible dependence of our analysis on an implicit model of PN formation, and the interpretation of our results.

The first question is readily answered by recasting equation (5) into a more general form,

$$\mathcal{N} \propto \int_{M_l}^{M_u} m^{-n} \exp\left\{-\frac{[T-\tau(m)]}{\lambda}\right\} dm \,,$$

where m^{-n} is the IMF. The behavior of $\mathcal{N}$ for changes in the IMF exponent n is determined by $d\mathcal{N}/dn$:

$$\frac{d\mathcal{N}}{dn} \propto \int_{M_l}^{M_u} \ln(m) m^{-n} \exp\left\{-\frac{[T-\tau(m)]}{\lambda}\right\} dm \,.$$

The ln (m) factor in the integrand suggests that $\mathcal{N}$ is not strongly dependent upon the IMF. Provided changes in n are modest (say, 10%–20%), we would expect very little change in our analysis.

The second question is less precise and, because of the format of our analysis, requires that the answer be generated in two parts. In § II*a* we consider the mass effect in a naive fashion; notably, we presume that the number of PN is simply proportional to the number of red giants. Implicitly, we are analyzing a family of PN models which limit the mass range of stars able to form PN. We are at the same time establishing the dependence of κ, the relative number of PN per unit stellar mass, as a function of the stellar mass limits for PN formation. Section II*b* establishes the abundance dependence of κ. Because we have used a statistical argument, we have not determined that physical mechanism which links abundance to PN formation. Therefore, our analysis implicitly investigates a large family of models which may or may not require PN formation to be dependent upon heavy element abundance.

The results of our analysis are that the mass and metallicity functions are separable to a first order of approximation, that heavy element abundance significantly affects PN formation, that PN formation is linearly dependent upon heavy element abundance, and that stars with masses less than or approximately equal to 0.9 $M_{\odot}$ can form PN. However, the strongest conclusion that can be made in view of the models we have adopted for the IMF, BRF, and history of element enrichment is that heavy element abundance is the most significant factor in PN formation.

An interesting consequence of the effect metallicity has on PN formation is the following. In those systems which are deficient in gas and which have very little pre-enrichment, even if they are relatively metal-rich, at present the relative number of PN, κ, will be smaller than that for a system of comparable metallicity but which has greater gas content. Thus the nuclei of galaxies and their bulges should be depleted of PN relative to the main luminous bodies of the galaxies. This result accounts for the observation by Lawrie and Ford (1978) that the nuclei of NGC 221 and NGC 224 are depleted of PN, i.e., $\kappa_{\text{nuclei}} < \kappa_{\text{galaxy}}$. Similarly, it has been known for some time that the bulge in our Galaxy is deficient of PN compared to the disk (Feast 1968; Perek 1968; Seaton 1968). These observations are in support of metallicity dependence for PN formation.

There is considerable independent observational data which implies that heavy element abundance affects PN formation. Barker (1978*a*, *b*, *c*), in analyzing line intensities for more than 35 PN, found O, Ne, and S abundances to be independent of kinematic groupings of the nebulae. This implies that metal-rich stars form PN. A very specific example is K648. While M15 is underabundant in iron by 100 relative to the Sun, K648 is only a factor of 10 underabundant in oxygen (Peimbert 1973), suggesting that metal-rich stars form PN. Many PN abundance measurements (Allen and Czyzak 1968, 1978; Torres-Peimbert and Peimbert 1978; Shields 1978; Peimbert and Torres-Peimbert 1971; Osterbrock 1970) indicate that PN are formed from metal-rich stars.

Apart from the obvious implications concerning PN formation, the metallicity effect may influence the course of chemical evolution in a galaxy and the amount of matter sloughed off by evolved low intermediate mass stars. The amount of matter in PN sheels is unknown, since it is subject to observational uncertainties and likely an intrinsic range of masses. We assume, however, that the mean mass of a PN shell is 0.2 $M_\odot$. The amount of enriched material ejected into the interstellar medium will then be $dM/dt = 0.2\mathscr{M}\kappa\epsilon^{-1}$ $M_\odot$ yr^{-1}, where $\mathscr{M}$ is the galaxy mass in solar masses, κ the number of PN per unit stellar mass, and ϵ the average life span of a PN: $\epsilon \sim 10^4$ yr. For a galaxy like our own, $dM/dt \sim 0.16\,M_\odot$ yr^{-1}. This value is approximately a factor of 20 below the rate at which material is consumed at the present rate of star formation (Miller and Scalo 1979). However, it may be the case that all stars, metal-rich and metal-poor, lose at least a few tenths of a solar mass over their lifetimes (Sweigart and Gross 1978), but that metal-rich stars, those which are more likely to form PN, are prone to losing more mass than their metal-poor companions. The result is that metal-poor stars will lose some of their mass, while metal-rich stars will lose a larger fraction of their mass; the metal-rich stars thereby accelerate the rate of element production in a galaxy by contributing a larger fraction of element enriched material to subsequent generations of stars. Galaxies having low metallicity and hence low rates of PN formation will therefore have their chemical evolution retarded compared to galaxies with greater metallicity. Likewise, galaxies which evolved quickly, ellipticals for example, will have a greater fraction of their mass retained by stars and not recycled to the interstellar medium.

Increased mass loss by metal-rich stars has other ramifications as well as accelerated chemical evolution. It is possible that very high mass stars lose mass more readily if they are metal-rich; therefore, one would expect a decline in the SN rate as metallicity increases. The number of neutron stars and white dwarfs formed may also be related to metallicity. Metal-rich stars may be able to lose a significant fraction of their mass and evolve to white dwarfs, even though their initial mass may have been well above the Chandrasekhar limit. Metal-poor stars, however, would presumably lose only a small fraction of their original mass and may be compelled to form neutron stars if they started life above the Chandrasekhar limit. We are pursuing these issues further.

V. CONCLUSIONS

From a simple but representative model for star formation and element enrichment in galaxies, we have constructed a function which estimates the relative numbers of PN in stellar systems of differing abundance and gas content. We compare the observed and predicted numbers of PN for a number of galaxies in the Local Group. Good agreement between observed and predicted values is obtained if PN formation is linearly dependent upon heavy element abundance and if a few galaxies have been pre-enriched—their mean element abundance near the onset of star formation was $\sim 10\%$ of their present mean abundance. Our analysis also argues in favor of a prolonged burst of star formation in the SMC of $2\text{–}3 \times 10^9$ yr duration. The metallicity dependence of PN formation also accounts for the deficiency of PN in the nuclei of NGC 221 and NGC 224 and in the bulge of our Galaxy.

While other effects cannot be completely excluded, we conclude that heavy element abundance is the principal factor influencing PN formation.

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APPENDIX

The number of Cepheids in the SMC is 1154 compared to 1109 in the LMC. (Payne-Gaposchkin 1974). The number of Cepheids per unit mass is 1.1×10^{-6} Cepheid per $M_\odot$ in the SMC and 1.1×10^{-7} Cepheid per $M_\odot$ in the LMC. The rate of star formation, developed in § II*a*, indicates that there should be five times the number of Cepheids per unit mass in the SMC compared to the LMC; therefore, it would seem that star formation in the SMC is approximately twice as vigorous as described in § II*a*. The additional fraction of stars, formed during the burst of star formation, which are possible candidates for PN is

$$2 \times \left(\frac{\text{Number of PN candidates}}{\text{Number of Cepheids}}\right) = 2 \times \frac{\int_{M_B}^{5} m^{-2.35} dm}{\int_{4.5}^{10} m^{-2.35} dm}, \tag{A1}$$

where M_B is the main sequence turnoff mass corresponding to the onset of the burst of star formation. If $M_B \approx 1.56\ M_\odot$, i.e., the burst has endured for 2.6×10^9 yr, then the total number of progenitor stars for PN is 10 times greater than that indicated in §§ II and III.

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