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The maximum energy of cosmic rays accelerated by supernova shocks

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Summary. The aim of this paper is to evaluate the maximum energy E_{max} that particles subjected to the process of diffusive shock acceleration can acquire during the lifetime of a supernova remnant. The rate of acceleration depends on the particle diffusion coefficient, which is determined by the level of hydromagnetic wave energy present at a scale comparable to the particle Larmor radius. We study the variations of the diffusion coefficient as a function of momentum, space, and time.

In the most optimistic case, the diffusion mean free path is everywhere comparable to the particle Larmor radius; then $E_{max} \sim 10^5$ GeV/n. Considering a more realistic behaviour of the diffusion coefficient, we obtain $E_{max} \lesssim 10^4$ GeV/n. Thus, supernova shock acceleration cannot account for the observed spectrum of galactic cosmic rays in the whole energy range 1–10^6 GeV/n.

Key words: cosmic-ray acceleration – shock waves – hydromagnetic waves

I. Introduction

In recent years, the idea that galactic cosmic rays are accelerated by supernova shock waves propagating in the interstellar medium has become very popular (see review by Axford, 1981; and introduction of Lagage and Cesarsky, 1983, hereinafter referred to as LC). The mechanism of acceleration by diffusive shock waves is especially attractive mainly because it matches the slope of the galactic cosmic ray spectrum corrected from interstellar propagation effects. However many authors have recognized that one of the main difficulties of this mechanism is that it cannot accelerate particles beyond a certain energy E_{max}. Widely different estimates of E_{max} exist at present in the literature, ranging from ~ 10 GeV (Völk et al., 1981) to 10^6–10^7 GeV (Krymsky et al., 1979a). In between one finds 300 GeV (Blandford and Ostriker, 1978), 3000 GeV (Federenko, 1982), 10^5 GeV (Cesarsky and Lagage, 1981; Ginzburg and Ptuskin, 1981); and this list is not exhaustive. Consequently, we have found it worthwhile to reconsider this problem in more detail than has been done previously; our purpose here is to establish an upper limit to the energy that can be acquired by particles trapped in the vicinity of a supernova shock.

In Sect. II, we introduce most of the ingredients needed for this study: the rate of energy gain by particles interacting with a supernova shock; the evolution of a supernova remnant; the interstellar turbulence, responsible for the scattering of cosmic rays throughout the galaxy. We find that, if only this turbulence were available, the acceleration process would be so slow that during the lifetime of a supernova remnant a particle can at most acquire a few GeV.

In fact, in the vicinity of the shock, the level of turbulence is expected to be much above that of an average region of the interstellar medium. Upstream of the shock, the flux of cosmic rays interacting with the shock is highly anisotropic, and thus very unstable to the generation of hydromagnetic waves. These waves are in turn amplified by the shock, so that the downstream region must also be highly turbulent. Section III is devoted to the study of this self-generated turbulence, and its effect on cosmic ray diffusion. The rate of wave generation increases with the cosmic ray flux, and thus depends on the rate of injection of particles in the acceleration mechanism. We find that when the damping of the turbulence is ignored, the steady state solutions predict a cosmic ray diffusion coefficient at the shock which is below its lowest possible value, $D_{min} = (1/3)\, r_l v$ (where r_l is the Larmor radius and v the particle velocity), as soon as the injection flux is a few times the flux of the preexisting cosmic rays. We then examine some mechanisms that can hamper the wave growth. We also take approximately into account the fact that the waves take a finite and sometimes a long time to grow, especially far from the shock.

In Sect. IV, we evaluate the maximum energy, E_{max}, that can be attained in a variety of cases. A firm upper limit to E_{max} is obtained by assuming that the particle diffusion coefficient has everywhere its lowest possible value, D_{min}; then, for the most favourable scheme of supernova expansion, $E_{max} \sim 10^5\, Z/A$ GeV/n where Z is the charge of the accelerated particle and A its mass number. Introducing the space dependence of the diffusion coefficient derived in Sect. III, reduces E_{max} by a factor of order 10. We also consider other effects of lesser importance.

A discussion of our results is given in Sect. V, where we conclude that supernova shocks cannot explain the spectrum of galactic cosmic rays in the whole energy range 1–10^6 GeV/n.

II. Main ingredients

There are essentially two effects tending to limit the energy gained by particles interacting with a supernova shock: the finite lifetime of the shock and its curvature. Because the acceleration process is inherently slow, particles take a long time to diffuse back and forth across the shock; thus the energy attained by particles injected right at the beginning of the supernova remnant expansion and lucky enough to remain trapped in the vicinity of the shock, is limited.

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The diffusion coefficient, D, of the cosmic rays increases with energy. When the characteristic length of diffusion of the particles (of order D/u_1 where u_1 is the shock velocity) becomes comparable to the shock radius, the shock can no longer be considered plane. Particles which are interacting with a spherical shock loose some energy when travelling downstream, and have the possibility of escaping from the shock upstream; as a consequence, the acceleration mechanism is partially or totally quenched. At first we neglect the shock curvature and consider a parallel shock, i.e. such that the direction of the magnetic field is perpendicular to the plane of the shock; this direction is represented by the x axis, with x positive downstream. The cosmic rays are treated as test particles in the sense that their possible influence on the shock structure is not considered. Other geometries, and other assumptions on the role of cosmic rays, are considered briefly in Sect. IV.

To compute the energy reached by particles which are trapped in the vicinity of the shock throughout its lifetime, we use the sequential approach introduced by Krymsky et al. (1979b). If the diffusion coefficient of the particles, D_j and the gas velocity in the shock frame, u_j, ($j=1$, 2 means upstream, downstream) are constant in space and time, the mean time taken by a particle to cycle through the shock is:

$$T_{\text{cycle}}=\frac{4}{v}\left(\frac{D_1}{u_1}+\frac{D_2}{u_2}\right). \tag{1}$$

Approximating

$$\frac{dp}{dt}\sim\frac{\Delta p}{T_{\text{cycle}}}=\frac{u_1-u_2}{3}\frac{p}{\frac{D_1}{u_1}+\frac{D_2}{u_2}}, \tag{2}$$

where Δp is the mean gain of momentum in one cycle, and integrating, we obtain p as a function of time, t. In the case $D \neq D(p)$ it is possible to solve exactly the transport equation for the isotropic component of the particle distribution function f:

$$\frac{\partial f}{\partial t}+u_j\frac{\partial f}{\partial x}=\frac{\partial}{\partial x}\left(D_j\frac{\partial f}{\partial x}\right)-\frac{1}{3}p\frac{\partial f}{\partial p}(u_1-u_2)\delta(x), \tag{3}$$

where δ is the Dirac function. The matching conditions at the shock are: continuity of f and

$$D_2\left.\frac{\partial f}{\partial x}\right|_2-D_1\left.\frac{\partial f}{\partial x}\right|_1=\frac{1}{3}p\frac{\partial f}{\partial p}(u_1-u_2) \tag{4}$$

(Krymsky et al., 1978). An approximate asymptotic solution can be found when D depends on p (e.g. Axford, 1981). The results are in excellent agreement with Eq. (2). When u_1 is a function of time but does not vary much during one cycle, the appropriate approach is obviously to use Eq. (2) (Cesarsky and Lagage, 1981; LC). To integrate Eq. (2) we must specify the values of u_1 and D.

u_1 is the velocity of a supernova shock wave in the interstellar medium. Our present view of the interstellar medium is that it consists of a mixture of several phases, characterised by different temperatures and densities, but similar pressures. A hot and diffuse phase (HIM: density $n_e\sim 3\ 10^{-3}\ \text{cm}^{-3}$, temperature $T\sim 7\ 10^5$ K) is believed to occupy a large fraction of the galactic volume (McCray and Snow, 1979); diffuse clouds of density $\sim 10\ \text{cm}^{-3}$, $T\sim 100$ K are numerous; denser clouds may contain a large fraction of the total mass, but they are rare. The strength of the magnetic field in the hot interstellar medium is unknown; the available measurements of the interstellar field either refer to cold, neutral clouds (Zeeman splitting; $B<50\ \mu$G; Troland and Heiles, 1982) or, probably, to ionized regions much colder and dense than the HIM (pulsar dispersion measures; $B\simeq 3\ \mu$G; Heiles, 1976 and references therein). If the fields of the clouds and of the HIM are well connected, the HIM field should not be much lower than that of diffuse H II regions (McKee, 1981); we adopt $B_H=1\ \mu$G, but we will scale our results according to B_H. If heat conduction between clouds and intercloud material is inhibited by magnetic fields, so that cloud evaporation does not occur (Cox, 1979), a supernova remnant evolves as in a uniform medium of the HIM type. In a medium of density as low as $\sim 3\ 10^{-3}\ \text{cm}^{-3}$, the supernova shock decays into a magnetosonic wave before the end of the adiabatic phase of expansion, so that radiative losses are always negligible. Consequently, we consider only two phases: a blast wave phase, lasting until the shock has swept up a mass equivalent to that ejected by the star, and a Sedov type adiabatic phase, lasting until u_1 is equal to the sound velocity,

$$s=[\gamma P_g/\varrho+B_H^2/(8\pi)]^{1/2}, \tag{5}$$

where P_g is the gas pressure, ϱ the gas density, and γ the adiabatic index. If M_e is the mass ejected by the supernova in solar masses, and E_{51} the kinetic energy released in the explosion in units of 10^{51} erg, we have

$$u_1\sim 10^9\left(\frac{E_{51}}{M_e}\right)^{1/2}\ \text{cm/s} \tag{6}$$

for $t<t_1$, with

$$t_1\sim 1.5\ 10^3\ \frac{M_e^{5/6}}{[n_e/(3\ 10^{-3}\ \text{cm}^{-3})]^{1/3}\ E_{51}^{1/2}}\ \text{yr}. \tag{7}$$

At later times

$$u_1\sim 4.1\ 10^{10}\left(\frac{E_{51}}{n_e/(3\ 10^{-3}\ \text{cm}^{-3})}\right)^{1/5} t_{\text{yr}}^{-3/5}\ \text{cm/s} \tag{8}$$

for $t_1<t<t_{\max}$,

$$t_{\max}\sim 6\ 10^5\left(\frac{E_{51}}{n_e/(3\ 10^{-3}\ \text{cm}^{-3})}\right)^{1/3}\ \text{yr}. \tag{9}$$

Typically $M_e=0.5\,M_\odot$, $E=5\ 10^{50}$ erg for a supernova of type I (SN I) and $5\,M_\odot$, 10^{51} erg for a supernova of type II (SN II) (Chevalier, 1977).

If the evaporation of clouds engulfed by the shock is important, the evolution is different (McKee and Ostriker, 1977; Cowie et al., 1981). In the two first phases, the cloud evaporation is saturated; the supernova evolves as if it were propagating in a uniform medium of density $\sim 0.1\ \text{cm}^{-3}$. After $2.2\ 10^4\ E_{51}^{1/3}$ yr, corresponding to a shock radius of $\sim 29\,E_{51}^{1/3}$ pc, the evaporation becomes unsaturated; then u_1 decreases more slowly with time:

$$u_1\sim 2.8\ 10^9\ t_{\text{yr}}^{-2/5}\ \text{cm/s}. \tag{10}$$

At $t\sim 5.5\ 10^5$ yr, the shock decays into a sound wave.

In the framework of the quasi-linear theory, the diffusion coefficient of cosmic rays of momentum p in a weakly turbulent field is determined by the amount of hydromagnetic waves of wavelength comparable to the Larmor radius r_l of the particles:

$$D\sim\frac{4}{3\pi}\frac{r_l v}{\mathscr{F}}, \tag{11}$$

where v is the particle velocity and $\mathscr{F}(p)\,d(\text{Log}\,p)$ is the energy density in waves resonating with cosmic rays of momentum in the range p to $(p+dp)$, normalised to the ambient magnetic energy density, $U_M=B^2/(8\pi)$ (Wentzel, 1974 and references therein). We first consider the interstellar turbulence[1]. The interstellar medium is known to be very turbulent at long scales ($\geqq 10$ pc); it is plausible to assume that some of this turbulent energy is transferred to smaller and smaller scales, giving rise to a spectrum of turbulence (Cesarsky, 1971, 1975). If the energy is transferred to short scales in the time it takes for a turbulent eddy to cross its length at its own velocity, and in the absence of dissipation, the equilibrium spectrum of turbulence is the familiar "Kolmogorov spectrum": $\mathscr{F}_{i.}$ proportional to $p^{2/3}$. If the energy transfer rate is as proposed by Kraichnan (1965) in his theory of MHD turbulence, then $\mathscr{F}_{i.}$ is proportional to $p^{1/2}$. In neutral regions of the interstellar medium, the dissipation of waves due to collisions of charged and neutral particles is faster than the turbulent transport rate predicted by either of these theories, and such a spectrum cannot develop. But in the hot phase, wave damping is much weaker (Foote and Kulsrud, 1979), and the turbulence spectrum may be there.

Various types of observations of scintillations have been reviewed by Armstrong et al. (1981); these authors deduce the possible existence of an interstellar turbulent spectrum that is a power law: $\mathscr{F} \propto p^{\alpha}$ but they cannot distinguish between the Kolmogorov and the Kraichnan slope. If cosmic ray diffusion in the galaxy is governed by resonant particle-wave interactions, the observed variation of the ratio of secondary to primary particles with energy may simply reflect the existence of this interstellar turbulence spectrum. In the context discussed here, the recent observations of iron secondaries by HEAO 3 C2 in the 0.7 GeV/n to 25 GeV/n range (Koch-Miramond, 1981) indicate that the cosmic ray mean path length is proportional to $(p/Z)^{1/2}$, implying that $\alpha \sim 0.5$ as expected if the turbulence spectrum is of the Kraichnan form. The HEAO 3 C2 data on the boron abundance, however, seem to require a somewhat steeper dependence of the mean path length, and therefore a smaller value of α. On the other hand, including compilations of higher energy (up to $E \sim 100$ GeV/n) data from balloon observations, Protheroe et al. (1981) and Webber (1982) find a mean path length proportional to (energy per nucleon)$^{0.4}$. In the following we take

$$\mathscr{F}_{i.} = \mathscr{F}_0 \, [p/(\mathrm{GeV}/c)]^{1/2}. \tag{12}$$

$\mathscr{F}_0$ is a badly determined constant, whose value is probably in the range $(10^{-4}$–$10^{-6})B_{-6}^{-\frac{1}{2}}$ (see also Ginzburg and Ptuskin, 1976). Throughout this paper, we take $\mathscr{F}_0 = 10^{-5} B_{-6}^{-\frac{1}{2}}$, where B is in units of 10^{-6} G. The impact of the values of $\mathscr{F}_0$ and of α on the determination of $E_{\max}$ is discussed in Sect. IV. If only the interstellar turbulence were present, the cosmic ray diffusion coefficient would be as given by Eq. (11), with $\mathscr{F} = \mathscr{F}_{i.}$. Integrating (dp/dt) of Eq. (2) over the supernova lifetime, we find that in this case $E_{\max}$ is only a few GeV/n. Fortunately, the cosmic rays increase considerably the level of turbulence in the shock environment; the next section is devoted to studying this effect.

III. Turbulence in the shock vicinity

1. *Waves generated by the cosmic rays*

Upstream of the shock, cosmic rays streaming away from the shock at a velocity greater than the Alfvén velocity generate resonant hydromagnetic waves. The equation describing the evolution of the wave energy density upstream is (Bell, 1978):

$$\frac{\partial \mathscr{F}}{\partial t} + u_1 \frac{\partial \mathscr{F}}{\partial x} - \sigma \mathscr{F} + \Gamma \mathscr{F} = 0, \tag{13}$$

where σ is the growth rate and Γ the damping rate of the waves. Bell has given the steady state solution of Eqs. (3) and (13), for a parallel shock in the absence of external turbulence, and when damping effects are negligible. In particular, he found:

$$\mathscr{F}_B(x) = \frac{a}{x_0 - x}, \tag{14}$$

$$f_B = f_\infty + \frac{(f_0 - f_\infty)}{(1 - x/x_0)}, \tag{15}$$

with

$$a = \frac{4}{3\pi} \frac{v}{u_1} r_l, \tag{16}$$

$$x_0 = \frac{1}{\pi^2} \frac{U_M}{v_A p^4 (f_0 - f_\infty)} r_l, \tag{17}$$

where f_0 and f_∞ are respectively the values of f at the shock and at infinity. Taking into account the underlying interstellar turbulence (LC):

$$\mathscr{F} = \mathscr{F}_{i.} + \frac{\mathscr{F}_{i.}}{\left(1 + \dfrac{\mathscr{F}_{i.}}{\mathscr{F}_B(x=0)}\right) \exp\left(-\mathscr{F}_{i.} \dfrac{x}{a}\right) - 1}. \tag{18}$$

The minimum number of cosmic rays injected in the mechanism are the pre-existing cosmic rays, for which $f_\infty = f_g p^{-4.75} H(p - p_0)$ $(\mathrm{erg\,s})^{-3}$. We assume that the actual number of particles injected at p_0 exceeds this number by a factor η. Let us define the compression ratio of the shock: $r = u_1/u_2$ and the quantity $q = 3r/(r-1)$. In the following we write $\delta = q - 4$; r and δ, as a function of time, are shown in Fig. 1 (see also Axford, 1981). For $\delta < 0.75$ and $p >$ a few p_0, the time independent solution of the transport Eq. (3) (e.g. Blandford and Ostriker, 1978) is:

$$f_0 - f_\infty \cong \frac{4+\delta}{0.75-\delta} \eta f_g p_0^{\delta - 0.75} p^{-(4+\delta)}. \tag{19}$$

Obviously, the flux of particles injected in the mechanism is determined by combination of the parameters η and p_0. In the following, we take $p_0 = 100$ MeV/c, which corresponds to $f_g = 1.3\ 10^{-34}$ in C.G.S. units. At the beginning of the expansion $\delta = 0$ and

$$x_0 = (10^{14}/\eta)\ [p/(\mathrm{GeV}/c)]\ [n_e/(3\ 10^{-3}\ \mathrm{cm}^{-3})]^{1/2}\ \mathrm{cm}$$
$$\mathscr{F}_B(x=0) = 0.45 \eta / u_{1,9}, \tag{20}$$

where $u_{1,9}$ is the shock velocity in units of 10^9 cm/s. The corresponding variation with x of the diffusion coefficient associated with $\mathscr{F}$ is plotted on Fig. 2. The curve is very well approximated by $D = u_1 (x_0 - x)$ for $|x| < b$, and $D = D_{i.}$ for $|x| > b$, with b given by:

$$b = x_0 \left(\frac{\mathscr{F}_B(x=0)}{\mathscr{F}_{i.}} - 1\right). \tag{21}$$

Even when only pre-existing cosmic rays are injected in the mechanism, this theory predicts that, in the vicinity of the shock, $\mathscr{F}$ is close to or greater than 1. It may be that dissipative mechanisms are at work, so that the energy density in waves settles at a lower level. In the next subsection, we investigate this possibility.

1 For details and background on this turbulence, see Cesarsky (1980, 1982)

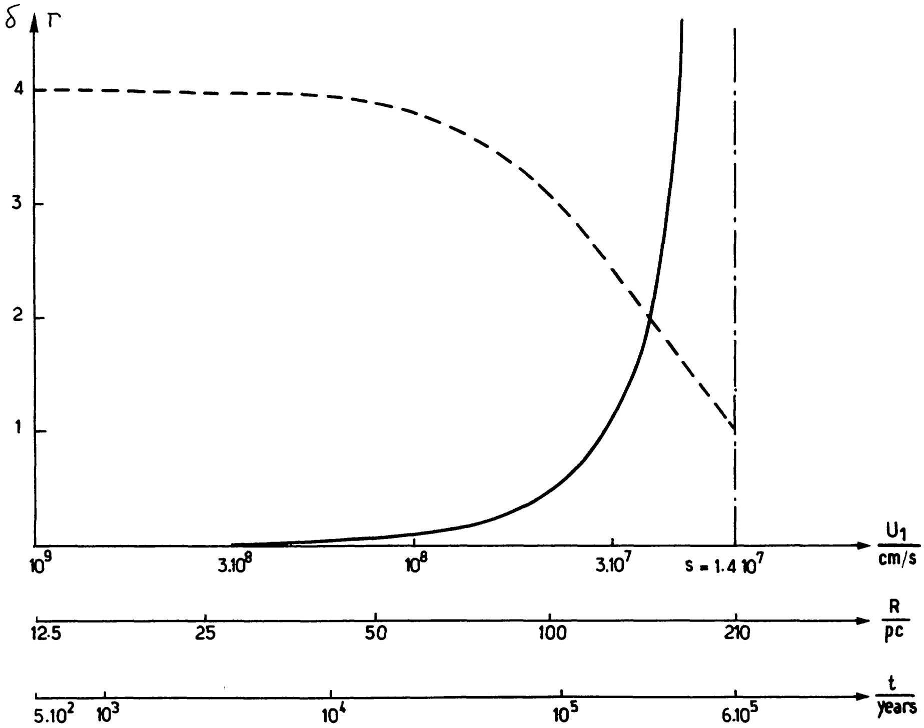

Fig. 1. Evolution of the compression ratio, r (dotted line) and of the difference, δ, between the spectral index q and its value for a strong shock, 4 (full line)

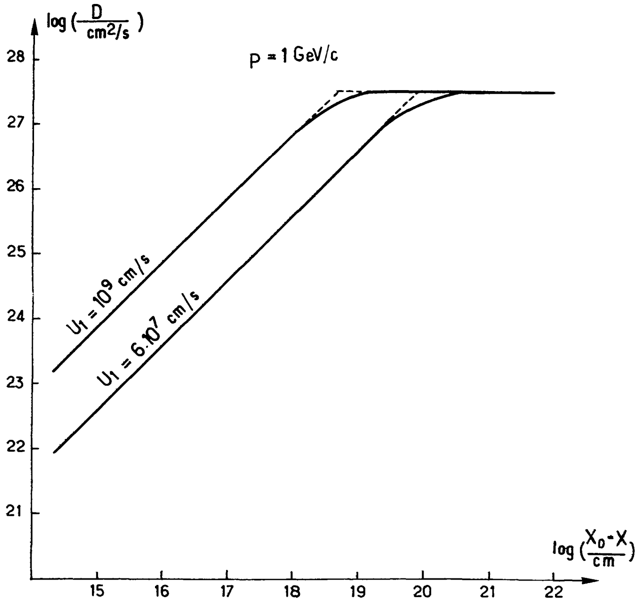

Fig. 2. Variation with the spatial coordinate, x, of the diffusion coefficient upstream when the turbulence generated by the pre-existing cosmic rays in a steady state is added to the interstellar turbulence. Full line: exact curve. Dotted line: approximation used

2. *Effect of wave damping mechanisms*

Let us now re-consider Eq. (13), for the case when $\Gamma \neq 0$. We consider some possible damping mechanisms, in turn.

a) Effect of pre-existing cosmic rays

Away from the shock, the pre-existing cosmic rays are quasi isotropic in the frame of the underlying plasma (we neglect here the small anisotropy related to their diffusion in the galaxy). Consequently, they tend to damp the waves generated by the cosmic rays returning from the shock. The effective growth rate of cosmic rays is then:

$$\sigma - \Gamma_{cr} = \frac{4\pi}{3} \frac{v_A p^4}{U_M} \left(\frac{v}{\mathscr{F}} \frac{\partial f}{\partial x} + \frac{\pi}{2} \frac{eB}{c} v_A \frac{\partial f}{\partial p} \right) \tag{22}$$

(Kulsrud and Pearce, 1969; Skilling, 1971). In the time independent case, and assuming a strong shock ($r=4$), we find that the damping rate of waves resonating with cosmic rays of momentum p exceeds the growth rate at all distances greater than:

$$d_1(p) = 1.68 \eta x_0 (u_1/v_A)(p/p_0)^{0.75}. \tag{23}$$

(Note that, since x_0 is proportional to $1/\eta$, d_1 is independent of η.) The length d_1 is shorter than b only for $p < 10\, u_{1,9}^{-1.6}$ GeV/c. This process does not help to limit $\mathscr{F}$ close to the shock; but at low energy it does limit the wave growth away from the shock, so that for $|x| \gtrsim d_1$ the diffusion coefficient remains at D_i.

b) Saturated non-linear Landau damping

Streaming cosmic rays emit waves preferentially along the field lines. In the hot component of the interstellar medium, where the Alfvén velocity ($v_A \sim 40\, B_{-6}$ km s^{-1}) is lower than the thermal velocity of ions of mass M_p ($v_p = (kT/M_p)^{1/2} \sim 76\, [T/(7\; 10^5\; \mathrm{K})]^{1/2}$ km s^{-1}), Alfvén waves travelling along the magnetic field are only dissipated by the non-linear Landau damping mechanism, i.e. by interactions of thermal particles with beat waves created by two coupled waves. The damping rate due to interactions with waves of similar wavelengths is given by (Völk

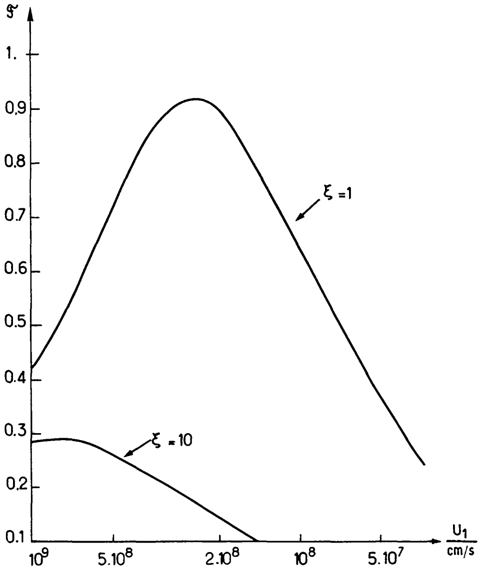

Fig. 3. Variation with the shock velocity, u_1, of $\mathscr{F}$ (the wave energy density) for two values of the damping parameter, ξ, when only preexisting cosmic rays are injected

and Cesarsky, 1982 and references therein)

$$\Gamma_{ns}(k) \sim \frac{1}{4}\sqrt{\frac{\pi}{2}}\, k v_p \mathscr{F}(p=eB/kc), \tag{24}$$

(assuming that the wave spectrum contains an equal number of waves with right hand side and left hand side polarization). In the presence of a spectrum of waves, Γ_{ns}/k has to be integrated over all waves of wavenumber lower than k:

$$\Gamma_{ns}(k) = \frac{1}{4}\sqrt{\frac{\pi}{2}}\, k v_p \int_{p=eB/kc}^{\infty} \mathscr{F}(p')\, d\,\mathrm{Log}\, p'. \tag{25}$$

However if their amplitude is high, the waves act as potential wells that can trap the thermal particles, limiting the amount of energy that they can extract from the waves (Kulsrud, 1978). Völk and Cesarsky (1982) find that, if $\varepsilon = (kv_p)/(\Omega_p \sqrt{\mathscr{F}}) \ll 1$ (where Ω_p is the Larmor frequency of thermal protons), the damping rate is reduced to:

$$\Gamma_s(k) = \frac{1}{2}\sqrt{\frac{\pi}{2}}\, w \left(\frac{v_p k}{\Omega_p}\right) [\mathscr{F}(p=eB/kc)]^{1/2}. \tag{26}$$

w, of order $(-\mathrm{Log}\, \varepsilon)$, depends on the shape of the spectrum. For the interstellar spectrum of turbulence assumed here, $\varepsilon = 7.6\ 10^{-2}/[p/(\mathrm{GeV/c})]^{3/2}$; the amplitude of the waves is even higher in the shock environment, so the saturated rate is relevant everywhere. In first approximation the growth of waves, σ, is proportional to $1/(x_0 - x)$, while Γ_s is proportional to $1/(x_0 - x)^{1/2}$; the two terms are equal at a distance

$$d_2 = 4\ 10^{21} \left(\frac{7}{w}\right)^2 \frac{u_{1,9}^2}{B_{-6}} \left(\frac{7\ 10^5\ \mathrm{K}}{T}\right) [p/(\mathrm{GeV/c})]^3\ \mathrm{cm}. \tag{27}$$

For $u_1 > 10^8$ cm/s and $p > 1$ GeV/c, d_2 is greater than b. So the non-linear Landau damping mechanism, if it saturates as in Völk and Cesarsky (1982), is irrelevant in this problem.

However, the calculation of Völk and Cesarsky (1982) also assumes that $\mathscr{F} \ll 1$; consequently, it may not be applicable to the problem at hand. It is plausible that when $\mathscr{F}$ is large, the trapping of thermal particles in wave packets becomes unstable, so that the unsaturated damping rate applies again. Thus, it seems worthwhile to consider the effect of unsaturated non linear Landau damping [Eqs. (24) or (25)], keeping in mind that, in any case, this rate is representative of any dissipation mechanism involving two-wave interactions.

c) Unsaturated Landau damping; non linear two-wave interaction

We consider the effect of a non linear mechanism whose damping rate is:

$$\Gamma(k) = \frac{1}{4}\sqrt{\frac{\pi}{2}}\, \xi k v_p \mathscr{F}(p=eB/kc), \tag{28}$$

where ξ is an undetermined factor. If $\mathscr{F} = 1$, for waves interacting with cosmic ray protons of energy less than 10^5 GeV, the integral in Eq. (25) increases the damping rate above the value given by Eq. (24) by a factor $\lesssim 14$.

Let us now consider the solution of Eq. (13) where Γ is as given in Eq. (28). As Γ is only important for $|x| \ll b$, we disregard the cumbersome expression (18) and only write the equivalent of expression (14) corresponding to the boundary condition $\mathscr{F}(x \to \infty) = 0$. We find:

$$\mathscr{F}_d(x) = \frac{a}{x_1 - x}, \tag{29}$$

$$f_d(x) = f_\infty + \frac{(f_0 - f_\infty)}{1 - (x/x_1)}, \tag{30}$$

where a is as given in Eq. (16) and

$$x_1 = x_0 \left(1 + \frac{1}{3}\sqrt{\frac{2}{\pi}}\, \xi \frac{V_p v}{u_1^2}\right). \tag{31}$$

If we consider a strong shock ($r=4$, $\delta=0$) we have, at the shock:

$$\mathscr{F}_d(x=0) = \frac{0.45\eta}{u_{1,9}\left(1 + \frac{6\ 10^{-2}}{u_{1,9}^2}\, \xi \sqrt{\frac{T}{7\ 10^5\ \mathrm{K}}}\right)}. \tag{32}$$

If $\eta = 1$–4 and $\xi = 1$–10, the damping rate (28) ensures that $\mathscr{F}$ remains smaller than 1 (Fig. 3). But at larger η, $\mathscr{F}$ become $\gg 1$.

What happens then? It may be that the waves can effectively grow to extremely high amplitudes. Since the waves are amplified by the shock, the problem is even worse downstream. Then, we note that the theory of diffusive shock acceleration of cosmic rays, based on approximations adequate only for the case of weak turbulence, is possibly being applied out of its range of validity in the case of the acceleration of galactic cosmic rays by supernova shocks. The effect of high amplitude waves on the mechanism of particle acceleration is difficult to guess. The small scale turbulence may lead to an amplification of the mean field strength seen by high energy particles, and therefore to a reduction of their acceleration time. But the low energy particles may get trapped in the waves generated by the high energy particles; this would quench the acceleration process at low energy (Völk and McKenzie, 1981) and reduce the small scale turbulence, at least at a few characteristic trapping lengths of the shock. Alternatively a process not considered here may limit the wave amplitude, even at large η. A

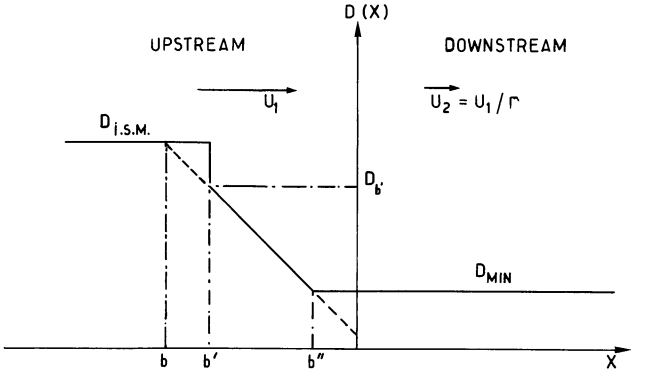

Fig. 4. Sketch of the spatial variation of the diffusion coefficient

realistic answer to this problem can only be obtained through numerical simulations which are much beyond the scope of this paper. In the following we assume that a damping mechanism ensures that $\mathscr{F}$ remains <1, whenever our formulation leads to $\mathscr{F} > 1$. Then the diffusion coefficient upstream is constant and equal to $D_{\min}$ up to a distance b'' from the shock,

$$b'' = \frac{D_{\min}}{u_1} - x_0 . \tag{33}$$

Downstream, we take $D = D_{\min}$ everywhere; we have shown in LC that the spatial variation of D, downstream, can be neglected.

3. Time-dependent effects

Up to this point, we have considered the time dependence of the particle acceleration, assuming that the wave field is as given by its final equilibrium value. By doing so, we have been over-estimating the wave field, for, in fact, the waves take a finite time to grow. To assess fully the importance of time-dependent effects, one should solve the non linear coupled Eqs. (3) and (13). This set of coupled equations is extremely difficult to solve, even using numerical methods. Consequently, we have only attempted to find upper bounds to the time dependent wave fields; we then consider the impact of these bounds on $E_{\max}$.

To evaluate, roughly, the influence of the growth of the waves, we compare the growth time, taken in the steady state

$$1/\sigma = (x_0 - x)/u_1 \tag{34}$$

to the age of the supernova, t. At $x = 0$, for a strong shock and for a supernova in the Sedov phase the wave growth time is shorter than the supernova age as long as:

$$p(t) \lesssim 10^4 \eta t_{\mathrm{yr}}^{2/5} \text{ GeV}/c . \tag{35}$$

In the following we will see that $p_{\max}$ is about 10^4 GeV/c, so that condition (35) is always fulfilled. But the growth of the waves is retarded more and more as the distance $|x|$ increases because the cosmic ray flux diminishes. Thus beyond a distance:

$$b'_1 \sim u_1 t , \tag{36}$$

the wave energy remains at its average interstellar level.

Another reason for a cut-off of the cosmic ray generated turbulence at large $|x|$ is that cosmic rays returning from the shock take a long time to reach regions that are far upstream. In a diffusive convective medium of constant diffusion coefficient, the mean time taken by a particle to reach a distance $x < L = 3D/u_1$ is $\langle t \rangle = x^2/(3D)$; while if $x > L$, this time becomes x/u_1. At the beginning, D is equal to $D_{i.}$ and L, equal to b, is large; thus an upper limit to b', b'_2, is given by the first formula:

$$b'_2 = (3 D_{i.} t)^{1/2} . \tag{37}$$

At later times, the diffusion coefficient has considerably diminished in the shock vicinity, so that it takes a time (x/u_1) for newly accelerated cosmic rays to reach a distance x from the shock upstream. Thus, roughly, $b' = b'_1$.

IV. Maximum energy

1. Firm upper limit

Let us first calculate $E_{\max}$, assuming that D takes everywhere the value $D_{\min}$. In this case, it is easy to integrate analytically Eq. (2). Neglecting the cloud evaporation, we obtain during the blast phase,

$$p(t) \sim p_0 + 47 Z B_{-6} \frac{E_{51}}{M_e} t_{\mathrm{yr}} \text{ GeV}/c , \tag{38}$$

where Z is the particle charge and p_0 the initial value of p. Thus at the end of the blast phase, neglecting p_0

$$p(t_1) = p_1 = 7.1\ 10^4 \frac{Z B_{-6} E_{51}}{[n_e/(3\ 10^{-3}\ \mathrm{cm}^{-3})]\, M_e^{1/6}} \text{ GeV}/c \tag{39}$$

for a SN I, $p_1 \sim 5.5\ 10^4\, Z$ GeV/c and $\sim 5.3\ 10^4\, Z$ GeV/c for a SN II. During the second phase:

$$p(t) = p_1 + 4\ 10^5 Z \left(\frac{E_{51}}{n_e/(3\ 10^{-3}\ \mathrm{cm}^{-3})} \right)^{2/5} \cdot B_{-6} (t_{1,\mathrm{yr}}^{-1/5} - t_{\mathrm{yr}}^{-1/5}) \text{ GeV}/c . \tag{40}$$

The variation of p as a function of t for a SNI is depicted in Fig. 5. The maximum energy attained, at the end of the evolution is:

$$E_{\max} = c p_{\max} \sim 10^5 Z B_{-6} \text{ GeV}. \tag{41}$$

For a SN II, $E_{\max}$ is of the same order. We note that $E_{\max}$ depends weakly on $t_{\max}$; it is only diminished by a factor 1.5 if $t_{\max}$ is shorter by a factor 100. This is because, as is clear from Fig. 5, most of the acceleration occurs in relatively early phases of the supernova expansion. We also note that $E_{\max}$ is proportional to Z; thus, this theory predicts a break in the cosmic ray spectrum; beyond the break, the cosmic ray flux (at a given total energy), should be progressively enriched in heavy elements. If clouds evaporate inside the remnant, $E_{\max}$ is reduced to $\sim 4\ 10^4 Z B_{-6}$ GeV. Thus, the slowing down of the supernova shock, due to cloud evaporation, leads to a decrement by a factor ~ 2.5 of the maximum energy that a cosmic ray can extract from its interaction with one supernova shock.

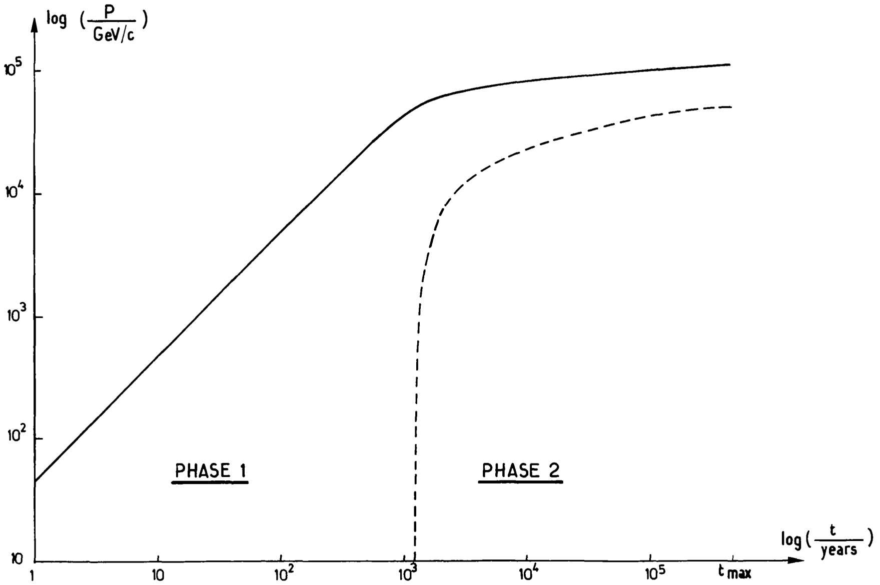

Fig. 5. Variation with time of the momentum, p, of a proton accelerated by a SN I shock, evolving in a medium of density $3\ 10^{-3}\ \mathrm{cm}^{-3}$ without cloud evaporation. Full line: injection at the beginning of the blast wave phase. Dotted line: injection at the beginning of the Sedov phase

Table 1. Energy reached by a proton accelerated all over the lifetime of a SN I remnant, for different values of the upstream diffusion coefficient; the downstream coefficient is taken as D_{min}

Diffusion coefficient	η	b'	E_{max}/(GeV) hot gas only	E_{max}/(GeV) cloud evaporation
$D_{i.s.m.}$			5.8	1.7
$D_{i.s.m.}$ + D_{CR}	1	b_1'	$7.3\ 10^3$	$1.7\ 10^3$
		b_2'	$1.7\ 10^4$	$4.9\ 10^3$
	10	b_1'	$2.1\ 10^4$	$4.7\ 10^3$
		b_2'	$4.9\ 10^4$	$1.5\ 10^4$
	100	b_1'	$2.4\ 10^4$	$5.8\ 10^3$
		b_2'	$5.7\ 10^4$	$1.7\ 10^4$
D_{min}			$1.1\ 10^5$	$4.3\ 10^4$

In the previous calculation we have considered a strong shock for which the compression factor is equal to 4. But as the shock slows down, it weakens; r varies as:

$$\frac{1}{r}=\frac{\gamma-1}{\gamma+1}+\frac{2}{(\gamma+1)\,(u_1/s)^2}. \tag{42.a}$$

For $\gamma=5/3$, the formula becomes:

$$r=\frac{4}{1+3\,(s/u_1)^2}. \tag{42.b}$$

We have integrated numerically Eq. (2), taking into account the variation of r with time; but the result does not differ appreciably from (39) and (40), since by far most of the acceleration occurs when $r=4$ (Fig. 1).

2. Influence of the spatial dependence of the diffusion coefficient

We now consider a diffusion coefficient varying with x, as found in Sect. III. We have seen that at a given time and a given energy, the cosmic ray diffusion coefficient has a lower bound which is given by the curve sketched in Fig. 4. Downstream, D is considered independent of x and equal to D_{min}. Upstream, D has a plateau (at least for $\eta>1$–4) up to a distance b'', because of the damping mechanism that we have assumed. Then, D takes its normal slope up to a distance b', where D is $D_{b'}$. Beyond b', the cosmic rays have not had enough time to generate waves, so that D is constant and equal to the interstellar diffusion coefficient.

The particle acceleration time, when D depends on x, has been calculated in LC (with $b''=0$, but the generalisation to $b''\neq 0$ is straightforward). Two approaches have been used; a macroscopic approach consisting in finding the solution of the transport Eq. (3); a microscopic approach, using the solution of an analogous random walk problem to calculate the mean time of a cycle through the shock. T_{cycle} is now:

$$T_{cycle}=\frac{4}{v}\left\{\frac{D_2}{u_2}+\zeta\,\frac{D_1}{u_1}\right\}, \tag{43}$$

where

$$\zeta=1+\exp\left(-\frac{b''}{x_0+b''}\right)\left[\left(\frac{D_{i.}}{D_{b'}}-1\right)+\mathrm{Log}\,\frac{1+b'/x_0}{1+b''/x_0}\right]. \tag{44}$$

The time spent upstream is multiplied by ζ. The fact that $u_1=4\,u_2$ diminishes the influence of the second term in brackets in Eq. (43); if ζ is 6, T_{cycle} is only multiplied by a factor 2.

We can calculate an upper limit to E_{max} by integrating $dp/dt \sim \Delta p/T_{cycle}$, assuming that D is as given in Fig. 4. The results are given in Table 1, for different values of the injection parameter η, and for different cases: cut-off at b_1' or b_2', cloud evaporation effects included or neglected. The values of E_{max} vary between $2\ 10^3$ GeV and $6\ 10^4$ GeV, depending on the parameters. A typical value is 10^4 GeV. The influence of η is weak; when η goes from 1 to ∞, E_{max}

only increases by a factor 3, because b'' tends to (D_{min}/u_1) and ζ saturates. The effect of cloud evaporation on the evolution of the supernova decreases E_{max} by a factor of order 3. We have also taken into consideration the influence of the other parameters of the problem: ξ, entering in the damping rate [Eq. (28)], $\mathscr{F}_0$, determining the level of the interstellar turbulence and α, the slope of the interstellar turbulence spectrum. In Table 1, ξ is set equal to 1. In this case the damping has only little influence on E_{max} because it is important only for $u_1 \lesssim 10^8$ cm/s (Fig. 2), when much of the acceleration has already occurred. For $\xi=10$, the effect of the damping decreases E_{max} by a factor 1.6. As for $\mathscr{F}_0$, its effect depends on the cut-off taken. If $b'=b'_2$, $D_{b'}$ is close to $D_{i.}$ and the variation of E_{max} with $\mathscr{F}_0$ is weak (logarithmic); E_{max} increases by a factor ~ 2, when $\mathscr{F}_0$ varies from 10^{-6} to 10^{-4}. But in the case b'_2, $(D_{i.}/D_{b'})$ becomes important; E_{max} increases by a factor ~ 10, when $\mathscr{F}_0$ increases by 100 in the range considered. So that for $\mathscr{F}_0=10^{-4}$, E_{max} is unchanged if b' goes from b'_1 to b'_2. As for α, when it goes from 0.6 to 0.3, E_{max} decreases by a factor ~ 3.5.

3. More complicated models

a) Non parallel magnetic field

When B makes an angle $\theta \left[\text{less than } \tan^{-1}\left(\frac{u}{v}\right)\right]$ with the normal to the shock, it is always possible to go to a frame where the electrical field vanishes on both sides of the shock (Hudson, 1965). In such a frame the transport equation becomes:

$$\frac{\partial f}{\partial t}+u_j\frac{\partial f}{\partial x}=\frac{\partial}{\partial x}\left[(\cos^2(\theta_j)D_{\parallel,j}+\sin^2(\theta_j)D_{\perp,j})\frac{\partial f}{\partial x}\right] -\frac{1}{3}(u_1-u_2)p\frac{\partial f}{\partial p}\delta(x), \quad (45)$$

where $D_\parallel$ $(D_\perp)$ is the diffusion coefficient parallel (perpendicular) to the field. The matching conditions between the upstream and downstream regions are the same (Drury, 1983). In the case $D_\parallel = D_{min}$, $D_\perp \sim D_\parallel$ so that Eq. (45) is the same as Eq. (3); thus

$$\frac{dp}{dt}\sim\frac{u_1-u_2}{3}\frac{p}{\frac{D_{1,min}}{u_1}+\frac{D_{2,min}}{u_2}}. \quad (46)$$

But here we have to take into account the fact that the magnetic field is compressed by the shock: $B_2=B_1[\cos^2(\theta_1)+r^2\sin^2(\theta_1)]^{1/2}$. Averaging over the angle (assuming no correlation with the shock direction) we obtain $\langle E_{max}\rangle \sim 2E_{max}(\theta=0)$.

b) Curvature effects

Roughly, for a constant diffusion coefficient, the effect of the curvature of the shock is negligible when D_1/u_1 (the upstream length scale of the cosmic ray distribution) $+D_2/u_2$ is shorter than the radius R of the supernova (Krymsky et al., 1979; Prischep and Ptuskin, 1981; see also Drury, 1983). If D_1 is equal to D_2 and u_1 is of the Sedov type, the constraint on D becomes:

$$D<\frac{4.2\ 10^{30}}{(1+r)}t^{-1/5}. \quad (47)$$

The finite lifetime the shock ensures that:

$$D<\frac{2.1\ 10^{30}}{(1+r)}t^{-1/5}\left[\left(\frac{t}{t_1}\right)^{1/5}-1\right]\Big/\mathrm{Log}\left(\frac{p}{p_1}\right). \quad (48)$$

The first inequality is always fulfilled when the second one is. When $D=D_{min}$ and thus depends on p, criterion (47) is perhaps too severe; and it leads to the condition: $p_{max}\lesssim 1.2\ 10^5$ GeV/c. We conclude that curvature effects, while bringing about a great deal of complication into the problem, are probably never crucial.

c) Non-linear effects

Throughout this paper, we have neglected the effect of the cosmic rays, and of the waves they generate, on the shock. One effect of these may be to widen the shock, thus slowing down the acceleration of low energy cosmic rays; but the shock will always appear to be thin to the very high energy cosmic rays that we consider here. If relativistic cosmic rays, with $\gamma=4/3$, are the dominant energy carrier on both sides of the shock, the compression ratio can attain values as high as 7. The efficiency of the mechanism is then increased, in the sense that the predicted slope of the momentum spectrum of the accelerated particles softens from -4 to -3.5. But then the rate of acceleration (dp/dt) is slower by a factor 1.5 [Eq. (2)] and E_{max} is reduced.

V. Conclusion

The integral spectrum of cosmic ray protons appear to be very close to a power law in the energy range ~ 10–10^5 to 10^6 GeV (Gregory et al., 1981; Webber, 1982 and references therein). In this paper, we have studied the problem of the acceleration of cosmic rays by supernova remnants, taking into account only one of the many nonlinearities involved: the fact that cosmic rays are scattered by waves they have generated themselves. We have found that the maximum energy that cosmic rays can attain from their interaction with one supernova shock is at most in the 10^4 GeV range. Consequently, despite its success at accounting for various aspects of the cosmic-ray data (e.g. Axford, 1981; Blandford and Ostriker, 1980), the theory of acceleration of galactic cosmic rays by supernova remnants encounters here a serious obstacle.

Blandford and Ostriker have conjectured that high energy cosmic rays, being more mobile than low energy cosmic rays, have a greater chance of being reaccelerated in the galaxy through encounters with many supernova shocks. For low energy cosmic rays, the mean velocity, v_{cr}, in interstellar space, is smaller than the shock velocity; the mean time for encounters with shocks is:

$$t_{sh}=\frac{3V_G}{4\pi\Sigma R^3}, \quad (49)$$

where V_G is the volume of the confinement region of cosmic rays ($\gtrsim 1.4\ 10^{12}$ pc^3) and Σ the rate of supernova explosions ($\sim(1/30)$ yr^{-1}). If R, the shock radius, is equal to its maximum value $R\sim 200$ pc and $\sim t_{sh}$ is $\sim 10^6$ yr. With a cosmic ray mean age of 10^7 yr at 1 GeV, there would be ~ 10 reaccelerations at low energy. The cosmic-ray mean velocity with respect to the gas v_{cr} increases as $E^{1/2}$; at high energies, $v_{cr}>u_1$ and

$$t_{sh}=\frac{3V_G}{4\pi\Sigma R^3}\frac{u_1}{v_{cr}} \quad (50)$$

is proportional to $E^{-1/2}$, like the escape time. Therefore, the number of accelerations becomes energy independent. In addition, a high energy particle can only be "re-accelerated" if it encounters a young enough shock; for instance, a cosmic ray of energy 10^4 GeV can only oncrease its energy by a factor 1.2, at most, if it encounters a supernova shock of radius $R \sim 30$ pc. The mean number of encounters with supernovae of radius $R < 30$ pc is very small, < 1. Thus, the reacceleration process does not seem to alleviate the problem of the maximum energy attained.

Axford (1980) suggested that shocks of supernovae exploding at the border of the galactic disk may be accelerated when propagating in the halo. The behaviour of shocks propagating in a medium with a density gradient has been calculated by Chevalier and Gardner (1974). They show that the shock speeds up only if the temperature of the medium obeys:

$$T_0 > 10^7 E_{51} \left(\frac{n_e}{10^{-3}\ \mathrm{cm}^{-3}}\right)^{-1} \left(\frac{H}{100\ \mathrm{pc}}\right)^3 \mathrm{K}, \quad (51)$$

where H is the gas scale height. For the halo, $T \sim 10^5$–10^6 K, and H is several kpc (de Boer and Savage, 1981). So the halo does not affect much the evolution of supernova shocks.

Another type of shock is present in the galaxy: those surrounding stars with strong stellar winds. Of these the most energetic are related to Wolf-Rayet stars. It has been suggested that stellar wind terminal shocks may accelerate cosmic rays (Cassé and Paul, 1980); but whether this acceleration is a continuous or an intermittent process is open to debate (see Völk and Forman, 1981, and discussion in Cesarsky and Montmerle, 1983). For the case of a stellar wind terminal shock, the configuration is inverted: the shocked gas is in the region outside the circular shock. The shock is bounded, on the inside, by the agitated stellar wind gas; on the outside, by the gas having passed through the shock. The turbulence level may be quite high on both sides, independently of cosmic rays; so we take $D = D_{\min}$ everywhere. Assuming that the acceleration is continuous and taking a terminal velocity, w_s, of $3\ 10^8$ cm/s and a lifetime of 10^5 yr, the constraint due to the finite lifetime of the shock yields $E_{\max} \sim 5\ 10^6 ZB_{-5}$ GeV, where B is the magnetic field strength at the shock (B could be 10^{-5} G if the magnetic field at the surface of the star has a strength ~ 100 G, and if the radius of the shock is $\sim 10^6$ times that of the star). The curvature condition yields $E_{\max} \lesssim 5\ 10^5 ZB_{-5} \left(\frac{R_c}{5\ \mathrm{pc}}\right)$ where R_c is the radius of the terminal shock radius. We know, from energy budget arguments, that stellar winds alone probably cannot maintain the galactic cosmic ray pool. Still, it seems interesting to speculate that the winds could contribute in a sizeable way to the very high energy cosmic ray pool. This might explain the apparent bump in the spectrum at $\sim 10^6$ GeV.

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