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MASSES OF WHITE DWARF PROGENITORS FROM OPEN CLUSTER STUDIES

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ABSTRACT

Photographic photometry has been used to survey areas in three intermediate-age open clusters, M6, M34, and Praesepe, for white dwarfs. Photometric candidates have been found in M34 and Praesepe, clusters with turnoff masses of 3 and 2 $\mathcal{M}_\odot$. The results of these studies, when considered in the light of previous results in other cluster surveys, indicate that the upper mass limit for white dwarf progenitors is near 5 $\mathcal{M}_\odot$.

Subject headings: clusters: open — stars: evolution — stars: stellar statistics — stars: white dwarfs

I. INTRODUCTION

Evolution to a white dwarf is probably the most common of fates for stars of low and intermediate mass, a frequent alternative to the explosive deaths of more massive stars. The high space density of white dwarfs, 1.4×10^{-3} pc^{-3} for $M_v \leq 12.75$ (Green 1980), substantiates the commonness of this final state for virtually all stars with masses less than 2 $\mathcal{M}_\odot$.

While white dwarfs are constrained to maintain a degenerate structure with a mass up to 1.44 $\mathcal{M}_\odot$, the Chandrasekhar mass limit, most DA white dwarf masses cluster narrowly around a value of 0.6 $\mathcal{M}_\odot$ (Koester and Weidemann 1980). There can be no severe upper limit imposed on the original masses of their precursor stars, however, due to our inadequate understanding of mass loss in evolved stars.

Were it not for the complication of post–main-sequence mass loss, the broad picture indicated by Paczyński's (1970) stellar evolutionary models would describe a convenient break up of stars into several mass ranges: in the mass range below about 3 $\mathcal{M}_\odot$, stars eject their envelopes as planetary nebulae and cool as white dwarfs; stars more massive than 8 $\mathcal{M}_\odot$ ignite carbon quiescently. Intermediate-mass (3–8 $\mathcal{M}_\odot$) stars develop highly degenerate carbon cores, ideal potential sites for thermonuclear explosions once the point of carbon ignition is reached. Models for carbon detonation supernovae, first explored by Arnett (1969), have been refined in attempts to avoid complete disruption of the core, and have been replaced to some extent by models which ignite the degenerate carbon by a subsonic deflagration which leaves part of the core intact (Mazurek and Wheeler 1980; Nomoto, Sugimoto, and Neo 1976). Subsequent studies of post–main-sequence evolution have shown that mass loss is probably an important factor. Fusi-Pecci and Renzini (1975) and Reimers (1975) developed theoretical formulations for steady mass loss, at a rate of 4×10^{-13} (L/gR) $\mathcal{M}_\odot$ yr^{-1} in the latter work, where L, g, and R are in solar units. Cassinelli (1979) has summarized observationally derived mass loss rates for several types of stars; rates ranging from 10^{-9} to 10^{-5} $\mathcal{M}_\odot$ yr^{-1} have been observed for giant-branch stars, with increasing mass loss rates for the more luminous giants. Several authors have explored these mass loss rates in an attempt to infer an upper mass limit for progenitors of white dwarfs, $\mathcal{M}_{wd}$. Mass loss retards the growth of the degenerate core and make it possible for larger stars to avoid carbon detonation under degenerate conditions. Scalo (1976) derived a value of 4–7 $\mathcal{M}_\odot$ for $\mathcal{M}_{ws}$, and Mengel (1976) a value of 4–6 $\mathcal{M}_\odot$. As Mengel pointed out, the more slowly growing core is additionally stabilized against instability if rotation is also present, and even higher values for $\mathcal{M}_{wd}$ are conceivable.

These explorations into the consequences of steady mass loss on the giant branch indicate that stars initially much more massive than the Chandrasekhar mass limit might well end up as white dwarfs. Periodic bursts of mass loss due to thermally driven relaxation oscillations at high luminosities may also play a role (Smith and Rose 1972; Wood 1974). Tuchman, Sack, and Barkat (1978) have evolved at 6 $\mathcal{M}_\odot$ Population I model which loses enough mass in pulsational oscillations to avoid carbon detonation.

Another approach to determining the parent masses of white dwarfs is to attempt to establish a statistical link between various stellar remnant species and appropriate mass ranges of their main-sequence precursors. The locally determined birthrate for white dwarfs is 2×10^{-12} pc^{-3} yr^{-1}, within a factor of 2 (Weidemann 1979). The birthrate for planetary nebulae is somewhat less well determined, and is subject to alterations due to distance scale uncertainties. An upward revision in the distance scale was proposed by Cudworth (1974), based on an analysis of statistical parallaxes of planetary nebulae. Weidemann (1977) argued for an increase by a factor of 1.3 in planetary nebula distances, which implies a

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decrease by a factor of $(1.3)^4$ in the derived birthrate. Such a revision in Cahn and Wyatt's (1976) birthrate, based on a sample selected from a survey of more than 600 planetary nebulae, results in a birthrate of 1.4 to 2.1×10^{-12} pc^{-3} yr^{-1}. The similarity between the white dwarf and planetary nebula birthrates bears out the presumed sequential connection between them.

Estimates of the birthrates of the remnants of higher mass stars have also been reexamined recently. The rather high pulsar birthrate of 1×10^{-10} pc^{-2} yr^{-1} proposed by Taylor and Manchester (1977) has been revised downward by a factor of 2 by Hanson (1979) following an examination of selection biases in the pulsar proper motion data. Shipman and Green (1980) have compared these lower pulsar birthrates with the stellar birthrates of Miller and Scalo (1979). Their conclusion is that the proposed lower pulsar birthrate can be well matched to the deaths of main-sequence stars over a rather narrow mass range, or by all stars larger than 6 $\mathscr{M}_\odot$. A firm lower mass limit for pulsar progenitors would presumably imply a firm upper limit for $\mathscr{M}_{wd}$.

Aside from the stellar evolutionary and statistical studies, another possible avenue of research is to attempt to ascribe ages to those few white dwarfs that are in presumably coeval systems with main-sequence stars. Wegner (1973) has analyzed several white dwarfs in common-proper-motion pairs, deriving remnant masses and cooling times for the white dwarfs from their spectra and ages for the main-sequence companions. An evolutionary mass for the white dwarf precursor was inferred from the difference between the main-sequence star's age and the white dwarf cooling time. He found evidence for prior mass loss in all cases, with some white dwarf parent masses greater than the Chandrasekhar mass limit.

It might be hoped that the study of white dwarfs in open clusters would provide a larger statistical base for the study of white dwarf progenitor masses. If star formation were coeval in the cluster, the original masses of the stars which are now white dwarfs may be assumed to have been more massive than the stars now leaving the main sequence, with masses $\mathscr{M}_{to}$. This type of study was pioneered by Sandage (1957) in the course of probing the differences between the field star and the open cluster luminosity functions. The luminosity functions constructed for open clusters resemble the initial luminosity function derived from the field star statistics and lifetimes, not the field star luminosity function which reflects a continuing history of star formation. Sandage used his field-star initial luminosity function to extrapolate each cluster's luminosity function to higher masses and was able to predict the number of stars which might now be found as remnants in several clusters.

Ideally, this method depends on the ability to confirm the membership of spectroscopically identified white dwarfs, and to accurately describe the main-sequence population on which the extrapolation is based. The Hyades cluster remains the best studied object for this sort of analysis. In spite of its large areal extent, greater than 20° in diameter (van Bueren 1952), it has been extensively surveyed, and it is close enough that any member white dwarfs are accessible to both astrometric and spectroscopic investigation. Nine white dwarfs in the Hyades field were included in the lists of Eggen and Greenstein (1965) and were presumed possible members. Some of these were confirmed as members in the course of concentrated astrometric surveys of the central 41 square degrees of the cluster by van Altena (1969) and Hanson (1975). Van Altena confirmed the membership of three subluminous stars outside his survey area, and eight within it, for a total of 11 Hyades white dwarfs.

Several of these stars merit closer inspection. One of the central members, vA 673 (HZ 9, EG 38), appears to have a composite spectrum of a DA white dwarf and an M dwarf. Four other central area members, vA 54, 71, 216, and 391 have $B - V$ colors redder than 0.8, hardly expected for a reasonably young white dwarf when compared to the cooling times calculated by Sweeney (1976). Furthermore, these five stars do not conform to the color-magnitude sequence expected for the Hyades distance, when compared to the $(M_v, B - V)$-relation of Sion and Liebert (1977).

Liebert (1975) examined all four red subluminous stars in a spectrophotometric investigation, and identified all four objects as normal dM or field subdwarfs. Liebert suggests that photometric errors may have obscured the lower main-sequence nature of two of these supposedly red degenerate members. The two apparent M dwarfs have high membership probabilities in both Hanson's and van Altena's studies; the two apparent F or G subdwarfs have very low (less than 30%) probabilities of membership in Hanson's study. These results strongly suggest that the member white dwarf population of Hyades should be counted as 7, not 11.

This implies a substantial revision in the results of studies such as that of van den Heuvel (1975), who compared the white dwarf sample in the Hyades to an estimated main-sequence population derived from surveys of the larger cluster area of Wayman, Simms, and Blackwell (1965), as van Altena had confirmed the membership of white dwarfs outside his immediate survey area. Van den Heuvel set a fairly firm upper limit of 5 $\mathscr{M}_\odot$ for masses of white dwarf progenitors, based on $N_{wd} = 11$. A reduction in N_{wd} to 7 lowers this firm upper limit for $\mathscr{M}_{wd}$ to about 4 $\mathscr{M}_\odot$. Elimination of these very red stars also removes the motivation to assign very low (0.1 $\mathscr{M}_\odot$) remnant masses to some of the Hyades dwarfs, as suggested by Chin and Stothers (1971).

Compared to this fairly massive estimate for $\mathscr{M}_{wd}$, the Pleiades cluster seems to provide evidence for an even more massive white dwarf progenitor. Luyten and Herbig (1960) found that LB 1497 has the spectrum of a white dwarf, and a proper motion consistent with cluster membership. Its membership was confirmed in an astrometric survey conducted by Jones (1973).

The age and turnoff mass of the Pleiades merit closer inspection in view of the apparent membership of a white dwarf. The age has recently been discussed by Stauffer (1980), who compared the age implied by the evolutionary lifetime of the brightest stars—6 or 7 $\times 10^7$ years—to the age implied by a comparison of the faintest members to pre-main-sequence contraction isochrones, about 6×10^7 years. The age indicated by the turn-on point on the lower main sequence, however, is much larger; this discrepancy has yet to be resolved.

The photoelectric photometry of Johnson and Mitchell (1958) of stars indicated by Hertzsprung (1947) to be members or probable members, is illustrated in Figure 1. Several theoretical isochrones are illustrated for comparison, constructed for a composition of $Y = 0.28, Z = 0.02$, and for ages ranging from 20 to 500 million years. The oldest isochrone is adapted from the work of Ciardullo and Demarque (1979*a*). The younger sequences and the zero-age main sequence are based on models with similar composition for 3, 5, 7, and 9 $\mathcal{M}_\odot$ stars published by Becker (1981). The transformation to the observational plane was accomplished with color-temperature–bolometric corrections tabulated by Ciardullo and Demarque (1979*b*). A true distance modulus of 5.54 and a value of $E_{B-V} = 0.048$ were derived from a Strömgren photometric study in the cluster by Crawford and Perry (1976); the theoretical sequences illustrated in Figure 1 have been modified accordingly for reddening and absorption equal to $3E_{B-V}$.

An age of 60–70 million years is apparently consistent with the comparison of isochrones with the photoelectric data. Again referring to the evolutionary models of Becker, an age of 60 million years corresponds to a turnoff mass of 5 $\mathcal{M}_\odot$, so the strongest interpretation of the membership of LB 1497 in the Pleiades is that its progenitor star, if it was a single star, must have been at least as massive as 5 $\mathcal{M}_\odot$.

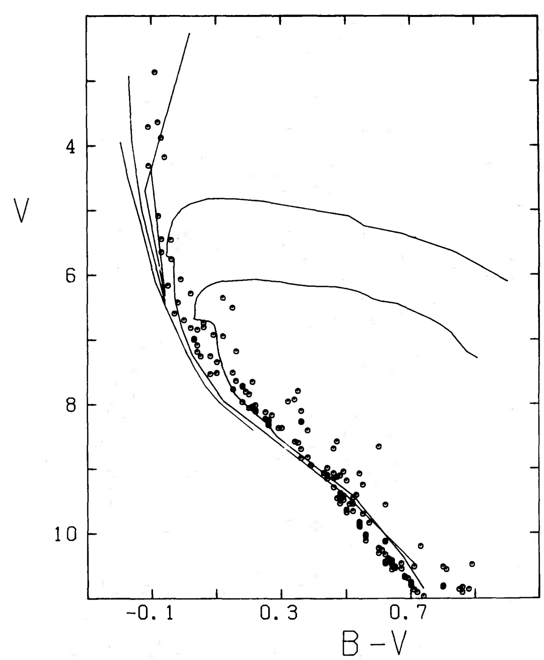


FIG. 1.—Photoelectric photometry by Johnson and Mitchell (1958) of Pleiades members, compared to isochrones with ages 0, 20, 40, 60, 200, and 500 million years. The apparent distance modulus is 5.68, the color excess 0.05.

In the wake of these interesting results, several open clusters have been surveyed for white dwarf candidates. Luyten (1958–1962) has identified blue stars in several clusters, but generally the photometric information is insufficient to narrow down the large numbers of stars to viable candidates. Several clusters ranging in turnoff mass from 2 to 25 $\mathcal{M}_\odot$ were selected by Hartwick and Hesser (1978), with several blue objects at appropriate magnitudes identified in each cluster.

A survey by Romanishin and Angel (1980) was confined to four clusters with turnoff masses ranging from 3 to 6 $\mathcal{M}_\odot$. One candidate in NGC 2168, a cluster with a turnoff mass of $\sim 5\ \mathcal{M}_\odot$, was measured photoelectrically. Its apparent magnitude and colors were found to be consistent with those of a member white dwarf. Romanishin and Angel concluded from their analysis of the incidence of blue objects in the fields of their clusters that $\mathcal{M}_{wd}$ is most probably 7 $\mathcal{M}_\odot$, with a firm lower limit of 5 $\mathcal{M}_\odot$ set by the photoelectric candidate in NGC 2168. Koester and Reimers (1981) have spectroscopically examined several of the photometric candidates identified by Romanishin and Angel in NGC 2287 and NGC 2422. Two candidates in NGC 2287, a cluster with a turnoff mass of about 4 $\mathcal{M}_\odot$, have been identified as white dwarfs with the appropriate luminosities for the cluster distance.

Surveys of this sort are initially photometric in nature; spectroscopic confirmation of candidates as white dwarfs or astrometric confirmation of membership is quite difficult for such faint objects. In one photometric survey of the nearby young cluster IC 2602 by Anthony-Twarog (1981), several promising photometric candidates were found to be reddened O and B subdwarfs, not white dwarfs, when examined spectrophotometrically. This young (turnoff mass 6.5–9 $\mathcal{M}_\odot$) cluster is rather poor, and the null result of the photographic search for white dwarfs did not shed much light on $\mathcal{M}_{wd}$; a value of 7 $\mathcal{M}_\odot$ is the lowest derived upper limit for $\mathcal{M}_{wd}$ from this research.

II. DESIGN OF THE PRESENT SURVEY

To widen the base of ages and types of clusters studied, a *UBV* photographic survey was proposed to search for white dwarfs in clusters with turnoff masses between 2 and 9 $\mathcal{M}_\odot$. *UBV* photographic photometry was chosen for the survey depth it permits, and because the discrimination provided by two colors allows a better selection of candidate objects than reliance on ultraviolet brightness alone.

Using the (M_v, $B - V$)-relation of Sion and Liebert (1977), one can see that white dwarfs are rarely brighter than $M_v = 10$, and may be as faint as $M_v = 13.5$ while still bluer than $B - V = 0.4$. In the cooling models of Sweeney (1976), white dwarfs bluer than this color might be expected to be younger than 5 billion years (for white dwarf masses of 0.51–0.89 $\mathcal{M}_\odot$). For these reasons, only clusters with apparent distance moduli less than 10 were considered for inclusion in the survey.

A further restriction is imposed by the age range of astrophysical interest, namely 20 to 700 million years, corresponding to turnoff masses of 9–2 $\mathcal{M}_{\odot}$. Ages for clusters were initially determined from the lists of Harris (1976), which employs a method of age ranking based on photometric and spectroscopic indicator. The ages and turnoff masses which were ultimately used in the analysis of each cluster were determined by a comparison of *UBV* photometry to theoretical isochrones transformed to the $(M_v, B - V)$-plane.

Several of these comparisons are illustrated in Figures 2, 3, and 4 for the three program clusters M6, M34, and Praesepe. Values for the color excess were obtained from published photoelectric studies, noted in Table 1. Uniform distance moduli are more difficult to acquire; several published values are available in some cases. These are noted, as well as the adopted values, which provided the best fit of data to isochrones. Sources for the photoelectric data used are also noted in Table 1. These clusters, together with IC 2602, comprise the program of this survey.

The assignment of ages to clusters based on theoretical isochrones is one of several necessary dependences on theory in this otherwise largely empirical technique. In addition to the use of a mass-luminosity relation based on the Yale stellar evolutionary models (Mengel *et al.* 1979) and the models of Becker (1981), some consideration must be made of the expected time scale for white dwarf cooling. The cooling models of Sweeney (1976) for

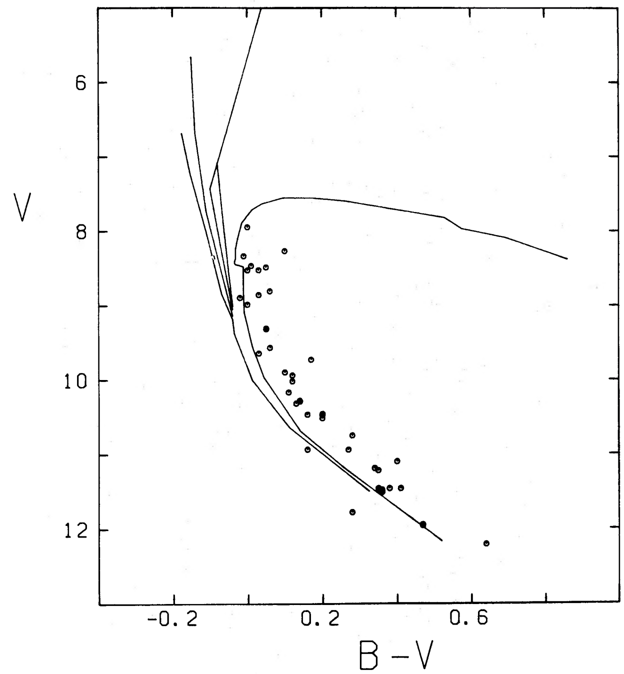


FIG. 3.—Photoelectric photometry of stars in M34 by Johnson (1954) compared to isochrones with ages 0, 20, 40, 60, 200, and 500 million years. A color excess of 0.07 and apparent distance modulus of 8.41 have been adopted.

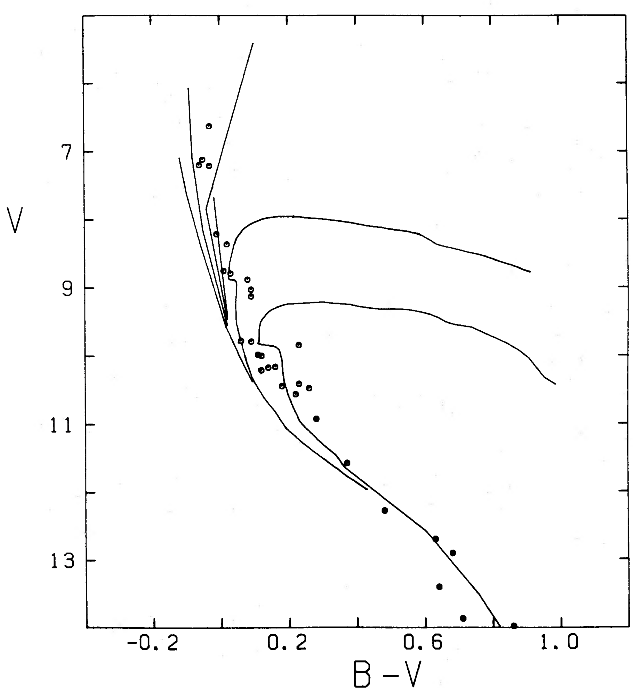


FIG. 2.—Photoelectric photometry by Eggen (1961) and Talbert (1965) of stars in M6, compared to isochrones with ages 0, 20, 40, 60, 200, and 500 million years. The apparent distance modulus is 8.82, the color excess 0.13.

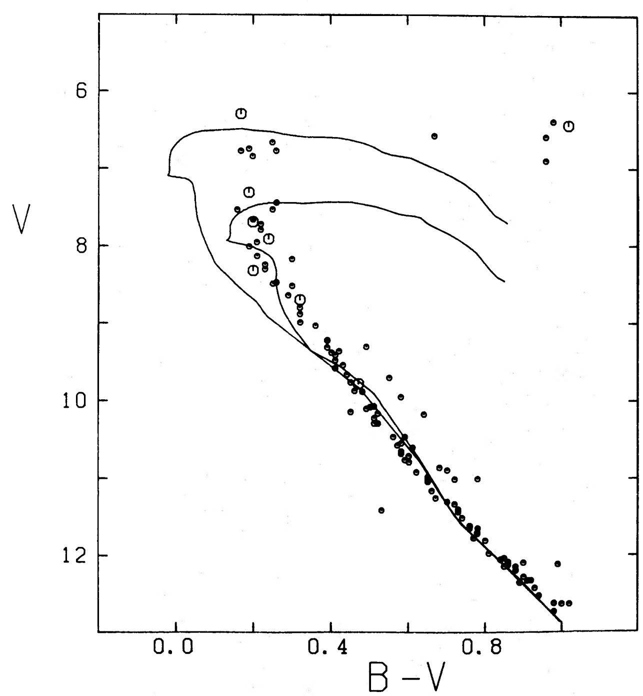


FIG. 4.—Johnson's (1952) photoelectric photometry of Praesepe stars, compared to isochrones for ages 500 and 1000 million years. The adopted distance modulus is 6.10, and the color excess is 0.00. The larger symbols denote stars which are known to be binaries.

TABLE 1
CLUSTER PARAMETERS

Cluster	Source	$(m-M)_0$	E_{B-V}	$\mathscr{M}_{to}$	Age ($\times 10^6$ yr)
Pleiades	(1)	5.54	0.05		
Adopted	...	5.54	0.05	5	60
M6	(2)	8.5	0.13		
	(3)		0.16		
Adopted	...	8.43	0.13	6.2	40
M34	(4)	8.6	0.09		
	(5)	8.2	0.09		
	(6)	8.35	0.11		
	(7)	8.20	0.07		
Adopted	...	8.20	0.07	3	200
Praesepe	(8)	6.0	0.00		
	(9)	6.1	0.00		
Adopted	...	6.10	0.00	1.8	700

NOTE.—Sources for the photometric parameters are as follows: (1) Crawford and Perry 1976. (2) Eggen 1961. (3) Talbert 1965. (4) Johnson 1954. (5) Johnson 1957. (6) Cester *et al.* 1977. (7) Canterna *et al.* 1978. (8) Johnson 1952. (9) Crawford and Barnes 1969.

carbon-core, hydrogen-atmosphere white dwarfs with remnant masses 0.2–1.2 $\mathscr{M}_\odot$ have been employed. The effective temperature-color-bolometric correction relations compiled from Shipman's (1972) study have been applied to Sweeney's cooling sequences in order to transform them to the $(M_v, B-V)$-plane. These sequences are illustrated in Figure 5, where the dashed lines show sequences for single masses, and the bold lines join the sequences along equal time lines. The position of color-magnitude sequences for white dwarfs of Sion and Liebert (1977) is also shown by lighter, unbroken lines. One should not expect to find white dwarfs of typical mass that have cooled to colors redder than $B-V=0.40$ in 10^9 years or less. Any photometric candidates should be evaluated to see whether they correspond to the expected color-magnitude sequence appropriate for the cluster distance, and whether they are within color limits provided by the available cooling time, which is at most the age of the cluster.

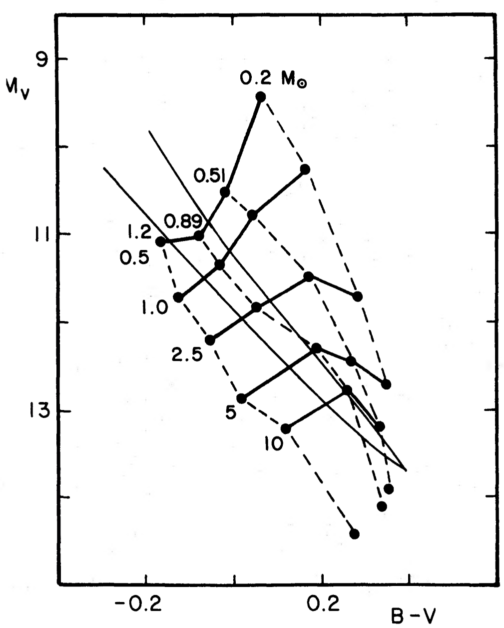


FIG. 5.—The results of Sweeney's (1976) models for the cooling of white dwarfs, transformed to the observational color-magnitude plane by means of the color temperature–bolometric corrections of Shipman (1972). The dashed lines show cooling sequences for dwarf masses as labeled, while the bold lines indicate equal time loci, labeled in units of 10^8 years. The Sion and Liebert (1977) white dwarf sequence is illustrated by lighter, unbroken lines.

In Sandage's (1957) study, as in the other subsequent studies, the white dwarf populations in clusters were compared to an extrapolation of the cluster main-sequence population based on some standard luminosity function. If N_{wd} is the number of white dwarfs in the cluster, N_{ms} the number of main-sequence stars brighter than M_*, and $\mathscr{N}(M)$ the number of stars brighter than absolute visual magnitude M predicted by some standard luminosity function, then:

$$N_{wd} = N_{ms}\left[\frac{\mathscr{N}(M_{to}) - \mathscr{N}(M_{wd})}{\mathscr{N}(M_*) - \mathscr{N}(M_{to})}\right].$$

M_{wd} and M_{to} are the unevolved absolute visual magnitudes corresponding to the upper mass limit for white dwarf progenitors, and the turnoff mass.

Taff (1974) has compiled a luminosity function from *UBV* data for 62 open clusters, limiting the counts from each cluster to sample the unevolved main sequence below the turnoff, down to 1 mag above the plate limiting *V* magnitude. While it might be preferable to have proper-motion selected samples to form the luminosity function in order to reduce the contamination from field stars, Taff's cluster luminosity function does avoid any possible differences between cluster and field-star luminosity functions, and his function compiled from all 62 clusters has been used to compute $\mathscr{N}(M)$, the integral luminosity function, in all cases.

The use of a fiducial luminosity function to extrapolate the presently observed main-sequence population presumes that all open clusters reflect evolved states of continuous and similar initial mass functions. This may not be true if star formation is bimodal, as suggested by Eggen (1976), or if cluster populations are not truly coeval (Herbig 1962; Stauffer 1980).

The use of this cluster luminosity function scaling technique can be instructively illustrated by reference to the Pleiades cluster. The photoelectric data of Johnson and Mitchell (1958) of stars designated by Hertzsprung (1947) as members and probable members, list 102 stars brighter than $M_v = 4$, if the apparent distance modulus is 5.68. The cluster age of 60 million years corresponds to a turnoff mass of $5\mathscr{M}_\odot$; an examination of the mass-luminosity relation derived from Becker's (1981) theoretical models indicates an unevolved absolute visual magnitude of -0.5 for a 5 $\mathscr{M}_\odot$ star. Thus, N_{wd} should be predicted by:

$$N_{wd} = N_{ms}\frac{\mathscr{N}(-0.5) - \mathscr{N}(M_{wd})}{\mathscr{N}(4) - \mathscr{N}(-0.5)}.$$

TABLE 2

PREDICTIONS FOR N_{wd}

$\mathscr{M}_{wd}$	M_{wd}	$\mathscr{N}(M_{wd})$	Pleiades: $\mathscr{M}_{to} = 5$	M6: $\mathscr{M}_{to} = 6.2$	M34: $\mathscr{M}_{to} = 3$	Praesepe: $\mathscr{M}_{to} = 1.8$
2.0	1.8	0.16240	...	...	...	3 ± 2
2.5	1.1	0.11710	...	...	...	7 ± 5
3.0	0.8	0.10130	...	...	...	8 ± 6
3.5	0.3	0.07897	...	...	5 ± 2	10 ± 8
4.0	0.0	0.06730	...	...	8 ± 3	12 ± 8
4.5	−0.3	0.05654	...	...	10 ± 3	13 ± 9
5.0	−0.5	0.05147	...	...	11 ± 4	13 ± 9
5.5	−0.7	0.04640	1 ± 1	...	12 ± 4	14 ± 10
6.0	−0.9	0.04192	3 ± 2	...	13 ± 4	14 ± 10
6.5	−1.0	0.03978	3 ± 2	0 ± 1	14 ± 4	14 ± 10
7.0	−1.2	0.03551	4 ± 2	1 ± 1	15 ± 4	15 ± 10
7.5	−1.3	0.03373	5 ± 2	1 ± 1	15 ± 4	15 ± 10
8.0	−1.5	0.03091	6 ± 2	1 ± 1	16 ± 4	15 ± 11
8.5	−1.6	0.02950	6 ± 2	1 ± 1	16 ± 4	15 ± 11
9.0	−1.7	0.02809	6 ± 3	2 ± 1	16 ± 4	15 ± 11
9.5	−1.8	0.02680	7 ± 3	2 ± 1	17 ± 5	15 ± 11

M_{wd} corresponds to the unevolved absolute visual magnitude for a star with mass $\mathscr{M}_{wd}$. Predictions for several values of $\mathscr{M}_{wd}$ are listed in Table 2 for the Pleiades as well as for the other clusters in this study. To these estimates for N_{wd}, an error arising from the statistical uncertainty in the main sequence population estimator must be applied, $N_{ms}^{-1/2}$. Comparison of these estimates to the observed number of white dwarfs in a cluster must be made, however, within the uncertainty imposed by the smallness of N_{wd} itself, $N_{wd}^{-1/2}$. One sigma uncertainties are tabulated along with all the predicted N_{wd} in Table 2. The observed number of white dwarfs in the Pleiades, 1, indicates that $\mathscr{M}_{wd}$ is less than 7 $\mathscr{M}_\odot$ if one sigma limits are considered, and less than 8 $\mathscr{M}_\odot$ if 2 sigma limits are considered, in the comparison between predicted and observed values for N_{wd}.

III. OBSERVATIONAL MATERIAL

The method of this survey, like that of previous investigations, was a survey of *UBV* photographic plates. For M34, and one field northwest of the center of the Praesepe cluster, plates were obtained at the prime focus of the Kitt Peak National Observatory 4 m telescope in January of 1978. In the case of M6, the Cassegrain camera on the Cerro Tololo Inter-American Observatory 1.5 m telescope was used with the CTIO 16 cm sequence extending wedge. The wedge produces secondary images approximately 4.5 mag fainter than and ~ 32″ distant from the primary images. The magnitude difference between primary and secondary images was solved for as a free parameter in the reduction of photometry for each plate; individual values for Δm are listed in Table 3, which fully describes all of the plate material. A mean value of $\Delta m = 4.63 \pm 0.51$ from five plates was employed in the final reduction of the M6 photographic data.

Although *UBV* photoelectric surveys have been published for all three clusters, extension to fainter magnitudes was necessary to calibrate the photographic photometry. For M34 and Praesepe, photoelectric studies by Johnson (1954, 1952) provided the bases for photoelectric sequences. The observations of Eggen (1961) and Talbert (1965) in M6 extend to $V \sim 15$. A comparison of *V*-magnitudes for 12 stars observed by both authors, however, reveals a systematic zero point difference $(V_T - V_E) = 0.08$. Talbert's values for *V*-magnitudes have been transformed to the system of Eggen's *V*-magnitudes prior to use as photoelectric standards in the M6 field.

Supplementary photoelectric observations were obtained for all three cluster fields. Additional stars in M6 were observed at CTIO in the April/May seasons of 1978 and 1979, on the 0.9 and 1 m telescopes using S-20 photocathode photometers and standard filters. The equatorial network of *UBV* standard stars published by Landolt (1973) was used to transform the observations to the standard system. The new photoelectric standards are described in Table 4, and some of their positions are indicated in Figure 6. Charts for other photoelectric sequence stars may be found in Vleeming's (1974) study, or in Antalova's (1972) study which shows the position for all stars on the numbering system of Rohlfs, Schrick, and Stock (1959), the system used by Eggen (1961) and Talbert (1965). Further observations were obtained at Kitt Peak for the two northern clusters, using the No. 1 0.9 m telescope and the Mark I photometer on the 2.1 m telescope with S-20 photocathode photometers, in January of 1979 and 1980. In addition to the Landolt *UBV* standard stars, the stars observed by Johnson (1952) in Praesepe were used to transform the observed values to the standard system. *BV* photometry was also obtained with the video camera on the 2.1 m telescope, using the photoelectric observations in NGC 5053 by Sandage, Katem, and Johnson (1977) and in NGC 2419 by Adams (1980) for calibration. The new photoelectric standards for M34 and the field in Praesepe are described in Tables 5 and 6, and their positions illustrated in Figures 7 and 8.

TABLE 3

PLATE MATERIAL

	Plate Number	Exposure (min)	Emulsion	Filter	Date (mo/day/yr)	Δm
Praesepe						
($\alpha_{1950} = 8^h36^m24^s$,						
$\delta_{1950} = 19°54'00''$)	2712	30	IIaD	GG495	01/07/78	
	2713	45	IIaO	UG2	01/07/78	
	2714	35	IIaO	GG385	01/07/78	
	2715	35	IIaO	GG385	01/07/78	
	2716	45	IIaO	UG2	01/07/78	
	2717	25	IIaD	GG495	01/07/78	
M6						
($\alpha_{1950} = 17^h36^m49^s$,						
$\delta_{1950} = -32°11'1''$)	2855	120	103aO	UG2	05/07/78	4.66
	2861	60	103aD	GG14	05/09/78	4.55
	2862	40	103aO	GG385	05/09/78	4.66
	2868	40	103aO	GG385	05/09/78	4.66
	2869	120	103aO	UG2	05/09/78	4.62
	2876	60	103aD	GG14	05/10/78	4.80
M34						
($\alpha_{1950} = 2^h38^m18^s$,						
$\delta_{1950} = 42°34'00''$)	2679	20	IIaO	GG385	12/31/77	
	2680	30	IIaO	UG2	12/31/77	
	2686	25	IIaD	GG495	01/02/78	
	2687	25	IIaD	GG495	01/02/78	
	2688	30	IIaO	GG385	01/02/78	
	2709	40	IIaO	GG385	01/07/78	
	2718	45	IIaO	UG2	01/08/78	

For the Praesepe field, the white dwarfs EG 59 and EG 60 were also used as photoelectric sequence standards (Eggen and Greenstein 1965).

U and *V* plates were examined in a blink comparator to select a sample of stars for further photometric study as white dwarf candidates. Blink surveys in M34 and in the Praesepe field yielded 131 and 431 stars, respectively. A blink survey of plates of the M6 field did not yield any conspicuously ultraviolet-bright candidates. Since the subjectivity of a blink survey for faint objects may be further vitiated by a significant color excess, it was decided to select one-quarter of the cluster area for an unbiased photographic study. A total of 692 stars were selected in evenly spaced rings in each of the quadrants in the cluster for measurement: quadrant 1 for the inner

TABLE 4

NEW PHOTOELECTRIC SEQUENCE STARS IN M6

Number (1)	Vl. (2)	R (3)	V (4)	$B-V$ (5)	$U-B$ (6)	σ_V (7)	σ_{B-V} (8)	σ_{U-B} (9)
12041....	...	32c	12.74	0.59	0.10	0.03	0.02	0.04
13078....	10 W	...	13.28	1.06	0.83	0.02	0.01	0.03
15033....	...	...	11.91	0.36	0.17	0.03	0.02	0.04
15034....	...	...	12.22	0.94	0.49	0.03	0.02	0.04
15043....	...	...	14.56	1.18	...	0.03	0.02	...
15044....	...	...	12.70	0.24	0.04	0.03	0.02	0.06
21039....	95 W	...	14.14	1.07	0.43	0.02	0.02	0.04
21040....	...	98	12.29	0.59	0.06	0.02	0.02	0.04
23011....	...	...	13.20	0.50	0.17	0.03	0.02	0.04
23027....	...	127	10.33	0.19	0.18	0.02	0.02	0.05
25018....	127 W	...	11.37	0.78	0.22	0.03	0.02	0.04
32002....	...	130	10.98	0.30	0.21	0.03	0.02	0.05
34092....	...	10	12.97	1.51	1.16	0.02	0.02	0.05
42087....	92 E	...	12.27	0.63	0.12	0.03	0.02	0.05
43045....	...	108	13.01	0.39	0.27	0.03	0.02	0.05
43058....	87 E	...	15.47	0.89	...	0.04	0.02	...
43059....	...	110	14.29	0.86	0.41	0.03	0.02	0.05
43121....	...	...	11.82	0.38	0.32	0.03	0.02	0.04

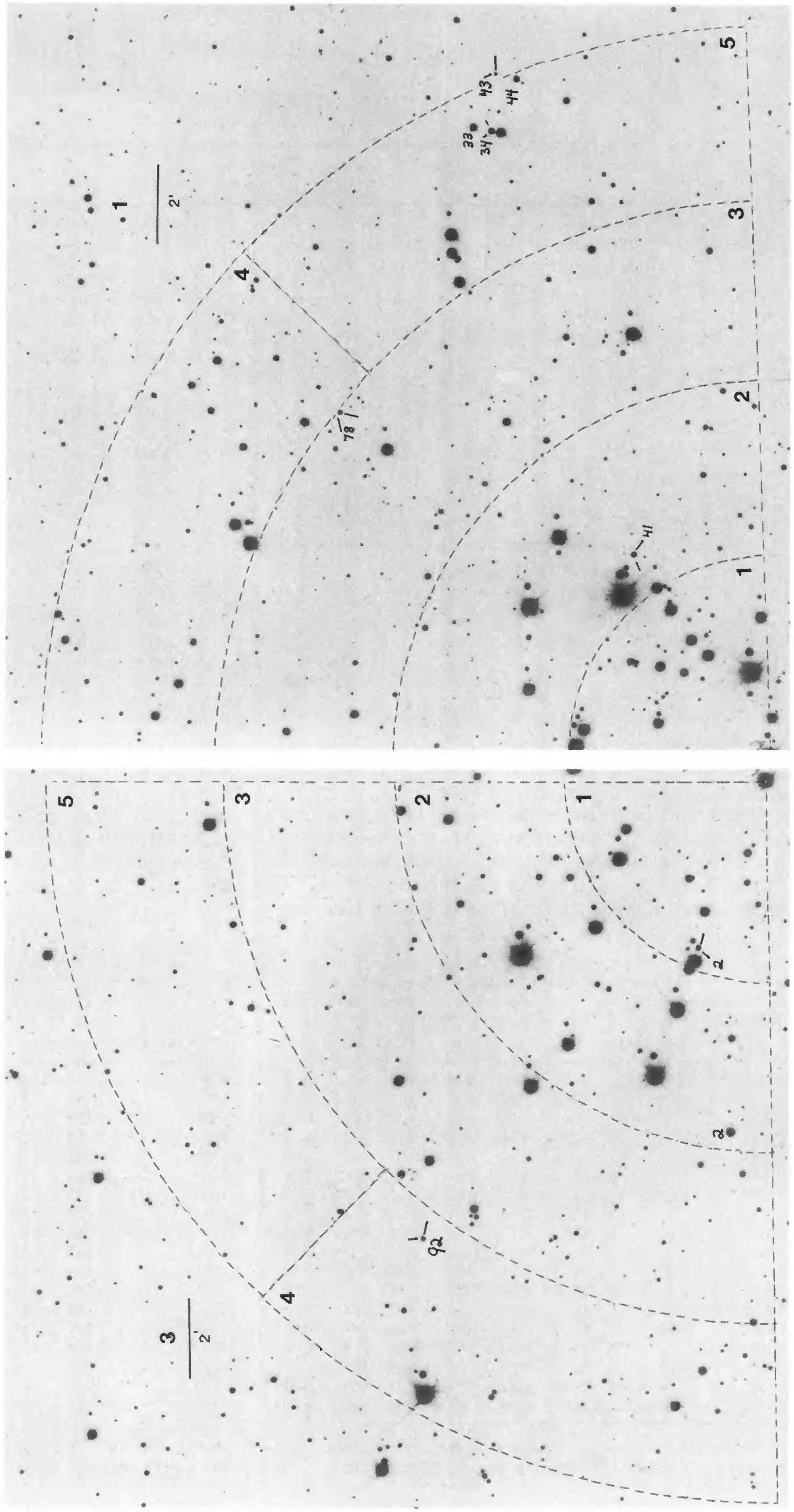
NOTE.—Numbers in columns (2) and (3) refer to the identification charts from Vleeming 1974 and Rohlfs *et al.* 1959, respectively.

TABLE 5

NEW PHOTOELECTRIC SEQUENCE STARS IN M34

Number (1)	Br. (2)	V (3)	$B-V$ (4)	$U-B$ (5)	σ_V (6)	σ_{B-V} (7)	σ_{U-B} (8)	Source (9)
16003 ...	...	16.05	0.93	1.27	0.02	0.05	...	VC
16004 ...	...	18.14	0.84	...	0.02	0.05	...	VC
23017 ...	98	12.17	1.10	0.32	0.05	0.07	0.05	PE
31050 ...	120	12.55	0.65	0.15	0.05	0.06	0.051	PE
32079 ...	125	11.99	1.05	0.76	0.05	0.06	0.05	PE
35005 ...	...	11.91	0.54	0.08	0.02	0.06	0.05	PE
35007 ...	...	19.84	1.27	...	0.02	0.05	...	VC
35009 ...	...	16.85	1.33	(1.46)	0.02	0.05	0.07	VC/PE
35010 ...	...	18.13	0.70	...	0.02	0.05	...	VC
35011 ...	...	20.13	1.13	...	0.02	0.05	...	VC
35013 ...	...	19.28	1.21	...	0.02	0.05	...	VC
35045 ...	...	14.79	0.84	0.33	0.04	0.07	0.05	PE
35058 ...	...	15.78	1.02	0.55	0.05	0.06	0.05	PE
42073 ...	...	19.12	1.64	...	0.02	0.05	...	VC
42074 ...	...	20.83	0.61	...	0.02	0.05	...	VC
42076 ...	...	18.48	0.71	0.02	0.02	0.05	...	VC
42133 ...	131	12.79	0.62	0.08	0.04	0.07	0.05	PE
42143 ...	...	15.09	0.62	0.07	0.05	0.07	0.06	PE

NOTE.—Numbers in column (2) refer to the identification of Bruggemann 1935. Notations in column (9) refer to photoelectric photometry and video camera photometry, respectively.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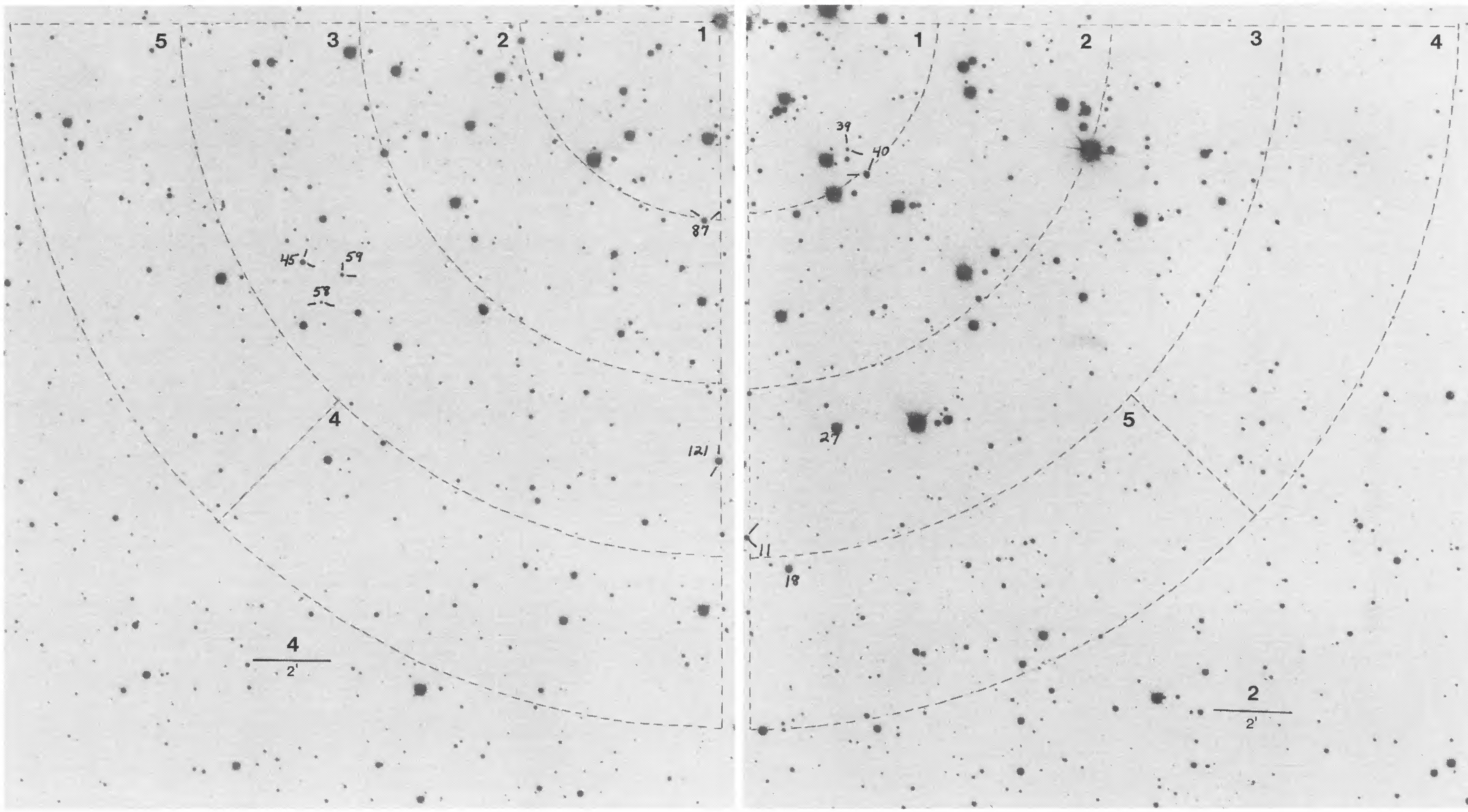
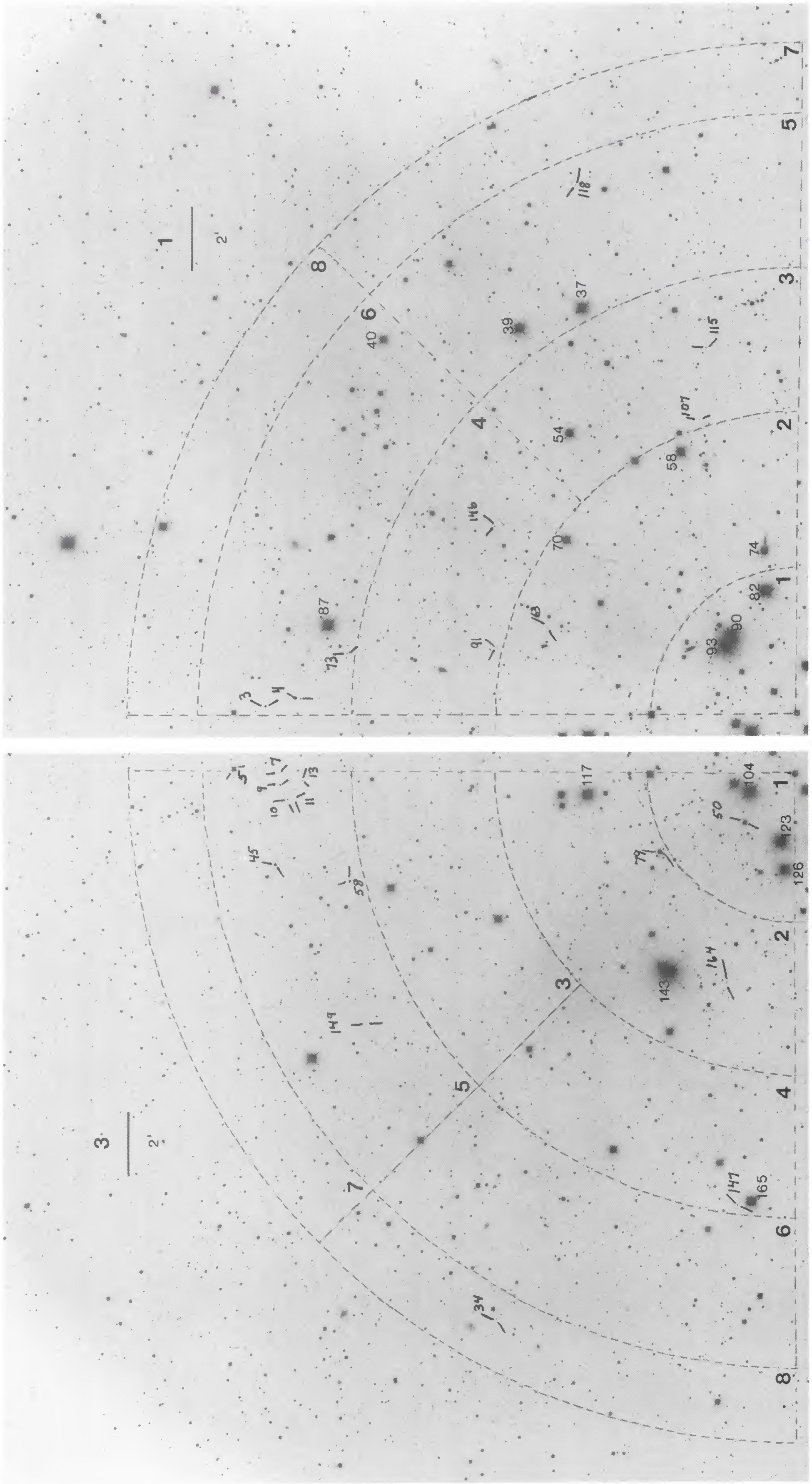


FIG. 6.—Identification charts for stars in M6. North is up, east to the left. The secondary images produced by the sequence-extending wedge can be seen west and slightly north of the primary images. The first digit of an identification number for a star refers to the quadrant in which it is found; the second digit, to the ring segment within the designated quadrant. The scale is indicated on the charts.

1982ApJ...255..245A



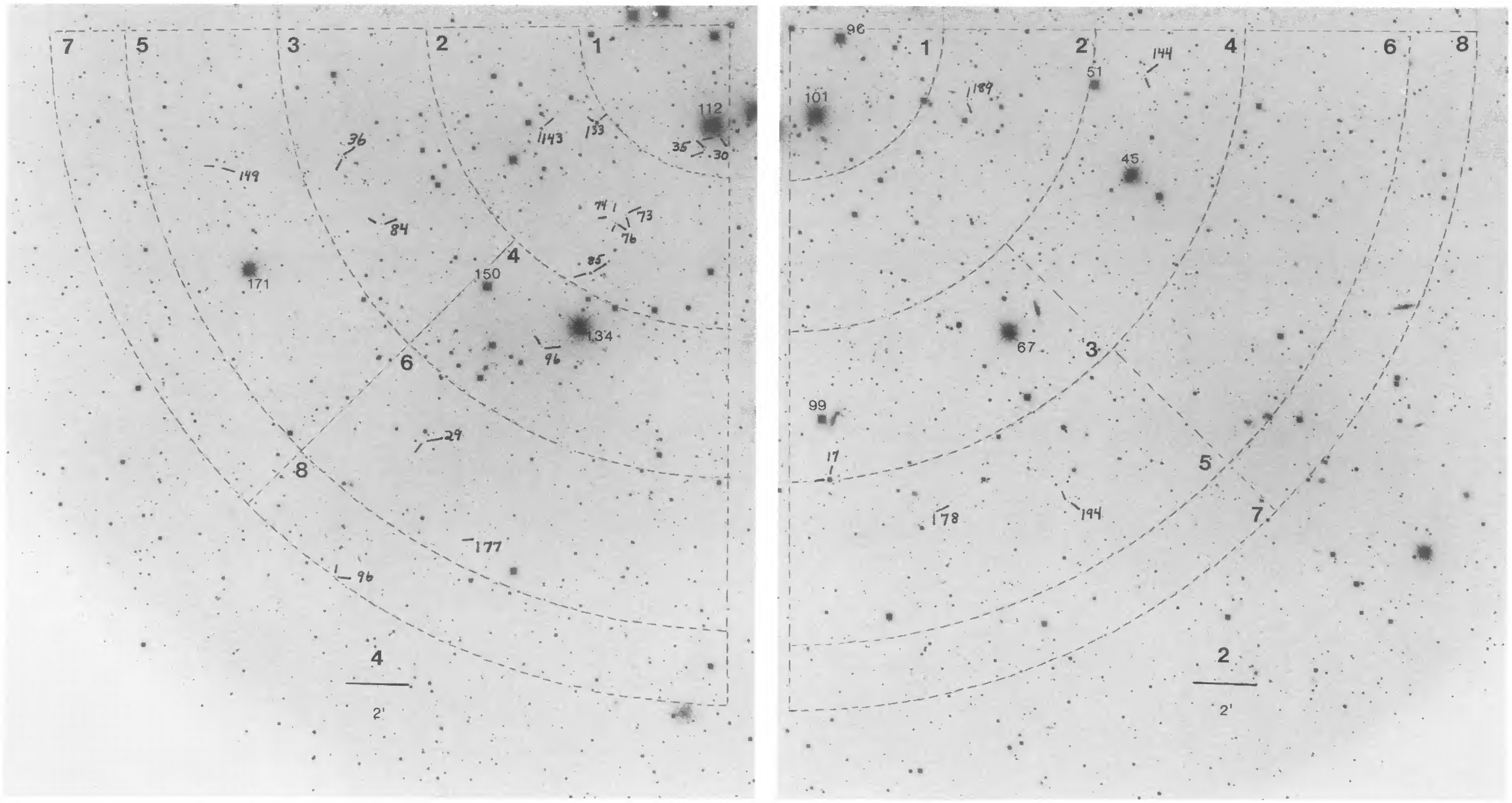
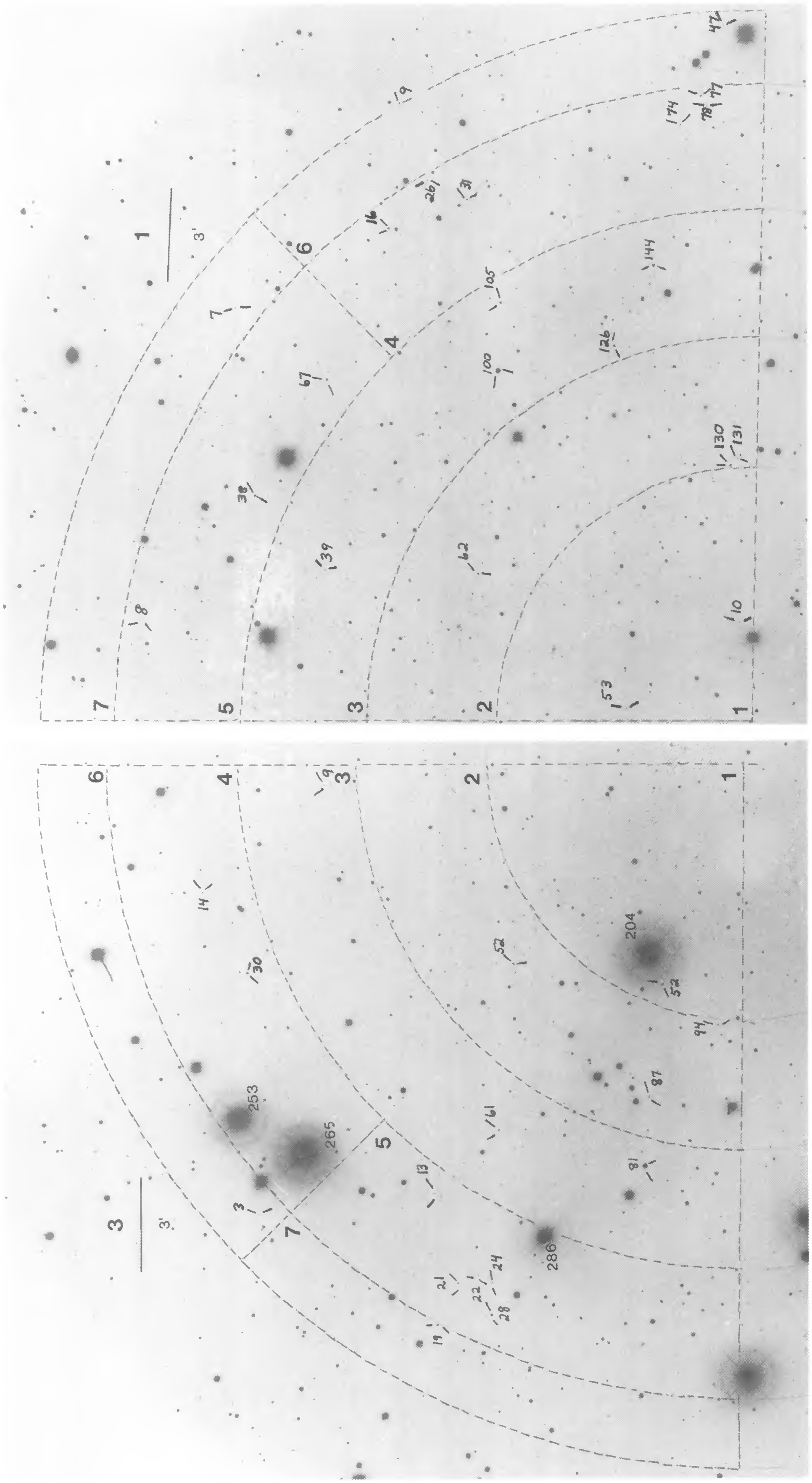


FIG. 7.—Identification charts for stars in M34. North is up, east to the left; the inner rings are 5′ wide, the outermost ring $2\frac{1}{2}'$ wide. The first and second digits of a star's identification number refer to the quadrant and ring in which it is found. Bold numbers show stars from Dieckvoss's (1954) study.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286
204

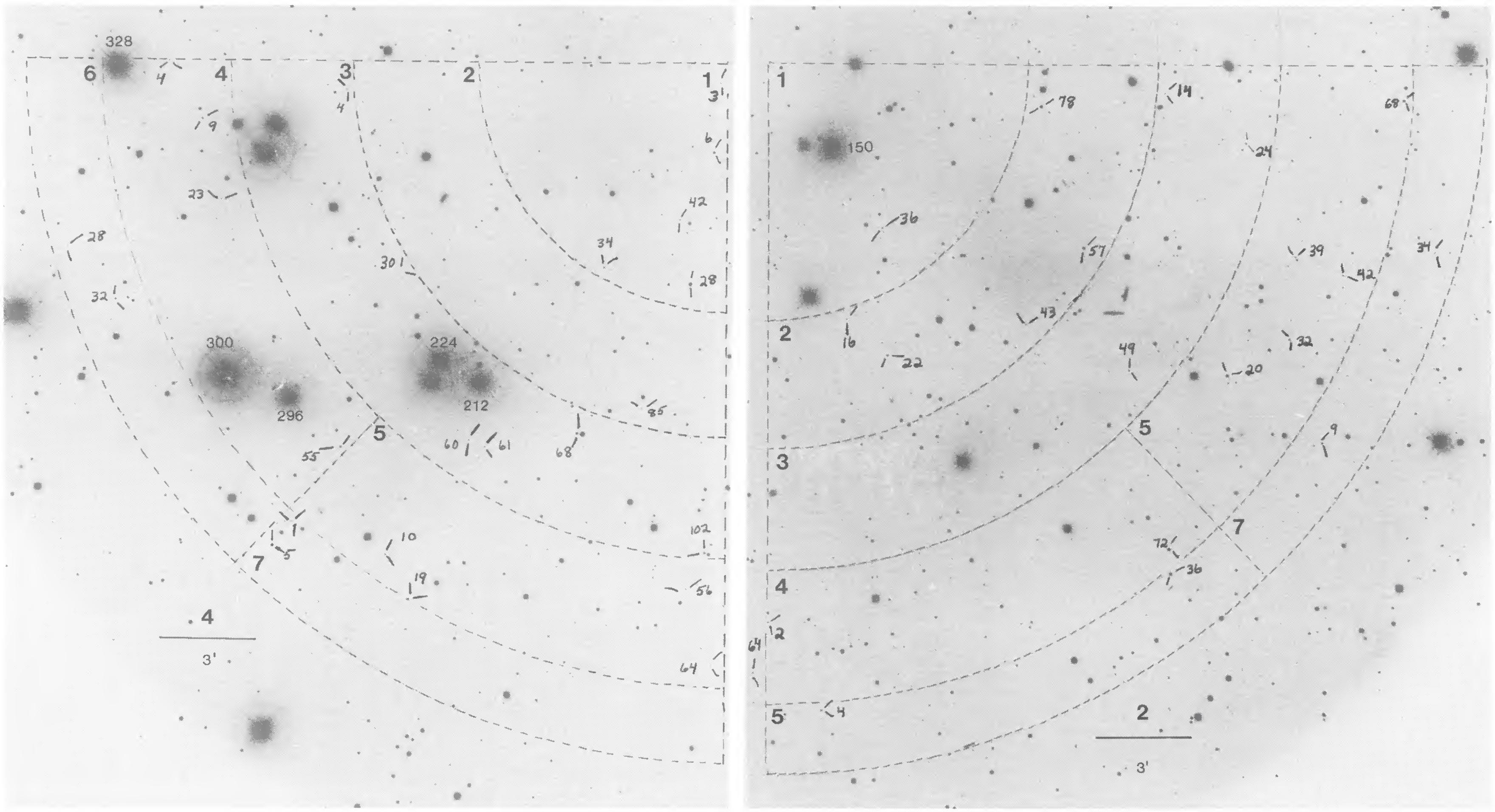

FIG. 8.—Identification charts for stars in Praesepe. North is up, east to the left, with the chart scale indicated. The first and second digits of a star's identification number refer to the quadrant and ring in which it is found. Some of the stars in Klein-Wassink's (1927) study are noted by bold numerals.

TABLE 6

NEW PHOTOELECTRIC SEQUENCE STARS IN PRAESEPE

Number (1)	V (2)	$B-V$ (3)	$U-B$ (4)	σ_V (5)	σ_{B-V} (6)	σ_{U-B} (7)	Source (8)
22057..........	16.83	0.82	0.49	0.05	0.09	0.10	PE
22078..........	16.23	0.85	0.42	0.05	0.09	0.10	PE
33081..........	13.12	1.05	0.90	0.05	0.09	0.10	PE
35021..........	17.51	1.52	...	0.03	0.06	...	VC
35022..........	20.22	0.87	...	0.03	0.06	...	VC
35024..........	19.26	0.58	...	0.03	0.06	...	VC
35028..........	16.90	0.73	...	0.03	0.06	...	VC
37019..........	18.00	0.89	...	0.03	0.06	...	VC
46032..........	17.22	1.07	0.19	0.05	0.09	0.10	PE
47005..........	16.51	1.42	1.53	0.05	0.09	0.10	PE

NOTE.—Notations in column (8) refer to photoelectric photometry and video camera photometry, respectively.

ring, quadrant 2 for the second ring, quadrant 4 for the third ring, and quadrant 3 for the outer ring.

The 131 blink candidates in M34 were measured on the Cuffey iris photometer at Yale University. The blink-selected sample in Praesepe and the randomly selected sample in M6 were measured on the KPNO PDS microdensitometer in 1980 January, using a 10 μm pixel, with 60 by 60 pixel scans. The PDS scans were processed using the Stetson (1979) image centering and photometric programs, which yield a magnitude index μ. The photographic magnitudes for the photoelectric sequence stars were fitted to a third order polynomial function of the iris measurement or the μ magnitude index by a least squares regression. The photographic magnitude is related to the photoelectric value by the color equation:

$$m_{\rm pg} = m_{\rm pe} + k \text{ C.I.} ,$$

where C.I. is the color index appropriate for the plate type, and k is the color term. Color terms were determined by an examination of the residuals in magnitude as a function of color for the photoelectric sequence stars. For M6, a color term of -0.10 was used for the V plates. Color terms for V, B, and U plates in M34 were found to be -0.10, 0.07, and 0.10, with values of -0.16, -0.07, and 0.00 for the Praesepe plates. A comparison of standard values for the photoelectric sequence stars to values derived from the photographic reductions yielded the following rms deviations for V, $B-V$, and $U-B$: 0.09, 0.12, and 0.12 for M6; 0.08, 0.11, and 0.05 for M34; and 0.12, 0.20, and 0.13 for Praesepe.

The color-magnitude and color-color diagrams for the photographic sample in each of these three clusters are shown in Figures 9–11 and 12–14. For the color-magnitude diagrams, the (M_v, $B-V$)-sequences for white dwarfs derived by Sion and Liebert (1977) are illustrated for comparison, following application of the distance modulus and color excess noted in Table 1. In the color-color diagrams, several sequences are illustrated: the main-sequence color-color relation of FitzGerald (1970), the Arp (1961) blackbody sequence, and the log $g = 7$ white dwarf sequence of Koester, Schulz, and Weidemann (1979). These sequences have been reddened

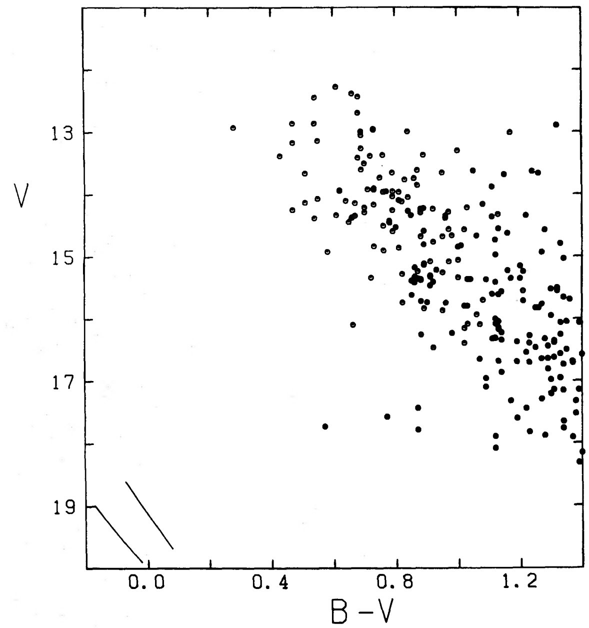

FIG. 9.—Color magnitude diagram for the program stars in M6. The (M_v, $B-V$)-relation of Sion and Liebert (1977) is shown for comparison.

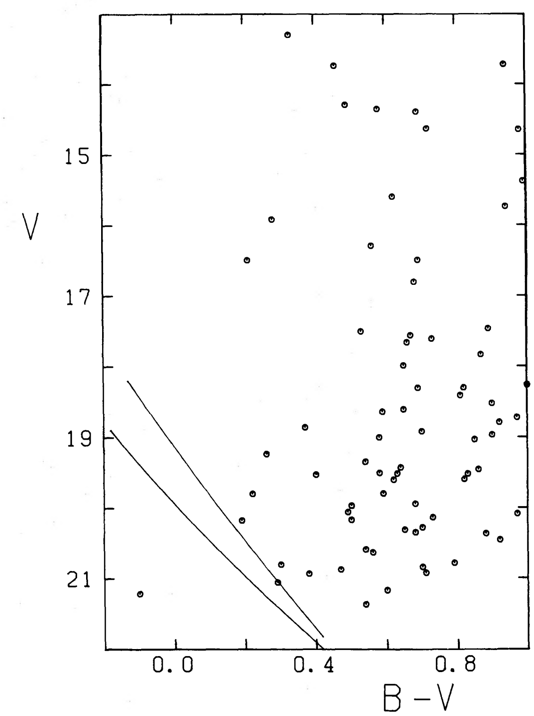

FIG. 10.—Color-magnitude diagram for the program stars in M34, with the (M_v, $B-V$)-relation of Sion and Liebert (1977) shown for comparison.

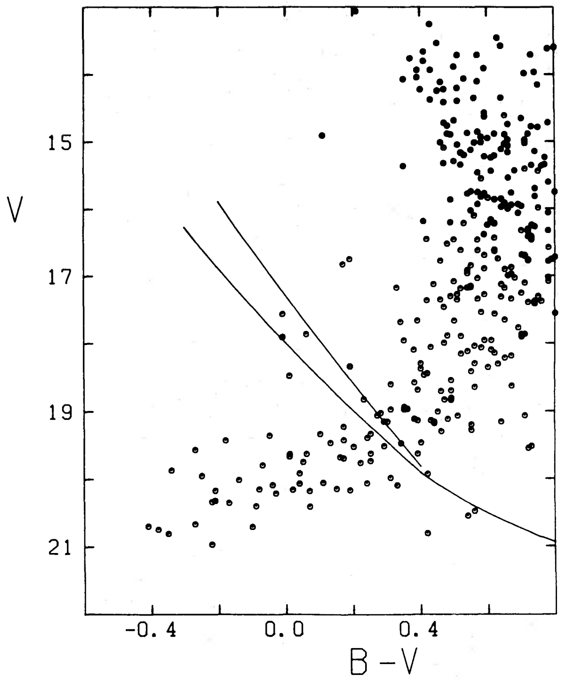

FIG. 11

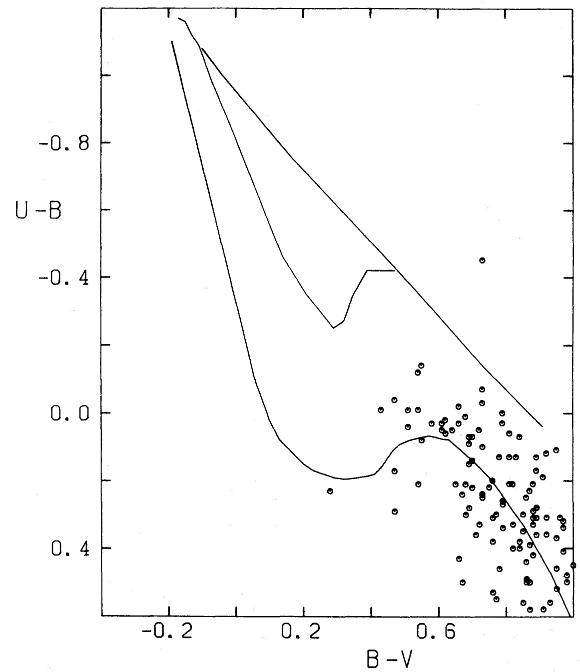

FIG. 13

FIG. 11.—Color-magnitude diagram for the program stars in Praesepe, with $(M_v, B-V)$-relation of Sion and Liebert (1977) shown for comparison.

FIG. 13.—Color-color diagram for the program stars in M34. The comparison sequences described in Fig. 12 are also shown; $E_{B-V} = 0.07$.

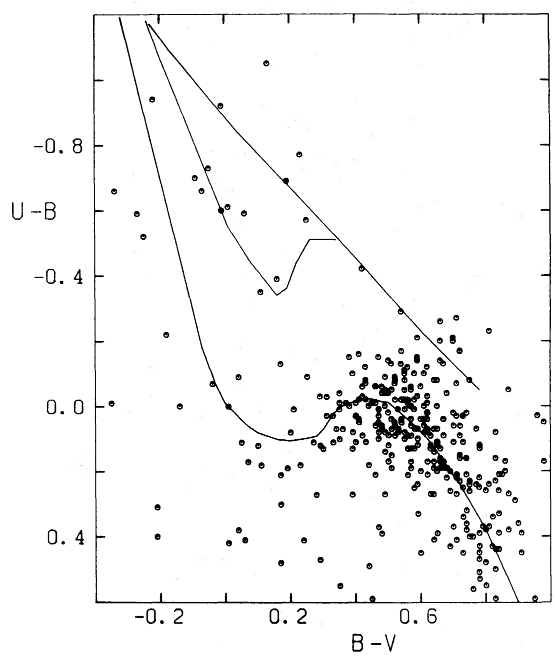

FIG. 12

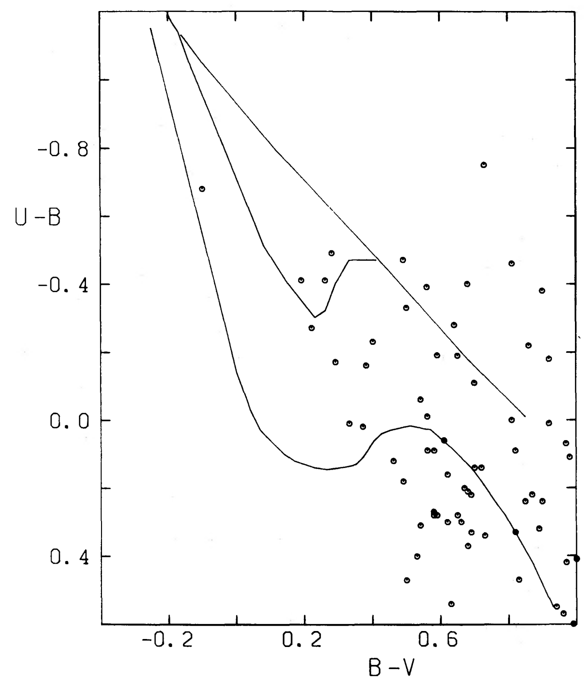

FIG. 14

FIG. 12.—Color-color diagram for the program stars in M6. Comparison sequences are for main sequence stars from FitzGerald (1970), the black body sequence of Arp (1961) and the white dwarf sequence of Koester, Schulz, and Weidemann (1979). Color excesses of $E_{B-V} = 0.13$ and $E_{U-B} = 0.72\ E_{B-V}$ have been applied to these sequences for comparison to the data.

FIG. 14.—Color-color diagram for the program stars in Praesepe. The comparison sequences described in Fig. 12 are also shown.

by the appropriate color excesses to facilitate comparison with the photometric data.

These initial results may be usefully compared to the expected numbers of field white dwarfs which might be present in the photographic fields. These expected numbers can be computed for a given area, for any apparent magnitude. If the luminosity function of Green (1980) for white dwarfs with $M_v \leq 13$ is employed, the number of white dwarfs in a field of w steradians is

$$N(V) = w\left(\frac{1000}{3}\right)10^{0.6V}\int_{10}^{13}\phi(M_v)10^{-0.6M_v}dM_v \,.$$

This reduces to a number of white dwarfs per square degree

$$N(V) = 10^{0.6V}(1.087 \times 10^{-11})\,.$$

For each of the three program clusters, the number of stars in apparent magnitude intervals has been computed for the accessible field area, and compared to the observed numbers; the results may be found in Table 7.

IV. ANALYSIS OF THE CLUSTER SURVEYS

It is now appropriate to consider the results of the individual cluster surveys in greater detail. Consideration should be given to the expected numbers of nonmember white dwarfs, the expected member population of white dwarfs, and the possible effects of photometric and areal limitations and of mass segregation.

a) *M6* (*NGC 6405*)

First, to consider the expected number of white dwarfs which might have evolved from the cluster, it is necessary to evaluate the richness of the cluster from the main-sequence population. Since the photoelectric surveys of Eggen (1961) and Talbert (1965) sample only a subset of the cluster stars, the larger photographic study by Vleeming (1974), covering 0.31 square degrees, was used to supplement the photoelectric observations. The field area of the present photographic survey is (0.31 ÷ 4) square degrees. From these data, the integral luminosity function, $\mathcal{N}(M_v)$, the number of stars brighter than M_v, has been constructed and compared with the composite cluster luminosity function compiled by Taff (1974) in order to verify a similarity of slope between the two. A main-sequence magnitude limit $M_* = 2.2$ was selected to minimize the effect on the luminosity function of incompleteness and field star contamination. From the compiled data, $N_{ms} = 73$; i.e., 73 stars are brighter than $M_* = 2.2$.

For an apparent distance modulus of 8.82, an age of 40 million years was inferred from the cluster color-magnitude diagram, corresponding to a turnoff mass of 6.2 $\mathcal{M}_\odot$. The absolute magnitude for an unevolved star of mass 6.2 $\mathcal{M}_\odot$ is $M_{to} = -0.9$, based on the models of Becker (1981). The number of white dwarfs which might be expected in the cluster, if stars up to mass $\mathcal{M}_{wd}$ form white dwarfs, can be calculated, as in § II. The values for $\mathcal{N}(M)$ were evaluated from the composite cluster luminosity function for all clusters of Taff. The estimates for N_{wd} for various values of $\mathcal{M}_{wd}$ may be found in Table 2, along with the uncertainties composed of the contribution due to the uncertainties in N_{ms} and the uncertainty due to the smallness of N_{wd} itself. The estimates listed in Table 2 have been divided by 4, in order to compare them to the results of a survey covering one-fourth of the Vleeming survey area.

Since an examination of the M6 color-magnitude and color-color diagrams reveals no promising photometric white dwarf candidates, several possible selection effects must be considered. Photometric limitations are the first obvious choice; the survey appears to cut off at $V = 18$. The sample of stars had been selected from an examination of blue plates, and an investigation of the luminosity function of stars as a function of B magnitude indicates completeness to $B \sim 19.4$. The apparent V magnitude limit at 18 results from the B limit and the predominantly red colors of the stars. Survey completeness for all stars with $B - V$ less than 0.0 to $V \sim 19.5$ may be considered to represent the sample characteristics of this survey. A young hot, white dwarf ought to have been selected in the photometric sample. There were no stars in the photographic sample for which U or B magnitudes were measurable but not V magnitudes, as would be the case for a blue object whose V magnitude was near or below the V magnitude limit. It does not appear that insufficient photometric depth can account for the lack of any faint blue objects.

A more serious limitation to the survey may be the restricted area of the sample. While the measured segments were chosen to sample the cluster randomly and without any radial bias, only one quarter of the cluster area of 0.31 square degrees could be measured. The number of white dwarfs predicted by the cluster population is small, and is not significantly different from zero for some assumed values of $\mathcal{M}_{wd}$.

The possibility of mass segregation needs to be considered as well, in light of the small masses of remnant stars. An estimate of the time scale for energy equipartition is given by the reference relaxation time as defined by Spitzer and Härm (1958):

$$T_{ref} = (8.3 \times 10^5)(Nr^3/m)^{1/2}(\log N - 0.3)^{-1}\ \mathrm{yr}\,,$$

where r is the radius in parsecs, N the number of stars, and

TABLE 7

PREDICTED AND OBSERVED NUMBERS OF BLUE OBJECTS IN CLUSTER FIELDS

V	M6	M34	Praesepe
15	0.00 (0)	0.00 (0)	0.00 (2)
16	0.00 (0)	0.02 (2)	0.02 (0)
17	0.01 (0)	0.07 (0)	0.09 (3)
18	0.05 (0)	0.30 (0)	0.38 (10)
19	0.20 (0)	1.10 (2)	1.50 (23)
20	0.80 (0)	4.30 (3)	6.00 (34)
21	3.30 (0)	17.30 (4)	24.00 (6)
Field area (square degrees)	0.077	0.400	0.550

m the mean mass in solar units. The apparent cluster radius is about 2.5 pc; and if N is estimated at 200 and the mean mass as 1, as would be expected for a typical mass function, the reference relaxation time is 23 million years. Since this is only half the age of the cluster, it seems unlikely that substantial mass segregation of the cluster population has taken place. This has been verified by an examination of the mean radial distances from the cluster center for stars in the photographic sample, binned in half-magnitude intervals. There is no systematic tendency for the size of the radial distribution to increase with magnitude, which indicates that mass segregation is still minimal in the cluster.

It does not seem, therefore, that the effects of mass segregation or sample limitations can be used to explain the lack of photometric white dwarf candidates in the accessible field, and the null result must be compared to the predictions listed in Table 2, with the principal limitation of this survey that of small areal coverage in the cluster in light of the small numbers of white dwarfs predicted by the luminosity function of the cluster. The discovery of no blue candidates in M6 may be regarded as consistent (to within ± 1 sigma) with a value for $\mathcal{M}_{wd} < 9\ \mathcal{M}_{\odot}$. No limit can be placed on $\mathcal{M}_{wd}$ if 2 sigma limits are considered.

b) *M34* (*NGC 1039*)

The richness of the main sequence of M34 has been evaluated from the results of a proper-motion study by Dieckvoss (1954). His study covered a field of 0.6 square degrees to an apparent visual magnitude of 12.5, finding 66 probable cluster members brighter than $M_v = 3.1$. From the comparison of photoelectric photometry to isochrones illustrated in Figure 3, an age of about 200 million years is indicated, corresponding to a turnoff mass of 3.0 $\mathcal{M}_{\odot}$, $M_{to} = 0.8$.

Using Taff's (1974) cluster luminosity function to evaluate $\mathcal{N}(M_v)$, and with $N_{ms} = 66$, N_{wd} can be computed. The estimates for the expected number of white dwarfs in M34 have been scaled by a factor of $\frac{2}{3}$ to simulate the anticipated results for a field of 0.4 square degrees, rather than the larger area in which Dieckvoss found his 66 numbers. These predictions, with their associated uncertainties, may be found in Table 2.

The blue-star finding list of Luyten (1961) was consulted for comparison following the $U - V$ blink survey. Of the 19 stars Luyten selected, three were not within the usable field of the 4 m plates. Two of Luyten's stars had been noted in the blink survey to be exceptionally blue, and six more had been selected in the survey as well. Eight of Luyten's stars were not selected in the blink survey. These eight stars were reexamined, and the four bluer of them were added to the photometric sample; all four turned out to have $B - V$ colors greater than 0.5.

If the rms errors for the photographic photometry in M34 are consulted, two candidates—34147 and 48096—are within one sigma of the expected sequences for cluster white dwarfs in both the color-color and color-magnitude diagrams. Another two stars—35149 and 43084—are within two sigma in all photometric parameters. Table 8 lists the photometric data for all stars in the photographic program which were found to have V magnitudes ≥ 16 and $B - V$ colors ≤ 0.5, as well as all the stars identified by Luyten (1961) as faint blue objects.

TABLE 8

PHOTOGRAPHIC PHOTOMETRY IN M34

Number	LB	V	$B-V$	$U-B$	N_V	N_B	N_U
12063........	3576	19.97	0.50	−0.33	1	3	2
13107........	3571	18.92	1.41	0.27	1	2	1
13115........	3569	18.97	0.90	−0.38	1	3	2
14091........	3577	20.14	0.73	−0.75	1	3	2
14146........	3574	18.42	0.81	−0.46	1	3	2
15118........	3567	17.96	1.99	...	1	3	0
16073........	...	20.06	0.49	−0.47	1	3	2
22189........	3575	19.80	0.59	−0.19	1	3	2
24144........	3570	19.33	1.06	−0.10	1	3	1
25078........	...	20.17	0.50	0.47	1	3	1
25194........	3572	18.31	0.82	0.33	1	3	1
32164........	...	19.53	0.40	−0.23	1	3	2
34147........	3583	19.80	0.22	−0.27	1	3	1
35149........	...	21.06	0.29	−0.17	1	3	1
38034........	...	21.22	−0.10	−0.68	1	3	1
41030........	...	16.49	0.21	0.76	1	2	1
41035........	...	18.86	0.37	0.02	1	3	2
42085........	...	15.92	0.28	−0.49	1	3	1
43036........	...	20.80	0.30	...	1	1	0
43084........	...	19.23	0.26	−0.41	1	3	2
44096........	3581	20.35	0.68	−0.40	1	3	1
45149........	...	20.93	0.38	−0.16	1	3	1
46029........	...	20.87	0.47	...	1	3	0
46077........	3582	20.59	0.54	−0.06	1	3	1
48096........	...	20.17	0.19	−0.41	1	3	1

The cooling calculations of Sweeney (Fig. 5) indicate that for white dwarfs with ages less than 250 million years, $B - V$ colors of less than approximately 0.2 might be expected. All four of the stars noted in the preceding paragraph have $B - V$ colors less than 0.2, if photometric errors are allowed for. By consulting Table 7, it may be noted, however, that the overall numbers of blue stars ($B - V \leq 0.4$) found in the M34 field are barely different from the expected number in each apparent magnitude interval contributed by field white dwarfs.

The possible effects of mass segregation again deserve consideration, since low mass remnants might assume a spatially more relaxed distribution than the brighter stars in the cluster. The formula for the reference relaxation time of Spitzer and Härm (1958) may be used to estimate the time scale for relaxation. If values of $N = 200$, $m = 1\ \mathcal{M}_{\odot}$, and $r = 3$ pc are used, then T_{ref} is estimated as 40 million years, substantially less than the age of M34. As was done for M6, the mean radial distances from the cluster center were calculated for stars designated by Dieckvoss as members, binned in magnitude intervals. The effects of mass segregation in a cluster might be manifested by a larger spatial distribution for fainter, hence less massive, main-sequence stars. There is no obvious correlation between the absolute magnitude and the mean radius for stars in absolute magnitude intervals, and hence little reason to expect that a population of member white dwarfs would be distributed in a larger area than the brighter cluster members.

Since there are no conspicuous ways in which a significant fraction of member white dwarfs should have been missed, it is appropriate to compare the results of the photographic survey to the numbers of white dwarfs predicted by an analysis of the cluster luminosity function. Four candidates have been found with photometric parameters consistent with M34's distance, reddening, and age. As these four stars are not a significant excess over the anticipated contamination by field white dwarfs, the actual number of cluster white dwarfs may be smaller, and the indicated upper limits for $\mathcal{M}_{wd}$ might be overestimates. If four candidates are compared with the predictions listed in Table 2, $\mathcal{M}_{wd}$ appears to be less than 4 $\mathcal{M}_{\odot}$, and less than 6 $\mathcal{M}_{\odot}$ if two sigma limits are considered.

c) Praesepe

Johnson's (1952) photoelectric sample survey was selected from stars listed as members by Klein-Wassink (1927), who studied the proper motions of 577 stars in a field of Praesepe with a radius 55′. The number of members found that were brighter than $m_{pg} = 14$ was 188. Photographic magnitudes have been converted to V magnitudes by utilizing a linear relation between M_{pg} and V for 96 main-sequence stars for which Johnson measured photoelectric values. With this transformation, it may be determined that there are 69 Klein-Wassink members brighter than $M_v = 4$.

The age of Praesepe has been determined as 7×10^8 years, corresponding to a turnoff mass of 1.8 $\mathcal{M}_{\odot}$. With values for $N_{ms} = 69$, $M_* = 4$, and $M_{to} = 2.1$, N_{wd} can be predicted for various values of $\mathcal{M}_{wd}$. These estimates ought to be altered (*a*) to account for the possible areal incompleteness of the Klein-Wassink sample, (*b*) to account for the selectively larger areal distribution of white dwarfs compared to main sequence-stars, if the cluster is dynamically relaxed, and (*c*) to simulate the expected number to be found in a field which has roughly one-fifth the area of the Klein-Wassink field, since the field area afforded by the 4 m prime focus plate is considerably less than that of the entire cluster.

Praesepe is sufficiently old that dynamical relaxation has probably played a role in segregating the more massive stars to a more concentrated distribution than the smaller main-sequence stars or equally small mass remnant stars. If the Spitzer and Härm (1958) formula for the reference relaxation time is applied for values $m = 1$, $r = 3$, and $N = 200$, a time scale of 30 million years is indicated, less than $\frac{1}{20}$ the cluster age. The cooling time for white dwarfs with $B - V$ colors less than 0.3, as is the case for the three Eggen and Greenstein (1965) white dwarfs in Praesepe, is predicted by Sweeney (1976) to be 5 or 10×10^8 years, an order of magnitude greater than the reference relaxation time. Since the time scale for relaxation is short compared to the cooling time, member white dwarfs might be expected to behave dynamically like low-mass main-sequence stars.

There is considerable similarity between the Hyades and Praesepe, with respect to their color-magnitude diagrams, space motions (Eggen 1958), and metal abundance (Henry, Anderson, and Hesser 1977). This similarity has been utilized to estimate the effects of mass segregation and completeness. Preliminary estimates of white dwarf population based on $N_{ms} = 69$ would be based on the main-sequence statistics within a restricted area. Referring to the results of more extensive studies in the similar cluster, the Hyades, it must be considered that this survey area may greatly underestimate the actual main-sequence population. The van Altena (1969) and Hanson (1975) studies in the Hyades cover a square area 6°.4 on a side, finding 41 main-sequence members brighter than $M_v = 5.1$ and four white dwarfs in that area. The much larger survey by Wayman, Simms, and Blackwell (1965) found 112 main-sequence members brighter than $M_v = 5.1$ in an area 30 times as large. Three more Hyades blue white dwarfs, whose memberships were confirmed by van Altena, are within this larger area. If the Wayman *et al.* survey may be assumed to be complete, this central 6°.4 square area contains 0.37 ± 0.18 of the main-sequence stars brighter than $M_* = 5.1$ and 0.57 ± 0.36 of the blue white dwarfs.

The 55′ radius of the Klein-Wassink survey in Praesepe corresponds to a physical radius of 2.7 pc at the distance of the cluster. Since the Hyades is 3.63 times closer than Praesepe, a survey with radius 3°.3 would sample the same physical dimensions in that cluster. The subset of the van Altena Hyades survey which samples an area of this size contains 40 stars brighter than M_*, or 0.36 ± 0.18 of the main-sequence stars in the cluster and 0.58 ± 0.36 of the blue white dwarfs.

The white dwarfs as well as the main-sequence stars in both the Hyades and Praesepe might be expected to exhibit dynamically relaxed distributions of stars. The distribution of stars in each cluster has been explored by constructing the mean distances from the center of member stars in magnitude intervals, and these are displayed in Figure 15. A radial unit of 2.7 pc has been used, corresponding to angular distances of 200′ and 55′ for the Hyades and Praesepe, respectively. The radial distances which circumscribe half the total survey area in each cluster are indicated in the diagram. The giants (indicated by a symbol G) and brightest stars are centrally concentrated in both samples, with mean radial distances of about 1 pc from the cluster center. In the case of Praesepe, the four stars in the interval $M_v = 1$–1.5 are conspicuously more concentrated than the giants, the brighter apparent "stragglers," or the slightly fainter main-sequence stars. At least one of these four stars is known to be a binary. The four blue white dwarfs which are within the van Altena Hyades field, indicated by a B in the diagram, are not apparently as concentrated to the center as are the giants. The error bars indicate the standard error of the mean radial distance of the stars in each interval of magnitude.

Referring to the mean radial distances from the center for stars in different magnitude intervals in the Hyades and Praesepe, as depicted in Figure 15, several points are apparent. In all cases where known binaries are included in the sample for an interval of visual magnitude, the mean radial distances for that sample are smaller than the mean radial distances if the binaries are excluded. This

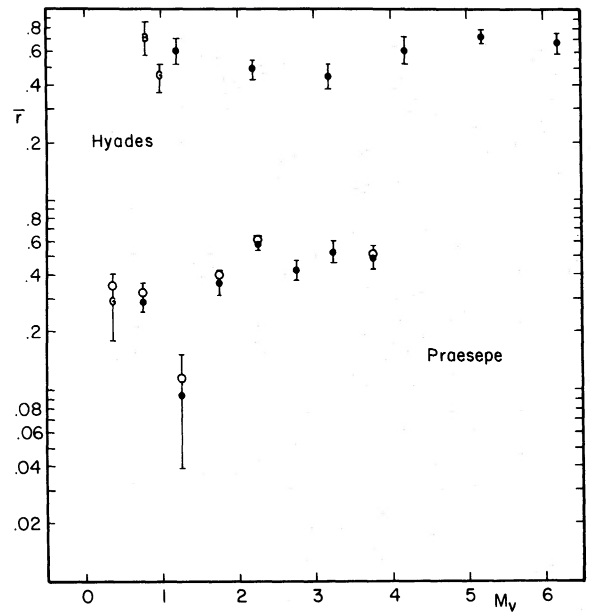

FIG. 15.—Mean radial distances from the cluster center for stars in the Hyades and Praesepe, binned in intervals of absolute magnitude. The unit of distance corresponds to a distance of 2.7 pc, 200′ in the Hyades and 55′ in Praesepe. Open circles denote mean radial distances calculated following the exclusion of known binaries, and the error bars reflect the standard error of the mean radial distance in each magnitude bin. Symbols G and B denote giants and blue subluminous stars.

reflects the dynamical tendency for binary systems to seek central positions in clusters (Aarseth 1979), borne out in observations for older ($> 10^8$ yr) clusters by Abt (1980). This may explain in part the central concentration of the giants; two of the Hyades giants are suspected binaries (Griffin and Gunn 1977).

Moreover, the Hyades white dwarfs (confining this observation to the four blue white dwarfs within the field area of van Altena's survey) are apparently not as concentrated to the center as are the giants. In fact, the mean radial distance of ~ 0.7 of the survey radius for the blue Hyades dwarfs is about what one would expect for a uniform distribution of stars across the survey areas.

It appears that the spatial distribution of stars in these two clusters are quite similar, and that the results of studies of the spatial distribution of stars in the Hyades, may be used to predict the proportion of Praesepe members which lie beyond the survey area of Klein-Wassink. It may be expected that the total number of main-sequence stars in Praesepe is larger than 69 by a factor of (112/41), in analogy to the ratio of the number of Hyades main-sequence members found in the much larger survey area of Wayman *et al.* to the number found in the van Altena survey area (41). This implies that the number of main-sequence stars in Praesepe would be 193 if a larger survey area were studied, and the resulting estimates of N_{wd} should also be larger by a factor of 2.8. Of these predicted white dwarfs, approximately 57% may be expected to lie within the original 55′ radius survey area, and it is expected that they will not be very centrally concentrated. Finally, the estimates have been further reduced by 0.20 to simulate predictions for a field 4.7 times smaller than the Klein-Wassink survey area.

Luyten (1962) first searched for Praesepe white dwarfs in a survey of the Palomar Sky Survey plates in the region, identifying some 1000 blue stars. Three of these were studied photoelectrically by Eggen and Greenstein (1965), in their survey of white dwarfs. These are LB 390, 1847, and 393 (EG 59, 60, and 61, respectively).

A comparison was not made between the blink-selected sample of stars and Luyten's finding list of blue stars until the completion of the photographic photometry. Of the 85 stars in Luyten's sample that intersect the survey area in Praesepe, 20 are described by Luyten as having a color index of less than 0.0. Of these 20 stars, 14 were found in the blink survey and measured, yielding a mean photographic $U - V$ color of -0.52. This suggests a completeness with respect to Luyten's survey of about 70%. One significant indication of incompleteness in the faint blue star search was the noninclusion of LB 393. Based on its loss in one of several steps of identification and subsequent measurement, and the estimate of completeness with respect to Luyten's sample, the numbers of white dwarfs found in the photographic sample should be upgraded by 45% to account for survey errors and incompleteness.

Based on the photometric results with respect to the fiducial sequences in the color-color and color-magnitude diagrams, there appear to be eight good *UBV* candidates for Praesepe white dwarfs, those stars designated with a "1" in Table 9 which describes the subset of the total photographic sample with the following photometric properties: $V > 16$, $B - V \leq 0.5$, and $U - B < 0.00$. Seven other candidates appear to be degenerate, but fall rather far from the white dwarf color-magnitude sequence expected for the cluster as illustrated in Figure 14. It should be noted that all but one of the eight best candidates are bluer than 0.40 in $B - V$. If the cooling calculations of Sweeney (Fig. 5) are consulted, it can be noted that white dwarfs younger than 10^9 years are not expected to be redder than $B - V = 0.35$.

The number of blue objects found in the Praesepe field must be compared to the number expected from contamination by foreground and background blue stars. From Table 7, only one or two field white dwarfs is expected at $V = 19$, and six at $V = 20$. From the photographic results, ten stars are bluer than $B - V = 0.4$ with $V = 19 \pm \frac{1}{2}$, and 13 in the magnitude interval 1 mag fainter. It is therefore reasonable to regard the eight photometric white dwarf candidates as a significant excess over any anticipated field star population, particularly for the candidates brighter than $V = 18$ since fewer than one field white dwarf of similar magnitude might be expected in the field.

Based on the 70% estimated completeness relative to the Luyten finding list of blue stars in Praesepe, the numbers of candidate white dwarfs ought to be upgraded by 45% to account for survey errors and incompleteness.

TABLE 9

Photographic Photometry in Praesepe

Number	LB	WDC	V	$B-V$	$U-B$	N_V	N_B	N_U
11010	5852	2	19.68	0.16	−0.39	2	2	1
11053	5866	...	18.83	0.47	0.36	2	2	2
12062	5845	...	18.10	0.38	0.00	2	2	2
12130	...	...	17.68	0.50	−0.14	2	2	2
12131	...	...	19.74	0.25	−0.09	2	2	2
13039	...	...	20.90	−0.88	−0.77	2	1	1
13100	...	...	13.54	0.45	−0.01	2	2	2
13105	5799	...	17.96	0.35	0.01	2	2	2
13126	5807	...	18.06	0.43	−0.08	2	2	2
13144	...	...	16.46	0.50	−0.04	2	2	2
14016	...	...	18.30	0.40	−0.01	2	2	1
14026	5778	...	19.63	0.01	0.00	1	2	1
14031	5779	...	19.87	−0.34	−0.66	2	1	2
14074	...	...	19.95	−0.24	−0.52	2	1	2
14077	...	...	18.58	0.38	−0.15	2	2	2
14078	...	...	20.01	−0.14	0.00	1	2	1
15008	5860	...	20.73	−0.62	−0.62	2	2	2
15038	...	1	18.48	0.01	−0.61	2	2	2
15067	...	...	17.67	0.39	−0.10	2	2	2
16009	5763	...	19.63	0.06	0.41	2	2	1
16047	...	...	20.81	−0.35	−0.01	1	1	1
17007	5798	2	19.47	0.13	−1.05	2	2	2
21036	5854	...	18.22	0.65	0.39	2	2	2
22016	5961	...	19.63	0.25	...	2	2	0
22022	1839	1	18.83	0.23	−0.77	2	2	2
22043	...	1	19.93	0.42	−0.42	1	2	1
23014	3800	2	20.40	−0.09	−0.70	1	1	1
23024	1832	...	19.03	0.28	0.27	2	2	2
23049	5810	...	19.12	0.48	0.39	2	2	1
24002	...	...	19.23	0.17	−0.13	2	2	2
24072	...	...	16.19	0.41	−0.16	2	2	2
25020	...	...	16.53	0.48	−0.06	2	2	2
25032	5788	2	19.36	−0.05	−0.73	1	2	2
25039	5780	...	19.48	0.34	...	2	2	0
25042	5770	...	19.34	0.10	0.12	1	2	1
25068	...	...	16.21	0.49	−0.05	2	2	2
26004	384	1	17.87	0.06	−0.59	2	2	2
26036	380	...	16.76	0.19	0.19	1	2	2
27009	1829	...	19.57	−0.27	−0.59	2	2	2
27034	5756	...	19.67	0.01	0.42	2	2	2
31052	1847	1	18.35	0.19	−0.69	2	2	2
32052	5889	...	18.42	0.55	0.10	1	2	1
32087	5934	...	20.34	−0.22	−0.94	2	2	1
32094	5910	...	19.43	0.17	0.30	2	1	1
33009	1842	2	19.80	−0.07	−0.66	1	2	2
33061	5946	...	19.16	0.29	−0.03	2	2	1
34014	5885	...	18.45	0.42	−0.06	2	2	2
34030	390	1	17.91	−0.01	−0.60	2	2	1
35013	5951	...	20.15	0.15	...	2	2	0
35024	5965	...	19.16	0.44	0.49	2	2	1
36003	...	...	21.35	−0.62	−0.70	1	2	1
41003	...	...	19.34	0.25	−0.58	2	2	2
41006	386	...	17.18	0.33	0.03	2	2	2
41022	5871	...	18.65	0.56	−0.01	2	2	2
41028	387	2	14.91	0.11	0.18	2	2	2
41034	5886	...	18.98	0.32	−0.03	2	2	1
42085	5877	...	20.81	0.42	...	2	1	0
43004	...	...	18.98	0.35	−0.01	2	1	2
43030	5936	2	20.06	0.11	−0.35	1	2	1
43060	...	...	18.84	0.49	−0.09	2	2	1
43061	5912	...	19.40	0.24	0.41	1	1	1
43068	5893	1	17.57	−0.01	−0.92	1	2	2
43102	...	...	16.46	0.42	−0.03	2	2	2
44004	5974	...	20.09	−0.04	−0.07	1	2	1
44009	5967	...	18.12	0.54	0.05	2	2	2
44023	...	...	20.07	0.04	−0.09	2	1	1
44055	...	...	17.47	0.47	−0.15	2	1	2
45010	5938	...	18.70	0.49	0.14	2	2	2
45019	1850	...	17.36	0.46	−0.06	2	1	2
45056	5872	...	18.55	0.49	0.27	2	2	2
45064	...	1	19.44	−0.18	−0.22	2	2	1
46028	5995	...	19.53	0.20	0.08	2	1	1
47001	5953	...	19.07	0.27	0.11	1	2	2

This would imply that 12 ± 3 candidate white dwarfs lie within the survey area of the Praesepe field.

There is little reason to expect that the field in Praesepe has sampled other than an unbiased 20% of the survey area defined by Klein-Wassink's study. The observed result of 12 ± 3 photometric white dwarf candidates can be compared to the predicted numbers of white dwarfs for this field area, found in Table 2. Only the predictions for $N_{\rm wd}$ based on an upper mass limit of 2 $\mathscr{M}_\odot$ can be ruled out by a comparison to the observed number, if one sigma limits are considered.

V. DISCUSSION

The results of studies in these additional three clusters have added a little information on the upper mass limit for white dwarf progenitors, and a few caveats. The value of two photometric colors to select photometric candidates is a considerable gain over dependence on ultraviolet excess alone. The possible confusion of white dwarfs with galaxies at high galactic latitudes, and with galactic O and B subdwarfs and dwarfs at low latitude, has been demonstrated; the best possible *UBV* photometry and photoelectric calibration is necessary, but still not sufficient, to unambiguously identify white dwarfs; spectroscopic confirmation is also necessary.

The results of this investigation and of several other studies in open cluster, are illustrated in Figure 16. Lower limits indicated by the turnoff mass in a few well studied clusters are indicated by the darkest shading. Diagonal and cross hatching indicate upper limits placed on $\mathscr{M}_{\rm wd}$ by 1 and 2 σ comparisons between observed and predicted numbers of white dwarfs in other clusters. The most recent results in NGC 2287 by Koester and Reimers (1981) have not been included in the figure, but would lend support to a firm lower limit of 4 $\mathscr{M}_\odot$ for $\mathscr{M}_{\rm wd}$ based on that cluster. The Hyades and Pleiades constitute a class apart, for the reason that the main-sequence stars and spectroscopically observed white dwarfs are astrometrically confirmed members. For most of the other clusters (excepting NGC 2287 and IC 2602), the evidence for cluster white dwarfs is only photometric, and for no other clusters have proper-motion studies confirmed the membership of candidate white dwarfs.

Based on the work to date, the upper mass limit for white dwarf progenitors appear to be 5 ± 1 $\mathscr{M}_\odot$. These limits are conservative attempts to reconcile results in the

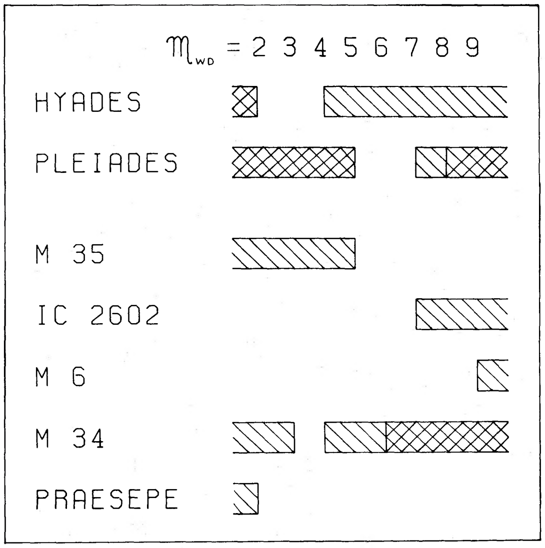

FIG. 16.—Diagram illustrating limits placed on $\mathcal{M}_{wd}$ by several open cluster studies. Lower limits are provided by turnoff masses of clusters with white dwarf members; astrometrically studied clusters show cross-hatched limits. Diagonal and cross hatching indicate upper limits placed on $\mathcal{M}_{wd}$ by 1 and 2 σ comparisons between observed and predicted numbers of white dwarfs in clusters.

Hyades and Pleiades. This result is consistent with the theoretical upper limits permitted by a consideration of steady giant branch mass loss, discussed in § I, and near the statistically derived lower limit of 6 $\mathcal{M}_{\odot}$ for pulsar progenitors of Shipman and Green (1980).

The value of the upper mass limit for white dwarf precursors is meaningful only within some framework of stellar evolutionary theory for single stars. The complications of binary evolution are only beginning to be explored. Webbink (1979) has pointed out that some types of mass transfer binaries with primary masses as high as 14 $\mathcal{M}_{\odot}$ can lead to a white dwarf. The observation of one white dwarf–M dwarf binary in the Hyades lends credence to this cautionary remark. It should be kept in mind, therefore, that the previously discussed results from open cluster studies are predicted on the assumption of primarily single-star evolution, and cannot preclude much more massive binary components as progenitors of white dwarfs.

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