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THE SCATTERING PHASE FUNCTION OF INTERSTELLAR GRAINS: THE CASE OF THE REFLECTION NEBULA NGC 7023

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ABSTRACT

IUE observations of HD 200775 and its associated reflection nebula NGC 7023 covering the spectral range 1300–3100 Å, supplemented by new ground-based measurements of the nebular brightness distribution at 3500, 4100, 4700, and 5500 Å, have been used as a basis for a determination of the scattering properties of the nebular dust grains. Derived values of the ratio of nebular to stellar flux, based on a comparison of *TD-1* and *IUE* observations of HD 200775/ NGC 7023 in the UV and on existing ground-based measurements of the visual nebular surface brightness distribution, and published data on the ratio of the far-IR luminosity of NGC 7023 to the bolometric luminosity of HD 200775 led to independent determinations of the average nebular optical depth in the UV and of the average grain albedo in the UV, $\langle a \rangle_{UV} \approx 0.54$. With other model parameters constrained by existing UV, visual, far-IR, and molecular observations, the wavelength dependence of the phase function asymmetry was derived through model fits to the average nebular surface brightness, observed at an offset of 22″.5 from HD 200775. The asymmetry factor g declines monotonically with decreasing wavelength, reaching a value of about $g = 0.25$ at 1400 Å. The dust albedo is found to increase to a level of $a \approx 0.6$ at $\lambda = 1400$ Å, after reaching a minimum of $a \approx 0.4$ near 2200 Å. The effect of possible H_2 fluorescence is considered and is found to lead to insignificant changes in the final results. The dust properties derived for the far-UV suggest that isotropically scattering particles of high albedo provide a significant contribution to interstellar scattering in the far-UV.

Subject headings: interstellar: matter — nebulae: individual — nebulae: reflection — ultraviolet: spectra

I. INTRODUCTION

The asymmetry factor of the phase function of scattering by interstellar grains is of far-ranging importance in the physics of interstellar space. Especially in the UV spectral region, where the interstellar radiation field determines the level of photoionization and photodissociation in diffuse interstellar clouds (Roberge, Dalgarno, and Flannery 1981) and where it affects the lifetimes of molecules in dark clouds (Bernes and Sandqvist 1977; Sandell 1978), the phase function asymmetry is the most critical factor which determines the penetration of radiation into these environments, particularly when the albedo is relatively high. Other calculations which depend on the asymmetry factor are the heating of grains in globules (Keene *et al.* 1980; Keene 1981) and the determination of the size distribution of the particles responsible for scattering in the ultraviolet (Witt 1979). Finally, the interpretation of the observations of the far-UV background radiation at high galactic latitudes (Paresce and Jakobsen 1980; Maucherat-Joubert, Deharveng, and Cruvellier 1980) in terms of extragalactic sources or simply as backscattered galactic starlight is largely dependent on the shape of the scattering phase function.

Past attempts to determine the phase function asymmetry of interstellar scattering have been reviewed most recently by Savage and Mathis (1979). There is substan-

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tial agreement concerning the scattering properties of grains in the visible, e.g., with an albedo $a = 0.6$–0.7, $g \approx 0.6$–0.7. The reported values for the asymmetry parameter $g = \langle \cos \alpha \rangle$ for the far-UV wavelengths $\lambda <$ 2000 Å, however, span the range from $g \approx 0.25$ (Witt 1977*a*), based on the study of the Merope reflection nebula, to $g > 0.9$ (Henry *et al.* 1978), derived from a search for diffuse galactic light (DGL) at intermediate galactic latitudes. The geometry of the Merope Nebula is not particularly certain (Jura 1979), which casts doubts upon the related result, while the null result of Henry *et al.* (1978) must be considered far from definitive in light of the very substantial differences between the reported intensity levels of the diffuse UV background radiation found in different observational surveys of the same portions of the sky (Paresce, McKee, and Bowyer 1980). The two independent studies of the extended Orion reflection nebulosity carried out with the *OAO 2* data by Witt and Lillie (1978) and with *TD-1* data by iMorgan, Nandy, and Thompson (1982) both lead to the identical conclusion that the phase function at $\lambda < 2000$ Å is significantly less forward throwing than the corresponding function in the visual, thus favoring the smaller value of g for the UV.

Among the methods available for the determination of the wavelength dependence of grain properties, the study of reflection nebulae offers certain distinct advantages.

1. In contrast to the DGL approach, one deals with a *wavelength-independent geometry* relating the scatterers with the source of illumination. This is a consequence of the fact that a suitable reflection nebula is illuminated by a single star, while the spatial distribution of the illumination source for the DGL varies with wavelength.
2. The spectrum of the illumination source of a reflection nebula can be determined directly with high accuracy.
3. While diffuse galactic light is difficult to measure, especially at higher galactic latitudes, reflection nebulae are relatively bright objects, which can be observed with relative ease at infrared, visual, and ultraviolet wavelengths.
4. The nature of the nebular light as scattered light in the visual and the UV can be readily established by spectroscopic means, whereas the diffuse scattered light background at high galactic latitudes may contain other components such as radiation from the hypothesized decay of cosmological neutrinos (Stecker 1980) or from H_2 fluorescence in interstellar clouds (Duley and Williams 1980; Jakobsen 1981).

An unfortunate feature of past analyses of scattered light intensities in the context of diffuse galactic light or reflection nebulae is the fact that a separate determination of grain albedo and phase function asymmetry was difficult, and the uncertainties in the determination of one of the quantities directly affected the determination of the other. In the present investigation of NGC 7023, we will introduce a new approach designed to essentially decouple the determinations of a and g. This is made possible through the availability of total nebular fluxes at far-infrared, visual, and ultraviolet wavelengths for NGC 7023 in addition to the more conventional data on nebular intensities and their radial distributions. In § II*a* of this paper we shall discuss new observations with the *International Ultraviolet Explorer* (*IUE*) of two nebular fields in NGC 7023 as well as new ground-based measurements of the surface brightness in NGC 7023 in the immediate vicinity of the illuminating star HD 200775. In § II*b* we shall derive total nebular fluxes from existing data on the NGC 7023/HD 200775 system obtained by the *TD-1, ANS,* and *IUE* satellites and by ground based observations. The relevance of far-IR fluxes from the nebula will be discussed. In § III we shall review the theory of the luminosity of reflection nebulae, and we shall derive models applicable to NGC 7023 both with regard to the ratio of nebular to stellar flux as well as to the nebular surface brightness distribution. Section IV contains the analysis of the observational data. Section V contains the discussion of the results, their potential uncertainties and implications, and the conclusions and summary of this investigation.

II. OBSERVATIONS

a) *Nebular Surface Brightness*

Detailed descriptions of NGC 7023, photographs of the nebula and its environment, and a review of previous studies are contained in the recent papers by Whitcomb *et al.* (1981) and Witt and Cottrell (1980*a*).

i) *IUE Observations*

Low-dispersion spectra of NGC 7023 and HD 200775 covering the 1250–3100 Å range were obtained with *IUE*, using the large ($\sim 10'' \times 20''$) aperture. A description of the instrument and its performance has been given by Boggess *et al.* (1978). The exposures used for this investigation are identified and described in Table 1. Figure 1 shows the relative location and orientation of the large *IUE* apertures with respect to HD 200775 for the NRBAW and the UK 207 programs. The pointing of the *IUE* aperture in the two programs was the result of slightly different objectives. In NRBAW we attempted to obtain maximum radial coverage consistent with observing a relatively bright portion of the nebula; in UK 207, the aperture was placed on a relative brightness peak near HD 200775 in order to maximize information about spectral shape, especially around 2200 Å. Using the *IUE* photometric calibration of Bohlin and Holm (1980) and aperture widths from Bohlin *et al.* (1980), the observed nebular fluxes were converted into intensities. In order to obtain good spatial resolution, yet maintain an adequate signal-to-noise ratio, the nebular spectra were extracted from the 55-line spatially

TABLE 1

IUE OBSERVATIONS OF NGC 7023 AND HD 200775

Object	Program ID	Image No.	Aperture	Exposure Time	Date
NGC 7023	NRBAW	LWR 5177	L	240 min	1979, day 208
NGC 7023	NRBAW	SWP 5966	L	240 min	1979, day 208
NGC 7023	UK 207	LWR 5488	L	275 min	1979, day 244
HD 200775 ...	NRBAW	LWR 5178	L/S	59 s/6 min	1979, day 208
HD 200775 ...	NRBAW	SWP 5967	L/S	1 min 39 s/6 min	1979, day 208

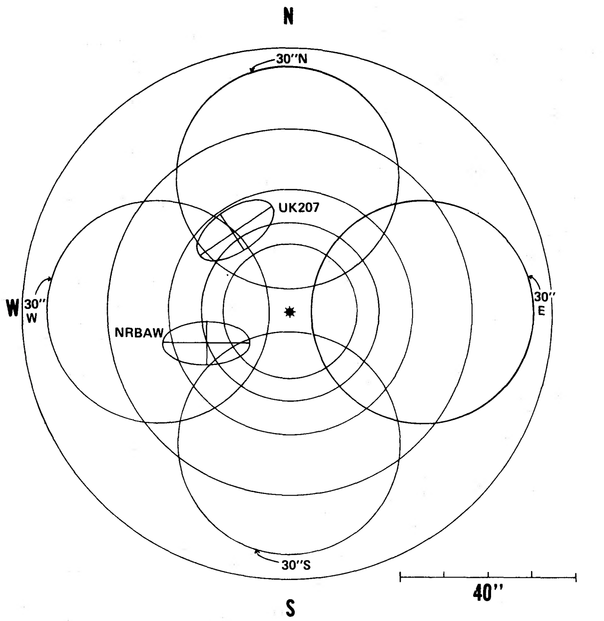

FIG. 1.—Distribution of observed regions near HD 200775 in NGC 7023. The four circular fields of 50″ diameter centered at 30″ E, N, W, and S are from Witt and Cottrell 1980*a*. Groundbased observations through five concentric apertures of 31″, 42″, 58″, 83″, and 118″ diameter are centered on HD 200775; and the *IUE* observations in fields NRBAW and UK 207 are indicated. The star represents the location of HD 200775.

resolved format, but then averaged over 100 Å wavelength bins. Wavelength resolution could be sacrificed, because the spectrum of NGC 7023 is essentially identical to that of HD 200775. Successive lines in the spatially resolved image format correspond to the spectrum of adjacent nebular strips approximately 2″.2 by 10″.2. The image quality of the *IUE* telescope is estimated to be 5″ full width at half-maximum.

To derive the radial intensity distribution from the NRBAW data, two corrections must be applied: (1) corrections for sensitivity variations of the instrument along the slit, and (2) corrections for instrumentally scattered light caused by the star near the aperture.

Clarke and Moos (1981) have investigated the relative sensitivity of the *IUE* instrument along the length of the large aperture at Lyα using the geocorona as a source of uniform intensity across the aperture. We repeated the study, using Lyα data from images SWP 5966, 5968, 5980, 5983, 6023, 7576, and 7578. Our results agree with the findings of Clarke and Moos within the $\pm 6\%$ errors. To investigate the relative sensitivity along the large apertures at wavelengths other than Lyα, the ideal source is a spatially uniform continuum. Lacking such a source, we studied the response of the *IUE* camera faces to trailed star exposures. By quantitative comparison of the line-by-line spatially resolved images of trailed star exposures near Lyα with the Lyα geocorona exposures, we established a transformation function, which allows the determination of a response profile to a uniform extended source through transformation of an observed response profile for a trailed star. The deviations from uniform response were found to be less than 10% over the central seven lines of the line-by-line file, which contain about 90% of the entire signal, at all wavelengths. Since the LWR camera has a physically different aperture, the most uncertain assumption is that the response to uniform illumination in LWR is the same as in SWP.

Little was known about instrumental scattering caused by stellar images located slightly outside the *IUE* apertures, when this study was undertaken. Therefore, following a plan originally devised by A. V. Holm, seven exposures of the scattered light produced by η UMa were obtained with each of the two *IUE* cameras, with the star placed at a range of offsets and position angles with respect to the center of the large *IUE* apertures. The distance dependence of scattered light was again determined from line-by-line spatially resolved data with each line integrated over 100 Å wavelength intervals, taking into account all known details about the aperture geometry. The wavelength dependence of scattered light was explored using the spectra of the scattered light integrated over the entire aperture.

The principal findings were as follows:

1. The scattered light intensity is independent of position angle for positions falling near the long axis of the large aperture. Limited data from positions near the short axis of the large aperture suggest that the relative intensity there may be about 50% larger for the same value of r.

2. The scattered light intensity as a function of distance can be well represented as a power law for the

distance range 10″–50″ between the illuminating star and corresponding points in the aperture. The intensity follows approximately a r^{-3} law.

3. In the SWP camera the relative intensity of scattered light decreases with increasing wavelength; the degree of the wavelength dependence increases with distance from the aperture. The LWR camera shows an essentially flat response with no significant wavelength dependence for the scattered light.

The findings on instrumentally scattered light are best represented by

$$\log\left(\frac{S_{\mathrm{scat}}}{F_*}\right)_{\mathrm{SWP}} = 8.85 - 2.80 \log r + (5.41\times10^{-4} - 6.39\times10^{-4} \log r)(\lambda - 1500) \quad (1)$$

and

$$\log\left(\frac{S_{\mathrm{scat}}}{F_*}\right)_{\mathrm{LWR}} = 9.40 - 3.21 \log r, \quad (2)$$

with the ratio S_{scat}/F_* in units of sr^{-1}, the distance r in arcsec, and the wavelength λ in Å. Respectively, S_{scat} and F_* are the measured scattered light intensity and the stellar flux at the *IUE*. Equations (1) and (2) apply to scattered light seen in line-by-line resolved image formats, with r being the distance between the star and a given intersection of a line with the long axis of the corresponding *IUE* aperture.

The details of the studies of instrumental response functions and the instrumental scattered light will be reported in the *IUE Newsletter*. Two circumstances helped to minimize potential problems with these corrections. First, the relative faintness of HD 200775 leads to a small scattered light correction for the NRBAW and the UK 207 fields, less than 18% and 10% of the total signal, respectively, and second, a scattered light observation was made with η UMa in nearly exactly the identical position of HD 200775 relative to the NRBAW field, thus allowing a correction to be made without use of interpolation or extrapolation.

The residual nebular intensities in the NRBAW field derived from images LWR 5177 and SWP 5966 obtained after subtraction of instrumentally scattered light are consistent with each other in two ways: (1) Reduced to a constant distance from HD 200775, the absolute intensity from LWR 5177 and SWP 5966 is continuous in the overlap region at 1850–1975 Å. (2) When plotted as a function of distance, both the SWP and LWR images reveal similar nebular fine structure, starting at a distance of ~20″. Therefore, the independent pointings for the SWP and LWR exposures apparently succeeded in placing the aperture on the same nebular area. Figure 2 displays the nebular intensity derived from SWP 5966 and LWR 5177 to illustrate the intensity gradient in

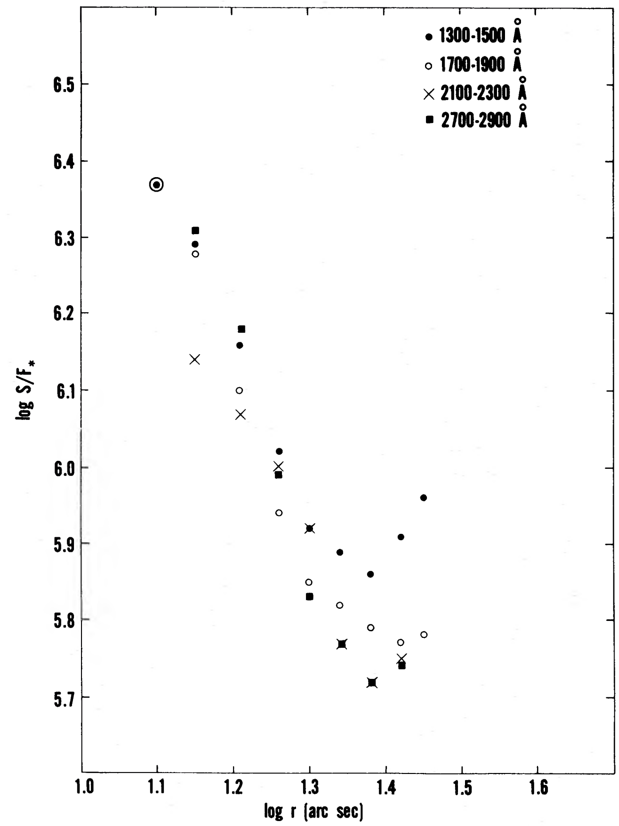

FIG. 2.—Spatially resolved surface brightness distribution in NGC 7023 near HD 200775 as seen in the NRBAW field. The 1300–1500 Å and 1700–1900 Å data are from the SWP 5966 image, the 2100–2300 Å and 2700–2900 Å data from the LWR 5177 image. The discontinuity in slope in the brightness data occurring near $\log r = 1.4$ is evidence that the relative positioning of the aperture in the two exposures was identical.

NGC 7023 near HD 200775 in four wavelength intervals. For later discussions, note that the gradient of the intensity in the range $1.15 \leq \log r \leq 1.35$ is steeper in the 2700–2900 Å range than it is in the 1300–1500 Å range.

The orientation of the UK 207 field does not permit a resolution of the radial intensity variation. As expected on the basis of visual photographs of NGC 7023 (Ney, Hatfield, and Gehrz 1980), the nebular intensity in the UK 207 field (at an average distance of $r = 22\farcs5$) is higher by more than a factor of 2 than the measured nebular intensity in the NRBAW field at $r = 22\farcs5$. However, the spectral shape is identical to that found in NRBAW. These small-scale intensity variations superposed on a large-scale nebular intensity gradient are not unexpected. The photographs of NGC 7023 by Ney, Hatfield and Gehrz (1980) show the existence of considerable nebular structure within 120″ of HD 200775. The data from NRBAW alone, as shown in Figure 2, give evidence of significant structure on a scale of a few arcsec. The surface brightness measurements in NGC

7023 at λλ5515, 4733, 4093 and 3470 Å by Witt and Cottrell (1980*a*) through circular diaphragms of 50″ diameter, centered on points 30″ north, west, south, and east of HD 200775 (see Fig. 1) also reveal differences in intensity of a factor of 2 or more between adjacent fields. Finally, Duncan, Harlan, and Herbig (1981) indicate that the type of nebular structure found in the inner regions of NGC 7023 appears to be typical of reflection nebulae illuminated by recently formed Ae/Be stars, a class of which HD 200775 is a member (Herbig 1960).

ii) *Ground-Based Observations*

To provide for a comparison of our UV data with the nebular intensity at visual wavelengths, the existing ground based photometry of NGC 7023 by Witt and Cottrell (1980*a*) needed to be extended to smaller offset angles and to higher angular resolution near the star. An ideal method to accomplish such measurements with a ground-based telescope is the photometry of the central star through a series of concentrically placed circular diaphragms of increasing radii. This approach is suitable where the nebular contribution to the total flux is large in relation to the photometric errors in the flux from the star. This condition is met in the case of NGC 7023. It has the added advantage that no corrections for instrumentally scattered light are necessary, because even the smallest diaphragm used already contains all stellar light. Corrections for sky background can be derived from a corresponding series of measurements at a nearby spot free of stars or nebulosity. Measurements of this type were made in NGC 7023 on 1981 August 26, using the No. 2 90 cm telescope of KPNO with a photon-counting photoelectric photometer and employing a set of diaphragms with diameters 30″.8, 42″.2, 55″.0, 83″.3, and 118″.0. The four-color filter set was the same as used earlier by Witt and Cottrell (1980*a*). The result of five independent measurement series in each of the four filters is shown in Figure 3, after corrections for sky contributions have been applied. The sky corrections ranged typically from less than 5% of the total signal in the largest diaphragm to about 0.4% in the smallest. It is important to note that the nebular flux within a 59″ radius of HD 200775 already amounts to 12% to 19% of the stellar flux in the wavelength regions from 5500 to 3500 Å, suggesting that the integrated flux of the nebula seen in still larger apertures at shorter wavelengths may indeed be comparable to that from the star itself (Witt and Cottrell 1980*b*).

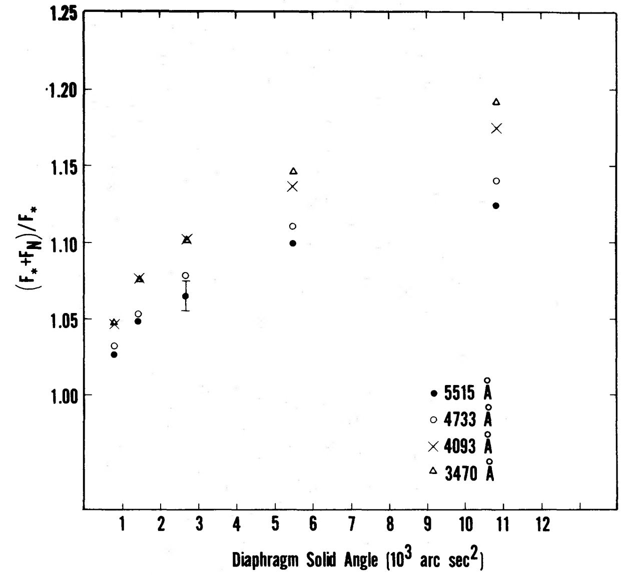

FIG. 3.—Ground-based observations of the combined stellar and nebular flux for HD 200775/ NGC 7023, as seen through the series of concentric apertures shown in Fig. 1. The error bar is typical for the data shown here and represents one standard deviation based on five series of independent measurements.

Average nebular intensities in annuli centered on the star can be derived from differences in the total flux through circular diaphragms of different size. Such results for NGC 7023 are listed in Table 2, where R_i and R_o are the inner and outer radii of the rings, respectively. The data are in good agreement with the earlier results of Witt and Cottrell (1980*a*), and they provide the needed extension of intensity measurements toward smaller offsets with relatively high radial resolution.

TABLE 2
FOUR-COLOR PHOTOMETRY OF NGC 7023 IN CONCENTRIC RINGS CENTERED ON HD 200775

R_i (arcsec)	R_o (arcsec)	log $S/F(\mathrm{sr})^{-1}$ 5515 Å	4733 Å	4093 Å	3740 Å
15.4	21.1	6.17±0.07	6.16±0.07	6.29±0.08	6.29±0.06
21.1	29.0	5.74±0.11	5.90±0.07	5.95±0.07	5.95±0.10
29.0	41.6	5.71±0.04	5.70±0.06	5.71±0.09	5.80±0.05
41.6	59.0	5.29±0.05	5.37±0.05	5.48±0.06	5.57±0.06

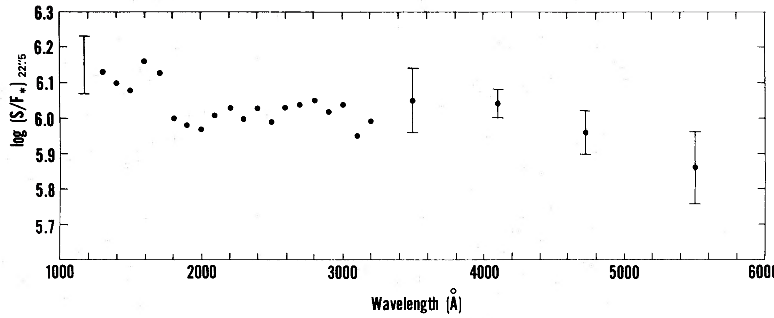

FIG. 4.—The average nebular surface brightness in NGC 7023 at an offset distance of 22″.5 from HD 200775 as a function of wavelength. The error bar on the left represents an uncertainty of ±20%, typical of the error in the ultraviolet measurements.

iii) *Combined Observations*

Before the ground-based data and the *IUE* surface brightness measurements can be compared, two matters must be resolved: (1) a common distance range must be identified which is equally covered in the ground based data and in the NRBAW and UK 207 measurements; and (2) the NRBAW and UK 207 should be averaged (ideally more observations in the 15″–25″ distance range around HD 200775 should be obtained) to make them more comparable to the ground-based data. We have identified a distance of $r = 22''.5 \pm 1.0$ as the most representative distance, set basically by the very limited distance range of the UK 207 exposure. The UK 207 and the NRBAW LWR data are averaged, using in each case the data from lines 28, 29, and 30 of the line-by-line resolved images, corresponding to the average distance of 22″.5. Since the shapes of the spectra derived from the UK 207 exposure and the NRBAW LWR exposure are essentially identical, the level of the NRBAW SWP spectrum was adjusted such that it would again join the average UK 207/NRBAW LWR spectrum continuously in the 1850–1975 Å region, exactly as it joined the NRBAW LWR spectrum alone before averaging. Intensities for $r = 22''.5$ for the spectral region observed from the ground are derived by straight interpolation between the data for the first two ring zones listed in Table 2. The resulting average nebular intensity S per steradian in units of the stellar flux F_* from HD 200775 as a function of wavelength at the radial distance $r = 22''.5$ is shown in Figure 4. The lack of a discontinuity in the level of S/F_* at the junction of *IUE* and ground-based data indicates that the average of the NRBAW and UK 207 observations is appropriate for comparison with the visible data. This lack of a discontinuity, however, is fortuitous and not a matter of design.

b) *Nebular and Stellar Fluxes*

i) *UV Observations*

Viotti (1976) published UV flux measurements of the star HD 200775, obtained with the S2/68 spectrometer aboard the *TD-1* satellite. Witt and Cottrell (1980*b*) claimed that these measurements were likely to be contaminated by a significant nebular contribution. This claim was based on several grounds: (1) the S2/68 aperture had a projected size of 12′×17′, sufficient to enclose the entire image of NGC 7023; (2) corresponding observations of HD 200775 with the *ANS* satellite with a 2′.5×2′.5 field had yielded significantly lower fluxes, consistent with a smaller nebular contribution; and (3) integrations of the nebular flux over the face of the nebula based on the measurements of surface brightness distributions in NGC 7023 at visual wavelengths by Witt and Cottrell (1980*a*) had yielded ratios of total nebular to stellar flux of the order 0.5. Higher values are to be expected at the larger optical depth in the UV.

With simple assumptions about the distribution law of the nebular surface brightness in NGC 7023, Witt and Cottrell (1980*b*) predicted ultraviolet fluxes for HD 200775, by correcting *TD-1* data for nebular contributions. The last two *IUE* observations listed in Table 1 determine UV fluxes for HD 200775. The residual nebular contribution in the *IUE* aperture can be estimated to be less than 3%, well within the 10%–15% uncertainty in the *IUE* absolute flux. Figure 5 shows the *IUE* results, the available *ANS* data for HD 200775, and Viotti's measurements. The last two sets of data have been reduced to the *IUE* flux scale, using the corrections of Bohlin *et al.* (1980), which are always less than 20%. As an independent check on Viotti's data, the *TD-1* S2/68 data reported for HD 200775 by Thompson *et al.* (1978) are shown as filled circles with error bars. These are in agreement with Viotti's measurements at $\lambda < 2000$ Å, but they appear to be systematically lower at $\lambda > 2000$ Å. Our new *IUE* data for HD 200775 fall about 20% lower than suggested by Witt and Cottrell (1980*b*), indicating that the total nebular flux at UV wavelengths included in the *TD-1* aperture was probably even larger than assumed.

If the difference between the corrected *TD-1* fluxes and the *IUE* fluxes is due to the nebular contribution, the ratios of nebular to stellar flux are in the range

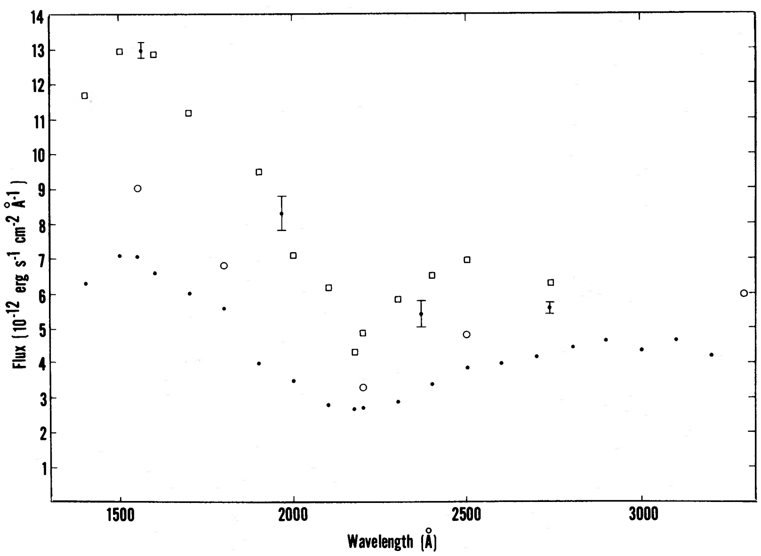

FIG. 5.—Total flux of HD 200775/NGC 7023 as seen through apertures of different sizes. All measurements have been converted to the *IUE* absolute flux scale. *Open squares*: *TD-1* observations by Viotti 1976; *filled circles with error bars*: *TD-1* data of Thompson *et al.* 1978; *open circles*: *ANS* data; *filled circles*: our *IUE* data.

0.7–1.0 for the spectral region 2400 Å$>\lambda>$1300 Å, with an observational average of $F_N/F_*=0.85$. The same procedure applied to the *ANS* data at 1550, 2200, and 2500 Å leads to the conclusion that the ratio of nebular flux included in the *ANS* aperture to that included in the *TD-1* aperture is ~ 0.33, a useful parameter describing the large-scale distribution of the UV surface brightness in NGC 7023.

ii) *Ground-Based Observations*

The ratio of nebular to stellar flux at visual wavelengths can be determined by integrating the nebular surface brightness distribution. We have used the data of Witt and Cottrell (1980*a*) for this purpose for the angular offset range $59'' \leq r \leq 300''$ and the data shown in Figure 3 to cover the range $0'' \leq r \leq 59''$. Thus, we have avoided the often uncertain extrapolation of surface brightness distributions found in the outer parts of the nebula to zero offset and have used an actual measurement, instead. The results of these integrations are listed in Table 3, along with the results obtained from the comparison of the *TD-1* data and the *IUE* fluxes for HD 200775. It must be emphasized that the ground-based measurements relate to a circular region of 10′ diameter, whereas the UV data correspond to a rectangular field of $12' \times 17'$. We estimate that nebular flux contributions from beyond $r=5'$ are less than 15%, based on an extrapolation from suitable models.

TABLE 3

OBSERVED RATIOS OF NEBULAR TO STELLAR FLUX IN NGC 7023

λ(Å)	F_N/F_*
5515	0.41
4733	0.43
4093	0.54
3470	0.56
2400–1300.......	0.85±0.15

Kurchakov (1968) also carried out a careful surface brightness study of NGC 7023 using narrow-band photographic means. His flux ratios F_N/F_* relating to the nebula within an offset radius of 3′.47 are: 0.19 at 6570 Å; 0.26 at 5960 Å; 0.34 at 4650 Å; 0.45 at 4070 Å; and 0.40 at 3750 Å. These results are in agreement with ours, if one takes the smaller field of integration into account and extrapolates.

iii) *Infrared Observations*

Far-infrared observations of reflection nebulae give information on the average nebular density, the nature and location of the illuminating source, and the degree to which stellar radiation is absorbed by the surrounding nebulosity (Emerson, Furniss, and Jennings 1975; Harvey, Thronson, and Gatley 1980; Whitcomb *et al.* 1981). For the study of NGC 7023 the recently published results of Whitcomb *et al.* (1981) are summarized: (1) Dust in NGC 7023 is illuminated and heated by HD 200775. (2) There is evidence for significant density variations on a small spatial scale, with the average density being higher than previously assumed. There appears to be a relative density maximum $\sim 50''$ north-west of HD 200775. (3) The total dust luminosity in the IR is about 1800 $L_\odot$, compared to a luminosity of 4000 $L_\odot$ for HD 200775, taken as a B3 IVe star. Thus, approximately 55% of the flux from HD 200775 escapes either directly or as scattered light from the NGC 7023/HD 200775 system. In § IV this fact is used to estimate average values for the nebular optical depth and the dust albedo in the UV.

Recently, Baschek *et al.* (1982) have completed a detailed analysis of HD 200775, concluding that a spectral classification B3 V is most appropriate. This has the effect of somewhat reducing the uncertainty of the bolometric correction -1.8 ± 0.4 used by Whitcomb *et al.* (1981) in deriving the luminosity of HD 200775. The quoted uncertainty was based on a possible range of spectral types from B2 IVe to B5e.

III. THEORETICAL DISCUSSION

a) *Luminosity of Reflection Nebulae*

The study of reflection nebulae would be greatly advanced by the identification of observable quantities which are largely independent of the details of the density distribution in the nebula and of the shape of the scattering phase function and which depend on global parameters such as the total nebular optical depth and the dust albedo. Obviously, the nebular intensity distribution does not satisfy this requirement, but the nebular luminosity and the ratio of total nebular to stellar flux derivable from it in the case of an isotropic configuration does come rather close.

In general, in a given spectral region the luminosity L_N of a reflection nebula due to scattering including multiple scattering, is

$$L_N = L_* P_S P_{ES} \tag{3}$$

where L_* is the luminosity of the illuminating star in the same spectral region, P_S is the probability that stellar photons are scattered once in the nebular volume, and P_{ES} is the probability that a once-scattered photon escapes from the nebula. In principle, the two probabilities P_S and P_{ES} can be evaluated for any arbitrary system involving nonconservatively scattering grains with an albedo $0 < a \leq 1$ and a phase function asymmetry $0 \leq g \leq 1$ by use of the Monte Carlo method.

Historically, the first study of the luminosity of reflection nebulae is credited to Roshkovskii (1965), who integrated the equations derived by Sobolev (1960) for the surface brightness distribution of a homogeneous spherical nebula with a centrally located star. While Sobolev's formulation for the intensity distribution specifically involves an asymmetric phase function of the form

$$\phi(\alpha) = 1 + x_1 \cos\alpha, \tag{4}$$

in the integrated form of Roshkovskii

$$\frac{L_N}{L_* e^{-\tau}} = \frac{a}{1-2a}\left[1 - e^{-(1-2a)\tau}\right], \tag{5}$$

a dependence on phase function is no longer apparent. Roshkovskii's work was superceded by the detailed Monte Carlo calculations of Kurchakov and Matyagin (1968*a*, *b*) and Kurchakov (1969) for homogeneous spherical nebulae involving three different phase functions covering a wide range of asymmetries. Finally, the radiative transfer in spherical circumstellar dust shells (Code 1973) is directly applicable to more extended homogeneous reflection nebulae. Code finds the excellent analytical approximation

$$\frac{L_N}{L_*} = \frac{2}{(1+\zeta)e^{\xi\tau} + (1-\zeta)e^{-\xi\tau}} - e^{-\tau}, \tag{6}$$

where

$$\zeta = [(1-a)/(1-ag)]^{1/2} \tag{7}$$

and

$$\xi = [(1-a)(1-ag)]^{1/2}. \tag{8}$$

In equations (5) to (8) the quantities a, g, and τ refer to the grain albedo, the phase function asymmetry, and the radial optical depth of the spherical nebula measured from the center to the outer edge.

We have extended the previous work by studying spherical nebulae with nonhomogeneous density distributions, using the Monte Carlo method and grains scattering with the Henyey-Greenstein (1941) phase function. For any spherical nebula with an arbitrary radial density distribution, we have

$$P_S = a(1 - e^{-\tau}). \tag{9}$$

Only the escape probability, P_{ES}, depends on the details of the geometry and the shape of the phase function and needs to be evaluated specifically. For our calculations we used the Monte Carlo approach to multiple scattering described by Witt (1977*b*) with appropriate modifications to the much simpler spherical geometries, as detailed by Witt (1982). Where applicable, our results were in agreement with the results of Kurchakov (1969) and Code (1973), while Roshkovskii's formula (eq. [5]) is poor for nonisotropic phase functions. In addition to homogeneous spherical nebulae, we also considered systems where the dust density increases toward the star to various degrees as well as configurations with central density depletions, as might be expected for cases where stellar wind effects, radiation pressure, and dust evaporation have been active for some time. While these

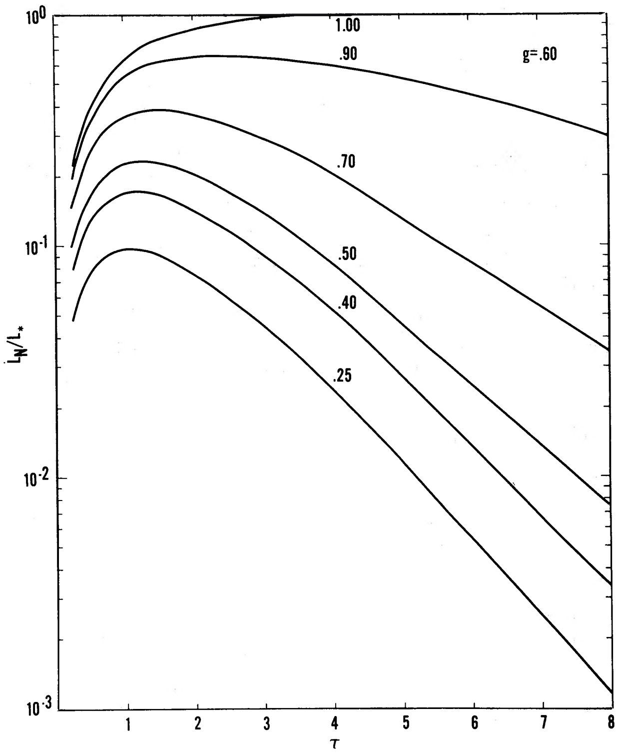

FIG. 6.—The luminosity of homogeneous spherical reflection nebulae in units of the luminosity of the centrally located illuminating star as a function of radial optical depth. The value of the albedo a is indicated next to each model curve. The models are computed for a forward-throwing phase function with $g = 0.60$.

different spherical cases display vastly different surface brightness distributions for identical values of a, g, and τ, the differences in total nebular luminosity are small. The variations are in the sense that a strongly centrally condensed nebula has a slightly higher luminosity than a homogeneous nebula, which in turn is slightly more luminous than a nebula having an extended central cavity. These differences are due entirely to the dependence of P_{ES} on geometry.

The variation of P_{ES} with g is surprisingly small, becoming noticeable only at large optical depths, where multiple scattering is more important. The same fact was noticed by van de Hulst (1980) for plane-parallel geometries with stars embedded in the plane of symmetry. The physical reason for this result in these symmetrical cases is easily understood, because the integration of nebular flux over the face of a nebula involves to first order an integral of the phase function over all scattering angles, and, by definition,

$$\int_{4\pi} \phi(\alpha)\, d\alpha = 1 \tag{10}$$

for all phase functions.

To the extent that a reflection nebula can be represented by any spherical geometry, be it dust limited or not, the *nebular luminosity is largely independent of the form of the radial density distribution or the shape of the phase function as long as the optical depth remains less than about 1.5.* Primary factors determining nebular luminosities are the total radial optical depth and the grain albedo. Figure 6 displays L_N/L_* for homogeneous spherical nebulae for $g = 0.6$ and a range of albedos $0.25 \leq a \leq 1.00$ as a function of τ. In Figure 7 the dependence of L_N/L_* on g is demonstrated for the case $a = 0.7$ and $0.1 \leq g \leq 0.8$.

A quantity of considerable practical interest for the study of reflection nebulae is the total probability P_{EST} for the escape of stellar photons, with or without scattering. For a spherical nebula with arbitrary radial density distribution we have

$$P_{EST} = P_S P_{ES} + e^{-\tau}. \tag{11}$$

Limiting values are $P_{EST} = 1$ for $a = 1$, $P_{EST} = e^{-\tau}$ for $a = 0$, and $P_{EST} = \exp[-(1-a)\tau]$ for $g = 1$. A range of P_{EST} values for spherical homogeneous nebulae with $0 \leq \tau \leq 8$, $0 \leq a \leq 1$, and $g = 0.6$ is shown in Figure 8. The quantity P_{EST} can be related to the combined stellar and nebular flux seen by a large-field spectrophotometer such as *TD-1*, as shown in Figure 5, while that portion of the stellar flux which is absorbed by the nebular particles, $1 - P_{EST}$, is related to the total far-IR nebular flux. Since a dependence on radial density distribution and phase function asymmetry enters P_{EST} only through

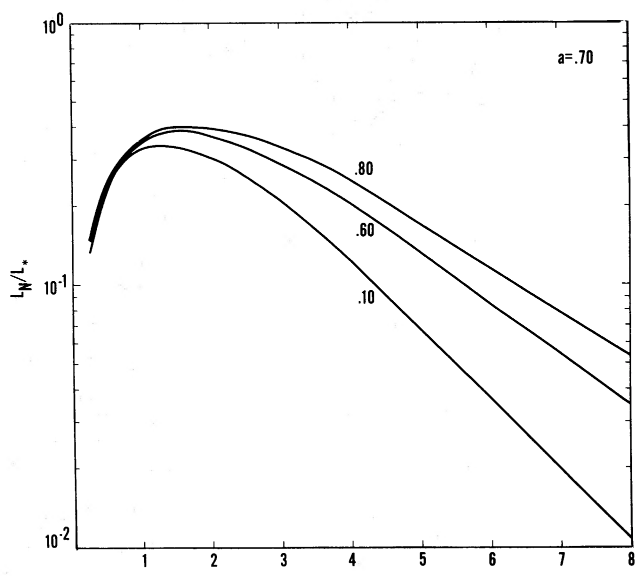

FIG. 7.—The dependence of the nebular luminosity on the phase function asymmetry g for the albedo $a = 0.7$ and three values of $g = 0.10$, 0.60, and 0.80.

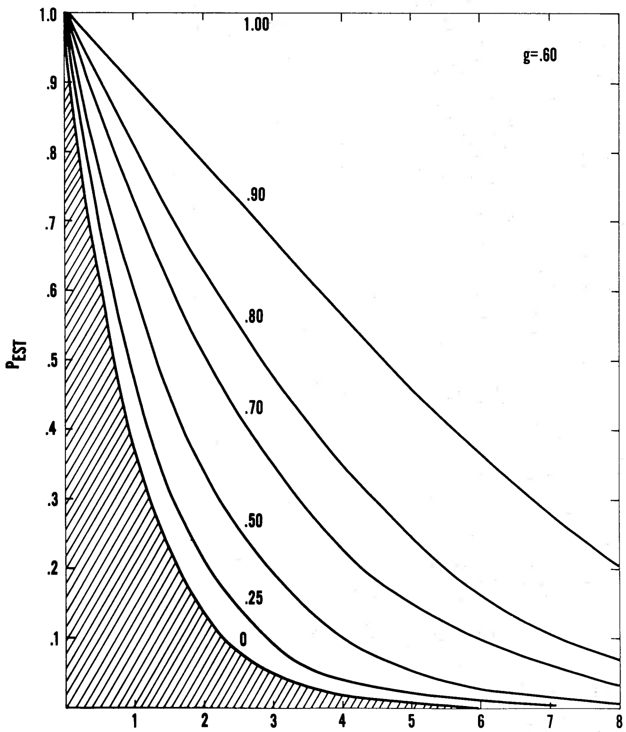

FIG. 8

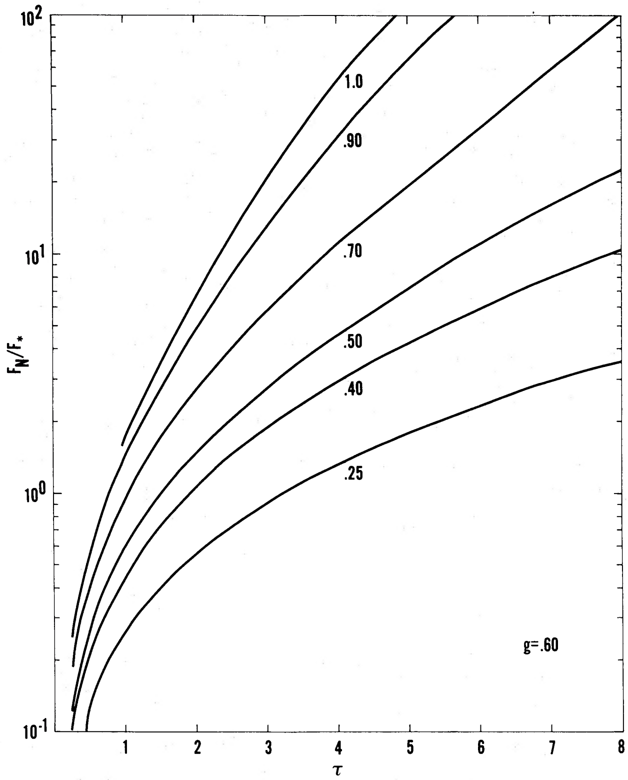

FIG. 9

FIG. 8.—The total probability of escape of stellar and nebular photons, P_{EST}, for $g = 0.6$ and $a = 0$, 0.25, 0.50, 0.70, 0.80, 0.90, and 1.00. The dependence on the radial optical depth of a homogeneous spherical nebula is shown.

FIG. 9.—The ratio of nebular to stellar flux as a function of optical depth. The value of the albedo for each model sequence is indicated next to each curve. All models apply to homogeneous spherical nebulae with dust scattering with $g = 0.6$.

the probability P_{ES}, P_{EST} also depends largely on a and τ and only weakly on g and the geometry. It therefore suffices to show the dependence of P_{EST} on albedo and radial optical depth for only one intermediate value of g, as is done in Figure 8. Given a well-determined ratio of the nebular far-IR luminosity to the stellar luminosity at UV and visual wavelengths for a case with a suitable geometry, this graph allows one to determine a relationship between albedo and optical depth consistent with such a ratio.

It is of equally great interest to explore theoretically the ratio of nebular to stellar flux as a function of optical depth and albedo. Our results are shown in Figure 9, again for a homogeneous spherical nebula since variations with different types of radial density distributions are negligible. The dependence of F_N/F_* on the phase function asymmetry g is in the sense of a slightly larger ratio F_N/F_* for higher values of g, but differences do not become significant until $\tau \geq 3$. In Figure 6 we saw that reflection nebulae with embedded stars become most conspicuous for $1 \leq \tau \leq 2$. Figure 9 indicates that in such nebulae the ratio F_N/F_* can easily be of the order of unity or higher, assuming a moderately high albedo. Note that a measured value of F_N/F_* at a given wavelength again defines a series of points in the a-τ plane, which can serve as a constraint for possible models employed to explain the observations. Detailed results of model calculations of the luminosity of reflection nebulae as well as intensity distributions will be reported elsewhere (Witt 1982).

b) *Nebular Surface Brightness Distributions*

Our Monte Carlo code for spherical nebulae (Witt 1982) provides for the calculation of surface brightness distributions as well as nebular luminosities. Three types of radial density distributions have been explored:

1. The first is characterized by a small core of radius R_1 of constant density, surrounded by an envelope where the density declines as $(r/R_1)^{-q}$, with $q \geq 0$, until the outer radius R is reached. Uniformly dense nebulae are included with this configuration for $q = 0$.

2. The second type exhibits a totally vacant inner volume of radius $R_1 < R$, surrounded by a shell in which the declining density can be described again by a power law with $q \geq 0$.

3. The third type of model has the gradually increasing and then decreasing density law specified by Schiffer and Mathis (1974).

The centrally condensed configuration may be a suitable representation of a nebula with a star in its final stage of formation, a stage suggested for HD 200775 by Herbig (1960), while the centrally depleted models may best describe somewhat more evolved reflection nebulae, e.g., NGC 2023 (Harvey, Thronson, and Gatley 1980) or H II regions (Schiffer and Mathis 1974), where the combined effects of radiation pressure, stellar winds, dust evaporation, or sputtering have led to a central cavity. While the details of the dust distribution close to the star is of great significance in determining the shape of the brightness distribution in the inner portions of the observable nebulae, the assumption of a spherical outer boundary is of major importance for the brightness distribution only at large angular offsets.

IV. ANALYSIS OF OBSERVATIONAL DATA

a) Derivation of a Specific Model for NGC 7023

A model for NGC 7023 must reproduce the new observations summarized in § II in a self-consistent manner and must meet all known boundary conditions for the HD 200775/NGC 7023 system. These constraints are that the total optical depth to HD 200775 is consistent with $E(B-V)=0.44$ (Witt and Cottrell 1980*b*) and the extinction curve determined for HD 200775 by Walker *et al.* (1980), and that the mean density is consistent with limits imposed by the angular size of the nebula, the observed reddening of HD 200775, CO observations of the larger molecular cloud, of which NGC 7023 is a part (Elmegreen and Elmegreen 1978), and recent far-IR observations (Whitcomb *et al.* 1981). Furthermore, the same geometrical model must successfully fit the large-scale brightness distributions observed at visual wavelengths by Grygar (1959), Kurchakov (1968), and Witt and Cottrell (1980*a*), with appropriate scaling in optical depth. Finally, the scattering properties of the grains in the visible, albedo a and phase function asymmetry g, should remain in the range found to be valid from visual studies of the diffuse galactic light (Mattila 1970) and bright dark nebulae (FitzGerald, Stephens, and Witt 1976), because the reddening of HD 200775 in the visible appears normal (Walker *et al.* 1980).

The following list summarizes the arguments for the assumed geometry of NGC 7023, resulting in an embedded illuminating star and scattered light affected mostly by the forward part of the phase function.

1. The galactic latitude $b \approx 14°$ is fairly high and the extinction in the UV is peculiar, suggesting that much of the observed extinction arises within NGC 7023 in front of HD 200775.

2. IR and CO observations indicate that HD 200775 is embedded in a molecular cloud. This cloud is a site of active star formation, with numerous T Tau stars (Weston 1953) and HD 200775 apparently only recently formed (Herbig 1960; Ney, Hatfield, and Gehrz 1980).

3. The IR observations of Whitcomb *et al.* (1981) imply a large covering factor for the grains that intercept the UV light and re-emit the energy in the IR.

4. The gradient of the scattered light intensity in the visual toward HD 200775 is steep, as expected in the case of forward scattering. This contrasts with the weak gradient in the Merope nebula, where the observed nebular radiation is mostly light scattered at large angles.

5. The ratio of $(S/F_*)_{22''\!.5}$ in NGC 7023 changes little with wavelength, as expected for an embedded star where the increase in extinction of F_* at 2200 Å is compensated by a corresponding decrease in the albedo.

6. HD 200775 is not behind the whole complex observed in CO because the extinction of $E(B-V)=0.44$ would be too low by a factor of 2–3, provided $R_V = A_V/E(B-V)=3.1$.

7. As discussed by Witt and Cottrell (1980*b*), HD 200775 is relatively cool and would be able to produce only a very small H II region in the immediate surroundings of the star. Dramatic dynamic effects such as observed, e.g., in the Trapezium in Orion, affecting the large-scale dust distribution, are therefore not to be expected.

b) The Nebular Optical Depth and Grain Albedo

In the absence of better information, Witt and Cottrell (1980*b*) assumed an optical depth $\tau_V = 1.35$ consistent with the total reddening of HD 200775. The observed surface brightness at visual wavelengths $\log(S/F)_{100''} = 4.97$ required a relatively low density of $n_{\mathrm{H}} = 170\ \mathrm{cm}^{-3}$, inconsistent with molecular CO data (Elmegreen and Elmegreen 1978) and recent IR observations (Whitcomb *et al.* 1981). The determinations of the nebular far-IR luminosity and of the UV nebular flux, combined with the methods discussed in § III, now allow a more direct determination of the nebular optical depth.

A star embedded in an extended molecular cloud such as HD 200775 will heat dust in a roughly spherical region. The far-IR radiation can readily escape from this volume, since Whitcomb *et al.* (1981) estimate an optical depth of 10^{-3} at 125 μm. The indirect determination of the total escape probability for UV and visible light on the basis of a spherical model should therefore be justified. Then, according to Whitcomb *et al.*, $1-P_{\mathrm{EST}} = 0.45$. This condition, combined with the results of Figure 8, leads to a specific locus in the a-τ plane, shown in Figure 10 as a dashed curve. This curve is arrived at with the assumption that $g=0.6$. Even if $g=0$, where

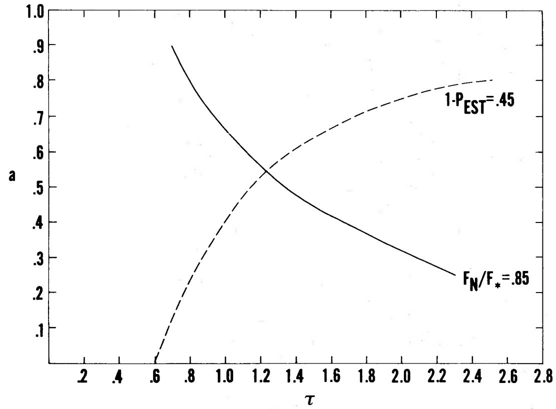

FIG. 10.—The determination of the average optical depth of NGC 7023 and of the average dust albedo in the UV based on possible solutions for the observed ratio of nebular to stellar UV flux $F_N/F_*=0.85$ and for the observed ratio of nebular far-IR luminosity to stellar luminosity of 0.45.

most of the UV absorption of stellar flux occurs, the curve would be shifted toward smaller optical depths by only about 10% in τ.

Since HD 200775 is a B3 star, most of the stellar energy emerges in the UV, where also the nebular optical depth reaches a maximum. If the solution is to be consistent with the balance of energy, the second locus in the a-τ plane, which is to intersect the dashed curve and provide a unique set of a and τ, must therefore be based on the nebular to stellar flux ratio in the UV. The data from Figure 5 suggest a conservative value of $F_N/F_*=0.85\pm0.15$ for the 2400–1300 Å spectral region. We will show below that a spherical nebula model, albeit with small-scale and large-scale density inhomogeneity and a certain amount of foreground extinction, does indeed best represent the surface brightness data available for NGC 7023. Furthermore, for a given quantity of dust a spherical configuration maximizes the conversion of stellar to scattered nebular flux, and an estimate of the albedo based on the ratio F_N/F_* for a given line-of-sight optical depth should result in a lower limit to a, even if spherical symmetry is lacking, provided that our line of sight to the star is typical for the average of all radial lines of sight. The spherically symmetric models of Figure 9 determine the locus in the a-τ plane corresponding to $F_N/F_*=0.85$. This locus is shown as the solid curve in Figure 10. If g is smaller than the assumed $g=0.6$, the solid curve would shift to larger optical depths, opposite to the corresponding shift of the dashed curve. The point of intersection would then move to higher values of a, leaving τ relatively unchanged. Since we show later that $g_{\mathrm{UV}}<0.6$, the solution of Figure 10, $\langle a\rangle_{\mathrm{UV}}=0.54$ and $\langle\tau_{\mathrm{UV}}\rangle=1.23$ is a lower limit to the average value of a in the ultraviolet and a determination of the average radial optical depth for the 2400–1300 Å region. Walker *et al.* (1980) have shown that the ultraviolet extinction for HD 200775 is lower and flatter than the average interstellar reddening curve; thus, optical depths in the range 0.55 to 0.9 should be expected for the visible region 5500–3500 Å. Given that $E(B-V)\approx0.44$ for HD 200775, part of the dust in the line of sight must not be directly associated with NGC 7023. This additional extinction could come from the same general region, however, since NGC 7023 is a $\sim12'$ diameter nebula in a $30'\times60'$ dark cloud.

c) *The Nebular Density Distribution*

Photographic data, ultraviolet and visual surface brightness measurements, and infrared observations all suggest that NGC 7023 is a region in which significant density variations occur from place to place on a scale of arc seconds to tens of arc seconds. Any value for the nebular density will, therefore, represent some average, which can be systematically affected by the nature of the method by which it was obtained. The nebular far-IR flux measured with a 50″ beam will originate predominately in high-density clumps near the star, where stellar radiation can be absorbed most effectively and completely. Also, high density regions behind the star with respect to the observer will contribute proportionately to the observed IR flux, because the optical depth is so small for IR radiation. The estimated density of NGC 7023 of at least 1×10^3 atoms cm^{-3} by Whitcomb *et al.* must, therefore, be viewed in this light as a likely upper limit to the actual average density.

The molecular observations of Elmegreen and Elmegreen (1978) suggest H_2 densities of 600 cm^{-3} for the dark cloud, of which NGC 7023 is a part. The prevalence of molecules is again favored by higher densities, and regions beyond HD 200775 contribute to the measured flux.

Since the optical depths at visual and ultraviolet wavelengths are of order unity and large compared to the IR optical depth, the scattered light observable by us arises primarily from that part of the nebula lying between the plane of HD 200775 and the observer. In an environment with significant density inhomogeneities, the observed scattered light must follow routes through regions with lower densities. Therefore, the corresponding density estimates that are based on scattered light data tend to be lower than the average and tend to relate to a more limited nebular volume.

On a large scale, Witt and Cottrell (1980*b*) found that the observed nebular surface brightness distribution is inconsistent with a spherical nebula of uniform density. Such models lead to brightness distributions following a power law $S/F\propto r^p$ with $p\approx-1.0$, whereas the existing ground-based data showed that $p\approx-1.5$, i.e., a considerably steeper distribution. Agreement with observations can be obtained by allowing the density to increase towards the star by about a factor of 2 between the outer edge and the central regions at $r=0.1$ pc.

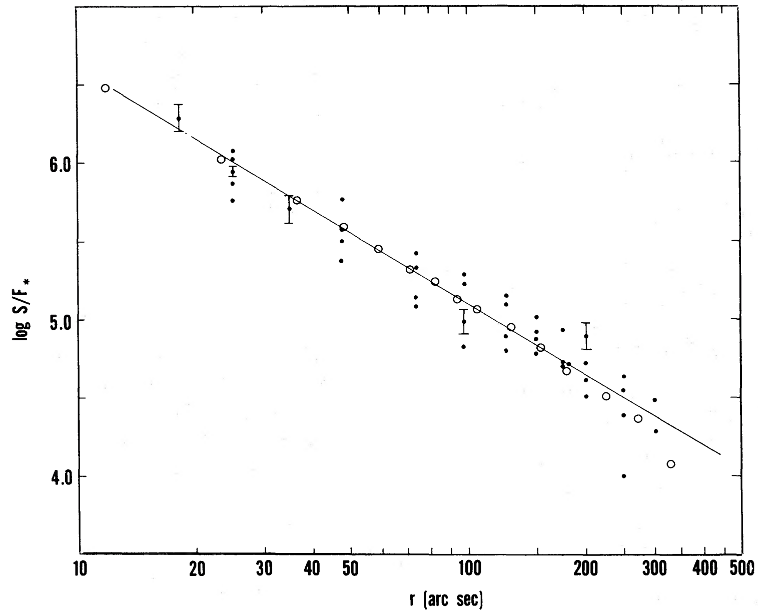

FIG. 11.—Fit of the adopted model for NGC 7023 to the surface brightness data at 4093 Å. The filled circles are data from Witt and Cottrell 1980*a*, supplemented with our results from Table 2. Representative error bars are indicated. The solid line is the least squares power-law fit to the data. The open circles represent our model predictions at 4093 Å with the parameters listed in Table 4. This fit requires a density structure with density increasing toward NGC 7023, as described in the text.

The actual density can be found by using the constraints of the distance to NGC 7023, the angular size of the nebula, the surface brightness at visual wavelengths, and the known scattering properties of dust in this spectral region. Following Witt and Cottrell (1980*a*) we assume a distance $d = 350$ pc, near the mean of several such estimates by several authors. We choose the photoelectric surface brightness measurements at 4093 Å of Witt and Cottrell (1980*a*) and adopt a value of $\tau = 0.77$ for this wavelength, consistent with the value for $\langle \tau_{\rm UV} \rangle$ found above and the extinction curve of Walker *et al.* (1980). We are in agreement with other determinations of dust scattering properties near 4000 Å if we assume $a = 0.6 \pm 0.05$, $g = 0.6 \pm 0.05$ (Lillie and Witt 1976). An excellent fit with the available brightness data can then be accomplished with a nebular radius $R = 1$ pc and a density law providing constant density for $0 < r \leq 0.1$ pc with the density then falling off as $(r/R_1)^{-0.26}$ for 0.1 pc $= R_1 < r \leq R$. Using the mean relation between $E(B-V)$ and hydrogen density of $N(\text{H I}+\text{H}_2)/E(B-V) = 5.8 \times 10^{21}$ atoms cm^{-2} mag^{-1} derived by Bohlin, Savage, and Drake (1978), an average H I+H_2 density of about 400 cm^{-3} is deduced, with about 300 cm^{-3} near the edge of NGC 7023 and about 500 cm^{-3} near HD 200775. These values are smaller by about a factor of 2 than the values derived from CO data and IR observations, as expected. Figure 11 demonstrates the agreement of this model and the data at 4093 Å, where the model values are shown as open circles following closely the least squares power-law solution representing the available data. Equally satisfactory fits are achieved with the identical model and the following parameters at 3470 Å: $\tau = 0.86$, $a = 0.55$, $g = 0.56$; 4733 Å: $\tau = 0.64$, $a = 0.60$, $g = 0.63$; 5515 Å: $\tau = 0.54$, $a = 0.65$, $g = 0.63$. The optical depths in NGC 7023 at the four visual wavelengths are consistent with one another through scaling via the HD 200775 reddening law of Walker *et al.* (1980) and $E(B-V) = 0.17$ in the nebula. This same reddening law then provides the following optical depths for the UV: $\tau(2500) = 1.04$, $\tau(2000) = 1.24$, $\tau(1700) = 1.16$, $\tau(1400) = 1.32$. These values are consistent with the average $\langle \tau_{\rm UV} \rangle$ derived from Figure 10.

d) *The Phase Function Asymmetry in the UV*

With the average dust albedo and the optical depth in the UV determined by the ratio of nebular to stellar flux and the ratio of nebular IR luminosity to stellar luminosity, and with the parameters of the geometrical model for the nebula fixed by requirements involving the nebular density and the brightness distributions at visual wavelengths, the phase function asymmetry for the UV is the final free parameter which may be varied to match the observed nebular intensity observed by *IUE* at $r = 22''.5$ and the ratio of nebular fluxes seen in the *ANS* and the *TD-1* apertures.

It is of greatest interest to determine the consequences of these constraints near the short-wavelength end of

TABLE 4

DERIVED DUST PARAMETERS FOR NGC 7023

λ(Å)	τ	a	g	$\log (S/F)_{22''.5}$	F_N/F_*[a]
5515	0.54	0.60	0.63	5.92	0.37
4733	0.64	0.56	0.63	5.96	0.41
4093	0.77	0.58	0.60	6.04	0.52
3470	0.86	0.55	0.56	6.05	0.54
2500	1.04	0.47	0.50	6.02	0.60
2000	1.24	0.42	0.40	5.99	0.61
1700	1.16	0.52	0.35	6.04	0.73
1400	1.32	0.60	0.25	6.13	1.01

[a] F_N/F_* is determined for the first four wavelengths for $r \leq 300''$, for the last four wavelengths for a $12' \times 17'$ aperture.

our observed range. At 1400 Å the observed conditions of $\log (S/F)_{22''.5} \approx 6.13$ and $F_N(ANS)/F_N(TD\text{-}1) \approx 0.33$ can be matched uniquely with only one pair of values: $a = 0.6$ and $g = 0.25$. If a were lower, g would need to be increased somewhat to reproduce the nebular intensity at $r = 22''.5$, but then the $F_N(ANS)/F_N(TD\text{-}1)$ would no longer be matched. The independent requirement $\langle a_{\rm UV} \rangle \geq 0.54$ sets a limit of $g \lesssim 0.35$.

If g decreases monotonically from $g(4070) = 0.6$ to $g(1400) = 0.25$, best fits are obtained with the values of scattering parameters listed in Table 4. In Figure 12 the model predictions based on these parameters are indicated as open circles. The variation of the albedo with wavelength is consistent with the existence of an albedo minimum of $a \approx 0.4$ near the 2200 Å extinction feature with a rise to higher values of $a \approx 0.6$ at both 1400 Å and 4100 Å, in agreement with Lillie and Witt (1976). The requirement that $\langle a_{\rm UV} \rangle = 0.54$ appears to be met satisfactorily by the values listed in Table 4. The increase of g, i.e., the change to a more forward throwing phase function, with increasing wavelength throughout the UV is supported independently by the observed steepening of the surface brightness gradient toward HD 200775 with increasing wavelength, as shown in Figure 2.

The last column of Table 4 lists predicted values for F_N/F_* for NGC 7023, which agree with the data in Table 3. Thus, the surface brightness of NGC 7023 measured by *IUE* is consistent with the total nebular flux as measured by *TD-1* (Thompson *et al.* 1978). It, therefore, appears to rule out the possibility that HD 200775 has undergone a substantial reduction in ultra-

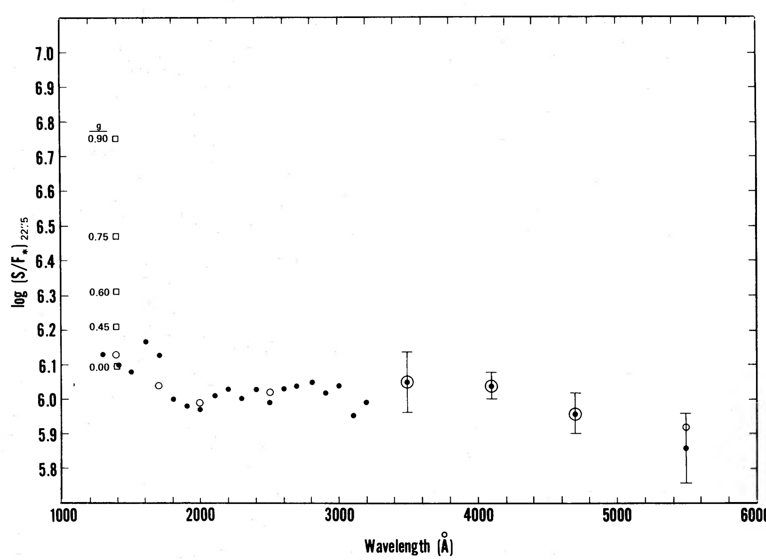

FIG. 12.—Determination of the phase function asymmetry. The filled circles are the data from Fig. 4. The open circles result from the adopted model for NGC 7023 with the scattering parameters listed in Table 4. The open squares indicate alternate models at 1400 Å using the parameters of Table 4 but adopting the values of g as indicated.

TABLE 5

SENSITIVITY OF NGC 7023 MODEL TO PHASE FUNCTION ASYMMETRY AT 1400 Å

Case	$\log (S/F)_{22''5}$	$F(\text{ANS})/F(\text{TD-1})$	F_N/F_*
Observed	6.13±0.08	0.33±0.05	0.85±0.15
Model, $\tau = 1.32$			
$a=0.6, g=0.00$ ...	6.10	0.35	0.88
$a=0.6, g=0.15$ ...	6.11	0.33	0.89
$a=0.6, g=0.30$ ...	6.14	0.36	0.98
$a=0.6, g=0.45$ ...	6.21	0.38	1.05
$a=0.6, g=0.60$ ...	6.31	0.43	1.15
$a=0.6, g=0.75$ ...	6.47	0.54	1.28
$a=0.6, g=0.90$ ...	6.75	0.76	1.54

violet flux over the interval of time over which the separate observations by *TD-1, ANS*, and *IUE* were undertaken. Note that the model predicts a ratio $F_N/F_* \approx 0.6$ for the 2000–2500 Å region, where Viotti's (1976) data and the observations of Thompson *et al.* (1978) of HD 200775/NGC 7023 differ, as shown in Figure 5. The data of Thompson *et al.* combined with our *IUE* fluxes for HD 200775 are in agreement with a ratio $F_N/F_* \approx 0.6$, whereas Viotti's results would require a larger ratio $F_N/F_* \approx 0.9$. Presumably, Thompson *et al.* revised the preliminary reduction of these data by Viotti.

Finally, our value of $g=0.25$ at 1400 Å may be compared to $g=0.9$ (Henry *et al.* 1978) and the range of $g=0.6$ to 0.9 found by Lillie and Witt (1976) for grains producing diffuse galactic light at high galactic latitudes. Using model parameters $a=0.6$ and $g=0.9$, our model predicts $\log (S/F)_{22''5} = 6.75$ at 1400 Å, more than 4 times the observed value, and $F_N(ANS)/F_N(TD\text{-}1) = 0.76$, more than twice the observed value. Such results are well outside our experimental errors for NGC 7023.

Figure 12 shows as open squares the expected values for $\log (S/F)_{22''5}$ at 1400 Å for $a=0.6$ and $g=0.00$, 0.45, 0.60, 0.75, and 0.90. Given an uncertainty in the observation of $\log (S/F)_{22''5}$ of ±0.08, no result in the range $0 \leq g \leq 0.45$ can be excluded. The additional observational constraints of $F(ANS)_{\text{UV}}/F(TD\text{-}1)_{\text{UV}} \approx 0.33$ and $(F_N/F_*)_{\text{UV}} \approx 0.85$ lead to the same conclusion, as is shown in Table 5.

V. DISCUSSION AND SUMMARY

The significance of our results and the nature of possible sources of error remain to be explored. The wavelength dependence of scattering properties of dust as derived from the observation of NGC 7023 and as listed in Table 4 is not new. The wavelength dependence of the albedo found here agrees with that found by Lillie and Witt (1976) in their study of the diffuse galactic light. Our variation of the phase function agrees with the results of Andriesse, Piersma, and Witt (1977) and Witt (1977*a*) for the Merope Nebula, the results of Witt and Lillie (1978) and Morgan, Nandy, and Thompson (1981) derived from independent studies of the Orion reflection nebulosity, and the preferred conclusion of Bohlin *et al.* (1980, 1982) for the interpretation of the UV scattered light in the Orion Nebula.

For the first time these conclusions have been reached for a case with less room for fundamental disagreement about the scattering geometry. The crucial assumptions in our analysis are that NGC 7023 represents a case of an embedded star and that the scattered light is produced mainly by material between us and the star. Hence, we base our analysis on the variation on the dominant forward half of the phase function. The other cases of reflection nebula studies, mentioned above, all depend on the assumption of large-angle scattering or backscattering in a geometry where most of the scattering material is either in the plane perpendicular to the line of sight, which includes the illumination source, or behind it. In such instances, the variation of the much smaller scattering amplitudes at large angles could easily be overwhelmed by the much stronger forward amplitude of scattering of suitably arranged foreground dust with a small optical depth. Merope, the stars of the Orion OB association, and the Trapezium all exhibit some reddening, and valid questions can be raised about the relative location of this foreground dust with respect to the stars in question.

It is of interest to explore the question why the diffuse galactic light study of Lillie and Witt (1976) yielded an albedo variation essentially identical to ours, but produced values for g in the range 0.6 to 0.9 for the UV. Those authors noticed that the ratio of diffuse galactic light to line-of-sight starlight is independent of phase function near galactic latitude $|b^{\text{II}}| = 17°$, and dependent only on albedo and optical depth. They determined the albedo variation with wavelength from the observed variation of this ratio from their observations at $15° < |b^{\text{II}}| < 20°$. For the measurement of the phase function asymmetry, they depended on the variations of the above ratio with galactic latitude. They assumed a latitude dependence for the line-of-sight starlight based on the interstellar radiation field model of Witt and Johnson (1973). Subsequent observations of this radiation field by Henry, Anderson, and Fastie (1980) and by Gondhalekar, Phillips, and Wilson (1980) demonstrated that the UV starlight decreased with increasing galactic latitude much more steeply than predicted by the earlier models. Hence, the ratio of diffuse to starlight, instead of declining as believed by Lillie and Witt (1976), is likely to be constant with latitude in the UV, consistent with a much smaller value of g than found by Lillie and Witt. With this correction, we are therefore in essential agreement with their results. A state of fundamental disagreement remains with the result of Henry *et al.* (1978) requiring $g \geq 0.9$. Since a relatively high value for the albedo is inescapable in view of the substantial scattered light fluxes observed from reflection nebulae, there appears to be no solution to bridge the discrepancy illustrated in Figure 12 but to assume that one or more of the fundamental assumptions of Henry *et al.* are not valid or that there is a fundamental difference in the nature of dust observed in the two experiments. Our conclusion that the phase function asymmetry of interstellar scattering approaches an isotropic form in the far UV leads to the result that most, if not all, UV background radiation observed at high galactic latitudes is starlight backscattered by dust, present at intermediate z-distances in our Galaxy. This is in essential agreement with the conclusion of Paresce *et al.* (1980) and of Maucherat-Joubert, Deharveng, and Cruvellier (1980).

There are some potential sources for errors in our results, stemming from several of our secondary assumptions. We find that only about one-half of the inferred extinction in HD 200775 is produced in that immediate vicinity around the star which gives rise to NGC 7023, yet we assume that the observed reddening law for HD 200775 valid for the total line-of-sight dust column is the appropriate law to scale the optical depth in NGC 7023. Since the observed law is peculiar, any contribution from "normal" dust along the line of sight would require the NGC 7023 optical depth scale to be still more peculiar.

The assumed distance of $d = 350$ pc has very little effect on the final results. All values of flux and luminosity ratios used are distance independent, and the determination of a and τ based on these values is, therefore, invariant. The possibility exists that NGC 7023 is as far as 450 pc away (Whitcomb *et al.* 1981). In our model we would need to increase the linear radius to some degree to match the observed angular size of the nebula, if $d = 450$ pc. This would reduce the density to an extent where it might become inconsistent with IR and CO observations. Values of $\log(S/F_*)$ for a given linear position on the nebular face would increase by $\log(450/350)^2 = +0.22$ for constant a, τ, and g, which would be largely balanced by an adjustment for the changed angular scale of $\log(450/350)^p = -0.16$, with $p = -1.5$ as observed for NGC 7023. A small reduction in density would provide the remaining change in $\log(S/F_*)$.

A potentially serious source of error is the uncertainty in the luminosity of HD 200775, stemming from a poorly determined bolometric correction. Taking BC $= -1.8 \pm 0.4$ as assumed by Whitcomb *et al.* (1981), we find the ratio of nebular IR luminosity to stellar luminosity $L_{\rm IR}/L_* = 0.45^{+0.17, -0.15}$, and therefore $1 - P_{\rm est} = 0.45^{+0.17, -0.15}$. The analysis of § IV*b*, then, yields values $\langle \tau \rangle_{\rm UV} = 1.23^{+0.30, -0.23}$ and $\langle a \rangle_{\rm UV} = 0.54^{-0.10, +0.13}$. Most importantly, however, the independent constraint $(F_N/F_*)_{\rm UV} = 0.85$ causes an increased value of τ to be associated with a decreased value of a or a decreased value of τ to be associated with an increased value of a.

The corresponding effects of these coupled uncertainties upon the predicted value of $\log(S/F)_{22''5}$ can be evaluated with the results reported by Witt (1977*c*) for reflection nebula models with embedded stars. In one extreme ($\tau_{\rm UV} = 1.53$, $a_{\rm UV} = 0.44$), $\log(S/F)_{22''5}$ increases by $+0.09$ in response to a larger value of τ but decreases by -0.06 because of a lower albedo, resulting in a net change of $+0.03$, which is not significant compared to the observational uncertainty in $\log(S/F)_{22''5}$. In the opposite extreme ($\tau_{\rm UV} = 1.00$, $a_{\rm UV} = 0.67$) the two effects balance to within a net change in $\log(S/F)_{22''5}$ of $+0.005$. In both cases the optimal value of g remains in the range $0.2 \leq g \leq 0.3$. Thus, while the uncertainty in the luminosity of HD 200775 has a measurable effect upon our estimates for the nebular optical depth and the grain albedo in the UV, it hardly at all affects our principal conclusion concerning the phase function asymmetry in the UV.

The values of $\log(S/F_*)$ in the UV are independent of the absolute calibration of *IUE*, since we are dealing always with flux ratios. The value of S is, however, affected by uncertainties in our knowledge of the solid angle subtended by the *IUE* large apertures. Recent indications are that the upper limits published by Bohlin *et al.* (1980) may be too large by 6% in case of SWP and too large by 8% in case of LWR. If true, our values of $\log S/F$ in the UV would need to be adjusted upward by about 0.03, which would have only a minute effect on the final results.

The derived sense of change of scattering parameters follows directly from the observed wavelength variation of $\log(S/F_*)$, the high average albedo in the UV, and the plausibility of a forward-scattering geometry. Different values for the nebular distances or a geometry other than spherical might shift the entire series of g-values to a higher or lower level, but the basic direction of the observed trend will not change.

The possibility that part of the nebular flux in the 1400–1800 Å region is due to H_2 fluorescence (Duley and Williams 1980) remains to be discussed. In a worst case, all stellar radiation in the 912–1100 Å region not absorbed by C atoms and grains could be converted by H_2 fluorescence to diffuse nebular radiation in the 1400–1800 Å band, which would increase the nebular flux in this spectral region by about 30%. A separate analysis was carried out with the assumption that $(F_N/F_*)_{1400} = 0.60$, allowing for a 30% fluorescence contribution. A reduction of all optical depths by 11% and a reduction of the average albedo in the UV by 15% are the result. In particular, at 1400 Å we find $a = 0.51$ instead of 0.60, but most importantly, the result of a near-isotropic phase function with $g \approx 0.25$ for the far-UV remains unchanged.

Note, that in Figure 4, $\log(S/F_*)_{22''5}$ at 1600 Å and 1700 Å appears to rise above the remaining spectrum at a marginally significant level. This increase is due to a rise in S, not to a decrease in F_*. The strongest H_2 fluorescence features are expected in this spectral region and our two data points at 1600 Å and 1700 Å may represent a marginal detection. Note, also that our model fits in Figure 12 remain close to the lower envelope to the data points in the 2000–1300 Å region.

As discussed in detail by Witt (1979), a trend toward decreasing values of g at shorter wavelengths in the UV suggests that the scattering in this spectral region is predominantly done by grains small compared to the wavelength and possessing an albedo approaching unity. Such a scattering process cannot be Mie scattering in the Rayleigh limit (size $\ll \lambda/2\pi$), because this would require $a \to 0$. Our results are consistent with the idea that nonclassical scattering as proposed by Platt (1956) is responsible for a major portion of scattering light seen at far-UV wavelengths. If such Platt particles can exist, their chance of survival in an environment rich in ultraviolet photons is far greater than that of equally small Mie particles, which are readily evaporated upon absorption of UV photons. The following points summarize the steps leading to our conclusions:

1. New low-dispersion spectroscopic observations from *IUE* in the 1300 Å to 3100 Å region of the reflection nebula NGC 7023 and of its illuminating star, HD 200775, are presented.

2. Nebular surface brightness values, corrected for instrumental scattering, have a spatial resolution comparable to similar ground-based observations. A consistent spectrum of $\log(S/F_*)$ for an offset angle of 22''5 covers the range 1300–5500 Å.

3. *IUE* flux measurements of HD 200775 are combined with existing observations of the NGC 7023/HD 200775 complex by *TD-1* and *ANS* to derive total nebular fluxes as a function of wavelength in the UV. Existing data in the visible extend the measured nebular flux to longer wavelengths.

4. A theoretical investigation of the luminosity of spherical reflection nebulae and the related ratio of nebular to stellar flux shows that these quantities are largely independent of the asymmetry of the phase function for scattering and are mainly determined by the dust albedo and the optical depth, thus making them suitable quantities for an independent albedo estimate.

5. The probability of escape for direct and scattered radiation is evaluated and is related to the observed ratio of far-IR luminosity of NGC 7023 to stellar luminosity of HD 200775. The IR result, combined with the ratio of nebular to stellar flux in the UV, determines the average optical depth and the average dust albedo in the UV.

6. A self-consistent model is derived for NGC 7023 which reproduces the observed spatial intensity distributions of the visible nebula and the nebular to stellar flux ratio at all wavelengths. The nebular density increases in the model by about a factor of 2 from the outer to the central region, with an overall average H density of 400 cm^{-3}.

7. The model is used to derive values for the phase function asymmetry consistent with the observed nebular brightness at $r = 22''5$ and with the established limits on albedo and nebular optical depth. The only consistent solution requires g to decline monotonically with decreasing wavelength, with g ranging from 0.63 at 5500 Å to 0.25 at 1400 Å, as summarized in Table 4.

8. The dust that is responsible for most of the scattering near 1400 Å must consist of high albedo grains with a near isotropic phase function. The existence of such particles has been proposed by Platt (1956).

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