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# A SIMULTANEOUS PHOTOMETRIC AND RADIAL VELOCITY STUDY OF SHORT-PERIOD SOUTHERN CEPHEIDS. IV. RADII AND MASSES

WOLFGANG GIEREN[1]
Physics Department, Universidad de los Andes, Bogotá, Colombia
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## ABSTRACT

Simultaneous $UBVRI_{KC}$ photometric and photoelectric radial velocity observations of 15 southern Cepheids have been used to derive the radii of the stars by the maximum likelihood method of Balona (1977). A new period-radius relationship for log $P < 1$ is constructed from these radii, and its slope is found to agree closely with the value of 0.70 derived by Cogan (1978) using theoretical arguments. It is shown that the constants of the observed period-radius relation agree also with those expected from the PLC relation of Sandage and Tammann and a ridge line $\langle B - V \rangle_0$ – log $P$ relationship.

Calculation and comparison of theoretical masses, pulsation masses, and Wesselink radius masses of the Cepheids show that theoretical and pulsation masses are basically in agreement now, with a scatter about $M_{\rm Pul}/M_{\rm Th} = 1$ being significantly lower than that found by Cox. However, a discrepancy is indicated in the sense that toward longer periods the pulsation masses might become increasingly smaller than theoretical masses.

The discrepancy between Wesselink radius masses and theoretical masses persists, the $M_{\rm Wes}$ being $\sim 20\%$ smaller than theoretical masses. Contrary to the result of Cox, the discrepancy persists also for Cepheids with $P < 7^{\rm d}$ if inhomogeneous model $Q_0$ values are used to calculate the theoretical masses, the $M_{\rm Wes}$ being still $\sim 15\%$ too small. The scatter of the Wesselink radius masses, however, is found to be significantly reduced in comparison to Cox's results. The present results do not suggest a strong necessity to introduce helium enriched surface layers of Cepheids as proposed by Cox *et al.* While the Wesselink radius mass discrepancy is slightly alleviated by the use of inhomogeneous models, the agreement between theoretical and pulsation masses seems to be slightly better if the homogeneous model $Q_0$ values are used.

For only one star of the present short-period sample, AZ Cen, there is reasonable evidence that it might be an overtone pulsator.

*Subject headings:* stars: Cepheids — stars: luminosities — stars: pulsation

## I. INTRODUCTION

In previous papers, simultaneously obtained photoelectric radial velocities (Gieren 1981*a*; hereafter Paper I) and $UBVRI_{KC}$ photometry (Gieren 1981*b*; hereafter Paper II) of 15 southern Cepheids were reported. These homogeneous data were recently used to search for binaries among these stars (Gieren 1982; hereafter Paper III). In the present study, we shall concentrate on the determination of the Cepheids' radii and masses; since the stars all have short periods ($3^{\rm d}$–$7^{\rm d}$), it will also be interesting to attempt to reach conclusions about their pulsation modes, since first harmonic pulsators are expected to occur among short period stars (Christy 1966; Stobie 1969).

The situation with respect to Cepheid radii has most recently been reviewed by Cogan (1978). He found appreciable inconsistencies between the results of different methods which can be used to derive Cepheid radii; in particular, there are frequently large inconsistencies between the radius results for the same stars from different authors who have used Wesselink's method or one of its modifications. The relatively large scatter of published Wesselink radii is reflected in an even larger and very unsatisfactory scatter among the Wesselink radii masses (Cox 1979; 1980). Among the well-known reasons limiting the precision of radius results which can be expected from Wesselink's method, there is one point which has up to now been largely neglected by observers: the necessity to obtain light, color, and velocity variations of Cepheids simultaneously, in order to avoid phase mismatch problems, which typically lead to 5%–10% radius errors if the phasing error is only 1% (Fernie and Hube 1967; Evans 1976). In view of typical period precision which is about $10^{-4}$ for most Cepheids, this figure is easily surpassed if observations of largely different epochs are combined.

With respect to how to apply best the basic idea of the Baade-Wesselink method, considerable progress has been made in recent years in developing more global

[1] Visiting Astronomer, South African Astronomical Observatory, Cape Town/Sutherland, South Africa; also at Observatorio Astronómico, Universidad Nacional, Bogotá, Colombia.

and statistically better justified approaches than the classic Wesselink analysis (Wesselink 1946), by the introduction of the maximum likelihood method of Balona (1977) and the CORS method (Caccin *et al.* 1981). In both papers, mean $\log R_0 - \log P$ relationships have been constructed from available data, but in most cases the authors had to rely upon data from different sources and epochs. Although, for example, the Balona $\log R_0 - \log P$ relationship has quite a low scatter (on average $\sim 0.045$ per star in $\log R_0$ about the mean relationship, or $\sim 10\%$ radius accuracy), the Cox (1979) Wesselink radius masses based on Balona's radii clearly show that in a number of cases, the radii of individual Cepheids must be substantially wrong, to account for the very large scatter found in his corresponding diagrams (Figs. 5 and 6). One basic motivation of the present work was therefore to obtain radii for a large number of Cepheids which are (*a*) freed from the presumably largest observational error source, the phase mismatch; (*b*) based on very accurate data and on different color indices; and (*c*) derived with a statistically fully justified method (the maximum likelihood method), in order to establish a more secure $\log R_0 - \log P$ relationship (although not necessarily with a smaller scatter than previously published relations because an appreciable intrinsic scatter about the mean relationship might be expected for Cepheids, due to the finite width of the instability strip), and to find out if the large scatter of Wesselink radius pulsation masses would partly disappear by use of adequate data and data analysis. These topics are discussed in §§ II and III, respectively.

In this context, the results of Paper III can be used to take account of binaries among the present sample of Cepheids. The new data are also used to obtain further conclusions with respect to the different mass anomalies defined by Cox (1979) (§ III). Finally, the new radius data and the photometric properties of the Cepheids established in Paper III are used to discuss their pulsation modes (§ IV). In the discussion of the $\log R_0 - \log P$ relationship and masses, one has to bear in mind that the present Cepheids are restricted to periods $< 7^d$; future observations of longer-period Cepheids are planned to extend the present research to the full period range of galactic Cepheids. However, the disagreement of Wesselink radii has been largest at the short periods (see Cogan 1978) where consequently more reliable radii are most urgently needed.

## II. RADII

### *a*) *Method and Solutions*

The maximum likelihood method described by Balona (1977) was applied to the present data to solve for the constants $R_0$, $A$, and $B$ in the relationship (linearized, assuming $r \ll R_0$)

$$V = A(B - V) - \frac{5 \log_{10} e}{R_0} r + B \,, \qquad (1)$$

where $V$ is the observed $V$ magnitude, $(B - V)$ a color index, $R_0$ the mean radius, and $r$ the displacement found by integrating the velocity curve according to

$$r = -\frac{kP}{R_\odot} \int V_r d\phi \,. \qquad (2)$$

Here $P$ is the period (in s), $R_\odot$ the solar radius (in km), $V_r$ the radial velocity (km s$^{-1}$), and $\phi$ the phase. The constant $k$ which corrects for geometrical projection and limb darkening, has been taken as 1.31 in all calculations, following Parsons (1972).

Fourier curves were fitted to the observed radial velocities of the Cepheids (Paper I) which were then integrated to obtain the displacement $r$ at the phases of the actual observations of $V$ and the corresponding color index, according to equation (2). The observed values of $V$, color index, and $r$ at given phases were then used to solve for $R_0$, $A$, and $B$ from equation (1) by the method of maximum likelihood, assuming standard errors of the $V$ and color index observations of 0.015 mag and 3 km s$^{-1}$ for the radial velocity observations (this is not very critical). Solutions were obtained using the four color indices $(B-V)$, $(U-B)$, $(V-R)$, and $(V-I)$. They are given in Table 1.

If one rearranges equation (1), one obtains

$$r = \frac{R_0}{5 \log_{10} e} [A(B-V) - V + B] \qquad (3)$$

which shows that $r$ can also be obtained from $V$ and the color index once the constants $R_0$, $A$, and $B$ are determined. In Figure 1, the displacement curve obtained from equation (2) is shown for the Cepheid R TrA (*smooth curves*); the points in these graphs are the values of $r$ calculated from equation (3), for the different color indices. The agreement between the points and the smooth curve is an indication of the quality of the estimates of $R_0$, $A$, and $B$, from the corresponding color index; the $\sigma$ given in Table 1 is the rms deviation about the displacement (i.e., the scatter of the points from eq. [3] about the smooth curve in solar radii). Weighted mean radii were obtained for each Cepheid by averaging the $R_0$ solutions from $(B-V)$, $(V-R)$, and $(V-I)$ with weights in inverse proportion to the $\sigma$ given in Table 1. The $(U-B)$ radius solutions are persistently much smaller than the solutions from the other color indices, indicating that $(U-B)$ is not a suitable index and should be discarded. The weighted mean radii $\langle R_0 \rangle$ of the Cepheids are given in Table 1 in column (6). $N_P$ and $N_R$ are the number of photometric and radial velocity observations, respectively. For AZ Cen (the star with the lowest number of photometric observations) no sensible radii were obtained from $(V-R)$ and $(V-I)$; consequently its $(B-V)$ radius is adopted as final mean radius. The $\sigma$ values of Table 1 show that the $(B-V)$ solution enters the final radius with the highest weight for all stars, followed by the $(V-I)$ solution for which the scatter is in the order of, but generally somewhat lower than, that of the $(V-R)$ solution (see also Fig. 1). The fact that for the $(V-I)$ solutions the scatter is always appreciably higher than

TABLE 1

Maximum Likelihood Solutions for the Constants $R_0$, $A$ and $B$

| Star (1) | Color (2) | $N_P$ (3) | $N_R$ (4) | $R_0$ (5) | $\langle R_0 \rangle$ (6) | $A$ (7) | $B$ (8) | $\sigma$ (9) | $\Delta r/\langle R_0 \rangle$ (10) |
|---|---|---|---|---|---|---|---|---|---|
| AZ Cen ................... | $B-V$ | 23 | 22 | 27.73 | 27.7 | 2.4856 | 6.9454 | 0.6335 | 0.036 |
| | $U-B$ | 23 | ... | 8.83 | ... | 6.1463 | 6.0153 | ... | ... |
| | $V-R$ | 20 | ... | 423.24 | ... | 3.8485 | 7.1359 | ... | ... |
| | $V-I$ | 20 | ... | −193.31 | ... | 2.1268 | 6.9773 | ... | ... |
| R TrA ..................... | $B-V$ | 43 | 27 | 27.51 | 28.2 | 2.2198 | 5.0341 | 0.1431 | 0.067 |
| | $U-B$ | 43 | ... | 14.94 | ... | 3.6434 | 4.9731 | ... | ... |
| | $V-R$ | 43 | ... | 28.74 | ... | 4.8629 | 4.6564 | 0.4596 | ... |
| | $V-I$ | 43 | ... | 29.50 | ... | 2.7995 | 4.3803 | 0.3356 | ... |
| SS Sct ..................... | $B-V$ | 42 | 27 | 34.52 | 34.8 | 2.1387 | 6.1413 | 0.2727 | 0.063 |
| | $U-B$ | 42 | ... | 23.05 | ... | 2.9905 | 6.3064 | ... | ... |
| | $V-R$ | 42 | ... | 37.73 | ... | 4.8077 | 5.5458 | 0.5891 | ... |
| | $V-I$ | 42 | ... | 33.29 | ... | 2.7081 | 5.2542 | 0.3489 | ... |
| AG Cru ................... | $B-V$ | 43 | 23 | 30.55 | 33.2 | 2.3787 | 6.3689 | 0.2033 | 0.087 |
| | $U-B$ | 43 | ... | 19.33 | ... | 4.8829 | 5.7666 | ... | ... |
| | $V-R$ | 43 | ... | 42.18 | ... | 4.6987 | 6.1524 | 0.8637 | ... |
| | $V-I$ | 43 | ... | 34.57 | ... | 2.6864 | 5.8973 | 0.5208 | ... |
| BF Oph ................... | $B-V$ | 34 | 22 | 35.81 | 38.0 | 2.1680 | 5.4407 | 0.2630 | 0.084 |
| | $U-B$ | 34 | ... | 24.85 | ... | 3.1470 | 5.5024 | ... | ... |
| | $V-R$ | 34 | ... | 38.27 | ... | 4.7712 | 4.9396 | 0.4291 | ... |
| | $V-I$ | 34 | ... | 41.58 | ... | 2.6782 | 4.7177 | 0.4586 | ... |
| V482 Sco .................. | $B-V$ | 46 | 24 | 47.25 | 50.4 | 2.1401 | 5.8421 | 0.3351 | 0.075 |
| | $U-B$ | 46 | ... | 27.75 | ... | 2.7674 | 6.0311 | ... | ... |
| | $V-R$ | 46 | ... | 53.37 | ... | 4.7863 | 5.1941 | 0.8417 | ... |
| | $V-I$ | 46 | ... | 54.95 | ... | 2.6494 | 4.9793 | 0.8001 | ... |
| S Cru ...................... | $B-V$ | 43 | 25 | 44.18 | 45.3 | 2.0336 | 5.0027 | 0.2821 | 0.082 |
| | $U-B$ | 43 | ... | 27.59 | ... | 2.9170 | 5.1233 | ... | ... |
| | $V-R$ | 42 | ... | 46.95 | ... | 4.7382 | 4.5640 | 0.5283 | ... |
| | $V-I$ | 42 | ... | 45.67 | ... | 2.5698 | 4.4068 | 0.4142 | ... |
| AP Sgr .................... | $B-V$ | 35 | 22 | 42.74 | 45.4 | 2.1536 | 5.1422 | 0.2061 | 0.097 |
| | $U-B$ | 35 | ... | 23.69 | ... | 3.0748 | 5.1378 | ... | ... |
| | $V-R$ | 34 | ... | 53.28 | ... | 4.8010 | 4.6920 | 0.5958 | ... |
| | $V-I$ | 35 | ... | 45.36 | ... | 2.6308 | 4.5376 | 0.5472 | ... |
| V350 Sgr .................. | $B-V$ | 34 | 22 | 37.68 | 41.7 | 2.2779 | 5.3520 | 0.1657 | 0.094 |
| | $U-B$ | 34 | ... | 24.46 | ... | 3.1486 | 5.4676 | ... | ... |
| | $V-R$ | 34 | ... | 52.11 | ... | 4.7877 | 4.8820 | 0.6105 | ... |
| | $V-I$ | 34 | ... | 44.19 | ... | 2.6679 | 4.6696 | 0.3929 | ... |
| V Cen ..................... | $B-V$ | 35 | 25 | 48.41 | 52.7 | 2.0971 | 4.9436 | 0.2970 | 0.082 |
| | $U-B$ | 35 | ... | 10.95 | ... | 3.6002 | 4.6135 | ... | ... |
| | $V-R$ | 35 | ... | 56.11 | ... | 4.6527 | 4.3932 | 0.5593 | ... |
| | $V-I$ | 35 | ... | 56.79 | ... | 2.5945 | 4.1624 | 0.4855 | ... |
| RV Sco .................... | $B-V$ | 32 | 23 | 46.31 | 49.7 | 2.1872 | 4.8719 | 0.2863 | 0.093 |
| | $U-B$ | 32 | ... | 30.45 | ... | 2.7873 | 5.0824 | ... | ... |
| | $V-R$ | 32 | ... | 56.47 | ... | 4.5670 | 4.3965 | 0.6060 | ... |
| | $V-I$ | 32 | ... | 50.16 | ... | 2.7070 | 3.9478 | 0.6409 | ... |
| S TrA ..................... | $B-V$ | 33 | 23 | 43.92 | 42.0 | 2.0644 | 4.8011 | 0.3145 | 0.107 |
| | $U-B$ | 33 | ... | 43.26 | ... | 2.2702 | 5.1885 | ... | ... |
| | $V-R$ | 33 | ... | 42.19 | ... | 4.9586 | 4.3094 | 0.9255 | ... |
| | $V-I$ | 33 | ... | 35.97 | ... | 2.6610 | 4.2408 | 0.9284 | ... |
| BB Sgr .................... | $B-V$ | 47 | 24 | 41.72 | 45.4 | 2.1464 | 4.7875 | 0.3170 | 0.086 |
| | $U-B$ | 47 | ... | 25.67 | ... | 2.3960 | 5.1974 | ... | ... |
| | $V-R$ | 45 | ... | 51.54 | ... | 5.1469 | 3.9784 | 1.1796 | ... |
| | $V-I$ | 45 | ... | 49.35 | ... | 2.8667 | 3.7509 | 0.6160 | ... |
| U Sgr ...................... | $B-V$ | 45 | 25 | 58.91 | 60.1 | 2.1236 | 4.3242 | 0.3737 | 0.093 |
| | $U-B$ | 45 | ... | 42.48 | ... | 2.1815 | 4.9281 | ... | ... |
| | $V-R$ | 45 | ... | 63.94 | ... | 4.8045 | 3.6640 | 0.8505 | ... |
| | $V-I$ | 45 | ... | 59.22 | ... | 2.7445 | 3.2661 | 0.7417 | ... |
| V496 Aql .................. | $B-V$ | 38 | 25 | 43.16 | 43.2 | 2.3315 | 5.0341 | 0.2827 | 0.067 |
| | $U-B$ | 38 | ... | 27.74 | ... | 2.2255 | 5.8167 | ... | ... |
| | $V-R$ | 37 | ... | 51.59 | ... | 5.7054 | 4.0145 | 1.261 | ... |
| | $V-I$ | 37 | ... | 38.57 | ... | 3.1104 | 3.8046 | 0.7237 | ... |

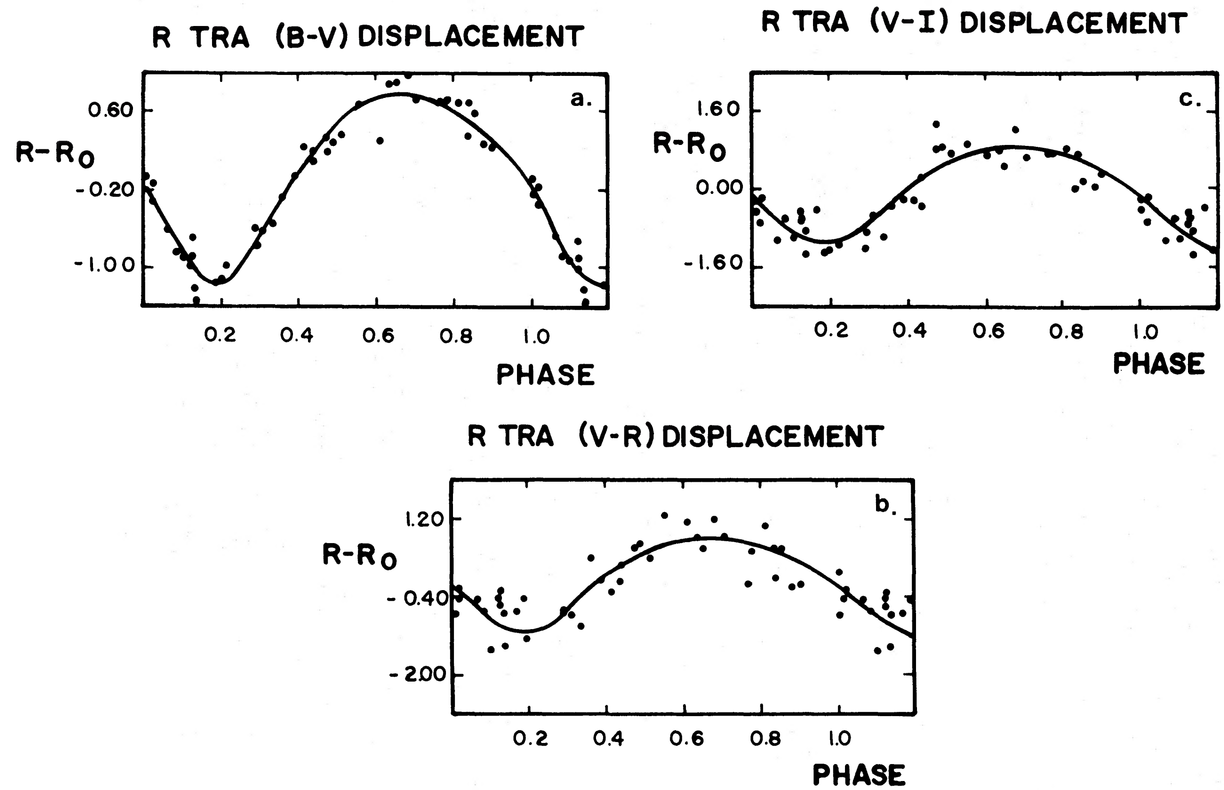


FIG. 1.—The radius displacement, in solar radii, obtained from eq. (2) (*solid line*) and the displacement calculated from eq. (3) (*points*), for R TrA and for the color indices $(B-V)$, $(V-R)$, and $(V-I)$.

for the $(B-V)$ solutions is somewhat surprising in view of the similar amplitudes and quality of these color curves (see Paper II) and contradicts to some extent expectations that $(V-I)$ might give more reliable Wesselink radii than $(B-V)$ due to fewer problems with line blocking in this wavelength range.

The consistency of the $R_0$ values from the different color indices is quite satisfactory. On average, the deviation of the two extreme values is $\sim 20\%$. (The extreme cases are V350 Sgr and AG Cru with 38%. It is interesting to note that both stars have companions; see Paper III). There is a slight systematic trend in the sense that the $(V-R)$ solutions give on average the largest $R_0$ values.

The displacement curves obtained from equation (2) were used to determine the total amplitudes $\Delta r/\langle R_0 \rangle$ of the radius variation as a percentage of the mean radius (see Table 1). Typical values are a few percent (only for S TrA does it exceed 10%), with a slight tendency to increase with period. Comparison with the values derived by Pel (1978) for 14 common stars from *VBLUW* photometry and the Kurucz (1975) model atmospheres reveals that his values are on average by a factor of $\sim 2$ larger than the present determinations. On the other hand, there is good agreement with the values given by Sollazzo *et al.* (1981). Our values show that $r \ll R_0$ is a reasonable approximation for the Cepheids of the present study and justify use of the linearized equation (1).

The values of the constant $A$ which for $(B-V)$ represents the slope of the visual surface brightness and $(B-V)$ relation are interesting because they contain information about the $(B-V) - \log T_e$ scale (e.g., Thompson 1975) and about possible blue companions (Balona 1977). From 39 Cepheids, excluding W Vir stars and known or suspected binaries, Balona (1977) found

$$A = 2.15 \pm 0.02 \text{ (s.e.)} .$$

The present 15 Cepheids yield

$$A = 2.20 \pm 0.12 \text{ (s.e.)} .$$

However, as shown by Balona (1977), the value of $A$ gets shifted in the presence of a photometrically significant companion star. Excluding, therefore, the two known binaries, AZ Cen and V350 Sgr, and the two very probable binaries, AG Cru and V496 Aql (see Paper III), the remaining 11 Cepheids yield

$$A = 2.13 \pm 0.05 \text{ (s.e.)} ,$$

in excellent agreement with Balona's value and Thompson's (1975) value of $2.11 \pm 0.03$. This supports earlier findings that the constant $A$ is very nearly identical for nonbinary Cepheids and suggests that at least for short-period Cepheids, a uniform temperature scale can be used. Consequently, as already noted by Balona (1977), $A$ is very useful as a sensitive detector of companions, especially of blue ones. This is demonstrated by the large values of $A$ for AZ Cen and AG Cru, and to a lesser extent for V350 Sgr, which all have blue companions, according to the results of Paper III. Even the size of the deviation of $A$ from its normal value of 2.13 agrees with the estimated brightnesses of the blue companions (AZ Cen, with the brightest blue companion of the present Cepheids, has the largest $A$ value of 2.49). The usefulness of $A(B-V)$ as a tool to detect blue companions is further confirmed by noting that $A(V-I)$ is very similar for both, nonbinary Cepheids and Cepheids with blue companions (see Table 1), in accordance with the expectation that $A(V-I)$ is left unaffected by a blue companion. An exception is AZ Cen, but the deviation of its $A(V-I)$ value from the normal value of $\sim 2.70$ may be caused by insufficient photometry. Thus, the present values of $A(B-V)$ derived from the maximum likelihood solutions provide an independent confirmation of the results of Paper III. The only slight disagreement exists in the case of V496 Aql where the present $A$ value suggests a blue companion, whereas the photometric properties discussed in Paper III hint at the presence of a G-type companion.

In Table 2, the new Wesselink radii are compared to other Wesselink determinations carried out by Sollazzo *et al.* (1981), Balona (1977), Thompson (1975), Woolley and Carter (1973), Fernie (1968), and others. In two cases (SS Sct and V350 Sgr), no former radius determinations were found. For some stars, the agreement is good (for instance, AG Cru, V482 Sco, and U Sgr), but for others it is very poor (BF Oph, RV Sco, and V496 Aql), and differences up to 80% (RV Sco) do exist. We conclude that the present determinations are the most reliable ones in the list, due to the following reasons:

(1) The present radii are the only ones where phase mismatch problems are definitively excluded.

(2) They are the only ones obtained as weighted average values from three different color indices (with the exception of the radius value of AZ Cen: in this case Stobie and Balona (1979) give a value based upon two color indices).

(3) A global and statistically justified technique has been used to derive the radii (this advantage is shared with the work of Balona (1977) and Sollazzo *et al.* (1981)).

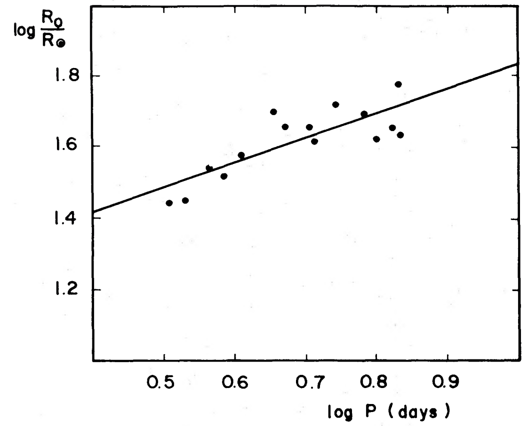


FIG. 2.—The period-radius relationship constructed from the weighted mean radii of the present 15 Cepheids. The line represents the best fit to the data.

### *b) The Period-Radius Relationship*

In Figure 2, the $\log R_0 - \log P$ diagram is constructed from the present mean radii, using the periods given in Paper I. A least-squares fit to the data gives

$$\log R_0 = 0.695 \log P + 1.139 \qquad (4)$$

with a standard deviation about this relation of 0.055 per star in $\log R_0$ (or $\sim 13\%$).

Excluding the binary Cepheids AZ Cen, AG Cru, V350 Sgr, and V496 Aql, a least-squares fit leads to the relation

$$\log R_0 = 0.683 \log P + 1.163 \qquad (5)$$

TABLE 2

RADIUS DETERMINATIONS OF THE PRESENT CEPHEIDS

| Star | Gieren (this work) | Sollazzo *et al.* 1981 | Balona 1977 | Thompson 1975 | Woolley and Carter 1973 | Fernie 1968 | Others |
|---|---|---|---|---|---|---|---|
| AZ Cen ............ | 27.7 | ... | ... | ... | ... | ... | 39.7 Stobie and Balona 1979 |
| R TrA ............... | 28.2 | ... | 35.8 | ... | ... | ... | ... |
| SS Sct .............. | 34.8 | ... | ... | ... | ... | ... | ... |
| AG Cru ............ | 33.2 | 30.6 | 33.2 | ... | ... | 37 | ... |
| BF Oph ............ | 38.0 | 40.7 | 50.4 | ... | 57.2 | ... | ... |
| V482 Sco .......... | 50.4 | 49.5 | ... | 49 | ... | ... | ... |
| S Cru ............... | 45.3 | 42.7 | 43.6 | 34 | 48.9 | ... | ... |
| AP Sgr ............. | 45.4 | ... | 36.8 | ... | ... | ... | ... |
| V350 Sgr .......... | 41.7 | ... | ... | ... | ... | ... | ... |
| V Cen .............. | 52.7 | ... | 48.4 | ... | 38.3 | ... | ... |
| RV Sco ............. | 49.7 | 63.0 | 44.4 | ... | 78.6 | ... | ... |
| S TrA .............. | 42.0 | ... | 54.6 | ... | ... | ... | ... |
| BB Sgr ............. | 45.4 | ... | 55.2 | ... | ... | ... | ... |
| U Sgr ............... | 60.1 | 57.9 | 55.3 | 60 | 61.3 | 54 | 53.8 Breger 1967 |
| V496 Aql ........... | 43.2 | 54.4 | 36.4 | ... | ... | ... | ... |

with a slightly enhanced scatter of 0.057 per star about this relation. Clearly, within the errors of the coefficients the two relations agree. We take this as a support of the earlier finding of Balona (1977) that in the case of Cepheids with companions the error in the maximum likelihood solution of equation (1) gets largely shifted to the constant $A$ and that $R_0$ is basically left unaffected. Using Table 2 of Balona's (1977) study and the spectral types of the companions of the present binary Cepheids estimated in Paper III, it turns out that for AG Cru, V350 Sgr, and V496 Aql the effect on their radii should be less than $\sim 6\%$; for AZ Cen the effect could be $\sim 10\%$. It thus appears that in the construction of the present $\log R_0 - \log P$ relation unresolved binaries are a minor source of error.

The Cepheid with the largest deviation in Figure 2 is V482 Sco. Supposing this star to be pulsating in the first harmonic mode, it would be displaced almost exactly onto the mean relationship, and the scatter of the new $\log R_0 - \log P$ relation (coefficients 0.717 and 1.116) would go down to the $\sim 10\%$ found by Balona (1977). However, as shown in § IV, there is no compelling evidence to suppose that V482 is a first harmonic pulsator, and the standard deviation of its radius from equation (4) remains within the 3 $\sigma$ limit.

A comparison with $\log R_0 - \log P$ relationships published by other authors can be made. Sollazzo *et al.* (1981) have recently published a paper where they give a compilation of earlier determinations (their Table 3; they omit the result of Parsons and Bouw [1971] who find $\log R_0 = 0.652 \log P + 1.173$) and discuss the relative merits of their CORS radius finding method with respect to Wesselink's method. They derive CORS radii for 30 Cepheids using the Pel (1976) Walraven photometry and individual radial velocity curves from the literature (mainly from Stibbs 1955) which are generally based on a low number of points. The generally large epoch difference between the Pel photometric observations and the velocity observations used makes their results quite vulnerable to phase mismatch errors. Nevertheless, they get a relation very similar to the present one ($\log R_0 = 0.675 \log P + 1.193$) with the smallest errors in the coefficients obtained up to now.

Sollazzo *et al.* (1981) argue that the Wesselink method probably does not produce slopes in the order of 0.65–0.70 which would agree with the theoretical value of 0.70 derived for the center of the instability strip by Cogan (1978), although in two earlier works based on Wesselink radii (Woolley and Carter 1973; Thompson 1975) slopes in this interval were obtained. The factors which lead them to doubt the validity of the results of Woolley and Carter (1973) and Thompson (1975), respectively, enter their own radius determinations as well (with the exception of better statistics based upon twice as many radii better distributed in period), especially the phase mismatch problem. However, the present results show that Wesselink radii obtained under the present conditions are apparently able to give a slope close to 0.70, as required by theory, at least for $\log P < 1$, but it has been this period range where the smaller slopes found in other Wesselink radii studies had their origin. This is most clearly seen in Evans (1976) where most of the short period Cepheids have (probably) too large radii, thus producing a (probably) too small slope in her $\log R_0 - \log P$ relation. Thus we do not expect that significant changes in the constants of the $\log R_0 - \log P$ relation will be introduced when the present work is extended to longer period Cepheids, but for the time being one should bear in mind that the validity of the present constants is restricted to $\log P < 1$. In this period range, however, the present work provides a reasonably large number of radii very evenly distributed in period.

Further, it should be emphasized that the present period-radius relationship is also in agreement with the constants expected from the currently accepted period-luminosity-color ($P$-$L$-$C$) relation of Sandage and Tammann (1969). The existence of a $P$-$L$-$C$ relation implies a mean $\log R_0 - \log P$ relationship as can be seen if one substitutes a ridge line $\langle B-V \rangle_0 - \log P$ relation into the equation

$$5 \log R/R_\odot = -F \log P + (A-G)(B-V)_0 + C - H \tag{6}$$

(for a derivation and a definition of the symbols, see Balona 1977). This can be done with very small error because the values of the constants $A$ and $G$ are very similar. Using

$$\langle B-V \rangle_0 = 0.46 \log P + 0.27 \tag{7}$$

given by Dean, Warren, and Cousins (1978), $A = 2.13$ from the present study, and the values of the constants $F$, $G$, $H$, and $C$ from Sandage and Tammann (1969), one obtains

$$\log R_0 = 0.649 \log P + 1.150 \tag{8}$$

which for $P < 7^{\mathrm{d}}$ yields radii which deviate from the present relationship (4) on average by $\sim 0.02$ in $\log R_0$, which is certainly within the standard deviation of both relations. It thus appears that we are about to obtain reasonable agreement between the theoretically predicted and observed period-radius relationship which in turn are both compatible with the $P$-$L$-$C$ relation.

Finally, since the present radii are derived from almost "ideal" observations, we can conclude that the scatter of $\sim 13\%$ observed in Figure 2 is close to reflect the limit of accuracy of radius results which can be expected from any $\log R_0 - \log P$ relation obtained from Baade–Wesselink radii. While some of the remaining scatter will be due to the principal limitations imposed by the assumptions of Wesselink's method, we believe that most of it is of an intrinsic nature (caused by the finite width of the instability strip, different evolutionary status of the individual Cepheids, etc.) and cannot be appreciably reduced by future work in this line.

## III. MASSES AND LUMINOSITIES

Cox (1979, 1980) has recently reviewed our actual knowledge about Cepheid masses and has discussed the discrepancies which still persist if one uses the different approaches to obtain Cepheid masses. He has intercompared the following:

1. Evolutionary masses $M_{\rm ev}$, which are obtained from evolutionary tracks (e.g. Becker, Iben, and Tuggle, 1977) and a mass-luminosity relation, assuming a certain chemical composition.

2. Theoretical masses $M_{\rm Th}$, which are obtained from the simultaneous solution of (*a*) $L = 4\pi R^2 \sigma T_e^4$ (definition of effective temperature), (*b*) an evolutionary mass-luminosity relationship, (*c*) the pulsation homology relationship

$$Q_0 = P_0 \left[\frac{M/M_\odot}{(R/R_\odot)^3}\right]^{1/2} ,$$

and (*d*) an expression for the pulsation constant $Q_0$:

$$\log Q_0 = f(M, R, L, T_e, \text{chem. composition}) .$$

From these four equations the four unknowns $M$, $R$, $L$, and $Q_0$ can be found. Using the mass-luminosity equation from Becker, Iben, and Tuggle for $Y = 0.28$ and $Z = 0.02$ and the Faulkner (1977) formula for $\log Q_0$, Cox (1979) has obtained theoretical masses for many Cepheids using (*a*) chemically homogeneous models, and (*b*) models which possess a helium enriched surface layer (inhomogeneous models) and are homogeneous with $Y = 0.28$, $Z = 0.02$ for layers deeper than 80,000 K.

3. Pulsation masses $M_{\rm Pul}$, which are calculated from the fundamental period $P_0$, a luminosity (from a *P-L-C* relationship), and a value of $T_e$, which yield a radius $R$. With this radius and a value of $Q_0$ (for a homogeneous or inhomogeneous model), one obtains $M_{\rm Pul}$ from the homology relationship.

4. Wesselink radius masses $M_{\rm Wes}$, which are obtained from the homology relationship using a Wesselink radius, the observed $P_0$, and a $Q_0$ value for a homogeneous or inhomogeneous envelope.

The main results found by Cox (1979), using extensive observational data available prior to 1979, the new distance scale proposed by Hanson (1977) which makes Cepheids by $\sim$0.26 mag brighter in $M_v$, and the temperature scale of Pel (1978) which has reduced Cepheid effective temperatures in comparison to formerly accepted scales, are:

1. Evolutionary and theoretical masses agree well, with a reasonably low scatter about $M_{\rm Th}/M_{\rm ev} = 1.0$.

2. The pulsation masses agree within the errors with the evolutionary masses, for homogeneous and inhomogeneous models, but the scatter of $M_{\rm Pul}/M_{\rm ev}$ is quite large ($M_{\rm Pul}/M_{\rm ev} = 0.97 \pm 0.25$ for homogeneous models, and $1.07 \pm 0.27$ for inhomogeneous models).

3. The Wesselink radius masses are too small compared to theoretical masses, the discrepancy increasing with period. The most disturbing fact, however, is the very large scatter in the plot $M_{\rm Wes}/M_{\rm Th} = f(P_0)$ (see Cox's Figs. 5 and 6). The discrepancy is slightly alleviated using inhomogeneous $Q_0$ values, but the scatter remains the same.

In order to carry out a similar comparison with the new data, we have recalculated pulsation masses and Wesselink radius masses (for both homogeneous and inhomogeneous model $Q_0$ values) for all present Cepheids, with the exception of V350 Sgr. The remaining 14 Cepheids have Pel (1978) $T_e$ values, and 11 of them have $M_{\rm Th}{}^{\rm HOM}$ and $M_{\rm Th}{}^{\rm IN}$ values given by Cox (1979). For the three Cepheids for which Cox does not give a theoretical mass (AZ Cen, SS Sct, and V482 Sco), a homogeneous model $Q_0$ value and a theoretical mass was calculated from the formulae given in Cox (1979). In detail, the new masses were calculated in the following way:

*Pulsation masses.* The fundamental periods given in Paper I and the Pel (1978) $T_e$ values were used. The luminosity was calculated from Sandage and Tammann's (1969) *P-L-C* relation, using the intrinsic $\langle B-V\rangle_0$ colors given in Paper II and correcting for the new distance scale adding $-0.26$ mag to $\langle M_v\rangle$). For the transformation of the $M_v$ into bolometric magnitudes, the Pel (1978) bolometric correction scale was used. The luminosities obtained in this way are called $L_{P\text{-}L\text{-}C}$. The radius was then calculated from $R/R_\odot = (L/L_\odot)^{1/2}(T_{e\odot}/T_e)^2$, using $T_{e\odot} = 5800$ K (Allen 1963). This radius is called pulsation radius $R_{\rm Pul}$. The corresponding pulsation masses for the homogeneous and inhomogeneous $Q_0$ values given by Cox (1979) were then calculated from the $Q_0 = P_0[M/R^3]^{1/2}$ relationship. For AZ Cen, SS Sct, and V482 Sco, the masses were obtained from the newly calculated $Q_0$ values.

*Wesselink radius masses.* The new maximum likelihood radii were used together with the $P_0$ of Paper I and the Cox (1979) $Q_0$ values for homogeneous and inhomogeneous envelopes. For AZ Cen, SS Sct, and V482 Sco, the newly calculated $Q_0$ values were used.

In addition to the PLC luminosities, Wesselink luminosities $L_{\rm Wes}$ were calculated for the Cepheids from their Wesselink radii and their effective temperatures (see Table 3, col. [14]).

All these new data for the present Cepheids, together with $Q_0$ values, theoretical masses, and theoretical luminosities for homogeneous and inhomogeneous envelopes from Cox (1979) are listed in Table 3. With these new data, a comparison of the different masses and luminosities can now be made.

### *a) Pulsation and Theoretical Masses*

Figure 3 displays the ratio of pulsation mass to theoretical mass against $\log P_0$, for the present 14 Cepheids, using homogeneous model $Q_0$ values. The mean value of this ratio is

$$\left\langle \frac{M_{\rm Pul}{}^{\rm HOM}}{M_{\rm Th}{}^{\rm HOM}} \right\rangle = 0.96 \pm 0.10 \text{ (s.d.)} .$$

Using the inhomogeneous model $Q_0$ values, one obtains

$$\left\langle \frac{M_{\rm Pul}{}^{\rm IN}}{M_{\rm Th}{}^{\rm IN}} \right\rangle = 1.09$$

with about the same scatter (i.e., the mass ratios get

TABLE 3

THEORETICAL, PULSATION, AND WESSELINK RADIUS MASSES FOR HOMOGENEOUS AND INHOMOGENEOUS MODEL $Q_0$ VALUES

| Star (1) | Period (2) | $T_e$ (K) (3) | $R_{\rm Wes}$ (4) | $Q_{\rm Th}{}^{\rm HOM}$ (5) | $M_{\rm Th}{}^{\rm HOM}$ (6) | $R_{\rm Pul}$ (7) | $M_{\rm pul}{}^{\rm HOM}$ (8) | $M_{\rm Wes}{}^{\rm HOM}$ (9) | $Q_{\rm Th}{}^{\rm IN}$ (10) | $M_{\rm Th}{}^{\rm IN}$ (11) | $M_{\rm Pul}{}^{\rm IN}$ (12) | $M_{\rm Wes}{}^{\rm IN}$ (13) | $L_{\rm Wes}$ (14) | $L_{\rm Th}{}^{\rm HOM}$ (15) | $L_{\rm Th}{}^{\rm IN}$ (16) | $L_{P\text{-}L\text{-}C}$ (17) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| AZ Cen .......... | 3ᵈ211 | 6324 | 27.7 | 0.0375 | 5.74 | 34.9 | 5.80 | 2.90 | ... | ... | ... | ... | 1084 | ... | ... | 1722 |
| R TrA ........... | 3.389 | 6000 | 28.2 | 0.0377 | 5.42 | 36.7 | 6.12 | 2.77 | 0.0395 | 5.32 | 6.71 | 3.05 | 911 | 1451 | 1349 | 1542 |
| SS Sct ........... | 3.671 | 5854 | 34.8 | 0.0379 | 5.49 | 37.8 | 5.76 | 4.49 | ... | ... | ... | ... | 1257 | ... | ... | 1486 |
| AG Cru .......... | 3.837 | 6207 | 33.2 | 0.0379 | 6.00 | 39.2 | 5.88 | 3.57 | 0.0397 | 5.88 | 6.45 | 3.92 | 1446 | 2104 | 1958 | 2014 |
| BF Oph .......... | 4.068 | 5827 | 38.0 | 0.0383 | 5.63 | 41.3 | 6.25 | 4.86 | 0.0403 | 5.50 | 6.91 | 5.39 | 1471 | 1662 | 1529 | 1738 |
| V482 Sco ........ | 4.528 | 5753 | 50.4 | 0.0386 | 5.53 | 41.8 | 5.31 | 9.30 | ... | ... | ... | ... | 2459 | ... | ... | 1690 |
| S Cru ............ | 4.690 | 5895 | 45.3 | 0.0386 | 6.07 | 44.2 | 5.85 | 6.30 | 0.0407 | 5.93 | 6.50 | 7.00 | 2190 | 2199 | 2021 | 2089 |
| AP Sgr ........... | 5.058 | 5708 | 45.4 | 0.0389 | 6.01 | 46.1 | 5.80 | 5.54 | 0.0412 | 5.86 | 6.50 | 6.21 | 1933 | 2120 | 1934 | 1995 |
| V Cen ............ | 5.494 | 5902 | 52.7 | 0.0390 | 6.51 | 50.7 | 6.57 | 7.38 | 0.0412 | 6.36 | 7.33 | 8.23 | 2978 | 2845 | 2603 | 2754 |
| RV Sco .......... | 6.061 | 5833 | 49.7 | 0.0394 | 6.67 | 52.1 | 5.98 | 5.19 | 0.0417 | 6.50 | 6.69 | 5.81 | 2527 | 3109 | 2832 | 2780 |
| S TrA ............ | 6.323 | 5625 | 42.0 | 0.0396 | 6.46 | 53.5 | 6.01 | 2.91 | 0.0421 | 6.29 | 6.79 | 3.28 | 1561 | 2770 | 2507 | 2535 |
| BB Sgr ........... | 6.637 | 5520 | 45.4 | 0.0399 | 6.39 | 53.1 | 5.41 | 3.38 | 0.0425 | 6.22 | 6.14 | 3.84 | 1691 | 2663 | 2398 | 2312 |
| U Sgr ............ | 6.745 | 5688 | 60.1 | 0.0398 | 6.72 | 53.6 | 5.36 | 7.56 | 0.0423 | 6.54 | 6.06 | 8.54 | 3341 | 3195 | 2892 | 2655 |
| V496 Aql ........ | 6.807 | 5490 | 43.2 | 0.0400 | 6.37 | 52.7 | 5.05 | 2.78 | 0.0427 | 6.19 | 5.76 | 3.17 | 1498 | 2630 | 2362 | 2228 |

EXPLANATION OF COLUMNS.—(1) Name of Cepheid. (2) Period (days). (3) Effective temperature from Pel 1978. (4) Wesselink radius from present work. (5) Homogeneous model $Q_0$ value (days) from Cox 1979 or present work. (6) Theoretical mass for homogeneous model from Cox 1979 or present work. (7) Pulsation radius. (8) Pulsation mass, for homogeneous model $Q_0$ value. (9) Wesselink radius mass, for homogeneous model $Q_0$ value. (10) Inhomogeneous model $Q_0$ value (days), from Cox 1979. (11) Theoretical mass for inhomogeneous model from Cox 1979. (12) Pulsation mass for inhomogeneous model $Q_0$ value. (13) Wesselink radius mass for inhomogeneous model $Q_0$ value. (14) Wesselink luminosity. (15) Theoretical luminosity, for homogeneous model $Q_0$ value from Cox 1979. (16) Theoretical luminosity, for inhomogeneous model $Q_0$ value from Cox 1979. (17) *P-L-C* luminosity.

NOTE.—Radii, masses, and luminosities are in solar units.

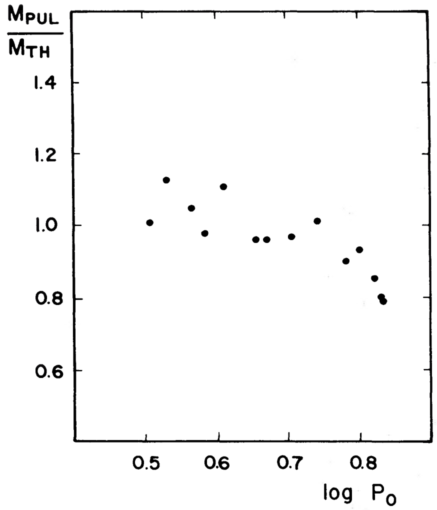


FIG. 3.—The ratio pulsation mass to theoretical mass as a function of the log of the fundamental period, for the present Cepheids. Homogeneous model $Q_0$ values have been used to calculate both masses.

shifted to larger values). The comparison of $M_{\rm Pul}$ and $M_{\rm Th}$ for the present Cepheids thus reveals the following features:

1. The two masses agree, within the errors, for both homogeneous and inhomogeneous model $Q_0$ values; the mean value of the ratios agree very closely with those found by Cox (1979). The agreement produced by the homogeneous $Q_0$ values is slightly better.

2. The scatter of the mean value ($\sim 10\%$) is much smaller than that found by Cox (1979).

3. A trend with period is present in the sense that $M_{\rm Pul}/M_{\rm Th}$ decreases with period.

### *b*) *Wesselink Radius and Theoretical Masses*

Figure 4 shows the plot $M_{\rm Wes}/M_{\rm Th}$ against log $P_0$ for the present Cepheids, using the homogeneous model $Q_0$ values. It is found that

$$\left\langle \frac{M_{\rm Wes}{}^{\rm HOM}}{M_{\rm Th}{}^{\rm HOM}} \right\rangle = 0.81 \pm 0.34 \text{ (s.d.)} .$$

Using the inhomogeneous model $Q_0$ values, the result is

$$\left\langle \frac{M_{\rm Wes}{}^{\rm IN}}{M_{\rm Th}{}^{\rm IN}} \right\rangle = 0.87 .$$

with a comparable scatter. It is observed that:

1. For the homogeneous model $Q_0$ values, the Wesselink radius masses are on average $\sim 20\%$ smaller than the theoretical masses. The scatter in Figure 4 is still large, but considerably smaller than in Cox's Figures 5 and 6. In particular, there is only one star deviating strongly in the sense of a too large Wesselink radius mass (V482 Sco, $M_{\rm Wes}/M_{\rm Th} = 1.68$), whereas in Cox's diagram, many such Cepheids with ratios up to 2 are found. On the other hand, six out of 14 Cepheids have $M_{\rm Wes}/M_{\rm Th}$ close to 0.5, which is also observed as approximate lower limit in Cox's figures.

2. No trend with period is apparent in the studied period range.

3. For the inhomogeneous model $Q_0$ values, the Wesselink radius masses agree slightly better with the theoretical masses (discrepancy $\sim 15\%$).

If V482 Sco is pulsating in the first harmonic mode, its Wesselink radius mass would decrease to 4.69 $M_\odot$ (assuming $P_1/P_0 = 0.71$) and $M_{\rm Wes}/M_{\rm Th}$ would be 0.85. This would increase the homogeneous model $Q_0$ values mass discrepancy to 25%, but the standard deviation of $M_{\rm Wes}/M_{\rm Th}$ would then reduce to

$$\left\langle \frac{M_{\rm Wes}{}^{\rm HOM}}{M_{\rm Th}{}^{\rm HOM}} \right\rangle = 0.75 \pm 0.24 .$$

We conclude that an appreciable number of Cepheids used to construct Cox's Figures 5 and 6 have erroneous Wesselink radii, to account for the very large scatter. In particular, many of his Cepheids with $P_0 < 10^{\rm d}$ seem to have radii which are too large. The present data show that the Wesselink radius mass discrepancy does also exist for $P_0 < 10^{\rm d}$ if one uses the inhomogeneous model $Q_0$ values, whereas Cox had found that $M_{\rm Wes}/M_{\rm Th} = 1.02$, in this case. While the Wesselink radius mass discrepancy does persist, a satisfactory aspect of the new data is that the scatter of $M_{\rm Wes}/M_{\rm Th}$ values is now much more acceptable, although still too large. Several possibilities exist to obtain a closer agreement between Wesselink radius masses and theoretical masses, although the period dependence of the discrepancy seen in Cox (1979) might pose a very serious problem in achieving this agreement. The possibilities include a larger factor $k$ in equation (2), and/or introduction of composition inhomogeneities

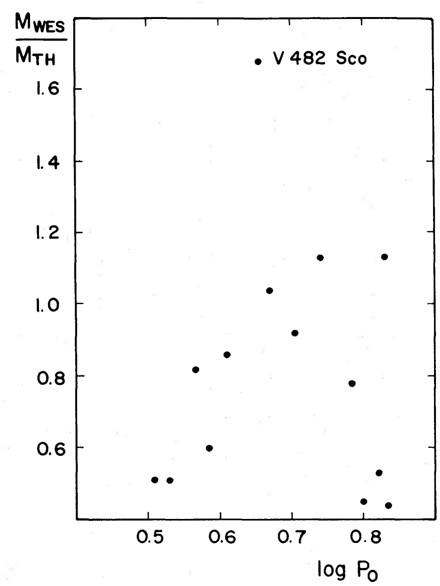


FIG. 4.—The ratio Wesselink radius mass to theoretical mass as a function of the log of the fundamental period, for the present Cepheids. Homogeneous model $Q_0$ values have been used to calculate both masses.

which would increase the $Q_0$ values even more than the inhomogeneous models used by Cox (1979); this, however, might destroy the presently good agreement between $M_{\rm Pul}$ and $M_{\rm Th}$ and might furthermore be very difficult to justify. The allowance for microturbulence changes during a Cepheid's pulsation cycle proposed by Evans (1980) would tend to make Wesselink radii smaller and would therefore increase the discrepancy. Clearly, the situation with respect to Wesselink radius masses is difficult to understand, and there is certainly no easy way out of the dilemma.

### *c) Theoretical and Wesselink Luminosities*

From 11 stars (excluding V482 Sco) which have theoretical luminosities given by Cox (1979), the present data yield

$$\left\langle \frac{L_{\rm Wes}}{L_{\rm Th}{}^{\rm HOM}} \right\rangle = 0.80 \pm 0.18 \ (\text{s.d.}) \ .$$

Using the theoretical luminosities based on the inhomogeneous models, one obtains

$$\left\langle \frac{L_{\rm Wes}}{L_{\rm Th}{}^{\rm IN}} \right\rangle = 0.87$$

with a similar standard deviation.

The behavior of the luminosities thus reflects the behavior of the corresponding masses: the Wesselink luminosities are on average $\sim 20\%$ smaller than the theoretical luminosities based upon homogeneous model $Q_0$ values, with a rather large scatter (although appreciably smaller than that of $M_{\rm Wes}/M_{\rm Th}{}^{\rm HOM}$), and the agreement gets slightly better if the inhomogeneous model $Q_0$ values are used.

### *d) Theoretical and P-L-C Luminosities*

For the homogeneous model $Q_0$ values, the mean ratio is

$$\left\langle \frac{L_{P\text{-}L\text{-}C}}{L_{\rm Th}{}^{\rm HOM}} \right\rangle = 0.94 \pm 0.07$$

For the inhomogeneous model theoretical luminosities, one obtains

$$\left\langle \frac{L_{P\text{-}L\text{-}C}}{L_{\rm Th}{}^{\rm IN}} \right\rangle = 1.02$$

with the same scatter.

Thus in both cases, theoretical and *P-L-C* luminosities agree, and the scatter is only $\sim 7\%$. Again, a slightly better agreement is obtained if the inhomogeneous model $Q_0$ values are used to calculate the theoretical luminosities.

## IV. PULSATION MODES

While we do not yet possess a method which would permit definitive conclusions about the mode in which a normal single-mode Cepheid is pulsating, we can obtain evidence from the following sources:

1. The location of the Cepheid with respect to blue edges in the H-R diagram, which depend on both chemical composition and mass, for a given mode of instability.

2. The location of the Cepheid with respect to the mean $\log R_0 - \log P$ relationship. Cepheids pulsating in the first harmonic mode will appear above the fundamental pulsator's line by $\sim 0.10$ in $\log R_0$. They consequently will have an anomalous large Wesselink radius mass if the observed period is used as the fundamental period in the homology relationship.

3. First harmonic pulsators may have more symmetric and lower amplitude light curves than Cepheids pulsating in the fundamental mode. This behavior is observed for the LMC Cepheids which are thought to be first harmonic pulsators (Connolly 1980), as well as for the galactic Bailey *c*-type RR Lyrae stars. It is also observed for the two most probable cases of galactic Cepheids pulsating in the first harmonic mode, the $1^{\rm d}.95$ Cepheid SU Cas (Gieren 1976; Iben and Tuggle 1975), and the $4^{\rm d}.23$ Cepheid AH Vel (Gieren 1980). Furthermore, first harmonic pulsators may be intrinsically bluer than fundamental mode pulsators of the same period (Stobie and Balona 1979).

In Figure 5, the position of the present Cepheids (except V350 Sgr) in the $\log L - \log T_e$ diagram is plotted with respect to the blue edges for pulsation in the fundamental and first harmonic modes given by Iben and Tuggle (1972) for 5 $M_\odot$ (which corresponds approximately to the masses of the present Cepheids) and $Y = 0.28$, $Z = 0.02$. Two luminosities are plotted for each Cepheid: its $L_{\rm Wes}$ and $L_{P\text{-}L\text{-}C}$ value (see Table 3). The effective temperatures are those given by Pel (1978).

It is seen from Figure 5 that the only two conceivable cases of overtone pulsators are AZ Cen and AG Cru. Both are known to have blue companions (see Paper III) which could have produced an overestimate of their effective temperatures, moving them closer to the blue edge for fundamental pulsation. For both stars, the present maximum likelihood radii are below the mean $\log R_0 - \log P$ relation, and the Wesselink radius masses are consequently smaller than the theoretical masses, thus providing no additional evidence for first harmonic pulsation. Amplitude and shape of light curve are very different for the two stars: whereas AZ Cen has the low amplitude symmetrical light curve expected from a first harmonic pulsator, AG Cru has a very asymmetric, large amplitude light curve for its short period. Whereas we can therefore probably discard the possibility that AG Cru is an overtone pulsator, the case is not so clear for AZ Cen. If one uses the mean radius of 39.7 $R_\odot$ obtained for AZ Cen by Stobie and Balona (1979), this star indeed becomes quite a serious candidate for being an overtone pulsator. It would be worthwhile to obtain another good Wesselink radius of AZ Cen to better understand its pulsation behavior.

From the radius and mass results of this work, there is another outstanding Cepheid, V482 Sco. As already discussed in §§ II and III, the assumption that this star pulsates in the first harmonic mode would make its radius and mass values more easily understandable. It is

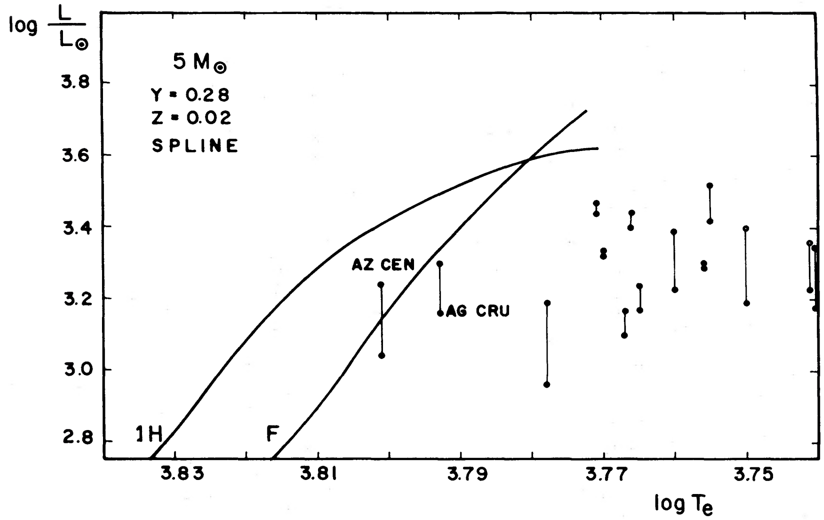


FIG. 5.—Location of the present Cepheids with respect to blue edges for pulsation in the fundamental and first harmonic modes. Stellar mass is 5 $M_\odot$ and composition is $(Y, Z) = (0.28, 0.02)$; adapted from Iben and Tuggle 1972. *Open circles:* PLC luminosities; *filled circles:* Wesselink luminosities.

important to note that the present radius value of 50.4 $R_\odot$ is in very good agreement with the other two available determinations of Thompson (1975) and Sollazzo *et al.* (1981) which have obtained 49 and 49.5 $R_\odot$, respectively. Furthermore, V482 Sco is one of the best-observed Cepheids of the present program. The position in the log $L$ − log $T_e$ plane, however, does not indicate the possibility of first harmonic pulsation, and the shape of its light curve does not favor first harmonic pulsation, either (although it is more symmetric than the light curve of AG Cru). Thus V482 Sco cannot be considered as a strong case for first harmonic pulsation and is more probably a fundamental mode pulsator.

There are two more stars among the present sample with rather symmetric, low-amplitude light curves, R TrA and SS Sct. For both stars, neither radius and mass values nor their location with respect to blue edges indicate the possibility of first overtone pulsation. Thus, it appears that the shape of the light curve alone is not a strong indicator of a Cepheid's pulsation mode.

In summary, we find that the only star of the present sample which might be an overtone pulsator is AZ Cen. This confirms previous results of numerous workers that pulsation in a pure first harmonic mode must be very rare among galactic classical Cepheids.

## V. CONCLUSIONS

The results of the present work show that we are probably close to obtaining agreement between observed and theoretical constants of the log $R_0$ − log $P$ relationship, whose slope seems to be close to 0.7. Using Baade-Wesselink radii obtained under ideal conditions, we can expect to be able to construct a log $R_0$ − log $P$ relation which yields radii with an $\sim$10% accuracy.

The results of § III show that theoretical and pulsation masses are basically in agreement now, with a scatter which is significantly lower than that found by Cox (1979). However, there might be a discrepancy toward longer periods in the sense that the pulsation masses become increasingly smaller than theoretical masses. This was not indicated in the Cox (1979) results. High quality data for longer period Cepheids are needed to clarify the question.

The discrepancy between Wesselink radius masses and theoretical masses persists, the $M_{\rm Wes}$ being $\sim$15–20% smaller than the theoretical masses. The discrepancy persists for stars with $P < 10^{\rm d}$ even if inhomogeneous model $Q_0$ values are used to calculate the $M_{\rm Th}$; the $M_{\rm Wes}$ is still $\sim$15% too small. The Cox (1979) results had indicated that in this case the $M_{\rm Wes}$ and $M_{\rm Th}$ might agree. Another important result is the fact that the scatter of the present Wesselink radius masses is found to be much smaller than that indicated in Cox (1979). We interpret this as evidence for the high reliability of the present radius determinations.

With respect to the helium enriched surface layers of Cepheids proposed by Cox *et al.* (1977), the present results do not suggest a strong necessity to introduce such a chemical inhomogeneity in the models. While the Wesselink radius mass discrepancy is slightly alleviated by the introduction of the inhomogeneous models, the agreement between theoretical and pulsation masses seems to be slightly better if the homogeneous models are used.

For only one star of the present sample, AZ Cen, there is reasonable evidence that it might be an overtone pulsator.

Among other things, future observational work should provide more reliable Wesselink radii for longer period Cepheids, based upon simultaneous accurate photometry and radial velocities, in order to extend the present investigation to the full period range of galactic Cepheids.

As long as we do not possess very reliable data, it will be difficult for theoreticians to decide how agreement can be brought about between the various discrepant results produced by stellar evolution and pulsation calculations.

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REFERENCES

Allen, C. W. 1963, *Astrophysical Quantities* (2d ed.; London: Athlone).
Balona, L. A. 1977, *M.N.R.A.S.*, **178**, 231.
Becker, S. A., Iben, I., J., and Tuggle, R. S. 1977, *Ap. J.*, **218**, 633.
Breger, M. 1967, *M.N.R.A.S.*, **136**, 61.
Caccin, B., Onnembo, A., Russo, G., and Sollazzo, C. 1981, *Astr. Ap.*, **97**, 104.
Christy, R. F. 1966, *Ap. J.*, **144**, 108.
Cogan, B. C. 1978, *Ap. J.*, **221**, 635.
Connolly, L. P. 1980, *Pub. A.S.P.*, **92**, 165.
Cox, A. N. 1979, *Ap. J.*, **229**, 212.
———. 1980, *Ann. Rev. Astr. Ap.*, **18**, 15.
Cox, A. N., Deupree, R. G., King, D. S., and Hodson, S. W. 1977, *Ap. J.* (*Letters*), **214**, L127.
Dean, J. F., Warren, P. R., and Cousins, A. W. J. 1978, *M.N.R.A.S.*, **183**, 569.
Evans, N. R. 1976, *Ap. J.*, **209**, 135.
———. 1980, in *Proc. Conference on Current Problems in Stellar Pulsation Instabilities*, ed. W. M. Sparks (Greenbelt, Md: NASA GSFC), p. 237.
Faulkner, D. J. 1977, *Ap. J.*, **218**, 209.
Fernie, J. D. 1968, *Ap. J.*, **151**, 197.
Fernie, J. D., and Hube, J. O. 1967, *Pub. A.S.P.*, **79**, 95.
Gieren, W. P. 1976, *Astr. Ap.*, **47**, 211.
Gieren, W. P. 1980, *Astr. Ap. Suppl.*, **39**, 153.
———. 1981*a*, *Ap. J. Suppl.*, **46**, 287 (Paper I).
———. 1981*b*, *Ap. J. Suppl.*, **47**, 315 (Paper II).
———. 1982, *Ap. J. Suppl.*, **49**, 1 (Paper III).
Hanson, R. B. 1977, in *IAU Symposium 80, The H-R Diagram*, ed. A. G. D. Philip and D. S. Hayes (Dordrecht: Reidel), p. 100.
Iben, I., Jr., and Tuggle, R. S. 1972, *Ap. J.*, **178**, 441.
———. 1975, *Ap. J.*, **197**, 39.
Kurucz, R. L. 1975, in *Multicolor Photometry and the H-R Diagram, ed. A. G. D. Philip and D. S. Hayes, Dudley Obs. Rept.*, **9**, 271.
Parsons, S. B. 1972, *Ap. J.*, **174**, 57.
Parsons, S. B., and Bouw, G. D. 1971, *M.N.R.A.S.*, **152**, 133.
Pel, J. W. 1976, *Astr. Ap. Suppl.*, **24**, 413.
———. 1978, *Astr. Ap.*, **62**, 75.
Sandage, A., and Tammann, G. A. 1969, *Ap. J.*, **157**, 683.
Sollazzo, C., Russo, G., Onnembo, A., and Caccin, B. 1981, *Astr. Ap.*, **99**, 66.
Stibbs, D. W. N. 1955, *M.N.R.A.S.*, **115**, 363.
Stobie, R. S. 1969, *M.N.R.A.S.*, **144**, 511.
Stobie, R. S., and Balona, L. A. 1979, *M.N.R.A.S.*, **189**, 641.
Thompson, R. J. 1975, *M.N.R.A.S.*, **172**, 455.
Wesselink, A. J. 1946, *Bull. Astr. Inst. Netherlands*, **10**, 91.
Woolley, R., and Carter, B. 1973, *M.N.R.A.S.*, **162**, 379.

WOLFGANG GIEREN: Departamento de Física, Universidad de los Andes, Apartado Aéreo 4976, Bogotá, D. E., Colombia