THE ASTROPHYSICAL JOURNAL, **267**:L97–L101, 1983 April 15


# DYNAMICAL CONSTRAINTS ON STAR FORMATION EFFICIENCY

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*Received 1982 October 5; accepted 1982 December 21*

## ABSTRACT

Dynamical constraints are placed on the local star formation efficiency during the formation of star clusters. Virial models are used to examine the expected changes in cluster velocity dispersion, density, and radius during the period when the gas not converted into stars is removed from the system. Comparison of observed initial and final states of open clusters indicate local efficiencies of about 30% if the gas dispersal is slow relative to the dynamical crossing time and 55% if the gas loss is rapid. Efficiencies are somewhat dependent on support mechanisms of molecular clouds, but it is argued that observations of young clusters are in accord with the assumptions of the models used here.

*Subject headings:* clusters: open — stars: formation

## I. INTRODUCTION

Although estimates of global star formation efficiency have been known for some time, usually being found to be on the order of 1%–10%, local efficiency estimates are difficult to obtain from direct observation of star-forming regions. This *Letter* takes a dynamical perspective in order to determine the local star formation efficiency during the formation of star clusters. It has become clear that the formation of stars occurs in molecular clouds, with some percentage of these stars being formed in bound clusters. Many of these clusters are now observed to be isolated in space, the remainder of the gas from which they formed presumably having dissipated through one of many suggested processes. This *Letter* describes simple virial models for the evolution of the cluster during that period of gas dissipation for the two cases of rapid and slow mass loss. These models, when combined with observed initial and final states of open clusters, constrain the local star formation efficiencies to be significantly higher than the global estimates.

## II. VIRIAL MODELS

Consider an isolated dynamical system of initial mass $M_i$, velocity dispersion $\langle v^2 \rangle_i^{1/2}$, and half-mass radii $R_i$. Let the system be in virial equilibrium, so that

$$\langle v^2 \rangle_i = kGM_i/R_i, \tag{1}$$

where $k$ is the proportionality constant appropriate for the spatial structure of the system. Now, allow an external energy source to remove a fraction $\varepsilon$ of the mass. In the limit where the mass loss occurs on a time scale which is long compared with the crossing time, the system remains in quasi-static equilibrium and the radius of the system is inversely proportional to the mass remaining (cf. Hills 1980). Thus

$$\frac{R_f}{R_i} = \frac{M_i}{M_f} = \frac{1}{1 - \varepsilon} \tag{2}$$

(where the subscript $f$ refers to the final equilibrium state). From equations (1) and (2), we find

$$\frac{\langle v^2 \rangle_f^{1/2}}{\langle v^2 \rangle_i^{1/2}} = 1 - \varepsilon$$

and

$$\frac{\rho_f}{\rho_i} = (1 - \varepsilon)^4,$$

where $\rho$ is the mass density.

Alternatively, if the mass loss is rapid compared with the crossing time, impulsive approximations can be used. In this case, the ratio of final to initial radius is (cf. Hills 1980)

$$\frac{R_f}{R_i} = \frac{1 - \varepsilon}{1 - 2\varepsilon}. \tag{3}$$

Note that the system is unbound for $\varepsilon > 0.5$. For $\varepsilon < 0.5$, we have from equations (1) and (3)

$$\frac{\langle v^2 \rangle_f^{1/2}}{\langle v^2 \rangle_i^{1/2}} = (1 - 2\varepsilon)^{1/2}$$

and

$$\frac{\rho_f}{\rho_i} = \frac{(1-2\varepsilon)^3}{(1-\varepsilon)^2}.$$

### III. DETERMINATION OF STAR FORMATION EFFICIENCY

The application of the above analysis involves the following simple star formation scenario. Consider a region of a giant molecular cloud (GMC) in which intense star formation occurs, forming at some efficiency a bound protocluster.[1] We assume the protocluster forms in equilibrium with the gravitational potential and retains the dynamical characteristics (velocity dispersion, size, etc.) of the gas from which it formed. At some point, the remaining gas will be removed from the system—either rapidly via stellar winds and/or supernovae or slowly via cloud evaporation. The cluster will then reach a final dynamical equilibrium determined by the gravitational energy of the stellar system alone. In this scenario, the percentage of gas removed, $\varepsilon$, is simply $1-\eta$, where $\eta$ is the star formation efficiency, and the above equations become

$$\frac{R_f}{R_i} = \frac{1}{\eta},$$

$$\frac{\langle v^2\rangle_f^{1/2}}{\langle v^2\rangle_i^{1/2}} = \eta \qquad \text{(slow case)}, \qquad (4)$$

$$\frac{\rho_f}{\rho_i} = \eta^4;$$

$$\frac{R_f}{R_i} = \frac{\eta}{2\eta - 1},$$

$$\frac{\langle v^2\rangle_f^{1/2}}{\langle v^2\rangle_i^{1/2}} = (2\eta-1)^{1/2} \qquad \text{(rapid case)}, \qquad (5)$$

$$\frac{\rho_f}{\rho_i} = \frac{(2\eta-1)^3}{\eta^2}.$$

We now consider the star formation efficiencies implied by each of these observable quantities.

#### *a*) *Velocity Dispersion*

The final velocity dispersion $\langle v^2\rangle_f^{1/2}$ represents the dispersion of clusters presently isolated from gas clouds. Kinematic data are available for the more massive clusters, such as the Pleiades ($\langle v^2\rangle^{1/2} \sim 0.5$ km s$^{-1}$, Jones 1970), Praesepe (0.4 km s$^{-1}$, Jones 1971), and M11 (1.7 km s$^{-1}$, McNamara and Sanders 1977). All of these clusters are sufficiently young (ages of, at most, several relaxation times) that internal dynamical evolution has not significantly affected their dispersions. Noting that M11 is an unusually massive cluster, we adopt 0.5 km s$^{-1}$ as a typical final velocity dispersion.

The initial velocity dispersion $\langle v^2\rangle_i^{1/2}$ is determined from the motions of the gas out of which the stars are forming, i.e., from the velocity dispersions of molecular gas in star-forming regions. An excellent example is the Mon OB1 molecular cloud surrounding the very young cluster NGC 2264. This GMC has been mapped in detail by Crutcher, Hartkopf, and Giguere (1978). The region consists of several cloud fragments, dominated by one large cloud (located behind the main body of the cluster) with a $^{13}$CO velocity dispersion of 1.5 km s$^{-1}$. Blitz and Thaddeus (1980) find a similar velocity dispersion in the star-forming region of the Rosette cloud. We then adopt 1.5 km s$^{-1}$ as a reasonable initial velocity dispersion.

With these observed values for the initial and final velocity dispersions, the first conclusion is that star formation efficiencies are certainly much higher than the global estimates of 1%–10%. If 90%–99% of the binding mass is removed rapidly, a protocluster of course becomes unbound. However, even if the mass is removed slowly, the final velocity dispersion would be at most 0.15 km s$^{-1}$. Alternatively, if one adopts a final velocity dispersion of 0.5 km s$^{-1}$, the formation region would have a velocity dispersion of greater than 5 km s$^{-1}$. No dispersions of this magnitude have been observed.

The observations actually indicate far higher star formation efficiencies. In the rapid mass loss case, the data suggest an efficiency of 56%. For slow mass loss, the data indicate an efficiency of 33%. Higher efficiencies would be required for the more massive clusters.

#### *b*) *Mass Density*

Similar conclusions are drawn from consideration of initial and final cluster mass densities. While theoretically this perspective is entirely analogous to using velocity dispersions, the observational data are independent. The best available data on initial cluster densities is the study of the $\rho$ Oph region by Wilking and Lada (1983*a*, *b*). Their near-infrared survey revealed a concentrated group of stars embedded in the cloud. After extensive analysis, they concluded that the total stellar component consists of roughly 200 $M_\odot$ in a 0.4 pc$^2$ area. This mass is comparable to that of many open star clusters (e.g., the Pleiades [700 $M_\odot$; Jones 1970] and the Hyades [300 $M_\odot$; Pels, Oort, and Pels-Kluyver 1975]), but the spatial extent is significantly smaller. Wilking and Lada estimate the core mass density (gas and stars) to be on the order of 800 $M_\odot$ pc$^{-3}$. The Pleiades,

[1] In the following, "protocluster" refers to the cluster prior to gas dispersal and "cluster" to the system after gas loss.

however, have a core mass density of 5 $M_{\odot}$ $pc^{-3}$ (van Leeuwen 1979). The difference is presumably due to the cluster expansion after gas loss.

Using equations (4) and (5), the initial and final densities suggest a stellar formation efficiency of 56% in the case of rapid mass loss. In the case of slow mass loss, the data suggest an efficiency of 28%. These values are remarkably similar to the results obtained from independent kinematic information; both suggest that local star formation efficiencies are high.

### c) *Radius*

The observations of Crutcher, Hartkopf, and Giguere (1978) and Blitz and Thaddeus (1980) in star-forming regions suggest a mass density of $\rho = 10^{-20}$ g $cm^{-3}$; this also agrees well with the total mass density found by Wilking and Lada (1983*a*, *b*) in $\rho$ Oph. If we adopt this value for the central mass density, adopt 1000 $M_{\odot}$ as the protocluster mass, and assume a mass distribution corresponding to that of a King model with $W_0 = 8$ (King 1966), we can compute protocluster half-mass radii as a function of star formation efficiency. Using equations (4) and (5) we can also compute final cluster half-mass radii. The results of such computations are given in Table 1.

We note first that efficiencies of less than 20% lead to excessively large final cluster radii. At a galactic radius of 10 kpc, the galactic gravitational field imposes a tidal limit on a 1000 $M_{\odot}$ cluster of only 14 pc (King 1962). The table shows that protoclusters formed at efficiencies of less than 20% would lose most members within one crossing-time after gas loss. Protoclusters formed at higher efficiencies, however, lead to cluster half-mass radii well within the tidal limit. These systems would easily survive post–gas-loss expansion and evolve into clusters with radii similar to the radii of observed open clusters. (M11, for example, has a half-mass radius of 5 pc [Mathieu 1983]).

Detailed kinematic, density, and size-scale arguments thus suggest high star formation efficiencies. But even simple arguments suggest that this must be the case, at least in regard to massive clusters. M11, for example, has a stellar mass on the order of 5000 $M_{\odot}$ (McNamara and Sanders 1977). Efficiencies of 1% imply a source cloud of $5 \times 10^5$ $M_{\odot}$, comparable to an entire GMC. More to the point, even an efficiency of 10% implies a source cloud of $5 \times 10^4$ $M_{\odot}$, roughly 25% of the gas in the largest GMCs observed. Given a usual star-forming region density of $10^{-21}$–$10^{-20}$ g $cm^{-3}$, the formation region would be roughly 30 pc in radius. No single molecular gas cloud of such density and extent has been observed. Furthermore, the tidal radius of M11 is only on the order of 20 pc. Clearly, the star formation efficiency during the formation of M11 was quite high.

TABLE 1

INITIAL AND FINAL RADII OF A 1000 $M_{\odot}$ CLUSTER

| $\eta$ | $R_i$ (pc) | Slow Mass Loss $R_f$ (pc) | Rapid Mass Loss $R_f$ (pc) |
|---|---|---|---|
| 0.01 ... | 7.8 | 780 | ... |
| 0.1 .... | 3.6 | 36 | ... |
| 0.2 .... | 2.9 | 14 | ... |
| 0.3 .... | 2.5 | 8.3 | ... |
| 0.4 .... | 2.3 | 5.7 | ... |
| 0.5 .... | 2.1 | 4.2 | ... |
| 0.6 .... | 2.0 | 3.3 | 6.0 |
| 0.7 .... | 1.9 | 2.7 | 3.3 |

## IV. DISCUSSION

### a) *Assumption of Initial Virial Equilibrium*

An essential assumption in the above analysis is that the systems from which we derive the initial conditions are in virial equilibrium. The assumptions here are in fact two-fold: (1) that the molecular clouds from which the clusters form are in equilibrium, and (2) that the protocluster remains in that equilibrium after star formation and retains the dynamic characteristics of the gas. We consider these two points separately.

1. The dynamical state of molecular clouds has given rise to much recent debate. The thermal energies of GMCs are insufficient to support the clouds, and it has been suggested that the $^{12}CO$ self-absorption features which have been observed toward several star-forming regions and which are often redshifted with respect to the line center are evidence for cloud collapse at velocities on the order of 1 km $s^{-1}$ (Leung and Brown 1977; Snell and Loren 1977). However, there is substantial evidence arguing against cloud collapse. First, while the thermal velocities are insufficient for cloud support, the $^{13}CO$ line widths are substantially larger than thermal, suggesting the presence of turbulence within the cloud. The turbulent kinetic energy is approximately sufficient to support the clouds (see, for example, the data of Crutcher, Hartkopf, and Giguere 1978 and Blitz and Thaddeus 1980). Second, blueshifted self-absorption features have been seen in NGC 2071 (Lada and Gottlieb, unpublished observation, 1974) and the Rosette molecular complex (Blitz and Thaddeus). Third, Zuckerman and Palmer (1974) note that, if the clouds are collapsing on a dynamical time scale, the global star formation rate would be an order of magnitude higher than observed. In conclusion, the evidence for cloud collapse is not very strong, and the observed $^{13}CO$ line widths imply internal motions quite sufficient to support the cloud dynamically.

2. We make the assumption in § III that a protocluster retains the velocity dispersion and size-scale of the gas from which it formed. This assumption would not be

valid if that gas were dynamically supported by *both* internal motions and other mechanisms (in particular, magnetic field pressure). As the protocluster would be supported by internal motions alone, within a crossing time it would establish a new configuration in equilibrium with the gravitational potential. This equilibrium would be achieved through a general contraction of the protocluster.

The theoretical picture regarding the dynamical support of GMCs is not clear. The primary alternative support mechanism would be magnetic field pressure. Magnetic fields do exist within the GMCs; however, the degree of coupling of these fields to the molecular gas is unclear (Guelin *et al.* 1977; but see Elmegreen 1979). One means to circumvent the theoretical problem is to examine the spatial extent of a young protocluster relative to the local molecular cloud. Consider again NGC 2264, located in front of the Mon OB1 cloud. The age of the cluster is estimated by Walker (1956) to be $3 \times 10^6$ years; Cohen and Kuhi (1979) find similar ages for the T Tauri stars associated with the cluster. Thus the age of the system is somewhat longer than the crossing time. In Figure 1, the 84 T Tauri stars discovered by Herbig (1954) are plotted on both the $^{12}$CO and $^{13}$CO maps of the region (Crutcher, Hartkopf, and Giguere 1978). There is a strong correlation of CO density with stellar density, implying that NGC 2264 formed in the near side of the Mon OB1 cloud. More importantly, there is no substantial contraction of the stellar system relative to the cloud; the stellar half-mass radius is approximately equal to the CO half-mass radius. However, it should be noted that the average reddening in NGC 2264 is only $E(B - V) = 0.08$, so that the cluster is no longer embedded in the cloud. It is thus possible that NGC 2264 may be somewhat evolved from its protocluster configuration.

Similar results are found in $\rho$ Oph. Wilking and Lada (1983*a*, *b*) estimate a lower limit of 0.6 million years for the age of the system, while most of the stars are in fact somewhat older (Cohen and Kuhi 1979). Again, the embedded stellar population shows no central concentration relative to the CO distribution. The essential point is that, while both of these systems have existed roughly a crossing time, neither shows any significant difference in the stellar and gas distribution. The implication then is that the stars and the gas are dynamically in a very similar state.

Finally, the star formation scenario may also be complicated by shock-induced star formation (e.g., Elmegreen and Lada 1977). Such processes may act to accelerate the stars relative to the internal gas motions. This would, however, only act to unbind the system, requiring still higher efficiencies to keep the system bound.

### *b*) *Time Scale of Mass Loss*

The dynamical crossing times for the dense star-forming regions considered here are on the order of $10^6$ years. Whether the gas dispersal is fast or slow relative to this time scale depends on the processes involved. Blitz and Shu (1980) argue that the lifetimes of GMCs are on the order of a few times $10^7$ years. In this time, OB associations will impart sufficient energy into a GMC to disrupt the cloud. If this is the means by which gas is removed from a protocluster, then the analysis for slow gas dispersal is appropriate and local efficiencies on the order of 30% are suggested.

However, a survey of young cluster populations (see, for example, Hagen 1970) shows that it is likely that the protocluster itself will be the source of OB stars. The associated expanding H II regions will remove the local gas on time scales an order of magnitude shorter than

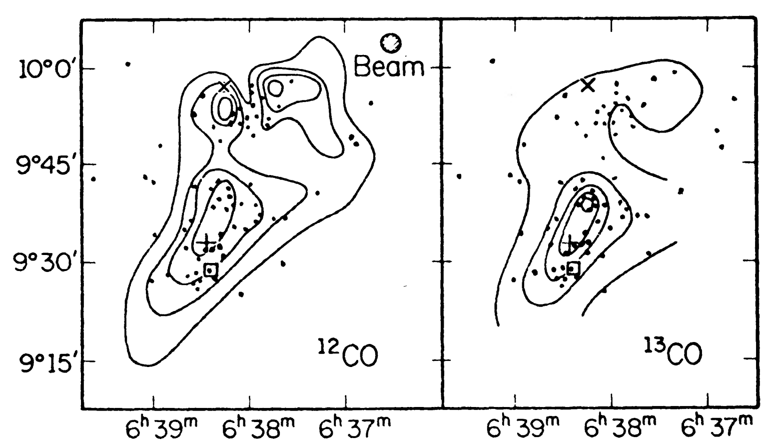


FIG. 1.—Mon OB1/NGC 2264. Contour levels are 4 K and 2 K for $^{12}$CO and $^{13}$CO respectively (taken from Crutcher, Hartkopf, and Giguere 1978). Dots are locations of T Tauri stars.

the dynamical time scale of the system. In this case, much higher local efficiencies (certainly greater than 50%) are required if the cluster is to survive.

## V. CONCLUSIONS

The essential problem with which this *Letter* is concerned is that the velocity dispersions and mass densities of open star clusters are of an order comparable to the velocity dispersions and mass densities of star-forming regions. This observation is difficult to reconcile with very low star formation efficiencies (1%–10%), since in such cases a large percentage of the binding mass is removed when the cloud dissipates. The expected result would be the expansion of the star cluster and significantly lowered velocity dispersions and densities, or the complete unbinding of the cluster.

Simple virial models have been developed to estimate local star formation efficiencies consistent with observed initial and final velocity dispersions, densities, and size scales. While the results are somewhat dependent on the rate of gas loss, the indicated efficiencies are high, on the order of 30% for slow mass loss and 55% if the rate of gas loss is rapid (relative to the crossing time). It is suggested that in most clusters OB stars will form and the gas loss will be rapid. We conclude then that, at least in the formation of bound star clusters, local star formation efficiencies are high, probably on the order of 55%.

The author would like to thank Leo Blitz, Bruce Elmegreen, Charles Lada, Joe Silk, and Stuart Vogel for valuable conversations. The referee also provided several helpful suggestions. Much appreciation is extended to Dave Latham for his hospitality at the Center for Astrophysics during the time when this work was done and to the National Science Foundation for financial support under grant AST 80-20606.

## REFERENCES

Blitz, L., and Shu, F. 1980, *Ap. J.*, **238**, 148.
Blitz, L., and Thaddeus, P. 1980, *Ap. J.*, **241**, 676.
Cohen, M., and Kuhi, L. V. 1979, *Ap. J. Suppl.*, **41**, 743.
Crutcher, R. M., Hartkopf, W. I., and Giguere, P. T. 1978, *Ap. J.*, **226**, 839.
Elmegreen, B. 1979, *Ap. J.*, **232**, 729.
Elmegreen, B., and Lada, C. J. 1977, *Ap. J.*, **214**, 725.
Guelin, M., Langer, W. D., Snell, R. L., and Wootten, H. A. 1977, *Ap. J. (Letters)*, **217**, L165.
Hagen, G. L. 1970, *Pub. David Dunlap Obs.*, Vol. **4**.
Herbig, G. H. 1954, *Ap. J.*, **119**, 483.
Hills, J. G. 1980, *Ap. J.*, **225**, 986.
Jones, B. F. 1970, *A.J.*, **75**, 563.
______. 1971, *A.J.*, **76**, 471.
King, I. R. 1962, *A.J.*, **67**, 471.
______. 1966, *A.J.*, **71**, 64.
Leung, C. M., and Brown, R. L. 1977, *Ap. J. (Letters)*, **214**, L73.
Mathieu, R. D. 1983, *Ap. J.*, in press.
McNamara, B. J., and Sanders, W. L. 1977, *Astr. Ap.*, **54**, 569.
Pels, G., Oort, J. H., and Pels-Kluyver, H. A. 1975, *Astr. Ap.*, **43**, 423.
Snell, R. L., and Loren, R. B. 1977, *Ap. J.*, **211**, 122.
van Leeuwen, F. 1980, in *IAU Symposium 85, Star Clusters*, ed. J. Hesser, (Dordrecht: Reidel), p. 157.
Walker, M. F. 1956, *Ap. J.*, **124**, 668.
Wilking, B. A., and Lada, C. J. 1983*a*, *Ap. J.*, in press.
______. 1983*b*, *Ap. J.*, in press.
Zuckerman, B., and Palmer, P. 1974, *Ann. Rev. Astr. Ap.*, **12**, 279.

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