

## ON THE EVOLUTION OF THOSE NUCLEI OF PLANETARY NEBULAE THAT EXPERIENCE A FINAL HELIUM SHELL FLASH<sup>1</sup>

ICKO IBEN, JR., JAMES B. KALER, AND JAMES W. TRURAN  
 University of Illinois

AND

ALVIO RENZINI  
 Università di Bologna

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### ABSTRACT

We suggest that some of the central stars of planetary nebulae experience a final thermal pulse after having achieved a white dwarf configuration and begun their descent along a cooling white dwarf sequence of nearly constant radius. A concrete theoretical calculation demonstrates that, during such a pulse, most of the hydrogen remaining in the star at pulse onset is incorporated into the helium-burning convective shell and completely burned, and that, following the pulse, the star swells briefly to red giant dimensions. The model then proceeds to burn helium on a long time scale, retracing in the H-R diagram approximately the same path that it followed while burning hydrogen during the initial excitation of the nebula, which has by now expanded considerably in extent.

We identify as being in the postpulse, quiescent helium burning phase the central stars of the planetary nebulae Abell 30 and Abell 78, and the central stars of a group of related high-excitation objects. These nebulae all have the large radii often found in conjunction with central stars of low luminosity that are thought to be cooling along the white dwarf sequence; however, they have the high luminosities that are characteristic of much smaller nebulae whose nuclei are thought to be proceeding for the first time through the planetary nucleus regime in the  $(\log L, \log T_e)$ -plane. Furthermore, both Abell 30 and Abell 78 show evidence for recent ejection of helium-rich matter from the surfaces of the central stars, and we speculate that the inferred absence of hydrogen at the surfaces may be due to either (1) a final hydrogen-burning episode during the peak of the final thermal pulse; (2) a mixing episode that may occur during the giant phase following the last thermal pulse, when a deep surface convective zone is expected to appear; or (3) a wind that operates during the high-luminosity quasi-static helium burning phase. The characteristics of R CrB stars, which we identify as erstwhile white dwarfs which have just experienced a thermal pulse and become giants, suggest that one or both of the first two mechanisms must occur in at least some cases; however, the magnitudes of mass loss rates that have been estimated for Abell 30 and Abell 78 are certainly large enough that the high helium and carbon abundances characteristic of intershell material will appear at the surfaces of the central stars when they are in the extended helium burning phase, even if neither of the first two mechanisms is effective.

*Subject headings:* nebulae: planetary — stars: interiors — stars: white dwarfs

The discovery of zones of matter in the inner portions of the planetary nebulae Abell 30 (Hazard *et al.* 1980) and Abell 78 (Ford and Jacoby 1983), which exhibit a He/H ratio that is at least 20, provides a significant challenge for theoretical and observational exploration. In this paper we put forth a possible explanation of the phenomenon, building on discussions by Renzini (1981) and Iben and Renzini (1982*a*) and on calculations by Schönberner (1979) and Iben (1982) which demonstrate

that a star can undergo a final thermal pulse while descending along the white dwarf cooling track and that much of the residual hydrogen envelope may be engulfed by the helium-burning convective shell.

In our calculations of the evolution of a model central star during the thermal pulse phase, we bypass the numerical difficulties associated with the explicit burning of hydrogen after its ingestion into the helium-burning convective shell and follow the evolution of the model during the quiescent helium-burning phase that takes place after the pulse has subsided. We find first that, during the pulse, most of the hydrogen which

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resides in a thin surface layer at the start of the pulse is absorbed by the helium-burning convective shell. Furthermore, sufficient nuclear energy is converted into excess thermal energy during the pulse that, when this excess thermal energy is subsequently converted into gravitational potential energy, the model star proceeds to expand to giant dimensions. The model then embarks on an extended phase of quiescent helium burning, during which it retraces very closely (and on approximately the same time scale) the same path in the H-R diagram which it traversed during the final hydrogen-burning phase (after first having ejected a planetary shell and after having achieved high enough surface temperature to light up the shell as a planetary nebula). Thus, the planetary shell brightens again, but now at a much larger radius than during the first phase of excitation.

The dashed curve in Figure 1 is an evolutionary track in the H-R diagram for a  $0.6 M_{\odot}$  hydrogen-burning model star after it has left the asymptotic giant branch (Iben 1982). The relevance to planetary nebulae of this track, and of similar tracks by Paczyński (1971), Gingold (1974), and Schönberner (1981), is discussed by Iben and Renzini (1982*a*). At the lower end of the dashed track, the model experiences a final helium-burning

thermal pulse that sends it to a position of low temperature and high luminosity, from which it returns to trace a second path through the plane at high surface temperatures. The track followed by the model during this high temperature, quiescent helium-burning phase is shown by a heavy solid curve. Time scales along the dashed curve are indicated by unslanted numbers, whereas time scales along the solid curve are shown by slanted numbers. Along the dashed track,  $t$  is defined as zero where the surface temperature of the central star reaches 30,000 K, about the time when the associated nebula first becomes visible as a planetary. The final pulse occurs about 19,000 years after this instant. The brightening portion of the postpulse track is very rapid, taking only about a century, and we begin  $t = 0$  again along the solid track at 30,000 K. Thus the age of the associated nebula at the end of the solid second track is  $43 + 19$  thousand years.

In Figure 2 we present blackbody He II Zanstra temperatures and luminosities for all central stars with known magnitudes for which  $\lambda 4686$  He II is observed in the nebula. The stars are plotted as numbers that give the radii of the associated nebulae in hundredths of a parsec, where the distance scale is that used by Cahn and Kaler (1971). Nebulae with radii above 0.175 pc are

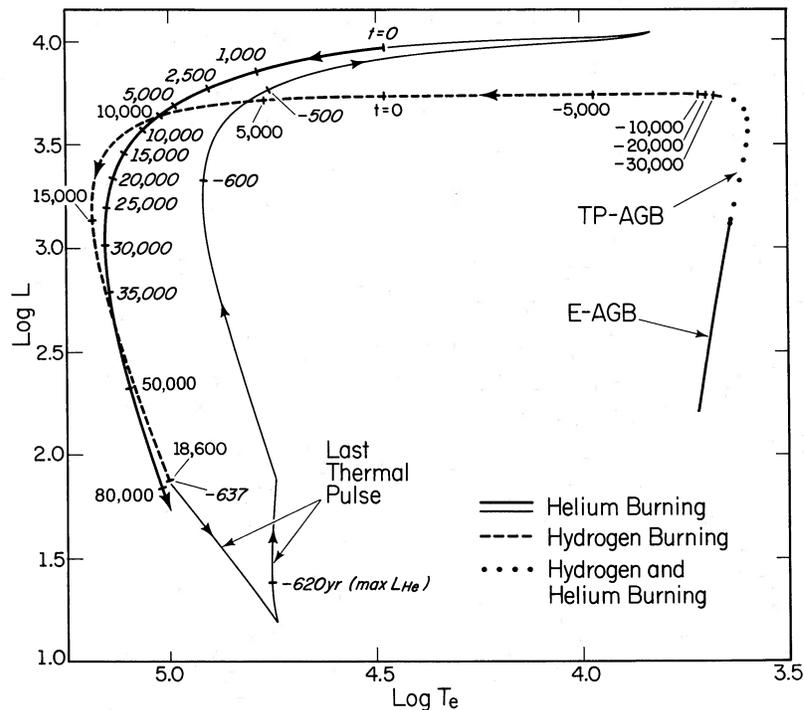


FIG. 1.—Post-horizontal-branch evolutionary track in the H-R diagram for a model star of mass  $M = 0.6 M_{\odot}$ . The locations of the early (nonthermally pulsing) AGB phase and of the thermally pulsing AGB phase are represented, respectively, by the solid line and the heavy dots at low surface temperature. Along the dashed curve, the model burns hydrogen quiescently, and times to reach points on this track beyond the point where  $T_e = 30,000$  K are denoted in years by unslanted numbers. Along the thin solid track which departs from the dashed track at  $t = 18,600$  yr the model experiences a thermal pulse. Along the heavy solid track following the thin solid track the model burns helium quiescently. Times to evolve to points along this track from the point where  $T_e = 30,000$  K are given in years by slanted numbers.

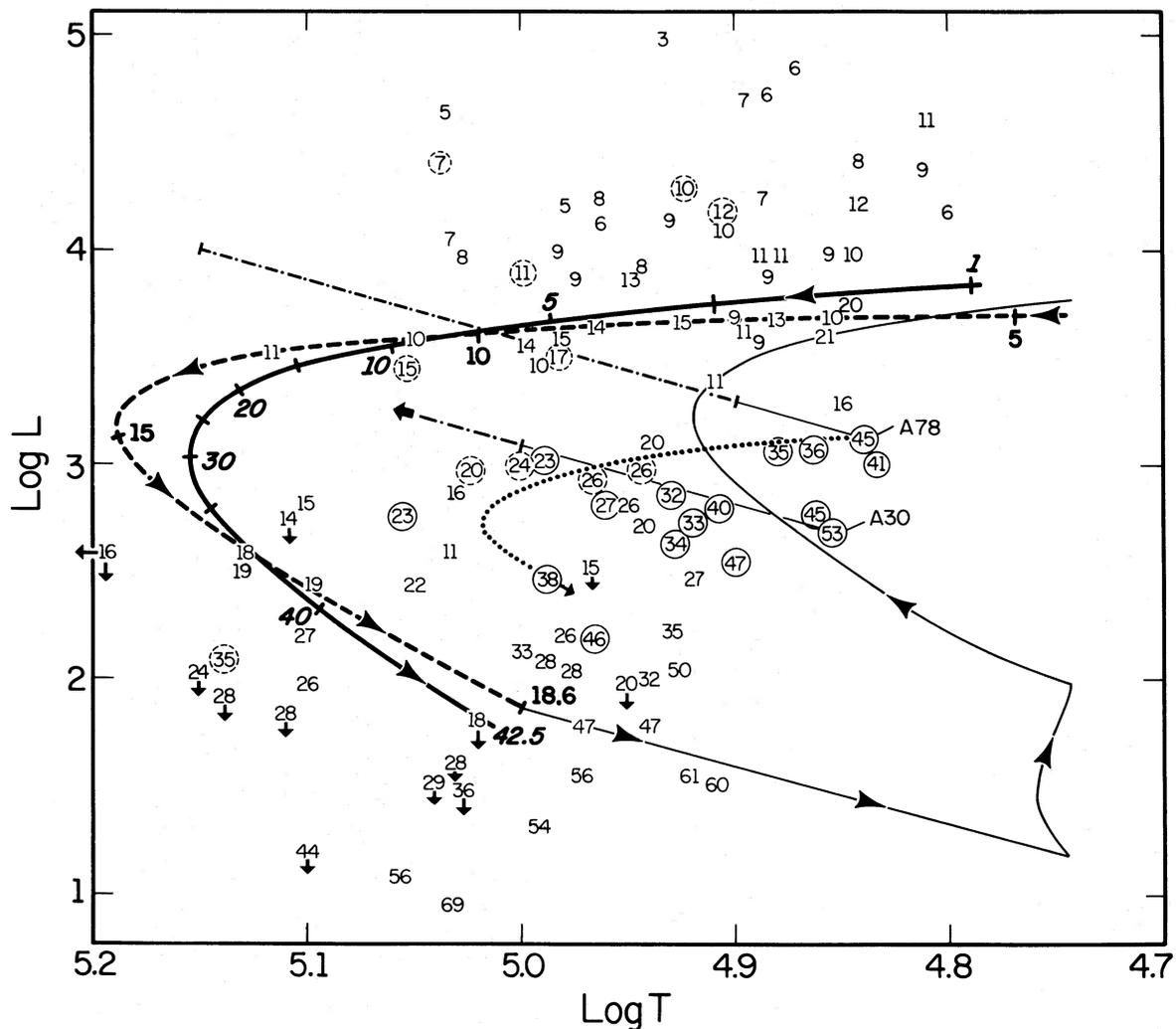


FIG. 2.—Planetary nuclei and evolutionary tracks in the  $(\log L, \log T)$ -plane. The positions of the central stars are expressed by numbers that give the nebular radii in hundredths of a parsec. The downward arrows indicate upper limits to both radius and luminosity. The circled figures represent lower limits to temperature and luminosity. The solid circles are Kaler's (1981) set of high-excitation nebulae. The solid lines ending at Abell 30 and Abell 78 (so labeled) are loci in  $T$  and  $L$  along which the star should lie. *Dash-dot lines*, likely temperature ranges for the central stars of A30 and A78; *dashed line*, Iben's (1982) track for a core of  $0.6 M_{\odot}$ ; *solid line*, the new postpulse track; *dotted line*, extrapolated track for a core of  $0.55 M_{\odot}$  from Schönberner and Weidemann (1981). Boldface numbers indicate ages in thousands of years along each loop, each starting  $t = 0$  at 30,000 K. Times on the helium-burning track are indicated by slanted numbers. The upward postpulse path takes less than 100 yr. The maximum luminosity and minimum surface temperature on the postpulse path are given at the right of the figure.

taken from Kaler (1983); the data on the smaller nebulae are from work still in progress. The downward arrows indicate stars for which the H and He II Zanstra temperatures are the same, which are optically thick, and for which distance, radius, and luminosity are probably overestimated. The circled numbers refer to nebulae for which helium is nearly fully doubly ionized, which implies that the nebulae are optically thin in the He<sup>+</sup> Lyman continuum, and that both  $L$  and  $T_e$  for the central star are lower limits. For these, the distances and nebular radii are correct insofar as the suppositions of the distance scale apply. The solid circles position the

central stars of the high-excitation nebulae considered by Kaler (1981), with Abell 30 and Abell 78 being flagged as A30 and A78, respectively.

For comparison with the theory, we reproduce in Figure 2 a portion of the  $0.6 M_{\odot}$  track from Figure 1 and also add a portion of the hydrogen-burning track for a  $0.55 M_{\odot}$  central star given by Schönberner and Weidemann (1981). Note first that the circles form an isolated group, and most lie close to but below the  $0.55 M_{\odot}$  track. They are almost certainly not on the upward, low-luminosity portion of the postpulse track which the  $0.6 M_{\odot}$  model star traverses in only about 100 yr.

Remember also that  $T_e$  and  $L$  for these stars are lower limits. The solid lines beginning at the A30 and A78 points are the loci along which they would lie as the  $\text{He}^+$  Lyman optical depth is increased. *IUE* spectra by Kaler and Feibelman (1983) indicate that<sup>2</sup> for A78  $\log T_e$  lies between 4.9 and 5.15, with a most likely value of 5.0. Greenstein (1981) considers  $\log T_e$  for A30 to be between 5.0 and 5.3. These ranges are shown by the dot-dash lines in Figure 2. Given the large size and the consequent ages of the A30 and A78 nebulae, we might expect the central stars to lie on the lower portion of a cooling track, along with the other large nebulae. *However, when moved up and to the left along their loci, they are found in the horizontal regime of the evolutionary tracks, in and among much smaller, presumably much younger, nebulae.* The central stars of A30 and 78 have Wolf-Rayet spectra (Heap 1982) and display powerful P Cygni profiles in the ultraviolet (Greenstein 1981; Heap 1979); both of these characteristics attest to mass loss and to great stellar luminosities. We conclude that these and perhaps some of the other stars with large nebular radii discussed by Kaler (1981) may be identified as being in the second, postpulse, quiescent helium-burning phase. The large nebular radii are qualitatively consistent with the evolutionary rates calculated for the model central stars. Renzini (1982) estimates that about 10% of all planetary nuclei should undergo a final shell flash and, since the time scale for helium burning is comparable to the time scale for hydrogen burning as a planetary nebula (PN) nucleus, we would expect roughly one out of 10 PN nuclei to be in the quiescent helium-burning phase. This is roughly consistent with the statistics we present here.

Several cautionary statements regarding the observations are in order. The distances, and the luminosities which these distances imply, follow from the assumption of constant nebular ionized mass, and some systematic error may well be present. However, after discussing the situation in detail, Kaler (1982) concludes that the problem is not serious. Second, analysis of the small nebulae with which the large high-excitation planetaries are compared may be affected by systematic effects involving optical depth and erroneous central star magnitudes. These problems have not yet been addressed. Third, although our arguments are based on relative nebular sizes, the expansion velocities for the large nebulae are not well known. From Bohuski and Smith (1974), the expansion velocity of two nebulae, NGC 1360 and Abell 36, is about  $27 \text{ km s}^{-1}$ , not enough to make them two or three times the size of their neighbors in the  $(\log L, \log T_e)$ -plane, but more observations are clearly needed.

<sup>2</sup>We follow the usual convention of stellar evolution, wherein  $T_e$  refers to the effective surface temperature of the star. It must not be confused with the electron temperature of the nebula.

Finally, there is the matter of evolutionary rate as a function of core mass. From Iben and Renzini (1982*a*), one may estimate that evolution along the  $0.55 M_\odot$  hydrogen-burning track requires about twice the time for evolution along the corresponding portion of the  $0.6 M_\odot$  track. Thus, for similar expansion velocities, nebulae associated with  $0.55 M_\odot$  nuclei in their highest luminosity phase can grow roughly twice as large as nebulae associated with  $0.6 M_\odot$  nuclei in their highest luminosity phase. Except by the argument used for A30 and A78, we cannot at present assign a central star unambiguously to either a hydrogen- or helium-burning track for a given stellar mass, and some of the circled stars in Figure 2 may have masses low enough that the nebulae have had time to grow exceptionally large. It is very important to determine independently the temperatures and luminosities of these stars by means of ultraviolet observations, so that we know where they lie relative to the evolutionary tracks, and so that we can distinguish planetaries containing low-mass central stars in the hydrogen-burning phase from those that are burning helium quiescently following a final thermal pulse.

As a further test, the theory predicts the existence of high-excitation planetaries with radii over 1 pc and with central stars that lie in the H-R diagram near the maximum temperature "turnaround" of the second loop. These nebulae will have extremely low surface brightnesses. One such may be the very large nebula found by Heckathorn, Fesen, and Gull (1983).

We now comment on the crucial matter of surface abundances. A critical constraint upon our model is that imposed by the observation that the ejected matter in the inner compact knots of A30 is extremely hydrogen deficient (Hazard *et al.* 1980) and carbon rich (Greenstein 1981). The precise mechanism by which hydrogen exhaustion of the upper region of the stellar envelope (from which the compact knots were recently ejected) might be effected has not yet been established. Indeed, in our model a surface layer of hydrogen-rich material persists throughout the entire evolution. At the start of the thermal pulse, all of the hydrogen in the star is confined to a thin surface layer of mass  $M_e \approx 2.7 \times 10^{-4} M_\odot$ . As the pulse develops, the outer edge of the convective layer engendered in the helium-burning region extends outward as far as the hydrogen-rich layer. Since hydrogen is not burning in this layer, there is no entropy barrier to inhibit the inward mixing of hydrogen, and the outer edge of the convective layer progresses still further outward in mass. In our particular model, this outward progression continues until the surface layer containing hydrogen is reduced to a mass  $M_e \approx 5 \times 10^{-5} M_\odot$ . Thereupon the pulse begins to subside and the outer edge of the convective layer recedes inward in mass. There exist several possible processes which have not been taken into account in our calculation and which may be expected to modify this result.

They are: (1) the thermonuclear conversion of hydrogen into helium during pulse peak; (2) convective dilution during the giant phase following pulse peak; and (3) wind-driven mass loss during the quiescent helium-burning phase.

1. *Hydrogen dilution and burning.*—We have assumed in our calculations that, during pulse peak, all of the hydrogen which is incorporated into the helium-burning convective shell (whose mass achieves a maximum value of  $\Delta M_{\text{CSH}} \sim 0.027 M_{\odot}$ ) is converted into  $^{16}\text{O}$  and neutrons *without the release of energy*. This is, of course, totally unrealistic, but the complexities introduced by attempting to follow the details of burning and convective mixing demand either a very extensive (but highly uncertain) treatment or dramatic simplification. We have chosen the latter course and have thereby been able to follow the subsequent evolutionary phases, which are to a large extent independent of the details of nuclear burning during pulse peak. This is because the total energy release associated with the exhaustion of the ingested protons represents less than 10% of the helium-burning output and the extent to which the star expands following pulse peak is sensitive only to the total amount of energy released during pulse peak. Furthermore, the evolution during the quiescent helium-burning phase is totally unaffected by the detailed course of the preceding thermal pulse.

However, what exactly happens to the hydrogen which is engulfed by the helium-burning convective shell during pulse peak can certainly have a profound effect on surface abundances. As hydrogen is convected downward (and thereby diluted in abundance) to regions of sufficiently high temperatures and densities, it will begin to burn via the  $^{12}\text{C}(p, \gamma)^{13}\text{N}$  reaction. The energy released by this and ensuing reactions may lead to the splitting of the original convective zone into two convective zones separated by a thin radiative zone, with helium burning at the base of the inner convective shell and hydrogen burning at the base of the outer one, as found (in another evolutionary context) by Sweigart (1974). Further, the increase in energy flux passing through the surface layers is expected to promote instability against convection even further into the hydrogen-rich surface material than in our calculation, perhaps even causing the outer edge of the outer convective shell to extend essentially to the very surface of the star.

At the base of the convective shell, the precise sequence of burning reactions which follows the conversion of some  $^{12}\text{C}$  into  $^{13}\text{N}$  depends both on the ratio of protons to  $^{12}\text{C}$  at the start of the burning phase in this shell and on the prevailing temperature. In the limit that the proton concentration is initially less than that of  $^{12}\text{C}$ , the burning will effectively terminate with the formation of  $^{13}\text{C}$ ; no new  $^{14}\text{N}$  will form, but preexisting

$^{14}\text{N}$  in the outer convective shell will survive. In the limit that the proton abundance far exceeds that of  $^{12}\text{C}$ , the reactions  $^{13}\text{C}(p, \gamma)^{14}\text{N}(p, \gamma)^{15}\text{O}(e^+ \nu)^{15}\text{N}(p, \alpha)^{12}\text{C}$  will build up fresh  $^{14}\text{N}$  and  $^4\text{He}$ . In either case, at the end of this hydrogen-burning phase the star will be left with the hydrogen-exhausted surface layer consisting of, among other things,  $^4\text{He}$ ,  $^{12}\text{C}$ ,  $^{13}\text{C}$ , and  $^{14}\text{N}$ . In general, we estimate that carbon will exceed nitrogen in abundance as a consequence of this phase, for the conditions which obtain in our particular model. However, if the number of protons introduced into the shell exceeds the number of  $^{12}\text{C}$  nuclei, the net result will be an abundance ratio  $\text{N}/\text{C} > 1$ .

Should hydrogen burning go to completion on a time scale short compared to the duration of the peak helium burning phase, the helium-burning convective shell may extend outward again into the matter which has just experienced hydrogen burning. The  $^{13}\text{C}$  that will thereupon be swept into the convective shell will disappear via the reaction  $^{13}\text{C}(\alpha, n)^{16}\text{O}$ , and the neutrons so released will produce heavy neutron-rich isotopes (on  $^{56}\text{Fe}$  as a seed), which will be subsequently distributed over the entire convective shell. The character of this process of neutron addition will be a sensitive function of both the  $^{14}\text{N}/^{13}\text{C}$  ratio in the shell and the effective temperature at which  $^{13}\text{C}$  burns. When  $^{14}\text{N}/^{13}\text{C} \geq 1$ , the  $^{14}\text{N}(n, p)^{14}\text{C}$  reaction will impose a restriction on the number of neutrons available for capture on heavy nuclei. The time scale of neutron release is essentially the lifetime of  $^{13}\text{C}$  with respect to the  $^{13}\text{C}(\alpha, n)^{16}\text{O}$  reaction. For the conditions appropriate to our model (at the base of the helium-burning convective shell,  $T \sim 3 \times 10^8$  K and  $\rho \sim 10^3$  g cm $^{-3}$ ), the neutrons are released on a time scale of order 100–1000 seconds. Ratios of neutrons to  $^{56}\text{Fe}$  seed nuclei of order 10–100 might reasonably be achieved. It thus appears that significant neutron exposures may be realized on a rather short time scale, assuming that  $^{14}\text{N}$  does not become a problem. It seems possible that some of the identified meteoric isotropic anomalies may have their origin in such an environment.

2. *Surface convective dilution.*—Another mechanism which may assist (in some stars) the transformation of the hydrogen-rich envelope into a helium-rich envelope involves the inward advance of a surface convective zone which appears and grows as the star expands to giant dimensions following pulse peak. As the  $0.6 M_{\odot}$  model star in the current study expands to giant dimensions, the base of a surface convective zone begins to extend inward through the hydrogen-rich layer, but formally achieves a maximum inward extent of only  $10^{-5} M_{\odot}$ , thereby failing to reach the base of the hydrogen-rich envelope. Thus, in our simplistic calculation, the central star of the planetary nebula retains a surface He to H ratio small compared to 1. However, process (1)

just considered may reduce the hydrogen-rich layer which survives convective engulfment during pulse peak to a mass much smaller than  $10^{-5} M_{\odot}$ , in which case surface convection during the giant phase will be effective in diluting this layer with fully processed material in its interior. Further, there is nothing sacrosanct about our choice of model mass, and it is known that, in asymptotic giant branch (AGB) model stars of core mass as large as  $0.7 M_{\odot}$ , the base of the convective envelope extends inward into the intershell region following each pulse (e.g., Iben 1975; Wood 1981). In such stars, for which the mass of the hydrogen-rich surface layer is large compared to the mass in the He-C layer below, the consequences of the ensuing outward mixing of carbon and helium are only mildly dramatic: a surface abundance of  $C > O$  is achieved, but hydrogen remains by far the most abundant element at the surface. In contrast, in a model star undergoing a final flash in the white dwarf configuration, the total amount of hydrogen-rich material that remains to mix with the outer edge of the C-He layer (which has a mass  $\Delta M_{C-He} \sim 0.03 M_{\odot}$ ) is from four to five orders of magnitude smaller than in the case of typical AGB carbon stars. It therefore seems quite natural to suppose that the end result of mixing during the giant phase in a model with  $M \geq 0.7 M_{\odot}$  will be a surface at which He and C are by far the dominant elements. When the effects of semiconvection (Iben and Renzini 1982*b*) are taken into account, the critical mass for the occurrence of this form of mixing is reduced to perhaps as low as  $\sim 0.62 M_{\odot}$ .

3. *Wind ejection.*—The occurrence of wind-driven mass loss associated with the central stars of planetary nebulae is currently inferred from the presence of P Cygni profiles in their spectra and reported mass loss rates range from  $10^{-10} M_{\odot} \text{ yr}^{-1}$  up to  $\sim 10^{-6} M_{\odot} \text{ yr}^{-1}$  (cf. Heap 1980; Pottash 1981; Perinotto, Benvenuti, and Cacciari 1981; Castor, Lutz, and Seaton 1981). Mass loss occurring during the shell hydrogen-burning phase of evolution at high luminosities may influence the time scale of evolution of the central stars of planetary nebulae at high luminosities, but it should only marginally affect the mass of the residual hydrogen-rich envelope at the onset of the final flash. The initial evolution along the white dwarf cooling track through the onset and early development of the thermal pulse proceeds on too rapid a time scale to allow substantial mass loss via a simple wind. Following the thermal pulse, while the shell helium-burning star retraces very closely, and on approximately the same time scale, its earlier hydrogen-burning path in the H-R diagram, mass loss may be effective in removing the last vestiges of hydrogen from the surface. Note that, unlike during the preflash evolution, stellar luminosity is now unaffected by the mass of the hydrogen-rich envelope. Neglecting processes (1) and (2), the development of a surface lacking any hydrogen requires that  $\sim 5 \times 10^{-5} M_{\odot}$  of material be removed

on a time scale of  $\sim 10^4$  years, implying a mass loss rate of only  $\geq 5 \times 10^{-9} M_{\odot} \text{ yr}^{-1}$ . This is well within existing estimates of mass loss rates, and we may be reasonably confident that mass loss during the high-luminosity portion of the postflash evolution will very likely remove the residual hydrogen-rich surface layer, thus exposing the helium- and carbon-rich intershell material.

The three mechanisms which we have described have in common their prediction that the resulting surface matter will be enriched in helium and carbon. They differ, however, in their predictions as to the surface nitrogen abundance. Both the thermonuclear conversion of the surface hydrogen into helium during the peak phases of the pulse and the process of convective dilution during the giant phase following the pulse peak are compatible with the presence of an observable nitrogen abundance. In contrast, wind-driven mass loss during the quiescent helium burning phase will ultimately lay bare intershell matter which contains no nitrogen (during a thermal pulse, N is totally destroyed in the intershell). It is thus possible to draw inferences concerning the mechanism of hydrogen depletion from direct observations of the compositions either of the central stars or of the inner shells of nebulae such as Abell 30 and Abell 78.

Most PN nuclei of the Wolf-Rayet type belong to the subclass WC (Heap 1982), with no detectable N lines. In the particular cases of A30 and A78, however, the powerful P Cygni profiles of [N v] show that N is present in the atmospheres of the central stars (they are classified as WCN stars by Heap 1982), and we suggest that this is evidence for either dilution and thermonuclear processing of hydrogen during pulse peak and/or dilution by surface convection during the giant phase following pulse peak. The central stars of A30 and A78 are efficiently losing mass, and the surviving N-containing skin may well be removed on a relatively short time scale. It would be most interesting to follow the subsequent evolution by finding the successors of the A30–A78 stage. We suggest that these stars may ultimately be identified with non-DA white dwarfs.

Surface abundances in R CrB stars may also be of relevance. The absence of hydrogen and the presence of high abundances of helium and carbon at their surfaces makes it tantalizing to associate this class of stars with the events discussed so far (Renzini 1979, 1981, 1982). They are giant stars, with  $L$  from a few times  $10^3 L_{\odot}$  to  $\sim 10^4 L_{\odot}$ ,  $T_e \sim 6000$  K, and  $R \sim 100 R_{\odot}$ . Further, they are known to be losing mass at fairly high rates (Alexander *et al.* 1972; Feast and Glass 1973). It is of interest that our model achieves and briefly retains luminosities and surface temperatures consistent with those characteristic of R CrB stars.

However, the duration of the lowest temperature phase in our particular model is probably too short to be consistent with the observed frequency of R CrB stars.

Since the maximum radius which a model will achieve is directly related to the violence of the preceding thermal pulse, and since the violence of this pulse is proportional to shell densities when the pulse is initiated, we expect that progenitors of R CrB stars are white dwarfs in which the final thermal pulse has been delayed to much lower luminosities than in the present instance. The additional time spent by the precursor white dwarf at low luminosities might be sufficiently long that the associated planetary nebula grows to such large dimensions (and correspondingly low densities) that it is no longer recognized. This may account for the fact that R CrB stars do not appear to be surrounded by nebular material, but it would be interesting to attempt a more serious search for the presence of very diffuse nebulosity about such stars.

The abundance at the surfaces of R CrB stars of elements other than helium and carbon, notably nitrogen and neutron-rich isotopes, can shed additional light on the formation process(es) of hydrogen-deficient stars of all types. The overabundance of Sr and Y in the R CrB star U Aqr (Bond, Luck, and Newman 1979) and that of Ba in XX Cam and in R CrB itself (Hunger, Schönberner, and Steenbeck 1982) strengthens our interpretation of an R CrB star as the result of a final thermal pulse which is initiated when the star is in a dwarf configuration. The difference in the abundance patterns of neutron-rich isotopes in these stars can also

naturally follow from the expected wide variation in the amount of hydrogen ingestion which these stars can have experienced.

In summary, two possible paths can, at this stage, be envisaged for the evolution of a PN nucleus past the final thermal pulse. In the first case, the pulse occurs well up on the white dwarf cooling track while the star is surrounded by a visible planetary nebula. The subsequent giant phase is of too short a duration to be observable, and the star next appears on the high-luminosity portion of the quiescent helium-burning track, where it may be identified with the nuclei of planetaries like Abell 30 and Abell 78. In the second case, the pulse occurs farther down on the track, when the surrounding nebulae would no longer be visible on existing surveys. The star remains longer in its giant state, where it is identified with the R CrB stars. Eventually, both of these fade to become non-DA white dwarfs. The R CrB stars should be examined with modern imaging techniques in order to test for the presence of faint surrounding planetary halos. We must further assess the frequency of occurrences of stars like A30 and A78, and try to identify their successors. Finally, although the detailed theoretical calculation of this series of events presents severe technical difficulties, exploratory calculations could be attempted using the simplified algorithm adopted by Sweigart (1974) in modeling the region where the ingested hydrogen is burned.

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*Note added in proof.*—The recent determination by Reay, Atherton, and Taylor (*IAU Symposium 103, Planetary Nebulae*, 1982) of velocities of 22–25 km s<sup>-1</sup> for the helium-rich knots associated with A30, coupled with the estimate by these authors of 1500 yr for the kinematic age of the knots, supports our hypothesis. The apparent age of the knots relative to the kinematically estimated age of ~20,000 yr for the primary nebula (from an assumed similar expansion velocity of 25 km s<sup>-1</sup>) indicates the occurrence of a secondary phase of rapid mass loss from the central star following

the phase which produced the primary nebula. The low expansion velocities of the knots suggest that the second ejection of mass must have occurred when the primary star had giant dimensions, just as in the case of the first ejection.

The simultaneous presence of nitrogen and absence of hydrogen in the knots (Ford and Jacoby 1983) supports the view that the final phase of hydrogen burning occurs in a convective shell detached from the main helium-burning convective shell, and that the mechanism which ejects the knots is for some reason limited to the ejection of material which lies above the base of the region that has undergone the final phase of hydrogen burning. The fact that A30 is one of the few central stars with nitrogen remaining at the surface (Greenstein 1981; Heap 1982) suggests both that a central star supports a wind during its condensed phase (while it is a luminous compact object) and that this wind must be a relatively mild one not to have exposed the region which has experienced extensive helium burning.

ICKO IBEN, JR., JAMES B. KALER, and JAMES W. TRURAN: Department of Astronomy, University of Illinois, 341 Astronomy Building, 1011 West Springfield, Urbana, IL 61801-3000

ALVIO RENZINI: Osservatorio Astronomico, C. P. 596, 40100 Bologna, Italy