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YELLOW SUPERGIANT REDDENINGS FROM *BVRI* DATA

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ABSTRACT

Using the Johnson *BVRI* system of photometry, it is shown that the expression

$$E_{B-V} = -0.255 + 1.727(R-I) - 0.475(B-V)$$

will reproduce the color excesses given by Parsons and Bell for stable F and G supergiants with an rms deviation of less than 0.03 mag.

The same expression is applicable to classical Cepheids at mean light, but if it is applied to Cepheid observations at some particular phase ϕ, a correction of the form

$$f = 1.2A(0.045 - 0.133\phi - 0.130\phi^2 + 0.263\phi^3)\,,$$

where A is the amplitude of the Cepheid's visual lightcurve, must be added. For Cepheids at mean light the rms deviation of the present results from those Parsons and Bell is again 0.03 mag, rising to 0.04 mag when observations at individual phases are used.

Color excesses so determined are listed for 41 Cepheids and 17 stable supergiants.

Subject headings: photometry — stars: Cepheids — stars: supergiants

I. INTRODUCTION

Due mainly to the work of Pel (1978) and Dean, Warren, and Cousins (1978), a large number of reliable color excesses have become available for southern hemisphere Cepheids. The situation for northern Cepheids is less satisfactory, despite an extensive literature on the subject, because (in part at least) discussions have been based on inhomogeneous and often inferior bodies of data. However, alleviation of this problem in the north has already begun: Feltz and McNamara (1980), for example, have produced a large collection of *uvby* material for Cepheids and have used it to determine a homogeneous set of reddenings, while Parsons and Bell (1975) have applied spectrum synthesis with model atmospheres to Lick six-color data to produce another body of homogeneous reddenings.

Most northern Cepheid photometry, however, continues to be done in the Johnson system rather than in the four-color or six-color systems. Moreover, with the advent of extended red-sensitive photomultipliers, it is becoming common to extend the photometry to R and I as well as UBV. An excellent homogeneous body of such data has recently been given by Moffett and Barnes (1980). The addition of R and I to UBV is an important step forward in the determination of color excesses for Cepheids. There are several reasons for this, the most important of which is that whereas Cepheid reddenings determined from the $(U-B)$ versus $(B-V)$ diagram are weak because the reddening line so nearly parallels the intrinsic sequence, much better determinations are expected from the $(B-V)$ versus $(R-I)$ diagram because there the two lines are far from parallel. It is the purpose of this paper to explore the use of $(B-V)$ and $(R-I)$ to determine a Cepheid's reddening as well as the reddenings of Cepheid-like stars, i.e., F and G supergiants.

This paper does not attempt any *ab initio* determination of reddenings. Instead it is based purely on the reddening scale of Parsons and Bell (1975), because their work encompasses both Cepheids and stable supergiants, *BVRI* data are available for almost all their stars, and their reddening scale is known to be in accord with the widely accepted scales of Pel (1978) and Dean *et al.*

To avoid confusion I emphasize that all *RI* data discussed here are on the Johnson and not the Kron-Cousins system.

II. METHOD AND DATA

The method to be adopted is essentially the analog in the $(B-V)$, $(R-I)$-plane of the Q-method introduced by Johnson and Morgan (1953) for the UBV system. In this case we define Q to be

$$Q \equiv (R-I) - (E_{R-I}/E_{B-V})(B-V)\,.$$

Inspection of existing intrinsic color calibrations (e.g., Johnson 1966), shows that Q is at least an approximately linear function of $(B-V)_0$ for supergiants with spectral types between about F0 and G8. Outside these limits Q becomes a sharply nonlinear function of $(B-V)_0$, and it is important to note that all the results of this paper are restricted to this spectral range. Within this range it is assumed that one may write

$$Q = a_1 + a_2(B-V)_0\,.$$

Combining these two equations then leads to

$$(B-V)_0 = a_3 + a_4(R-I) + a_5(B-V)\,,$$

and for convenience in making comparisons this may be rewritten as

$$(B-V)-(B-V)_0 = -a_3 - a_4(R-I) - a_6(B-V)$$

or

$$E_{B-V} = b_1 + b_2(R-I) + b_3(B-V)\,.$$

The procedure then is very simple: Values of E_{B-V} have been taken from the listing by Parsons and Bell (1975), *BVRI* data from the literature (explicit references given below), and the above equation solved for the best values of b_1, b_2, and b_3. Thereafter, values of E_{B-V} are computed for each star from the equation and compared to the original Parsons/Bell values to check on the accuracy of the method.

Traditionally one would have used multilinear regression analysis to obtain b_i, but this assumes that all the observational error resides in $(R-I)$ and $(B-V)$, which is not true. Under such circumstances Cogan (1979) and Martin, Warren, and Feast (1979) have pointed out that one should use the method of maximum likelihood, in which estimates of the uncertainties in each of the three quantities enter the determination of the b_i. This method has been followed here, although as a matter of interest a least-squares solution was made also. The latter gave significantly different values for the b_i, although the average residual in E_{B-V} was not significantly worse than that derived from the maximum likelihood method. However, if each of these equations was rearranged as an explicit expression for either $(B-V)$ or $(R-I)$, the maximum likelihood equation gave much smaller residuals than the least-squares equation, i.e., it had the better all-around balance.

The photometric *BVRI* data for stable supergiants were taken from Johnson *et al.* (1966). (I have not attempted to scour the literature for every available *BVRI* observation, preferring to take the data from a few major sources to ensure as far as possible homogeneity in the results.) The Cepheid photometry has come from Moffett and Barnes (1980) and Schmidt (1976), and details of how it has been used are discussed below.

III. RESULTS AND DISCUSSION

For reasons connected with the original intention of this work, the determination of the b_i was made from the stable supergiant data only. As will be seen, however, when the resulting equation is applied to the Cepheid data, no systematic zero-point shifts are discernible, thus verifying that the method is applicable to stable or non-stable stars.

For input to the maximum likelihood method I have assumed uncertainties in E_{B-V}, $(R-I)$, and $(B-V)$ of 0.04, 0.02, and 0.02 mag, respectively. These can only be educated guesses, but the results are not at all sensitive to the exact values used. With these figures, the data of Table 1 lead to

$$\begin{array}{lllll} E_{B-V} = & -0.255 + & 1.727(R-I) - & 0.475(B-V) & (1) \\ & \pm 0.022 & 0.109 & 0.057 & \text{(s.e.)}\,. \end{array}$$

The excesses computed from equation (1) are listed in the final column of Table 1. They are related to the original excesses given by Parsons and Bell by

$$\begin{array}{ll} E_{\rm PB} = 0.001 + & 0.965\, E_{(1)}\,, \\ \pm 0.010 & 0.063 \end{array}$$

showing that equation (1) represents satisfactorily the original data. The rms deviation between the two sets of excesses is 0.026 mag.

Although the deduction of spectral types from purely photometric data is always risky, it may on occasion be useful to know that the spectral types in Table 1 can be represented by

$$\begin{array}{ll} S = -1.5 + & 15.0(B-V)_0\,, \\ \pm 0.9 & 1.1 \end{array}$$

TABLE 1

STABLE SUPERGIANTS

HD	Spectral Type	$(B-V)$	$(R-I)$	$(E_{B-V})_{\rm PB}$	$(E_{B-V})_{\rm Eq.\,(1)}$
20902.........	F5 Ib	0.48	0.33	0.04	+0.09
193370.........	F5 Ib	0.65	0.43	0.16	+0.18
195295.........	F5 II	0.40	0.23	0.03	−0.05
171635.........	F7 Ib	0.61	0.31	0.01	−0.01
54605.........	F8 Ia	0.67	0.33	0.03	0.00
194093.........	F8 Ib	0.67	0.34	0.02	+0.01
31910.........	G0 Ib	0.92	0.46	0.11	+0.10
204867.........	G0 Ib	0.84	0.41	0.03	+0.05
222574.........	G0 Ib	0.81	0.42	0.05	+0.09
67594.........	G2 Ib	0.98	0.46	0.08	+0.07
74395.........	G2 Ib	0.84	0.44	0.07	+0.11
209750.........	G2 Ib	0.98	0.47	0.07	+0.09
63700.........	G3 Ib	1.25	0.55	0.13	+0.10
192876.........	G3 Ib	1.08	0.53	0.13	+0.15
206859.........	G5 Ib	1.18	0.56	0.13	+0.15
48329.........	G8 Ib	1.40	0.62	0.19	+0.15
217476.........	var.	1.55	0.85	0.48	+0.48

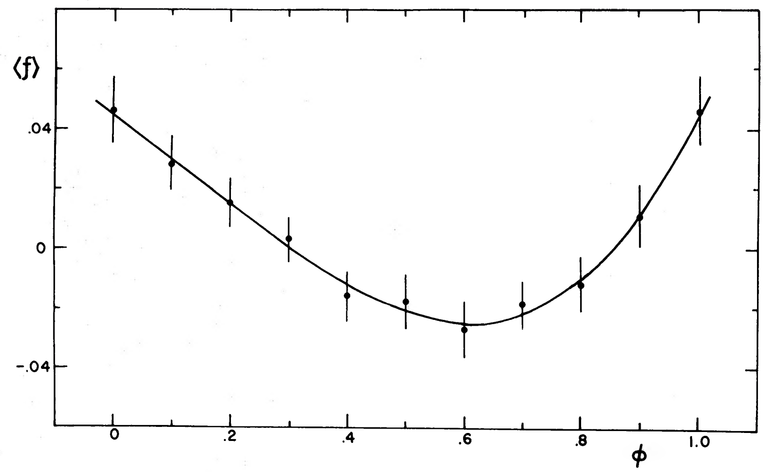

FIG. 1.—The average correction $\langle f \rangle$ that must be applied at individual phases of a Cepheid in order to obtain the reddenings listed by Parsons and Bell. The averaging has been done over all Cepheids in the Moffett and Barnes collection, and the error bars represent $\pm 1\ \sigma$ of each mean. The curve has the equation $\langle f \rangle = 0.045 - 0.133\phi - 0.130\phi^2 + 0.263\phi^3$.

where $S = 0$ corresponds to F0, $S = 5$ to F5, $S = 10$ to G0, etc. The rms deviation is 1.0 spectral subclasses. It is assumed that the star is already known to be an intermediate-type supergiant.

The first set of Cepheid data to be used was that of Moffett and Barnes (1980). These data are extensive and have excellent phase coverage, so that it is possible to plot each color curve and obtain smoothed, accurate colors at regular phase intervals. This was done in order to investigate any phase effects in the results. For each star, values of $(R-I)$ and $(B-V)$ were read off at phases (from maximum light) of 0.0, 0.1, 0.2, ... 0.9, and a value of E_{B-V} was computed for each of these phases from equation (1). This in turn was compared to the excess given by Parsons and Bell. A decided phase effect was indeed found, as Figure 1 shows. Here, for all the Cepheids in common to the two lists, is plotted the average correction at a given phase needed to give the Parsons/Bell value. Clearly a phase correction is needed. As one might expect, more detailed investigation revealed that the effect increases for stars of larger amplitude, and it was eventually found that the correction can be reasonably represented by

$$f = 1.2A(0.045 - 0.133\phi - 0.130\phi^2 + 0.263\phi^3)\,,$$

where A is the amplitude of the V light curve and ϕ the phase from maximum light.

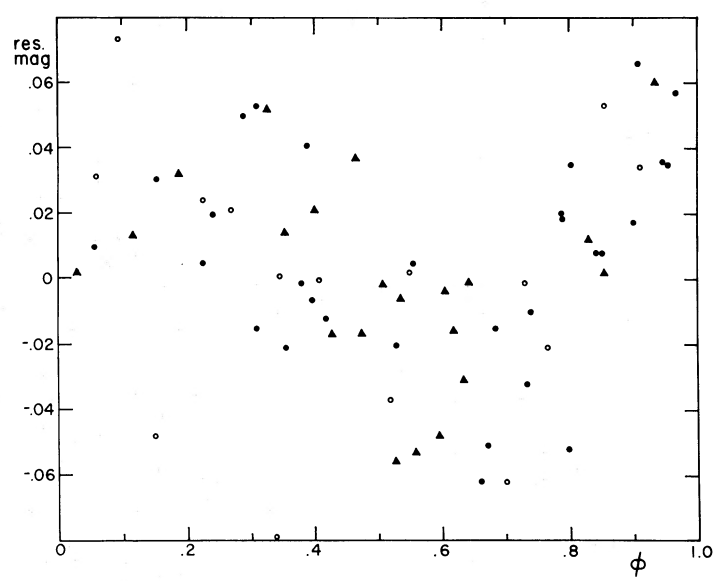

FIG. 2.—Test of Fig. 1 phenomenon by use of the independent data of Wisniewski and Johnson. Open circles refer to X Cyg, closed circles to δ Cep, and triangles to T Mon. Ordinates are the residuals between the excess calculated at a given phase and the excess averaged over all phases.

TABLE 2

CEPHEID REDDENINGS FROM VARIOUS DATA SETS

Star (1)	PB[a] (2)	MB[b] (3)	Sch[c] (4)
η Aql	0.15	0.18	...
U Aql	0.31	0.32	...
TT Aql	0.42:	0.40	0.46
FF Aql	0.15:	0.17	...
RT Aur	0.07	0.06	0.05
RX Aur	0.26	0.22	...
RW Cas	...	...	0.44
RY Cas	...	...	0.66
SU Cas	0.21	0.19	0.20
SZ Cas	...	...	0.78
CF Cas	0.48	...	0.55
CY Cas	...	...	1.04
DL Cas	0.48	...	0.49
δ Cep	0.08	0.11	...
CP Cep	...	...	0.72
X Cyg	0.22	0.22	...
SU Cyg	0.13	0.06	...
TX Cyg	...	...	1.13
CD Cyg	0.45	...	0.50
DT Cyg	0.05	0.00	...
ζ Gem	0.05	0.14	...
W Gem	0.25	0.25	...
Z Lac	0.29	...	0.38
T Mon	0.21	0.15	...
Y Oph	0.56	0.56	...
SV Per	0.37:	...	0.40
UY Per	...	...	0.82
VX Per	...	...	0.48
VY Per	...	...	0.91
AW Per	0.46	0.40	...
U Sgr	0.36:	0.40	0.41
W Sgr	0.14:	0.11	...
X Sgr	...	0.21	...
Y Sgr	0.18	0.16	...
S Sge	0.10	0.10	...
Y Sct	0.76:	...	0.71
EV Sct	0.61	...	0.57
SZ Tau	0.28	0.23	0.29
S Vul	...	...	0.81
T Vul	0.06	0.06	...
SV Vul	0.41	0.39	0.48

[a] Values given by Parsons and Bell 1975.

[b] Values determined from eq. (1) + f, using the smoothed photometry of Moffett and Barnes 1980.

[c] Values determined from eq. (1) + f, using individual observations by Schmidt 1976.

There seems no obvious explanation for this effect. It is not due to the fact that for a given amount of interstellar matter the excess varies with spectral type and so as a Cepheid goes from maximum to minimum its excess varies. This effect is given by $E = E_0[1 - 0.08(B-V)_0]$ (Dean *et al.*, eq. A2). E_0 is the reddening appropriate to a star of zero $(B-V)_0$, and E the reddening corresponding to any other $(B-V)_0$. If one takes typical values for the stars that were used (see Table 2), the change in excess from maximum to minimum is only of order 0.005 mag. Even for an extreme case like SV Vul, where both the amplitude and reddening are unusually large, the change is only 0.019 mag. Yet for the average case in Figure 1, the change is 0.07 mag.

Another concern would be whether this is an artifact of the photometry. In short, is there anything peculiar about the photometry of Moffett and Barnes that has produced an apparent phase effect? This can be ruled out almost immediately, because one would expect photometric peculiarities to appear at specific values or ranges of $(B-V)$ or $(R-I)$, yet stars having quite different color ranges show the same phase effect. Nevertheless, to be quite sure I have tested the data of Wisniewski and Johnson (1968) in the same way. These data are not only entirely independent of those from Moffett and Barnes, but were obtained with a different photomultiplier-filter system. For illustration I have taken the Wisniewski and Johnson data for X Cyg, T Mon, and δ Cep, chosen because they have pronounced (~1 mag) and similar V amplitudes. Equation (1) was applied to the individual points, and the residual, $E_{B-V} - \langle E_{B-V} \rangle$, from the star's average excess was formed. Results are shown in Figure 2. At this scale the scatter is large, but the same phase effect as shown in Figure 1 is clearly present. In fact the amplitude is more pronounced because these three stars have larger than average light amplitudes. Thus one concludes that this phase effect is not an artifact of the photometry, but is intrinsic to the Cepheid phenomenon itself. It is, in fact, directly associated with the circumstance that Cepheids describe loops in color-color plots, even in $(B-V)$ versus $(R-I)$, whereas equation (1) describes a straight line in that plot. Since, however, the loop phenomenon is poorly understood we shall not pursue the matter in this empirical investigation, but simply accept that Cepheid reddenings can be obtained by equation (1) plus f, as given above.

Table 2 lists Cepheids for which *BVRI* data are available in the above references. The first column of excesses are those given by Parsons and Bell. The second column lists excesses based on the Moffett/Barnes photometry and the use of equation (1) plus the correction f. They are averages over each of the 10 phases. The figures in columns (2) and (3) are related by

$$E_{PB} = \underset{\pm 0.015}{0.012} + \underset{0.060}{0.985}\, E_{MB} ,$$

showing that there is no systematic difference between the two sets.

To show that the smoothing process is unnecessary (i.e., that complete color curves need not be drawn), Schmidt's data have been left in the form of individual observations. The latter actually consists of $(R-I)$ and four-color $(b-y)$ data; $(b-y)$ was transformed to $(B-V)$ using the equation given by Schmidt himself as being appropriate for Cepheids. Equation (1) plus f was applied to the individual observations and phases listed by Schmidt in his Table 1, and then averaged over each Cepheid. The resultant values of E_{B-V} are given as the final column in Table 2. The relation between the Parsons-Bell values and those from Schmidt's data is

$$E_{PB} = \underset{\pm 0.035}{-0.021} + \underset{0.078}{0.994}\, E_{Sch} ,$$

again showing no systematic difference between the two sets of data.

Equation (1) (without f) is also applicable to Cepheids at mean light, i.e., $\langle B\rangle - \langle V\rangle$ and $\langle R\rangle - \langle I\rangle$. For 20 Cepheids common to PB and MB

$$E_{\mathrm{PB}} = \underset{\pm 0.015}{-0.010} + \underset{0.056}{0.990}\, E_{\text{mean light}} \,.$$

The rms deviation between the Parsons/Bell and Moffett/Barnes data is 0.037 mag, and that between Parsons/Bell and Schmidt's data is 0.042 mag. For Cepheids at mean light it is 0.033 mag. These figures are probably close to the actual uncertainty of an individual determination in the original Parsons and Bell listing, indicating that the method adopted here and the assumptions on which it rests [e.g., a linear relation between Q and $(B-V_0)$] are adequate.

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