THE ASTROPHYSICAL JOURNAL, 262:145–152, 1982 November 1

X-RAY ILLUMINATION OF GLOBULAR CLUSTER PUZZLES

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Received 1981 December 14; accepted 1982 May 10

ABSTRACT

We enumerate many of the infamous puzzles presented by globular clusters, regarding their gross features, their internal dynamics, and their X-ray emission. We then focus on the latter puzzles, in particular: Why do globular clusters provide a favorable environment for forming discrete X-ray sources? Are X-ray emitting globular clusters (XEGC) unusually near the galactic center, and if so, why? Why do the observed clusters never contain more than a single discrete X-ray source? A statistical analysis of the data allows plausible explanations of these puzzles. Adopting the model that globular cluster X-ray sources consist of binary systems formed by two-body tidal interactions, we find a significant positive correlation between the two-body binary formation time scale, t_{B2}, and the distance from the galactic center, R, for globular clusters *in general*. This is a major result of our paper, and we offer possible physical explanations. XEGC are found to have significantly small values of R and t_{B2}. Furthermore, t_{B2} and the inferred lifetime of X-ray sources may be used to estimate the probability of observing an X-ray source in a cluster, and in the Galaxy in general. We obtain rough quantitative agreement with the observed rate of occurrence of X-ray sources in globular clusters. If clusters are classed according to either R or t_{B2}, the probability of finding more than one X-ray source in a single cluster is less than 50%. Our data set consists of 116 clusters for which R is known, of which eight are XEGC, and 84 clusters for which t_{B2} can be computed, of which seven are XEGC.

Subject headings: clusters: globular — stars: binaries — X-rays: sources

I. GLOBULAR CLUSTER PUZZLES

Because they are thought to be among the oldest objects in the Galaxy, globular clusters provide important clues for determining the age and process of formation of the Galaxy, as well as for verifying theories of stellar evolution. Furthermore, globular clusters are a theorist's paradise for the study of stellar dynamics, with a number of stars (10^5–10^6) large enough to eliminate the importance of higher stochastic effects and small enough to allow time for interesting gravitational evolution, such as energy equipartition and core collapse. In short, there is a great deal to be learned from globular clusters, and the task is, in principle, within our reach.

Despite our continued attack, however, there remain quite a few unsolved puzzles regarding globular clusters. In this paper, we address those puzzles relating to the X-ray emission of globular clusters:

Why do globular clusters provide a particularly favorable environment for the formation of galactic X-ray sources, as first pointed out by Katz (1975)? The ~ 100 known globular clusters in our Galaxy contain only $\sim 10^{-4}$ of the mass of the Galaxy; yet ~ 10 of the ~ 100 known galactic X-ray sources lie in globular clusters.

Is it true that X-ray emitting globular clusters (XEGC) are located unusually near the galactic plane and/or galactic center? If so, why?

Why is never more than one luminous compact X-ray source observed in a globular cluster?

In addition to these puzzles, one can list: Why are globular clusters so weakly gravitationally bound compared to galaxies? Why are globular clusters remarkably similar in size and mass, considering the large range of scales between galaxies and globular clusters? Why is there a large variation in the numbers of clusters in otherwise similar galaxies? Why do globular clusters differ systematically in metal abundance from one galaxy to another? Why are binaries so rare in globular clusters? Why do observed spatial distributions of stars in globular clusters show no evidence of the core collapse predicted by dynamical, theoretical models? What is the explanation for the strikingly smooth and simple distribution observed for central relaxation times in globular clusters? How is it possible to form a compact object in a cluster without expelling it?

Almost certainly some of these puzzles are intimately related, with a common resolution, and it is for this reason that we have collected them together. We hope that a probe of the first three puzzles, those involving the XEGC, may ultimately bring us closer to an understanding of globular clusters in general.

The rest of the paper is organized as follows: In § II we discuss our adopted model for the nature, formation, and lifetime of X-ray sources in globular clusters; in § III we analyze the data in a search for correlations between binary formation time scales, central relaxation times, galactic locations, and X-ray emission; and in § IV we discuss our results.

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II. X-RAY EMITTING GLOBULAR CLUSTERS

The reader may consult the recent reviews by Lewin and Clark (1980) and by Grindlay (1981) for a comprehensive summary of observational data and ideas regarding the XEGC.

a) *Model of X-Ray Sources in Globular Clusters*

There are several independent lines of evidence to suggest that the globular cluster X-ray sources are close binary star systems, in which gas is transferred from a normal star to a compact companion, probably a neutron star. This model was suggested by Katz (1975) and by Clark (1975). (1) The binary nature of a number of X-ray sources outside of globular clusters has long been known. (2) The generally accepted nuclear flash model for X-ray bursters (e.g., Joss 1978 and references therein), many of which are in globular clusters, employs gas accumulation onto a neutron star. (3) Analysis of the observed black body-like spectrum during X-ray bursts (Swank *et al.* 1977; van Paradijs 1978) indicates a characteristic emitting radius of ~ 10 km, typical of the size of neutron stars. (4) Finally, a statistical analysis of the positions of the X-ray sources within the XEGC, assuming energy equipartition and employing the method of Lightman, Hertz, and Grindlay (1980), indicates the mass of a typical globular cluster X-ray source is between 2 and 6 times the mean mass of a star, at the 10% likelihood level (Grindlay 1981; Grindlay *et al.* 1982).

The absence of observed periodicities and eclipses from the globular cluster X-ray sources imposes certain requirements on the above model. The absence of periodicities, normally associated with a strong magnetic field inclined with respect to the neutron star rotation axis, may simply result from the decay of the magnetic field (e.g., Gunn and Ostriker 1970). Since globular clusters are old systems, most of their neutron stars might be expected to be quite aged. (However, recent results showing that X-ray bursters radiate above the Eddington limit [e.g., Grindlay *et al.* 1980] may require strong magnetic fields to confine the radiating gas.) The absence of eclipses could be due to the geometry of the binary systems, particularly if the compact object is fueled by Roche lobe overflow from a low-mass companion star (e.g., Joss and Rappaport 1979).

We will tentatively adopt the above model for globular cluster X-ray sources.

b) *Formation, Lifetime, and Probability of Occurrence*

Clark (1975) has suggested that the great age of globular clusters requires that globular cluster X-ray binaries be formed through the capture of stars in the cluster by single compact remnants of massive (cluster) stars, rather than by the evolution of primordial binaries. We will test Clark's hypothesis by searching for a correlation between the binary formation time scale and X-ray emission. The preferred mechanism here for binary formation is the two-body tidal interaction process first suggested by Fabian, Pringle, and Rees (1975). This process for forming binaries occurs at least 100 times more rapidly than the dissipationless three-body process, for parameters typical of globular clusters (see, e.g., Lightman and Shapiro 1978), and automatically produces a close binary of the type required to produce an X-ray source.

If we require a pericenter separation $l = 3r_*$ between the two approaching stars in order to dissipate sufficient orbital energy for forming a binary (e.g., Press and Teukolsky 1977), the time scale for two-body binary formation *per globular cluster*, with one star compact, is calculated to be

$$t_{B2} = 7 \times 10^{13}\ \mathrm{yr}\left(\frac{v_c}{1\ \mathrm{km\ s^{-1}}}\right)^{-2}\left(\frac{n_c}{1\ \mathrm{pc^{-3}}}\right)^{-1/2} \times \left(\frac{f}{0.03}\right)^{-1}\left(\frac{r_*}{R_\odot}\right)^{-1}\left(\frac{m_*}{M_\odot}\right)^{1/2} . \quad (1)$$

Here v_c and n_c are the central values of the root mean square velocity dispersion and stellar density in the globular cluster, r_* and m_* are the radius and mass of the captured field star, and f is the fraction of core stars that are compact remnants. Henceforth, we will set r_* and m_* to their solar values. To obtain equation (1) we included the effects of gravitational focusing (cross section goes as r_* and not r_*^2), used the virial theorem, and set the rate of binary formation per star averaged through the core equal to a tenth the rate at the center of the cluster. We refer the reader to the review of stellar dynamics in Lightman and Shapiro (1978).

The quantities v_c and n_c, or equivalently core radius and n_c, may be measured for each cluster. The fraction f is unknown, but may be estimated on the basis of the globular cluster mass function or by fitting dynamical models to the observed light distribution (Da Costa and Freeman 1976; Illingworth and King 1977; Gunn 1980). In a fit to the cluster M3, Gunn (1980) obtains a total remnant fraction of 0.03 for the dominant mass classes. In a fit to M15, Illingworth and King (1977) obtain a value $f \sim 0.01$ for the neutron star population. Hills (1976) has pointed out the likelihood of exchange encounters, in which a compact remnant would replace a member of a binary composed of normal stars. This process would increase the effective value of f. In any case, we will make the simplifying assumption that f is the same for all globular clusters and treat all compact remnants as neutron stars.

We will roughly approximate the lifetime of the X-ray source, τ_x, as the time to transfer 1 $M_\odot$ from the normal star to its compact companion. If we assume the efficiency of mass to energy conversion is $\sim 10\%$ (appropriate for a neutron star) and all of the energy produced is observed in the X-ray luminosity L_x, then

$$\tau_x = 7 \times 10^8\ \mathrm{yr}\left(\frac{L_x}{10^{37}\ \mathrm{ergs\ s^{-1}}}\right)^{-1} . \quad (2)$$

The probability P_x of observing an X-ray source in a given globular cluster is then, for $P_x \le 1$,

$$P_x \sim \tau_x / t_{B2} . \quad (3)$$

When $P_x > 1$, that quantity may be interpreted as the expected number of X-ray sources in the cluster. The expression given in equation (3) is closely related to, but more general than, Clark's (1975) estimate for the expected number of observed globular cluster X-ray sources, N_x. Letting N_R be the total number of binaries that have been formed and ξ be the fraction of normal stars that are now overflowing their Roche lobes, Clark obtains $N_x = N_R \xi$. If we make the identification $\xi = \tau_x/t_0$, where t_0 is the age of the cluster, and $N_R = t_0/t_{B2}$, then the two expressions become equivalent. If we assume that the available population of normal stars and hence τ_x do not vary much between clusters, then equation (3) yields $P_x \propto t_{B2}^{-1}$, a hypothesis that can be tested.

Finally, we will compute the probability that no single clusters contain more than one X-ray source. Let there be n globular clusters in the same class, defined by some common value of a significant parameter such as the two-body binary formation time scale or the distance from the galactic center. Let there be m X-ray sources independently distributed with equal probability among these n clusters. Then the probability $P(m, n)$ that every X-ray source is located in a different cluster is, for $m \leq n$,

$$P(m, n) = \frac{n!}{n^m (n-m)!} . \tag{4}$$

III. DATA ANALYSIS

Table 1 presents the data used in our analysis. The data for the distances R from the galactic center were taken from Harris and Racine (1979), with the exception of the last three clusters, where we made our own estimates using the observed angular coordinates and an assumed distance from Earth of 10 kpc. There are 116 clusters in all with tabulated R. The data for the central velocity dispersions, central number densities, and central relaxation times t_{rc}, were taken from Peterson and King (1977), with the exception of Liller 1 and Terzan 2, where the data were taken from Malkan, Kleinmann, and Apt (1980) using their most metal-rich (conservative) values. The central relaxation times appear in column (2). As Peterson and King do not measure velocity dispersions directly but fit observed light curves to equilibrium models, there is some uncertainty in the data. From the data for v_c and n_c and equation (1), we computed the two-body binary formation time for each cluster, given in column (4). There are 84 clusters in all with tabulated t_{rc} and t_{B2}. Blank spaces indicate that no data are available.

The last eight clusters in the data set are those we have designated as X-ray emitting globular clusters, XEGC. Specifically, all such clusters have X-ray luminosities $L_x \geq 10^{36}$ ergs s$^{-1} \equiv L_*$ in the 0.5–4.5 keV energy band. The luminosity L_* was chosen as follows: All other X-ray sources identified in globular clusters have $L_x \leq 0.03 L_*$ (Hertz and Grindlay 1982) and almost certainly form a separate class from the XEGC with $L_x \geq L_*$. Data for L_x were taken from Grindlay (1981) and are given in column (5), only for the eight XEGC. (NGC 104 has a positive X-ray detection $L_x = 0.03 L_*$, but is not one of the XEGC by our definition.) All of the XEGC contain a single discrete X-ray source (see Grindlay *et al.* 1982 for methods of verification and details of the X-ray images). Excluded from our set of XEGC are Terzan 1 and Terzan 5, with R values of 1.1 kpc and 1.2 kpc, respectively. These clusters have been tentatively identified with X-ray burst sources (Grindlay 1981; Makishima *et al.* 1981), but the persistent X-ray emission and source identification have not been ascertained with the *Einstein* satellite.

a) *Distribution in R*

We first test the hypothesis that the XEGC are unusually near the galactic center, an oft stated premise. There is an important selection effect that must be removed in testing correlations between X-ray emission and distance to galactic center. Because of obscuration and confusion, it is more difficult to detect clusters near the galactic center than far from the galactic center, at optical wavelengths. To eliminate this bias in a conservative manner, all XEGC *first identified* by X-ray emission must be excluded from the data analysis. In practice, this eliminates Liller 1 and Grindlay 1, *reducing the effective number of XEGC for this analysis from 8 to 6*. (For the analyses in §§ III*c* and III*d* below, not explicitly subject to this selection effect, the full data set will be used.)

We have computed the probability distribution $P_6(\bar{R})$ of the mean value of R of a random subset of six clusters drawn from the population of 108 non-XEGC. $P_6(\bar{R})d\bar{R}$ is the probability this mean would lie between $\bar{R}$ and $\bar{R} + d\bar{R}$. $P_6(\bar{R})$ is shown in Figure 1 and was

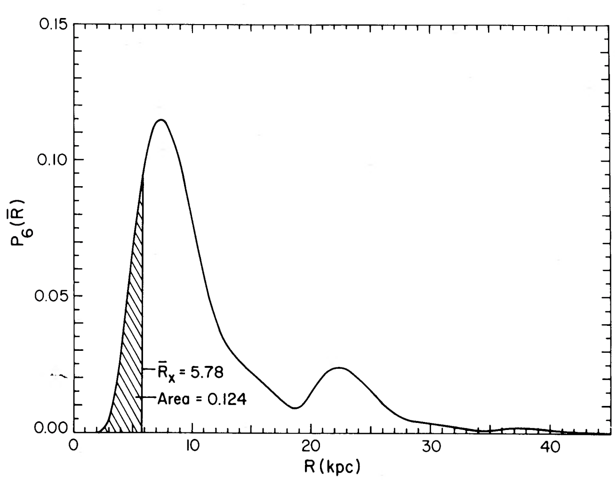

FIG. 1.—Probability distribution of the mean value of distance to galactic center $\bar{R}$ of a random subset of six globular clusters. This distribution is calculated from the 108 non X-ray emitting globular clusters, according to the filtered Fourier transform method described in the text. The mean value of R of the six X-ray emitting globular clusters is $\bar{R}_x = 5.78$.

TABLE 1

Globular Cluster Data

Cluster (1)	t_{rc} (yr) (2)	R (kpc) (3)	t_{B2} (yr) · f (4)	L_x/L_* (5)
104	8.3E+07	8.20	9.4E+07	
288	1.2E+09	12.30	8.0E+10	
362	7.1E+07	10.20	1.3E+08	
1261	4.9E+08	16.10	3.1E+09	
PAL 1	...	52.00	...	
1904	1.4E+08	20.00	1.4E+09	
2298	2.3E+08	17.90	9.8E+09	
2419	1.2E+10	101.30	2.3E+10	
2808	1.6E+08	11.50	7.1E+07	
PAL 3	4.2E+09	99.40	1.1E+13	
3201	2.9E+08	9.70	3.4E+09	
PAL 4	6.9E+09	96.30	4.5E+12	
4147	1.3E+08	20.20	1.2E+10	
4372	...	7.80	...	
4590	4.7E+08	10.20	3.2E+09	
4833	6.3E+08	7.60	2.6E+09	
5024	1.1E+09	18.10	2.5E+09	
5053	5.4E+09	16.20	6.8E+11	
5139	3.5E+09	7.10	4.7E+08	
5272	4.3E+08	12.40	6.8E+08	
5286	7.1E+07	7.50	2.5E+08	
5466	2.8E+09	15.30	1.5E+11	
5634	...	17.50	...	
5694	1.6E+08	26.10	1.3E+09	
4499	2.5E+09	15.40	1.6E+11	
5824	4.7E+07	17.10	8.7E+07	
PAL 5	...	16.50	...	
5897	3.0E+09	6.90	4.1E+10	
5904	2.4E+08	6.70	5.6E+08	
5927	2.2E+08	5.00	8.7E+08	
5946	...	5.30	...	
5986	4.4E+08	4.50	8.2E+08	
PAL 14	...	60.00	...	
6093	2.5E+07	3.20	1.4E+08	
6101	...	8.60	...	
6121	1.2E+08	7.00	1.6E+09	
6139	...	3.00	...	
6144	1.8E+08	2.80	1.4E+10	
6171	2.0E+08	4.30	1.3E+10	
6205	3.1E+08	9.10	1.0E+09	
6218	2.8E+08	5.10	4.4E+09	
6229	2.3E+08	30.60	2.7E+09	
6235	...	2.80	...	
6254	1.5E+08	5.50	1.1E+09	
6266	8.0E+07	3.20	1.5E+08	
6273	3.6E+07	2.40	5.3E+08	
6284	6.8E+06	2.00	3.4E+08	
6287	5.7E+07	1.70	4.9E+09	
6293	1.5E+07	2.00	2.8E+08	
6304	1.3E+08	3.70	1.3E+09	
6316	...	3.50	...	
6325	1.4E+08	2.50	2.4E+09	
6333	9.6E+07	2.70	6.2E+08	
6341	8.8E+07	10.00	2.5E+08	
6342	...	6.70	...	
6352	2.2E+08	4.30	5.7E+09	
6355	...	2.30	...	
6356	3.5E+08	8.60	3.3E+08	
6362	1.3E+09	5.60	1.5E+10	
6366	...	5.60	...	
6388	1.3E+08	6.30	2.7E+07	
6397	2.3E+07	7.10	1.1E+09	
6401	...	2.30	...	
6402	1.6E+09	4.40	2.0E+09	
PAL 6	...	6.10	...	
6426	3.1E+08	9.60	1.1E+11	
6440	3.0E+07	4.80	2.5E+07	
6453	...	2.10	...	
6496	...	2.40	...	
6517	1.7E+07	3.20	6.4E+07	
6522	1.4E+07	2.60	1.2E+08	
6528	3.5E+07	1.80	3.0E+08	
6535	...	5.30	...	
6539	...	6.90	...	
6541	6.7E+07	3.00	2.5E+08	
6544	...	4.40	...	
6553	3.4E+08	3.30	4.5E+08	
6558	...	1.00	...	
PAL 7	...	5.80	...	
6569	8.1E+07	1.60	6.6E+08	
6584	...	7.30	...	
6626	5.2E+07	3.10	2.4E+08	
6637	2.0E+08	2.20	6.5E+08	
6638	1.3E+08	1.90	1.5E+09	
6642	...	3.20	...	
6652	...	6.40	...	
6656	5.5E+08	6.10	4.3E+08	
PAL 8	...	22.00	...	
6681	2.3E+07	2.90	2.0E+08	
6715	1.4E+08	13.00	6.0E+07	
PAL 9	...	7.50	...	
6723	6.0E+08	2.70	4.1E+09	
6752	8.9E+07	6.00	3.6E+08	
6760	2.1E+07	6.30	1.2E+09	
6779	5.4E+08	9.70	4.4E+09	
PAL 10	...	7.90	...	
6809	9.6E+08	4.80	7.1E+09	
PAL 11	...	6.70	...	
6838	8.1E+07	7.60	9.6E+09	
6864	7.8E+07	11.70	1.9E+08	
6934	4.2E+08	12.00	1.2E+09	
6981	7.2E+08	12.90	1.6E+10	
7006	1.2E+09	32.10	1.1E+10	
7089	4.2E+08	10.50	2.2E+08	
7099	1.5E+07	7.60	2.0E+08	
PAL 12	...	15.60	...	
PAL 13	...	25.70	...	
7492	9.8E+08	21.30	5.3E+11	
1851	3.6E+07	16.40	9.6E+07	1.0
6441	7.6E+07	1.90	4.4E+07	3.7
6624	2.5E+07	1.40	2.5E+08	52
6712	3.5E+07	4.00	2.3E+09	1.2
7078	1.2E+08	10.30	9.6E+07	3.0
Ter 2	1.0E+07	0.70	8.0E+08	3.1
Lil 1	2.5E+07	0.90	1.1E+08	8.8
Gri 1	...	0.90	...	12

computed by filtering the Fourier transform $\Phi_n(k)$, defined by

$$\Phi_n(k) \equiv \int \exp\left[\frac{ik}{n}(R_1 + R_2 + \cdots + R_n)\right] \times p(R_1)p(R_2)\cdots p(R_n)dR_1 dR_2 \cdots dR_n\,, \quad (5a)$$

with $n = 6$. Here $p(R)$, the distribution in R, is taken to be a normalized sum of delta functions centered on the observed values of R, $\tilde{R}$, for the non-XEGC,

$$p(R) = \frac{1}{N}\sum_{i=1}^{N} \delta(R - \tilde{R}_i)\,, \quad (5b)$$

with $N = 108$. Filtering $\Phi_n(k)$, i.e., imposing $\Phi_n(k) = 0$ for $k > k_{\max}$, serves to smooth out features arising from the delta functions in $p(R)$. Inverting the filtered $\Phi_n(k)$ then yields $P_6(\bar{R})$. In Figure 1 we have used $k_{\max} = (0.25\ \text{kpc})^{-1}$, but our results are not sensitive to this value, as confirmed by a number of different trials.

Application of the central limit theorem to estimate $P_6(\bar{R})$ would have been disastrous here, yielding a Gaussian distribution with mean 11.7 and standard deviation 7.06. Not only does such a Gaussian have substantial amplitude at negative $\bar{R}$ but also it has the wrong shape. The correct bimodal nature of $P_6(\bar{R})$ (cf. Fig. 1) derives from beating between the three clusters at large R and the remainder. We have verified that as n approaches infinity, $P_n(\bar{R})$ does indeed approach a Gaussian.

The mean R of the six XEGC is $\bar{R}_x = 5.8$. From $P_6(\bar{R})$ we compute that the probability that $\bar{R}$ could be this small or smaller is 0.12. Thus, on this basis, it is unlikely the XEGC are part of the same population as the non-XEGC. (This test, when applied to the full set of eight XEGC, yields a probability of 0.018.)

We also used a completely independent statistical test, the Kolmogorov-Smirnov test (see, e.g., Lehman 1975) and ranked all 114 clusters according to R. If the XEGC clusters are typical of all clusters in their values of R, then their ranks should be uniformly distributed between 1 and 114. The cumulative, normalized distribution in ranks of the XEGC have a maximum deviation of 0.44 from a uniform distribution; the probability of this value, for six objects drawn randomly from a uniform distribution, is 0.071. This test again confirms the fact that the XEGC have significantly smaller values of R than the non-XEGC. (This test, when applied to the full set of eight XEGC, yields a probability of 0.005. A similar probability is obtained if Terzan 1 and Terzan 5 are included instead of Liller 1 and Grindlay 1.)

We performed the same Kolmogorov-Smirnov test on the globular clusters in M31, using optical data from Battistini *et al.* (1980) and Sargent *et al.* (1977) and X-ray data from van Speybroeck (1981). In M31 there are 323 globular clusters identified, 20 of which contain discrete X-ray sources. The XEGC have significantly small values of *projected* radius, R_p; the Kolmogorov-Smirnov test gives a probability of 9.8×10^{-3} that their ranks could have been drawn from a uniform distribution.

b) *Distribution in z*

We also performed the Kolmogorov-Smirnov test for the distribution in height above the galactic plane, z (data not shown in Table 1), yielding a probability of 0.22. Finally we performed a Kolmogorov-Smirnov test on the distribution of a "flatness-parameter" $F \equiv 3z^2/R^2$ and found that the XEGC do not have an abnormal distribution of this parameter.

From these tests, we conclude that the XEGC are indeed correlated with small R, and that their small values of z are probably a consequence of the first correlation.

c) *Correlations between R and t_{B2}*

Evidently, there is something about proximity to the galactic center that favors creation of an XEGC out of an ordinary globular cluster. To test the hypothesis that a relatively low value of t_{B2} is also correlated with the XEGC, and with small R in general, we consider the location of clusters in the t_{B2}-R plane, shown in Figure 2. *It is clear that the XEGC have relatively low values of t_{B2}.* A Kolmogorov-Smirnov test on the ranks of t_{B2} for the XEGC shows that the probability they could have been drawn from a uniform distribution is 0.036. *More generally, there is a positive correlation of R with t_{B2} for all 84 globular clusters for which data are available.* Using a Spearman's rank correlation test (see, e.g., e.g., Lehman 1975) for R versus t_{B2}, we find that the probability the observed correlation could be accidental is $1 - \text{erf}\ (2.2) \approx 2 \times 10^{-3}$, where erf ($x$) is the error function of x. (The Spearman's test assigns a rank in R and a rank in t_{B2} to each cluster, defines a function of the square of the difference in these two ranks, summed

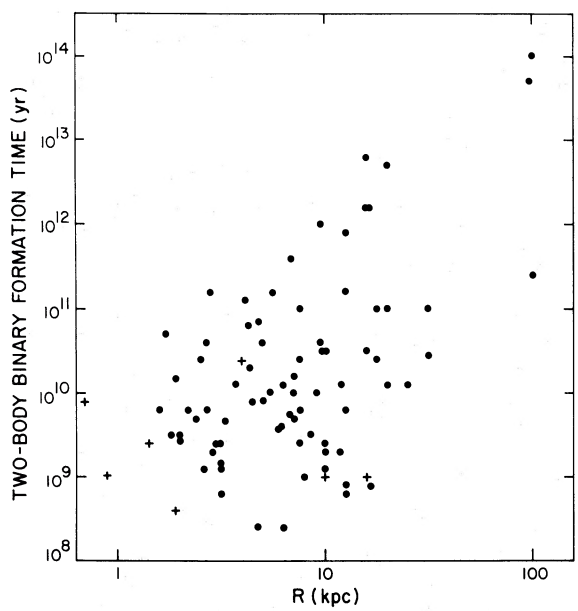

FIG. 2.—Scatter diagram of the two-body formation time versus distance from the galactic center, R, for 84 globular clusters. The positions of the X-ray emitting clusters and non X-ray emitting clusters are idicated by plus signs and filled circles, respectively. For this diagram we used a value $f = 0.1$ in eq. (1).

over all clusters and then computes the probability this function could be as large and positive or as large and negative as it is for arrays of ranks chosen randomly.)

One might suspect that some neglected effect is responsible for the observed correlation between R and t_{B2}. Most selection effects would be associated with the distance D of a cluster from Earth. We computed the correlation between D and t_{B2} and found that according to the Spearman's test the probability that the observed correlation could have been accidental is 0.06, a factor 30 larger than the corresponding probability for R versus t_{B2}. This result is consistent with that expected from a principal correlation of t_{B2} with R, and a residual correlation of R with D. Thus we can rule out a selection effect associated with D.

d) *Correlations between R and* t_{rc}

It was originally pointed out by Bahcall and Ostriker (1975) that the XEGC (only four were known at that time) had relatively low values of the central relaxation time, t_{rc}. In terms of v_c and n_c, t_{rc} may be written as (setting the logarithm term to a constant)

$$t_{rc} = 6 \times 10^9 \text{ yr} \left(\frac{v_c}{1 \text{ km s}^{-1}}\right)^3 \left(\frac{n_c}{1 \text{ pc}^{-3}}\right)^{-1}, \quad (6)$$

and is the time for energy exchange via cumulative two-body gravitational scatterings. Core collapse of a self-gravitating system occurs on a time scale $\sim 100 t_{rc}$, and such a collapse could produce conditions favorable for forming an X-ray source. Bahcall and Ostriker argued that clusters that had undergone core collapse were those likely to have formed massive black holes, which they took as a model for globular cluster X-ray sources. For a variety of reasons (see § II*a*) such a model now seems unlikely, but core collapse does very definitely decrease the time scale for two-body binary formation, as both v_c and n_c increase in core collapse. This would promote formation of an X-ray source according to our adopted model.

To test the hypothesis of a correlation between R and t_{rc} for globular clusters in general, we consider the location of clusters in the t_{rc}-R plane, shown in Figure 3 (van den Bergh 1980, considered a similar diagram.) There is a positive correlation. A Spearman's test for R versus t_{rc} reveals that the probability the observed correlation could be accidental is $1 - \text{erf}(3.4) \approx 2 \times 10^{-6}$.

IV. DISCUSSION AND CONCLUSIONS

a) *Bulge Population Association*

The first of our analyses reveals that the XEGC are significantly nearer the galactic center than the average globular cluster. If we define the "galactic bulge" as the region $R \leq 4$ kpc, then four out of 34 globular clusters in the galactic bulge are XEGC (again using the reduced, optically selected sample that excludes Liller 1 and Grindlay 1). If all of the bulge clusters were equally likely to harbor an X-ray source, then the probability that every X-ray source be located in a different cluster, as observed, is from equation (4), $P(4, 34) = 0.83$.

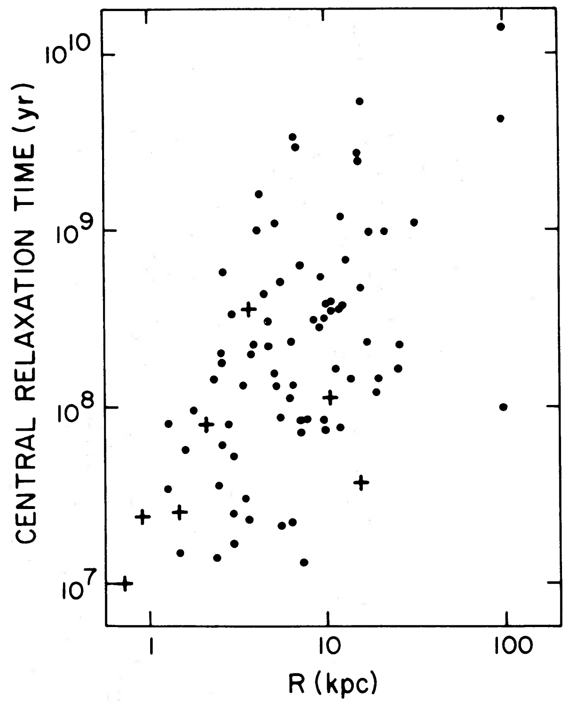

FIG. 3.—Scatter diagram of the central relaxation time versus distance from the galactic center, R, for 84 globular clusters. The positions of the X-ray emitting clusters and non X-ray emitting clusters are indicated by plus signs and filled circles, respectively.

If it is *only* the value of R, rather than some other parameter such as t_{B2}, that determines the probability of forming an XEGC, then one must search for an acceptable physical explanation associated only with R. One possibility is that clusters with small apocenter pass near the galactic center more frequently than other clusters, and that each passage may somehow create an X-ray source, e.g., by forming a binary or accumulating gas. However, if such sources are binary systems, then their lifetime (cf. eq. [2]) is longer than an orbital period for many clusters outside of the bulge, so we should expect to see many XEGC in that region also, and we do not.

Another possibility is that the relatively high metal abundances found in clusters nearer to the galactic center (e.g., van den Bergh 1980) are associated with likelihood of forming an X-ray source. This might be the case if such higher abundances indicate a higher initial population of massive stars and possibly a higher number of compact remnants (Grindlay 1981).

b) *Two-Body Binary Formation Rate Association*

If the model for X-ray sources in globular clusters discussed in §§ II*a* and II*b* is relevant, then the probability of observing an X-ray source in a cluster, P_x, should vary inversely with t_{B2} (cf. eq. [3]). Small number statistics make it difficult to reliably test this expectation, but we have made an attempt by binning the data into half decade logarithmic intervals, shown in Table 2. In each range of t_{B2}, the observed P_x is the number of

TABLE 2

PREDICTED VERSUS THEORETICAL PROBABILITIES OF CLUSTER X-RAY SOURCE OCCURRENCE

$t_{B2} \cdot (f/0.03)$ (1)	Total Number of Clusters (2)	Number of X-ray Clusters (3)	P_x Observed (4)	P_x Theoretical (5)
$>10^{11}$ yr	24	0	0	<0.007
3.2×10^{10}–10^{11} yr	16	1	0.063	0.011
10^{10}–3.2×10^{10} yr	17	1	0.059	0.033
3.2×10^{9}–10^{10} yr	17	2	0.12	0.11
10^{9}–3.2×10^{9} yr	8	3	0.38	0.33
$<10^{9}$ yr	2	0	0	>1

XEGC divided by the total number of clusters. It can be seen that the data are not inconsistent with $P_x \propto t_{B2}{}^{-1}$.

There is a further *quantitative* check on P_x. The last column in Table 2 computes the "theoretical" value of P_x, using equation (3), a typical L_x of 10^{37} ergs s^{-1} in equation (2), and an average value of t_{B2} for each range in the first column of Table 1. The values of f and L_x are not arbitrary, but are consistent with observations, so the theoretical values of P_x cannot be freely scaled. That these values do not differ substantially from the observed values (cf. cols. [4] and [5] of Table 2) gives further support to our adopted model for the nature and formation of X-ray sources in clusters. For example, binary formation by the three-body process alone yields less than one expected XEGC.

Table 2 also explains, within the adopted model, why two X-ray sources are never seen in the same cluster, in answer to one of our puzzles. If the probability of forming an X-ray source is uniform for all clusters in each range of t_{B2}, then equation (4) gives $P(3, 8) = 0.66$ for clusters in the next to last row, and higher values for the others. Thus either by this analysis, or the analysis treating all bulge clusters on equal footing as in § IV*a* above, it is not surprising that two X-ray sources are not found in any one cluster.

We can attempt to push the two-body binary formation model one step further. If all the X-ray sources in the galactic bulge region *outside* of globular clusters are also formed by this process, then the expected number observed should be (cf. eq. [3]) $N_{xG} \sim \tau_x/t_{B2G}$. Here t_{B2G} is the two-body binary formation time scale for the galactic bulge as a whole, and can be computed to be

$$t_{B2G} \sim 4 \times 10^8 \text{ yr}\left(\frac{n}{0.1 \text{ pc}^{-3}}\right)^{-1} \times \left(\frac{v}{250 \text{ km s}^{-1}}\right)\left(\frac{N}{10^{11}}\right)^{-1}\left(\frac{f}{0.03}\right)^{-1}, \quad (7)$$

where N is the total number of stars and n and v are characteristic densities and velocities, normalized to their expected values. (In eq. [1], appropriate for globular clusters, the virial theorem was used to express N in terms of n and v.) Using $L_x = 10^{37}$ ergs s^{-1} again as a characteristic value in τ_x, we obtain from equations (2) and (7), $N_{xG} \sim 1$. That this computed number, at least based on the assumed values of the parameters, is considerably smaller than the observed value of ~ 30 suggests that most bulge X-ray sources may require a mechanism of formation other than capture of a general field star by a compact remnant. For example, the evolution of white dwarf binary systems, e.g., cataclysmic variables, into compact neutron star binary systems does not produce a system with the high X-ray luminosities observed, as shown by Rappaport, Joss, and Webbink (1981). The origin of the galactic bulge X-ray sources outside globular clusters constitutes another puzzle. Such sources may, in fact, originate in globular clusters. In any case, this analysis and equation (7) do give some semiquantitative explanation for the fact that globular clusters, relative to their total mass, provide a favorable environment for the formation of X-ray sources, in answer to another of our puzzles.

There is always the possibility of an additional, underlying factor correlating with X-ray emission from clusters, other than those factors considered here. One possible such factor, core mass, may be ruled out, as quantitative analysis shows.

c) *Influence of the Galactic Center*

The positive correlations of t_{B2} and t_{rc} with R (cf. Figs. 2 and 3) motivate a physical explanation in which proximity to the galactic center produces a cluster with relatively low values of t_{B2} and t_{rc}. We offer two possibilities, involving either the initial conditions or the initial dynamical evolution. The relatively high densities of matter near the galactic center, at the initial time of formation of the clusters, may have produced clusters with relatively low values of t_{B2} and t_{rc}. Alternatively, the relatively strong shock-heating of clusters passing near the galactic center could have hastened the rate of collapse of such clusters by increasing the initial rate of core contraction (cf. Spitzer 1975). In either of these possibilities, the effective *initial* distributions of t_{B2} and t_{rc} among the clusters would have been R-dependent, thus modifying the analysis of Lightman, Press, and Odenwald (1978) regarding the distribution of central relaxation times.

In any case, the observed correlation of R with t_{B2} for globular clusters in general, together with our adopted model for globular cluster X-ray sources, affords an explanation to one of our puzzles.

In conclusion, the positive correlation found between distance from galactic center and two-body binary formation time for globular clusters in general, plausible cause-and-effect explanations for this correlation, and the strongly supported hypothesis that X-ray sources in globular clusters require binary star systems give a possible explanation of several of the puzzles regarding X-ray emitting globular clusters.

We thank Y. Avni, G. Rybicki, and L. van Speybroeck for useful discussions. This work was supported in part by NASA grants NAGW 246 and NAS 8-30751.

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