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AN UPPER LIMIT ON THE STOCHASTIC BACKGROUND OF ULTRALOW-FREQUENCY GRAVITATIONAL WAVES

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ABSTRACT

Using pulse arrival time data for PSR 1237+25, a pulsar with unusually regular timing behavior, we place a firm upper limit on the magnitude of any stochastic background of low-frequency gravitational radiation in the universe. At frequencies between 0.3×10^{-8} and 3×10^{-8} Hz, the equivalent energy density limits are well below those needed to close the universe on account of gravitational waves alone.

Subject headings: cosmology — gravitation — pulsars — relativity

Detweiler (1979) has shown that high-precision timing measurements of pulsars can serve as a sensitive test for the existence of ultralong-wavelength gravitational waves. Briefly, the method involves using the Earth and one or more distant pulsars as free masses, the separations of which are to be monitored by means of evenly spaced "clock ticks" transmitted electromagnetically from the remote objects back to Earth. The arrival times of these ticks at Earth are compared with those of a terrestrial standard clock; any observed irregularities can, in principle, yield measurements of the characteristics of passing gravitational waves. Pulsars are useful for this purpose, of course, because they provide the evenly spaced clock ticks in the form of naturally pulsed radio emission.

This method of searching for evidence of gravitational waves is similar in principle to those involving suspended masses whose separation is monitored by laser interferometry (e.g., Forward 1978; Weiss 1979) and the Doppler tracking of spacecraft (Armstrong, Woo, and Estabrook 1979). An important difference is that pulsar timing data provide useful sensitivities at extremely long wavelengths—of the order of several light-years, or frequencies of $\sim 10^{-8}$ Hz. In this *Letter*, we report on the application of Detweiler's (1979) method to some timing data on the pulsar PSR 1237+25 recently published by Downs and Reichley (1983). We are able to place a cosmologically interesting limit on the stochastic background of gravitational waves, and we compare this limit to similar results obtained by other methods.

The JPL pulsar timing observations (Downs and Reichley 1983) are a set of high-quality pulse arrival-time measurements for 24 pulsars, extending over nearly 12 years starting in 1968. We chose to use this data set for PSR 1237+25 because it contains a long span of measurements ($T = 11.3$ yr) taken at frequent intervals (375 measurements in all, or an average of one every 11 days); furthermore, PSR 1237+25 is a well-studied pulsar known to have unusually small, intrinsic timing irregularities (Cordes and Helfand 1980). We used the barycentric arrival times as tabulated by Downs and Reichley, which have already been corrected for all known effects involving the motions of the Earth and the pulsar, and we computed timing residuals from the equations

$$\phi(t) = \phi_0 + \nu_0 t + \tfrac{1}{2}\dot{\nu} t^2, \tag{1}$$

$$R(t) = [\phi(t) - N(t)]/\nu_0. \tag{2}$$

In these equations, $\phi(t)$ is the phase of the pulsar wave form at barycentric time t, in cycles; ν_0 and $\dot{\nu}$ are the pulsar frequency and its derivative; and $N(t)$ is the "pulse number" at time t, or simply the closest integer to $\phi(t)$. The residuals $R(t)$ represent the difference between observed and expected pulse arrival times, based on the pulsar position and proper motion given in Table 8 of Downs and Reichley (1983) and on the values used for ϕ_0, ν_0, and $\dot{\nu}$.

We solve for best fit values of ϕ_0, ν_0, and $\dot{\nu}$ by minimizing the summed squares of the 375 residuals. The resulting values of ν_0 and $\dot{\nu}$ were consistent with those measured by Gullahorn and Rankin (1978) and Helfand *et al.* (1980). The procedure yielded a set of postfit residuals $R(t_i)$, $i = 1, 2, \ldots, 375$, that form the basis of our subsequent analysis.

In the absence of strong gravitational waves, and if PSR 1237+25 were a "perfect" pulsar, we would expect to obtain timing residuals with a zero-mean Gaussian distribution reflecting the random uncertainties inherent in the measurements. The residuals should be uncorrelated, and their spectrum should be flat down to a frequency of about $2/T \approx 2/(11.3 \text{ yr}) \approx 0.6 \times 10^{-8}$ Hz. In fact, the observed residuals approximate these expectations very well: the amplitude distribution is close to Gaussian, and the power spectrum (illustrated in Fig. 1)

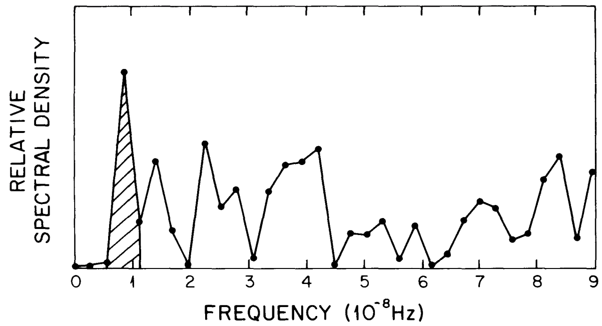

FIG. 1.—Power spectrum of the timing residuals for PSR 1237+25, after averaging them in bins of a length of 64.6 days. The shaded area corresponds to the fraction of the total fluctuation power concentrated at frequencies below about 10^{-3} Hz. The very low spectral density below 0.3×10^{-8} Hz is an artifact of having already removed a parabolic fit to the data.

is essentially flat over a 10 to 1 frequency range. The spectrum in Figure 1 was computed by binning the residuals into 64 equally spaced blocks, averaging the measurements within each block, and computing the discrete Fourier transform of the averages.

As pointed out by Detweiler (1979), the magnitude of observed pulsar timing residuals can be used to place an upper limit on the energy density of a stochastic background of gravitational waves. On the assumption that the gravitational wave energy spectrum is flat, is centered on some frequency $f > T^{-1}$, has a bandwidth equal to f, and is solely responsible for the nonzero timing residuals, Detweiler shows that the equivalent mass density in gravitational waves is given by

$$\rho = \frac{243\,\pi^3 f^4 \sigma^2}{208\,G}, \tag{3}$$

where G is the constant of gravitation and σ is the rms timing residual. For our 64 binned residuals, we find $\sigma = 240\ \mu$s. If we take $f = 0.7 \times 10^{-8}$ Hz, we obtain the limit $\rho \lesssim 8 \times 10^{-32}$ g cm^{-3}.

It is possible to set stronger limits and at the same time correctly ascribe the timing fluctuations to a particular range of fluctuation frequencies, by making use of the power spectrum computed from the binned data. For example, as shown by the shaded region in Figure 1, only about 14% of the total fluctuation power lies at frequencies below about 10^{-8} Hz. Therefore, for the band $f = (0.70 \pm 0.35) \times 10^{-8}$ Hz, the appropriately scaled equivalent rms residual is $\sigma = (0.14)^{1/2}(240\ \mu\mathrm{s}) = 90\ \mu$s. The corresponding energy density limit for gravitational waves is then $\rho \lesssim 1.0 \times 10^{-32}$ g cm^{-3}. This value and corresponding results for the frequency ranges $(2 \pm 1) \times 10^{-8}$ and $(6 \pm 3) \times 10^{-8}$ Hz are listed in Table 1. For comparison, we note that the density needed to close the universe in standard cosmological models is $\rho_c = 2 \times 10^{-29}$ (H_0/100 km s^{-1} Mpc^{-1}) g cm^{-3}, where H_0 is the Hubble parameter.

TABLE 1

UPPER LIMITS FOR GRAVITATIONAL RADIATION

Frequency Range (10^{-8} Hz)	Equivalent Mass Density, ρ (g cm^{-3})	Flux Density, F (ergs cm^{-2} s^{-1} Hz^{-1})
0.70 ± 0.35 ...	$< 1.0 \times 10^{-32}$	$< 3.9 \times 10^{7}$
2 ± 1.......	$< 1.3 \times 10^{-30}$	$< 1.7 \times 10^{9}$
6 ± 3.......	$< 2.6 \times 10^{-28}$	$< 1.2 \times 10^{11}$

Other methods of delimiting the energy density of gravitational waves in the universe have been discussed by Zimmerman and Hellings (1980). In Figure 2 we present an adaptation of their Figure 4, showing various experimental upper limits on the flux density of gravitational radiation, plotted as a function of frequency. As suggested in the figure, the limits come from analysis of orbital perturbations of Mercury and Mars, from Doppler tracking of spacecraft, from terrestrial seismic data, and from gravity-wave detectors of the "Weber bar" and laser-interferometer types. We also show the sloping line which gives the flux density of gravitational waves (with fractional bandwidth unity) that is equivalent to the critical mass density ρ_c. (See Zimmerman and Hellings 1980 for further details and for references to the original data.)

Upper limits to the flux densities of gravitational radiation can be computed from our equivalent mass densities from the relation $F = \rho c^3/f$, where c is the speed of light. These flux densities (averaged over each of the three frequency ranges) are listed in the last

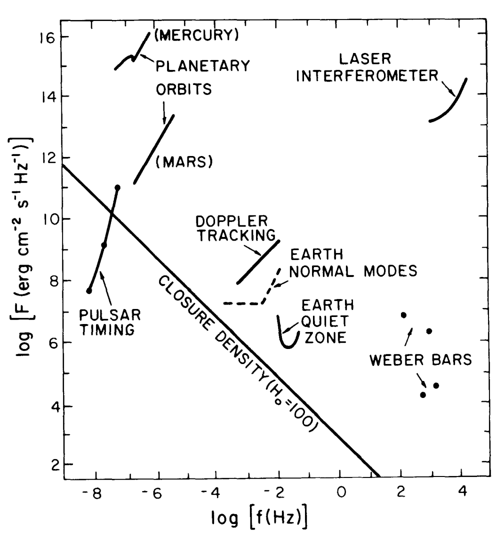

FIG. 2.—Experimental upper limits on the cosmic gravitational radiation background, compared with the flux density, required to close the universe (after Zimmerman and Hellings 1980). The results of this *Letter* are labeled "pulsar timing."

column of Table 1. As can be seen in Figure 2, two of these flux densities are well below the amounts needed to close the universe on the basis of low-frequency gravitational radiation alone. Our results appear to be the most stringent limits on cosmological gravitational radiation yet available. They demonstrate that, at the present epoch, the mass density of the universe is not dominated by gravitational waves of frequency $\sim 10^{-8}$ Hz.

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