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ON THE STATISTICS OF ZZ CETI STARS[1]

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ABSTRACT

The results of a continuing survey for additional pulsating white dwarfs are summarized. In particular, the color distributions of stars investigated for variability are discussed. In terms of the Greenstein $G-R$ color index, among the hydrogen-rich white dwarfs *only* variable stars have thus far been found in the range $-0.45 \leq (G-R) \leq -0.38$. This color range corresponds to an effective temperature interval of 13,000 K $\gtrsim T_e \gtrsim$ 11,000 K. The implication of this observational result is that the vast majority, and probably all, of the hydrogen-rich white dwarfs evolve to become ZZ Ceti variables during an evolutionary phase of their lifetimes. Completeness arguments and a discussion of possible selection effects support this conclusion. The consequences of this result for theoretical studies of ZZ Ceti stars are briefly examined.

Subject headings: stars: pulsation — stars: white dwarfs

I. INTRODUCTION

The ZZ Ceti stars are a class of luminosity-variable DA white dwarfs, the variations of which are due to nonradial gravity-mode (*g*-mode) pulsations. Over the past 5 years, relatively rapid progress has been made in delineating and understanding these variables. Photometrically, they exhibit virtually all of the various kinds of behavior one expects of variable stars, in general. There is a correlation between amplitude and the complexity of the light curve for these stars such that low-amplitude variables, those with peak-to-peak variations of less than about 0.05 mag, usually have very simple, stable, sinusoidal light curves. In fact, two small-amplitude ZZ Ceti stars, R548 and G117–B15A, are by far the most stable "astronomical clocks" in the sky with $|\dot{P}| < 10^{-12}$ and 10^{-13} for these two stars, respectively (Stover *et al.* 1980; Kepler *et al.* 1982). For stars with amplitudes in the range from about 0.05 mag through about 0.30 mag, the maximum amplitude of variation thus far observed for a ZZ Ceti variable, nonlinear effects become increasingly important. The first effect to appear with increasing amplitude is that the light curve become increasingly nonsinusoidal and the power spectrum of the light curve shows harmonics of the primary pulsation frequency (cf. McGraw 1980). At somewhat higher amplitude, nonlinear coupling to additional modes may occur.

Because the periods of these variables are short, ranging from about 100 s to 1200 s, many cycles of a variable may be observed in one night. These qualities, the clear demonstration of *all* of the expected linear and nonlinear effects in a pulsating star and the ability to be intensively observed, make the ZZ Ceti stars the best "practical laboratory" of pulsating stars in the sky, against which we may test our theories of pulsation, in general. For example, Fontaine *et al.* (1980) have recently observed a monotonic growth in amplitude of a pulsation mode of GD 385 leading to an estimate of the growth rate of about 10^{-6}. This result is in agreement with the theoretical computations of Dziembowski (1979) and, if confirmed, represents the first measurement of

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the complex eigenfrequency of a pulsation mode in a real star.

In addition to supplying an exquisite "laboratory in the sky" for the study of linear and nonlinear pulsations, there are yet other extremely useful aspects of the ZZ Ceti variables. For example, they may provide information on the rotational periods of white dwarfs, a quantity that is very difficult to estimate from line profile measurements because of the high pressure broadening present in the atmospheres of degenerate dwarfs. Rotational splitting, leading to characteristic equally spaced frequency peaks in the Fourier domain, has been invoked to explain the frequency structure of the light curve of BPM 30551, G29–38, G207–9, and R548 (McGraw 1977*a*). The inferred rotation periods, ranging from about half an hour to more than a day, are quite consistent with the few upper limits we have from line profile measurements (Greenstein and Peterson 1973; Greenstein *et al.* 1977) and from polarization measurements in magnetic white dwarfs (Angel, Borra, and Landstreet 1981).

Long-term observations of some of the more stable, small-amplitude ZZ Ceti stars should also provide us with a unique means of measuring the cooling time scales of white dwarfs (Robinson and Kepler 1980). Indeed, as a ZZ Ceti star evolves (cools), its period structure must change slightly in response to the changing physical conditions in its envelope. These differences in period structure could be detectable over periods of 10–20 years.

Recent advances in the theoretical understanding of ZZ Ceti stars lead us to believe that the study of these objects will also be extremely useful in probing the interiors of white dwarfs. Winget, Van Horn, and Hansen (1981) have recently shown that layered models of white dwarfs may act like mechanical filters. Modes that resonate with the thickness of the layer of hydrogen (and to a lesser extent that of helium) appear to require considerably less energy to be excited than other modes. Consequently, these might be expected to be preferentially driven in real stars. It is possible that detailed comparisons of the period structures of ZZ Ceti stars with future theoretical computations will allow us to derive estimates of the hydrogen content in white dwarfs.

Extensive observations have shown that the ZZ Ceti phenomenon is related to an evolutionary phase of cooling DA white dwarfs (cf. McGraw 1979). As a star cools, it eventually enters a region delimited by 13,000 K $\gtrsim T_e \gtrsim$ 11,000 K in which variability occurs. While no satisfactory theoretical understanding of this phenomenon has yet appeared in the literature, past observations have already provided significant insights into the problem. In particular, it is now well established that the presence of hydrogen in the atmospheres of ZZ Ceti stars is a necessary condition for variability. Recent observations have considerably added to our evolutionary picture of ZZ Ceti stars and have forced us into a somewhat revised understanding of these objects. In a recent review, McGraw (1980) has briefly summarized the results of our ongoing survey program for additional ZZ Ceti stars. In the present paper, we wish to examine in more detail the implications of the results thus far obtained in our survey.

Recent observational research on ZZ Ceti variables has primarily concentrated on three related topics: (1) the discovery of new variables, (2) a determination of the fraction of DA white dwarfs which become ZZ Ceti variables as they evolve through the instability region, and (3) the investigations of the period stability of low-amplitude variables. This last aspect has been done primarily by Robinson, Nather, and their collaborators at the University of Texas while we have embarked on a restricted program that addresses itself to the first two questions.

Our candidate stars for variability were selected on the basis of their DA spectral type as well as their Greenstein $G-R$ color index derived from multichannel-scanner observations. Some colors were taken from the literature (Greenstein 1976) while others were kindly communicated to us by Dr. J. L. Greenstein. Because the multichannel colors are more accurate and homogeneous than the Johnson broad-band colors on which previous surveys were generally based, we have had a high success rate at discovering new ZZ Ceti stars. In fact, the discovery rate has been 100% in the range $-0.45 \leq (G-R) \leq -0.38$ while no ZZ Ceti variable has been found outside this range (cf. Tables 1 and 2). Our survey has thus far led to the discovery of GD 385 (Fontaine *et al.* 1980), a star showing nonlinear amplitude-limiting effects on time scales of a month, of G191–16 and G185–32 (McGraw *et al.* 1981), respectively large- and small-amplitude variables that tend to

TABLE 1

COLORS OF KNOWN ZZ CETI STARS

Star	V	$B-V$	$U-B$	$G-R$
BPM 30551	15.42	+0.17	−0.50	...
R548	14.10	+0.20	−0.54	−0.43
BPM 311594 ...	15.03	+0.21	−0.66	...
HL Tau-76	15.20	+0.20	−0.50	−0.39
G38–29	15.63	+0.16	−0.53	−0.42
G191–16	15.98	...	...	−0.44
GD 99	14.55	+0.19	−0.59	...
G117–B15A ...	15.52	+0.20	−0.56	−0.45
GD 154	15.33	+0.18	−0.59	−0.43
L19–2	13.75	+0.25	−0.53	...
R808	14.36	+0.17	−0.56	−0.38
G226–29	12.24	+0.16	−0.62	−0.43
G207–9	14.64	+0.17	−0.60	...
G185–32	13.00	+0.17	−0.57	−0.42
GD 385	15.13	+0.19	−0.68	−0.43
G29–38	13.10	+0.20	−0.65	−0.43

TABLE 2

COLORS OF NONVARIABLE DA STARS

Star	$G-R$	$B-V$	Star	$G-R$	$B-V$
G130–49	−0.03	+0.25	LTT 7658	−0.29	+0.07
G1–7	−0.35	+0.23	G186–31......	−0.65	−0.07
G271–115 ...	+0.13	+0.34	G231–43......	−0.52	+0.15
F24..........	−0.33	−0.23	G261–43......	−0.62	+0.01
HZ 4	−0.58	+0.15	G130–5.......	−0.46	+0.15
HZ 2	−0.69	−0.05	GD 140.......	−0.70	−0.06
40 Eri B......	−0.57	+0.03	G138–56......	+0.04	+0.35
G87–7	−0.74	−0.08	G169–34......	−0.18	+0.24
G116–52	−0.63	+0.07	G181–B5B ...	−0.50	+0.17
G61–17......	−0.30	+0.22	G172–4.......	−0.21	+0.16
G238–44	−0.70	−0.09	GD 25	−0.15	+0.23
G66–32......	−0.48	+0.02	G67–23.......	−0.30	+0.20
G201–39	−0.18	+0.23	HZ 14	−0.80	−0.15
G141–54	−0.31	+0.20	G175–46......	−0.09	+0.24

act as counterexamples to the amplitude-complexity correlation noted above. It has also led to the discovery of G226–29 (McGraw *et al.* 1982), a remarkable new variable in the sense that it has the smallest amplitude ($\Delta m \lesssim 0.005$) and shortest dominant period ($P \sim 110$ s) yet seen in a ZZ Ceti star. In addition, our survey has led to the realization that most, if not all, DA white dwarfs become ZZ Ceti variables as they evolve through the effective temperature range 13,000 K $\gtrsim T_e \gtrsim$ 11,000 K. This is discussed in detail in § II, where we reconsider the color distribution of ZZ Ceti stars in light of the results we have obtained thus far in our survey. We then argue in § III that these results are statistically significant. Finally, in § IV, we briefly examine the implications of our suggestion on theoretical investigations of evolving white dwarfs in general, and of ZZ Ceti stars in particular.

II. THE COLOR DISTRIBUTION OF ZZ CETI STARS

To date there are 16 firmly established ZZ Ceti variables. These are listed in Table 1 along with their V magnitudes, Johnson $B-V$ and $U-B$ indices, as well as the Greenstein $G-R$ color index when available. (References to the variability discovery papers are given by McGraw 1980. For original references to color data, see the white dwarf catalog of McCook and Sion 1977.) Variability is found in narrow color ranges, $0.16 \leq (B-V) \leq 0.25$ and $-0.45 \leq (G-R) \leq -0.38$. Both of these color ranges correspond to a small effective temperature interval because both of these indices are primarily temperature indicators in the color range of ZZ Ceti variables. We now wish to discuss the distribution of the variables and known constant stars in terms of their colors. For completeness, Table 2 lists the DA stars having $G-R$ colors which, thus far in our continuing search for variability in white dwarfs, we have tested for variability and found constant. For each star the published $G-R$ and $B-V$ colors are given (see McCook and Sion 1977). The upper limit to the amplitude of variability for these stars is $\Delta m \approx 0.005$ (mag) in the period range from 20 s to about 2000 s.

Figure 1 shows the Johnson two-color diagram for 125 white dwarfs which have been investigated for variability. This is an updated version of a similar diagram presented by Robinson (1979). The 15 ZZ Ceti stars for which colors are available are shown as filled triangles. Nonvariable DA white dwarfs are plotted as open circles; the helium-rich DB white dwarfs, metallic line DF stars, carbon-rich C_2 and λ4670 stars, and hot DO white dwarfs are represented by the symbols B, F, 2, λ, and ∅, respectively. White dwarfs of unknown spectral type are designated by U. Typical uncertainties are also shown. The solid line is the locus of the theoretical DA sequence for $\log g = 8$, taken from Terashita and Matsushima (1969). Colors for models at T_e (10^3 K) = 7, 8, 12, 15, 20, and 25 are plotted as crosses.

From these results, there can be no doubt that the ZZ Ceti phenomenon is related to the presence of hydrogen at the surface of a white dwarf (DA spectral type) and occurs in a relatively narrow range of effective temperature. Because a white dwarf cools at almost constant radius, it will closely follow a track parallel to the solid line from left to right in the two-color diagram if it has $\log g \approx 8$, a typical value. As it cools, a DA white dwarf will eventually enter the instability region. An important problem is to determine the probability that this star becomes pulsationally unstable in this region. In other words, we would like to know if *all* DA white dwarfs become ZZ Ceti stars. If they do, this means that pulsational instabilities are triggered by a mechanism that depends only upon the effective temperature. If they do not, this implies that there is at least one other parameter that can make a DA star either stable or

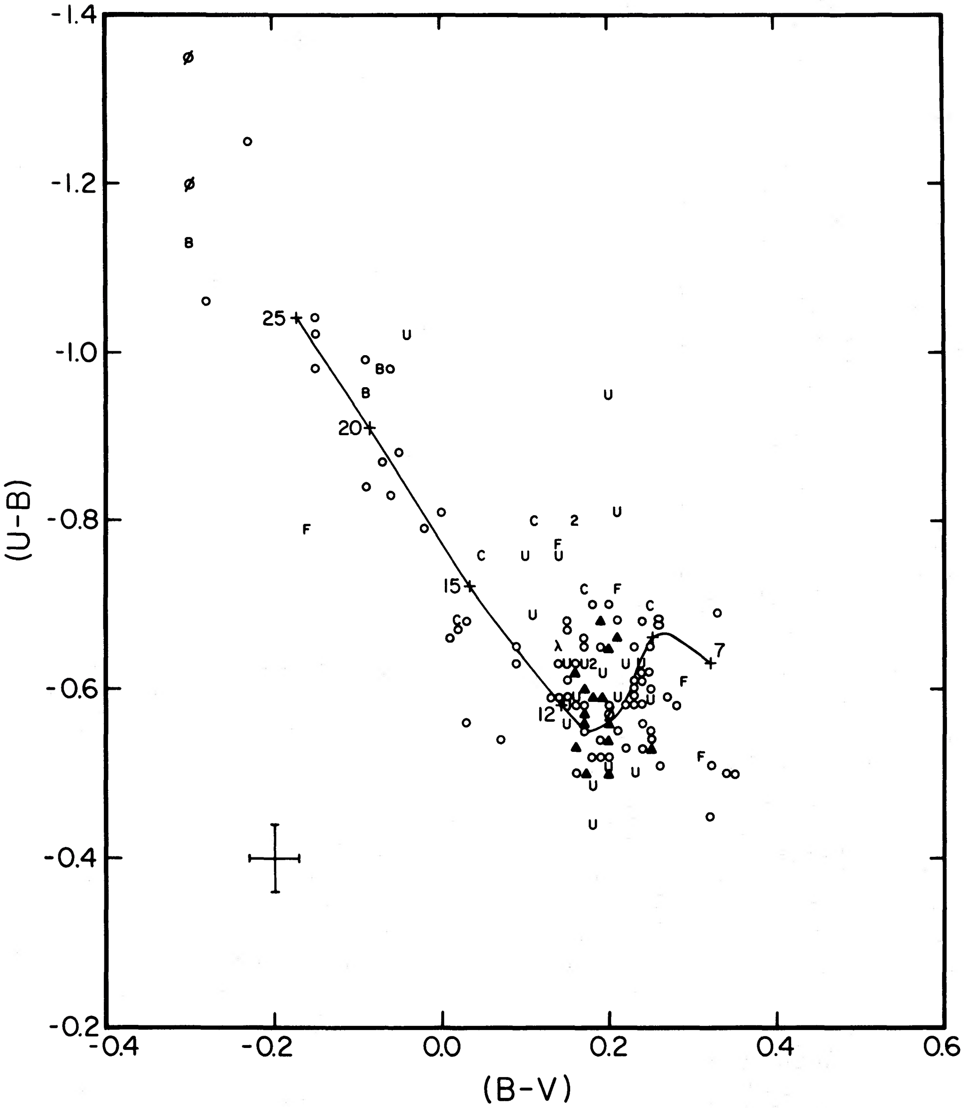

FIG. 1.—The Johnson two-color diagram for 125 white dwarfs investigated for variability. The ZZ Ceti stars are represented by filled triangles. Constant stars have spectral type DA(∘), DB(B), DF(F), C_2(2), λ4670(λ), DO(∅), and unknown (U). The solid line is the theoretical DA sequence of Terashita and Matsushima (1969) for log $g = 8.0$ models. Colors for models at $T_e(10^3\ \mathrm{K}) = 7$, 8, 12, 15, 20, and 25 are plotted as crosses. Typical uncertainties are illustrated.

unstable in the temperature range where ZZ Ceti stars are found. This additional parameter, however, would not be directly observable.

Without knowing specifically the exact nature of the mechanism responsible for pulsational instabilities in ZZ Ceti stars, it is still possible to answer the above question by making use of the numerous observations that have been accumulated thus far. We are interested in the distribution of all DA stars which have been searched for variability in terms of the effective temperature. For a white dwarf of given mass, the temperature axis of an H-R diagram, in which we shall trace its evolution, corresponds to a time axis since a white dwarf simply cools at very nearly constant radius as it evolves. In the *UBV* photometric system, $B-V$ is a measure of the temperature and, in the top panel of Figure 2, we therefore show the distributions of the constant stars (*continuous lines*) and the known variables (*hatched lines*) in terms of $B-V$ for all 97 DA's searched for variability and having Johnson's colors. An inspection of this histogram indicates that variables and nonvariables seem to coexist in the color range $0.16 \leq (B-V) \leq 0.25$ in the ratio of about 1 to 3. Taken at its face value, this result implies that roughly 25% of evolving

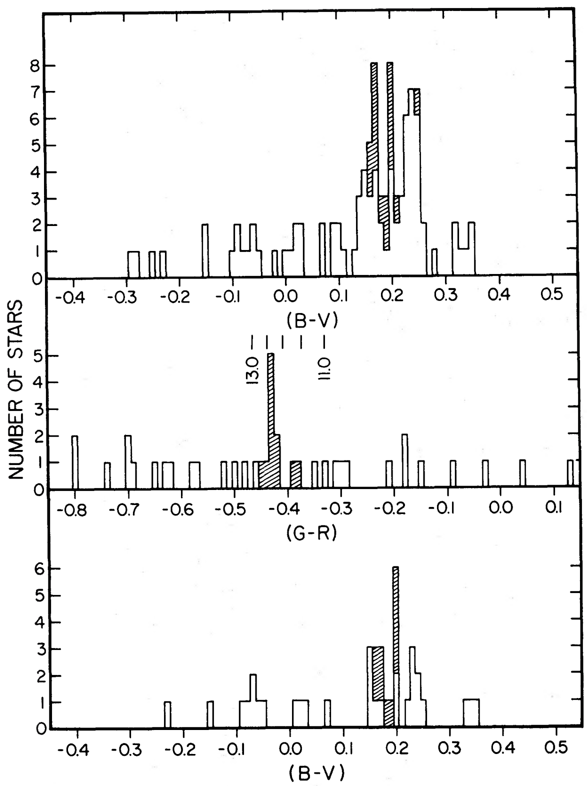

FIG. 2.—Color distributions of DA white dwarfs investigated for variability. Constant stars are represented by continuous lines; the contributions of the variables, by shaded areas. *Top panel*, distribution of 97 DA's having Johnson colors in terms of the $B-V$ index. *Middle panel*, distribution of 39 DA's having Greenstein's multichannel colors in terms of the $G-R$ index. *Bottom panel*, distribution of the same stars found in the middle panel in terms of the Johnson $B-V$ color index.

DA white dwarfs eventually become ZZ Ceti variables. If true, this means that temperature is a necessary but *not* sufficient condition for variability in white dwarfs.

There are reasons, however, to suspect the reliability of the above results. Indeed, it has been known for some time (Weidemann 1971) that the *UBV* color system is not well suited to derive atmospheric parameters for white dwarfs because of significant contamination of the various bandpasses by the wide absorption lines. This is particularly true of DA white dwarfs in the temperature region where ZZ Ceti stars are found because the hydrogen Balmer lines nearly reach their maximum strength there. Furthermore, the set of *UBV* observations has been collected from various sources in the literature and is therefore not homogeneous. Differences as large as 0.05 in the $B-V$ color indices for the same stars measured by different observers have been found.

The goal of our survey has been to avoid these difficulties by selecting candidate stars on the basis of a reliable, homogeneous, and accurate color system. Such a system is provided by Greenstein's (1976) multichannel color observations of white dwarfs. In particular, he has devised a set of color indices that are astrophysically meaningful for white dwarfs, of which $G-R$ is a good temperature indicator for DA dwarfs (Shipman 1979*a*). The middle panel of Figure 2 shows the distribution of all 39 DA stars with known $G-R$ colors which have been searched for variability (cf. Tables 1 and 2). In this histogram, *all* of the stars in the range $-0.45 \leq (G-R) \leq -0.38$ are variables. A temperature scale derived from Shipman's (1979*a*) calibration of Greenstein's colors is indicated above the region of variability. The tick marks corresponds to 500 K intervals. This calibration is from blanketed, pure hydrogen models with $\log g = 8.0$. The inferred temperature range for the variables is about 13,000 K $\gtrsim T_e \gtrsim$ 11,000 K. Except for one object, G191–16, which has no reliable $B-V$ color, the bottom panel shows the same stars found in the middle panel but distributed according to their $B-V$ colors. In terms of this temperature index, both variables and nonvariables again appear to coexist in the same temperature range. It is clear that the $G-R$ color is a better discriminant of variability than is the $B-V$ color.

We have tested the correlations between the $G-R$ color and the maximum pulsational amplitude and the period. In lieu of more complex procedures we have simply made histograms of $G-R$ color versus the maximum pulsational amplitude measured by eye from the light curves and $G-R$ versus a "mean" period derived by eye estimate from all available power spectra of the light curves. Thus far we have found no correlation. These results are, however, subject to change as it is extremely difficult to parameterize the complex amplitude and period structure of ZZ Ceti stars in a physically meaningful way. Should this be done, a correlation may emerge.

From our analysis, utilizing a more precise temperature indicator for white dwarfs, with lower observational uncertainties, it appears that the vast majority and, *quite possibly all*, of the DA white dwarfs become pulsationally unstable as they evolve through the temperature range from about 13,000 K to about 11,000 K. There is, however, no obvious correlation between the $G-R$ color and the pulsational properties of individual stars.

III. SELECTION EFFECTS

The fraction of DA white dwarfs which become ZZ Ceti stars while cooling through the color range $-0.45 \leq (G-R) \leq -0.38$ is important for determining whether physical parameters other than temperature are significant in determining the pulsational stability of DA stars. The observed fraction of stars which pulsate while passing through this color range can be affected by a number of selection effects. The net result of these factors is to make the observationally determined percentage of variable stars in the temperature range a *lower limit* to the true number of variables in the range.

The first effect is due to the fact that the $G-R$ colors have finite uncertainties. Thus, given definite color boundaries to the temperature range in which variability occurs, nonvariable stars with true colors within a few sigma of these boundaries may, by virtue of their observational uncertainties, appear to occur in the region of variability. We may estimate the magnitude of this potential effect by noting that the current observed width of the instability zone in $G-R$ is only 0.07 mag and that the quoted uncertainty (1 σ) on $G-R$ is 0.03 mag. Thus, should the instability region be bounded by definite colors, we could expect to find roughly half of the stars in the instability region to be constant by virtue of their being artificially shifted into the instability region by observational uncertainties, if our estimate of 0.07 mag for the intrinsic width of the instability region is approximately correct. Similarly, we could expect to find about as many variables outside the intrinsic instability region under the same assumptions. The only way of minimizing this effect would be to reobserve the colors of the stars in and around the region of variability to reduce their observational uncertainties. We can only suppose that we do not yet see this effect among the ZZ Ceti stars either because we have been inordinately fortunate or, more probably, because the relative *internal* uncertainties in $G-R$ are much lower than 0.03 mag. Our continuing search for new variables will help decide the importance of this effect.

The second selection effect which causes us to underestimate the number of variables in the instability region is our limited ability to detect pulsators with exceedingly low amplitude. The effect is pointedly driven home by the recent discovery of G226–29 as a ZZ Ceti variable (McGraw *et al.* 1982). This star, at $V=12.2$, is one of the brightest white dwarfs in the sky. On the basis of its $G-R$ color of -0.43, we predicted it to be a variable. The star was observed for variability several times with telescopes as large as 1.6 m, but variability could not be definitely confirmed. Only when this star was observed with the Multiple Mirror Telescope, utilizing both the light-gathering power of the telescope and the fact that sky noise (scintillation) is effectively averaged-out over the large mirror-to-mirror spacings, was variability definitely detected. The maximum amplitude of the pulsations of this star is 0.005 mag, and there may well be as of yet undetected ZZ Ceti stars with still smaller amplitudes. It is obvious that for fainter stars and/or smaller telescopes, pulsations of this small amplitude will be difficult or impossible to detect. This effect will again lead us to underestimate the number of variables in the ZZ Ceti instability region.

Coupled to small amplitude is another selection effect. It is currently believed that the small-amplitude variables closely approximate linear pulsators. For a linear nonradial *g*-mode pulsation, the instantaneous surface brightness distribution is described by a spherical harmonic, Y_l^m. Here l is the total number of node lines on the surface of the star, and m is the number of node lines passing through a symmetry axis imposed, for example, by the rotation of the star. The degeneracy parameter m can take on the $2l+1$ values $l \geq m \geq -l$. (For a graphical representation of some of these modes, see Van Horn 1980.) We are not yet able to identify individual pulsation modes in ZZ Ceti stars; thus we do not know if there is a preference for any particular combination of l and m. There are modes represented by certain combinations of l and m which, when viewed from the proper aspect, give rise to *no* luminosity variations. This is simply because the surface brightness distribution averaged over the visible hemisphere of the star is constant for all times. Basically, there are as many "bright" segments as "dark" segments visible at one time. This occurs, for example, for a star viewed equator-on, any time the sum $l+m$ is odd. Thus, naively, one could expect to be unable to detect about half of the low-amplitude, *linearly* pulsating ZZ Ceti stars. Clearly, the fact that we do detect a very high percentage of variables in the instability region implies that ZZ Ceti stars *do* have preferred pulsation modes and they are those which lead to luminosity variations. Nonetheless, this selection effect must be kept in mind when considering the lower limit to the number of DA white dwarfs which become ZZ Ceti variables.

The last selection effect we shall discuss here concerns the fact that all ZZ Ceti variables found to date are multiply periodic. This leads to "beats" in the light curve which, for many minutes, can cause a drastic reduction or even complete cancellation of the observed amplitude of the luminosity variations. An example of this effect is provided by the light curve of the star BPM 30551 (Hesser, Lasker, and Neupert 1976; McGraw 1977*b*) which for approximately 40 minutes can show virtually no photometric activity. When the multiple periods interfere constructively, however, the amplitude can exceed 0.25 mag. Forty minutes to 1 hour is, historically, about the length of an observation when one is searching for new ZZ Ceti variables. It is thus potentially possible to miss a variable due to this effect. Repeated observations or longer observing runs must be obtained to minimize this effect.

We have now discussed four selection effects, all of which act in a direction which makes the percentage of variables in the instability region a lower limit. Three of these effects can be minimized, at least in principle, by further, possibly extensive, observations. The aspect effect can never totally be eliminated, however. For this reason, the ratio of variable to constant stars in the ZZ Ceti instability region will always be a lower limit. With this in mind, we examine the observed distribution of white dwarfs.

TABLE 3

Observed and Expected Color Distributions of White Dwarfs

Region	Color Range	Count[a]	Expected[a]	Count[b]	Expected[b]
A	$(G-R) < -0.45$	61	67	46	46
B	$-0.45 \leq (G-R) < -0.38$	15	13	8	9
C	$-0.38 \leq (G-R) < -0.20$	22	24	16	16
D	$-0.20 \leq (G-R) < +0.20$	32	26	19	18
Total		130	130	89	89

[a]All white dwarfs.
[b]DA white dwarfs only.

In the catalog of McCook and Sion (1977) there are 130 white dwarfs with $V \leq 16.0$ and having $G-R$ colors, 89 of which are DA's. As can be seen in Table 3, the DA and non–DA stars show a nearly identical distribution with respect to $G-R$. The observed distribution is illustrated by the histogram in Figure 3 where the stars are binned in intervals of $|\Delta(G-R)| = 0.02$. If the white dwarf birthrate has been constant over the last 2.5 billion years, then the number of white dwarfs in a temperature range should depend simply on the time spent cooling through that range. For an individual white dwarf, the cooling time depends on its mass (massive white dwarfs cool slower than low-mass ones before crystallization sets in). This mass dependence cancels out, however, when calculating the relative number of stars in two color regions.

We have used the model atmospheres of Shipman (1979*a*) to relate the $G-R$ color to the effective temperature and the flux in the visible. A temperature-age relation was obtained from the 1 $M_{\odot}$, pure carbon white dwarf evolutionary calculations of Lamb and Van Horn (1975). Using this, we derive an expected distribution (normalized to the total number of observed stars), and can compare it to the observed distribution.

The dashed line in Figure 3 shows this theoretical distribution. By grouping stars into four color ranges (cf. Table 3), we expect nearly twice as many white dwarfs with $G-R < -0.45$ than in the color range $-0.45 \leq (G-R) < -0.20$. The higher surface brightness of the hotter stars makes them visible at larger distances. We also expect to see nearly five times as many stars with $G-R < -0.45$ as in the narrow ZZ Ceti range $-0.45 \leq (G-R) < -0.38$. Table 3 indicates that the observed distribution agrees well with the expected one.

Of the stars tested for variability, the middle panel of Figure 2 shows 15 with $G-R < -0.45$ and six in the color range $-0.38 < (G-R) \leq -0.20$. Again this agrees well with the relative numbers expected. The 11 stars shown in the color range $-0.45 \leq (G-R) \leq -0.38$ far exceeds the three stars expected, and the excess is of

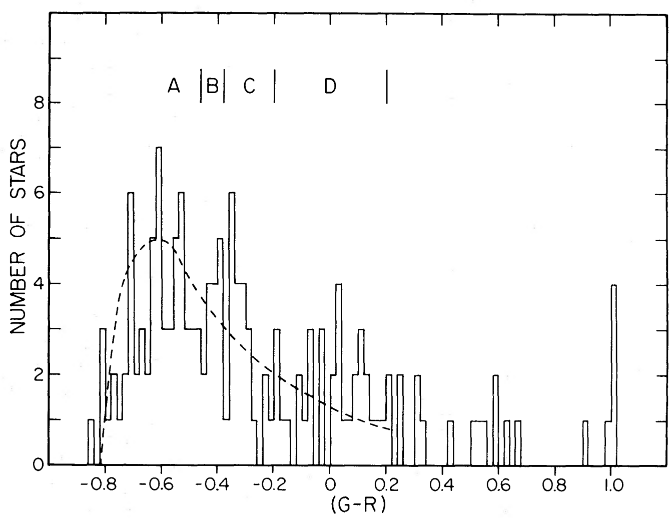

Fig. 3.—The histogram shows the distribution of all white dwarfs having $G-R$ colors as given in the catalog of McCook and Sion (1977). The dashed line is the theoretically expected distribution. Stars are grouped in the following color ranges: $G-R < -0.45$ (A); $-0.45 \leq G-R < -0.38$ (B); $-0.38 \leq G-R < -0.20$ (C); and $-0.20 \leq G-R < +0.20$ (D).

course due to preferentially choosing stars in that color range to test for variability.

That all 11 stars in the ZZ Ceti color band are variable implies that more than 94% of all DA white dwarfs become ZZ Ceti stars (at the 50% confidence level). That none of the 15 hotter stars is variable moreover suggests that fewer than 5% are variable at this point. This is strong evidence that most, if not all, DA white dwarfs become ZZ Ceti variables while cooling through the temperature range 13,000 K $\gtrsim T_e \gtrsim$ 11,000 K (1–5$\times 10^8$ years).

A point of further interest is associated with the effects of pulsations on the energy transport. Strong pulsations could enhance the energy transport and decrease the cooling time. This would lead to a deficiency of stars in the color range of ZZ Ceti stars. The number of white dwarfs with $G-R$ colors is, however, not presently sufficient to place interesting limits on this effect, though it is clear that there is no gross modification of the cooling time resulting from the pulsations.

IV. DISCUSSION

Our suggestion that all DA stars (i.e., some 75% of all known white dwarfs) become pulsationally unstable has important consequences on theoretical studies of white dwarf pulsations. We briefly examine the impact of this hypothesis.

In addition to effective temperature, there are at least two other obvious parameters that affect the pulsation properties of a ZZ Ceti model: the total mass of the star and the thickness of the hydrogen (and helium) layer in a stratified model. It has been claimed for some time (cf. Koester, Schulz, and Weidemann 1979, and references therein) that all DA white dwarfs have masses in a very narrow range centered on $0.58 \pm 0.10\ M_\odot$. If true (see Shipman and Sass 1980 for an opposite view), the parameter of mass is essentially fixed since the pulsation properties of models having only slightly different masses may not be widely different. On the other hand, the amount of hydrogen a DA white dwarf may have is largely uncertain. We still do not know if hydrogen (which, as we have seen, is a prerequisite for ZZ Ceti variability) is left over from previous evolution, has been added through accretion episodes, or both. There are theoretical considerations (Shaviv and Kovetz 1973) that indicate that only a quantity of hydrogen less than about 10^{-4} of the total mass of the star can have survived the planetary nebula phase from which most white dwarfs are believed to originate (Weidemann 1975). In addition, hydrogen burning in the post–main-sequence phases of stellar evolution does not normally cease until $M_{\rm H} \lesssim 10^{-5}\ M_\odot$. If the amount of hydrogen in a white dwarf very much exceeds this, it is well known that thermal runaway will occur. In such a case, the subsequent rapid evolution of the star is away from the white dwarf stage (Starrfield *et al.* 1976).

The question of obtaining a lower limit for the amount of hydrogen a DA star can have is more delicate. From optical observations alone, it is not even possible to determine a reliable atmospheric composition for such a star. This is because, in the visible, the opacity of hydrogen is much larger than that of helium. In fact, in ZZ Ceti atmospheres, one atom of hydrogen is as opaque as about 10^3 atoms of helium. One may thus expect that any atmospheric mixture that is less helium-rich than $n_{\rm He}/n_{\rm H} \sim 10^3$ will show a spectrum dominated by prominent hydrogen Balmer lines in the temperature range in which ZZ Ceti stars are found. Fortunately, however, EUV observations of a few hot DA's have been extremely useful in determining the atmospheric composition of these objects. They have shown that these stars have, in fact, atmospheres extremely rich in hydrogen (Auer and Shipman 1977). These authors and others (e.g., Shipman 1979*a*; Wesemael *et al.* 1980) are now even assuming a pure hydrogen composition in the outermost layers of these objects. This is quite consistent with the diffusion model of white dwarf envelopes (Fontaine and Michaud 1979; Alcock and Illarionov 1980; Vauclair, Vauclair, and Greenstein 1979) in which hydrogen, helium, and heavier elements physically separate under the influence of diffusion which leads to a stratified compositional structure early in the life of a white dwarf.

To interpret the EUV and soft X-ray emissions from the hot DA white dwarf HZ 43 in terms of Shipman's thermal model, the extent of the pure hydrogen surface layer must *at least* have a fractional mass $\sim 10^{-14}$ (Shipman 1979*b*; Heise and Huizenga 1980). Moreover, recent calculations by Arcoragi and Fontaine (1980) have shown that DA white dwarfs in general *must* have hydrogen fractional contents larger than 10^{-14} if the thin hydrogen layer is to survive against mixing due to turbulence generated in an underlying helium convection zone. In general, this theoretical lower limit can be expected to be even much larger.

In view of these considerations, it appears that the recent suggestion by Dziembowski and Koester (1981) that ZZ Ceti stars are stratified DA white dwarfs with a hydrogen fractional content of *less* than 2×10^{-13} is unlikely. In addition, observational results clearly rule out the possibility that *all* ZZ Ceti stars have such low hydrogen content *if* current mixing theories are correct. Indeed, if this were true, the vast majority, if not all, DA white dwarfs would mix below about $T_e \sim 10{,}000$ K as a result of the coalescence of a superficial hydrogen convection zone with a much more massive underlying helium convection zone (Koester 1976; Vauclair and Reisse 1977). That is, *no* DA white dwarfs should be seen below that temperature, which is clearly not the case as DA white dwarfs span the whole temperature range 4,000 K $\lesssim T_e \lesssim$ 70,000 K (Liebert 1980; Sion 1979). This conflicts with the requirement of Dziembowski and

Koester (1981) and suggests that alternatives to their He-zone driving mechanism should be investigated.

At present, the uncertainties of pre–white-dwarf evolution and of accretion rates are so large that the fractional hydrogen mass contents of DA white dwarfs are uncertain by at least 10 orders of magnitude ($-14 \lesssim \log q(\mathrm{H}) \lesssim -4$). Any theoretical attempt at understanding ZZ Ceti stars should therefore include an exploration of the pulsational properties of models in which the parameter $\log q(\mathrm{H})$ is allowed to vary over many orders of magnitude. Such studies are currently in progress (Winget *et al.* 1982; Winget 1981), and nonradial modes have been found to be excited in ZZ Ceti star models containing realistic amounts of hydrogen [i.e., $\log q(\mathrm{H}) \gg -14$].

It is possible that theoretical pulsation analyses allow us to narrow down the range of $\log q(\mathrm{H})$ that can be expected in DA white dwarfs. Indeed, the results of our observations imply that *all* evolutionary models entering the ZZ Ceti temperature range should become unstable. This, obviously, imposes constraints on the values of the model parameters such as $\log q(\mathrm{H})$, for example. Moreover, any successful theory of the ZZ Ceti stars must propose a driving mechanism that is very sensitive to the effective temperature, and that operates only between approximately 13,000 K and 11,000 K so as to be consistent with the observed blue and red edges of the ZZ Ceti instability strip. Clearly, the results of our observing program will have far-reaching consequences on the understanding of the evolution of white dwarfs.

Greenstein (1982) has reanalyzed his multichannel scanner data for DA stars in and near the ZZ Ceti instability zone.[3] While his results are in essential agreement with ours, he finds the instability zone occurs approximately 1000 K cooler than our results indicate. Part of this disagreement results from his use of a different absolute calibration of the scanner data. We agree more closely on the width (in temperature) of the instability zone, $\Delta T \approx 1400$ K. We point out that this ΔT corresponds to the change in temperature over which flux transport in DA stars changes from about 90% radiative to 90% convective (Fontaine and Van Horn 1976). This may be an important clue to the understanding of the driving mechanism in the ZZ Ceti stars in that, by analogy with Cepheids, for example, a partial ionization zone potentially capable of driving pulsations must precede the rapidly growing convection zone. The passage of this partial ionization zone through a critical layer in the star's envelope will create a "blue edge" to the instability region. The radiative portion of the partial ionization zone will be destroyed when the layer becomes completely convective, leading to a "red edge" for the instability region. Our suggestion is, then, that because of observational uncertainties and uncertainties in the absolute calibration of temperature, the observed temperature *range* of ZZ Ceti variability is probably a more concrete clue to the nature of the instability mechanism than the absolute value of the temperature of the instability zone.

[3]While this paper was being prepared, we learned that Dr. J. L. Greenstein has also independently studied the statistics of ZZ Ceti stars. His results appear in a complementary paper.

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