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# THE OOSTERHOFF PERIOD GROUPS AND THE AGE OF GLOBULAR CLUSTERS. IV. FIELD RR LYRAE STARS: AGE OF THE GALACTIC DISK

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## ABSTRACT

Individual period shifts, $\Delta \log P$, determined by comparing the periods and amplitudes of $\sim 125$ Bailey type *ab* RR Lyrae field stars with the period-amplitude relation for M3, are well correlated with Preston's $\Delta S$ index. The distribution of $\Delta \log P$ forms a continuum over the interval $+0.15 > \Delta \log P > -0.12$ which embraces the metallicity range of $0 \leq \Delta S \leq 10$. The slope of the correlation is $\partial \Delta \log P / \partial [\mathrm{Fe/H}] = 0.116$ which, by the argument in Paper III, is the condition that all RR Lyrae stars that obey the correlation have the same age to within the accuracy of the method at $\sim \pm 10\%$.

Because the high-metallicity RR Lyrae variables with $\Delta S \leq 2$ are known to be old disk objects, it follows from their great age that this component of the disk began to form at closely the same time as the halo globular clusters. A similar conclusion follows by noting that older giants than those in NGC 188 also exist in the disk. The field giants studied by Wilson, by Helfer, and by Janes in the luminosity range $0.5 \leq M_v \leq 1.5$ became progressively bluer than those in NGC 188, as [Fe/H] is decreased. The observed color gradient with metallicity of $\partial (B-V)_{0,g} / \partial [\mathrm{Fe/H}] = +0.44$ agrees with the color difference between giants in NGC 188 and the 47 Tuc in the composite *C-M* diagram at $M_v = +1$.

A summary diagram is given which shows again that the variation of [Fe/H] with age for various components of the Galaxy was rapid during halo collapse, but that the enrichment of the disk was slower as this structure was built over a longer time.

*Subject headings:* clusters: globular — stars: evolution — stars: RR Lyrae

## I. INTRODUCTION

Absolute magnitudes of the main sequence termination points for low-metallicity globular clusters are brighter than the turnoffs of more metal-rich clusters if the luminosities of their respective horizontal branches are placed at different levels according to their Oosterhoff-Sawyer period shifts (Sandage 1982, hereafter Paper III). If their horizontal branch levels are ordered by $\Delta M_{\mathrm{RR}} = 3 \Delta \log P$, expected from $P \langle \rho \rangle^{1/2} = Q$ in the sense that clusters with longer period variables have brighter horizontal branches, then the observed gradient of turnoff magnitudes with metallicity is $\partial M^{\mathrm{TO}} / \partial [\mathrm{Fe/H}] = +0.3$. This is also the required theoretical value if clusters with different metallicities have equal ages (Simoda and Iben 1968, 1970; Iben and Rood 1970; Ciardullo and Demarque 1977). From this it was concluded in Paper III that the sample of 30 clusters studied there have the same age to within $\sim \pm 10\%$, centered upon an absolute value of $t = (17 \pm 2) \times 10^9$ years.

Most of the clusters that could be age dated in this way are in the halo; hence because of their nearly equal age, it follows that the formation of these halo components was relatively rapid, taking less than $\sim 2 \times 10^9$ years, which is the accuracy of the method. Even the high-metallicity, old-disk globular clusters of NGC 6838 and 47 Tuc were found to have the same old age, and hence must also have formed during the initial rapid galactic collapse, not during the later more gradual building of the disk.

Gradual as the formation of the *complete* disk may have been, it must nevertheless have begun to form as soon as the first gas from the halo, not yet fragmented into stars (Hoyle 1953), met gas falling from other directions and, by dissipation, settled into the rotation plane (Eggen, Lynden-Bell, and Sandage 1962).

The minimum time for the beginning of suck disk formation is the free-fall period from the wider halo regions. As there is a hierarchy of free-fall times (Searle and Zinn 1978) depending on the mean density of the Galaxy inside a given spherical boundary centered on the nucleus (Sandage 1970, n.1), the disk has formed over an appreciable time; matter far out took longer to fall than that closer in. As the shortest fall time is of order $\lesssim 5 \times 10^8$ years, it is of considerable interest to know if the *oldest* disk stars are, in fact, nearly as old as the halo, to within this difference.

To this end, we apply here the age dating method of Paper III to the field RR Lyrae variables. The high-metallicity variables with $\Delta S \lesssim 3$ belong to the disk; their $W$ velocities are low, and the eccentricity of their orbits is small (Preston 1959; Spite 1960; Epstein 1969; Epstein and de Epstein 1973). And because these variables are undoubtedly among the oldest disk stars that exist, their age dating provides important information for this problem.

In this paper we continue the analysis of the Oosterhoff period shifts, applied here to the field RR Lyrae stars as a whole. It is shown in § II that the period shifts, determined by comparison with the period-amplitude relation for M3, form a *continuous* distribution over the large range $-0.12 < \Delta \log P < +0.15$, at constant amplitude. It is also shown that the correlation of [Fe/H] with these period shifts for field variables, discovered by Preston (1959, his Fig. 5), is the same correlation that applies to variables in the clusters. Furthermore, the correlation extends linearly to the field variables of highest metallicity ($\Delta S \leq 2$) not represented in the clusters themselves. It is this property that permits age dating of the disk variables.

It is shown in § III that much older disk stars exist than are present in NGC 188. The observed lower metallicities of the older disk giants place them blueward of the NGC 188 giant-star envelope; hence this envelope itself does *not* age date the oldest component of the disk.

Finally a new summary is given in § IV of the rapid variation of [Fe/H] with age in the early disk, and its much slower subsequent enhancement. The data are ages of globular clusters of different [Fe/H] determined in Paper III, together with the ages and [Fe/H] values of NGC 188, M67, and the Sun, and disk stars in the solar neighborhood.

## II. CORRELATION OF PERIOD SHIFTS WITH METALLICITY FOR FIELD RR LYRAE STARS

Preston (1959, Fig. 5) showed that field variables with different $\Delta S$ values populate different regions of the period-amplitude diagram. He divided his sample into three categories of metallicity and found that the period-amplitude relations were different for each. RR Lyrae stars of lowest metallicity have the longest period at a given amplitude, and vice versa. This discovery was the extension to the field variables of the same effect found by Arp (1955) for globular clusters; the two broad Oosterhoff period groups are also separated by metallicity. However, Preston's discovery in the field extended Arp's result to the highest metallicity variables with $\Delta S \leq 2$ that are not well represented, if at all, in the clusters. These variables have the shortest periods for any given amplitude, reaching values as small as $P_{ab} \approx 0.33$ days.

From the form of these results it had been widely believed that the period-amplitude-metallicity relations, shown as three broad groups in Preston's Figure 5 and as the two broad Oosterhoff groups in Arp's dichotomy, gave only a general indication of a correlation of period shifts with metallicity, but did not hold precisely for any individual star. It then came as a surprise that the ($\Delta \log P$, [Fe/H])-correlation was, in fact, good for the globular clusters (Paper III, Fig. 4). The scatter is scarcely larger than the observational errors in [Fe/H] and $\Delta \log P$ considered together.

From Paper I (Sandage, Katem, and Sandage 1981) and Paper II (Sandage 1981) it is, however, possible to understand this if (1) amplitude is a unique and very precise function of horizontal position in the RR Lyrae instability strip and (2) the absolute luminosity level of the horizontal branch is tightly correlated with [Fe/H] for reasons stemming from the equality of age of globular clusters of different metallicity (Paper III, § V).

We now show that a *continuous* correlation exists between metallicity and period shift at fixed amplitude for the field RR Lyrae stars, and that the relation has the same form as for the globular clusters. If the cause is the same as described above, it then follows by the argument of Paper III that all field RR Lyrae variables which obey this ($\Delta \log P$, [Fe/H])-relation have the same age to within limits set there. Recall that this will be the case if the gradient of $\Delta \log P$ on [Fe/H] is close to

$$\partial \Delta \log P / \partial [\mathrm{Fe/H}] \approx +0.1, \tag{1}$$

(Paper III, § V*b*). We now show that the field RR Lyrae stars satisfy equation (1).

The period-amplitude relation for M3 is adopted as fiducial, and $\Delta \log P$ is calculated for any given field variable from it. Using the linearized approximation to this M3 relation gives

$$\begin{aligned} \Delta \log P &\equiv \log P(M3) - \log P(\text{observed}) \\ &= -[\log P + 0.129\, A_B + 0.088]. \end{aligned} \tag{2}$$

This equation has been applied to the sample of field RR Lyrae stars that have (1) known $\Delta S$ values either from Preston (1959) or from other observers on the Preston system, and (2) photoelectric light curves from which the $A_B$ amplitudes can be found. Only variables with photoelectric photometry were considered. The sources for light curves are Spinrad (1959), Sandage (1960), Kinman (1961), Jones (1965), Harding and Penston (1966), Fitch, Wisniewski, and Johnson (1966), Clube, Evans, and Jones (1967), and Lub (1977).

The correlation of the calculated period shifts with metallicity for ~125 field variables is shown in Figure 1. The top panel shows the correlation with $\Delta S$, the bottom with [Fe/H] using Butler's (1975) calibration of

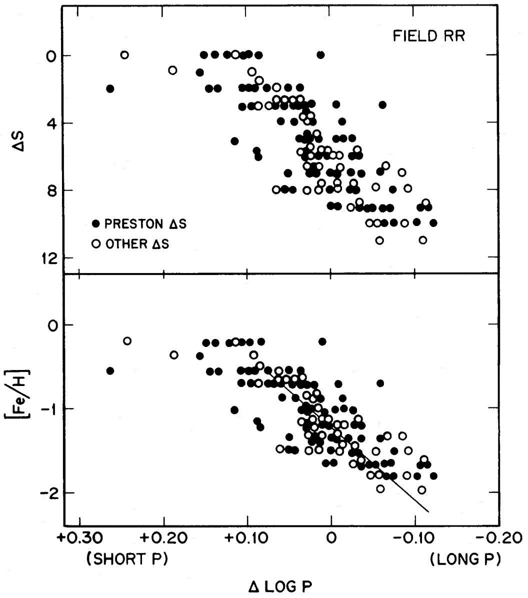

FIG. 1.—*Top panel*, correlation of $\Delta S$ with period shifts, determined using the M3 period-amplitude relation as fiducial, for field RR Lyrae stars that have photoelectric light curves. *Bottom panel*, same as the top using [Fe/H] from Butler's (1975) calibration of $\Delta S$ and metallicity. The line is the globular cluster correlation of Paper III, shifted by 0.2 in [Fe/H] for the difference between Butler and Zinn's calibrations.

$[\mathrm{Fe/H}] = -0.16\Delta S - 0.23$. The errors on the coordinates are $\varepsilon \Delta S \approx \pm 2$ and $\varepsilon \Delta \log P \approx \pm 0.02$, both of which are larger by a factor of $\sim 2$ than the corresponding errors for the equivalent parameters in the clusters (Paper III, Fig. 4). The reason is that only single determinations exist of $A_B$ and $\Delta S$ for individual field stars, whereas the period shifts and [Fe/H] for the clusters represent averages over many stars. Given the observed errors, the observed scatter in Figure 1 is of the order expected from them alone; hence the true $(\Delta \log P, [\mathrm{Fe/H}])$-correlation must also be nearly as tight for these field variables as for the clusters.

The line drawn in the lower panel of Figure 1 is the cluster correlation (Paper III, Fig. 4 and eqs. [2] and [13]), but shifted by $\Delta[\mathrm{Fe/H}] = 0.2$ for the known difference between the metallicity scales of Butler (1975) and of Zinn (1980). The slope of this line is $\partial \Delta \log P / \partial [\mathrm{Fe/H}] = 0.116$, which fits the data well enough.

The line has been drawn only to $\Delta \log P \approx +0.10$ and $[\mathrm{Fe/H}] \approx -0.5$ which is the limit of the globular cluster correlation. The next most important feature of Figure 1, after noting that the slope is the same as for the clusters, is that the correlation extends into the $\Delta S \leq 2$ high metallicity domain with the same slope. These RR Lyrae stars are clearly part of the same distribution, forming its high-metallicity end. It therefore seems plausible that the explanation developed in Paper III for the $(\Delta \log P, [\mathrm{Fe/H}])$-correlation that applies in the $-2.3 < [\mathrm{Fe/H}] < -0.5$ domain should also apply to them. Hence, rather than repeat the argument given there, we assert that the slope of the correlation in Figure 1 requires the ages of field RR Lyrae variables of all metallicities including those with $\Delta S \leq 2$ to be the same to the accuracy of the method, which is $\Delta t/t \approx \pm 10\%$.

## III. AGE OF THE OLDEST COMPONENT OF THE GALACTIC DISK

### *a) The High Metallicity RR Lyrae Stars*

The kinematic properties of the short-period $\Delta S \leq 2$ Bailey *ab* variables show them to be disk objects (Pavlovskaya 1953; Preston 1959; Spite 1960; Epstein 1969; Saio and Yoshii 1979). To illustrate this again, the $|W|$ velocity (perpendicular to the galactic plane) and the eccentricity of the galactic orbits for RR Lyrae stars are plotted versus $\Delta S$ in Figure 2, taken from the calculations of Saio and Yoshii (their Table 4). In the lower panel only RR Lyrae stars with proper motions marked A in that reference have been plotted. In the upper panel such variables have been separated into those whose $|W| \lessgtr 60$ km s$^{-1}$.

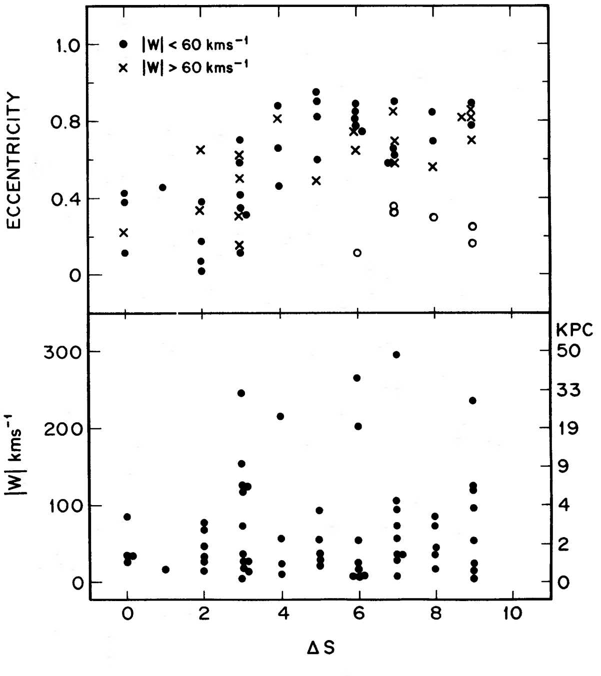

FIG. 2.—Correlation between $\Delta S$ and $W$ velocity perpendicular to the plane and eccentricity of the galactic orbit for RR Lyrae stars used by Saio and Yoshii (1979). The maximum height reached by stars with given $W$ values are shown along the right hand ordinate of the lower panel, as calculated in the cited reference. Open circles are low metallicity variables in nearly circular orbits, and may form a separate group.

In both these diagrams it should be remembered that the proper motions of the RR Lyrae stars are very uncertain; hence, errors in the ordinates are large. Nevertheless it is clear from Figure 2 that variables with $\Delta S \leq 2$ have small $|W|$ velocities and small orbital eccentricities; hence they belong to the galactic disk. The conclusion is strengthened further by noting that Saio and Yoshii used $M_v = +0.6$ as the absolute magnitude for RR Lyrae variables of all $\Delta S$ values, whereas if absolute magnitudes are assigned by $\Delta M_{\rm bol} = 3\Delta \log P$ as required by the Oosterhoff shifts, then $M_v$ for $\Delta S =$ 0–2 stars will be ~0.8 mag fainter than for $\Delta S = 10$. Hence $|W|$ and $e$ for them will be even smaller than shown in Figure 2.

Since these high-metallicity variables are disk objects, and since, by the arguments of § II, they are the same age as the lower metallicity variables and the globular clusters, it follows that the oldest component of the disk has an age comparable to that of the halo. The same conclusion can be reached using old disk field giants.

### *b*) *The Old Field Giants*

It is well known that the subgiant and giant branches of the old open cluster NGC 188 form an adequate lower envelope to the *C-M* diagram of field giants in the solar neighborhood (Sandage 1962; Wilson 1976, Fig. 6). An earlier demonstration that the lower envelope of the field star distribution was fainter than the sequences in the somewhat younger cluster of M67 had been given by Wilson (1959). Because it was also known that the M67 and NGC 188 stars have nearly the solar metal abundance, it could be concluded already in 1962 that there has been no appreciable enrichment of the

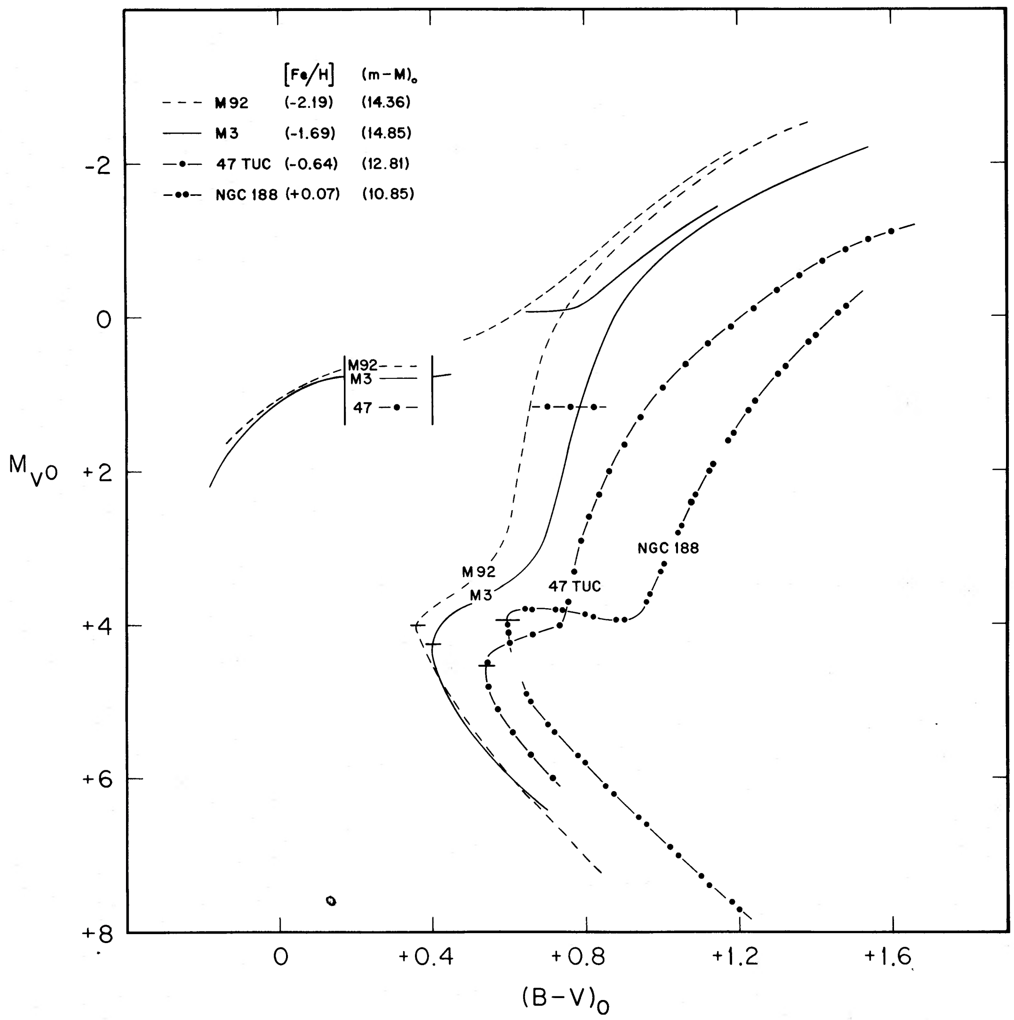

FIG. 3.—Composite *C-M* diagram for three globular clusters and the old open cluster NGC 188. Although NGC 188 is a factor of ~3 younger than the globulars, its giant branch still forms the lower envelope of the diagram.

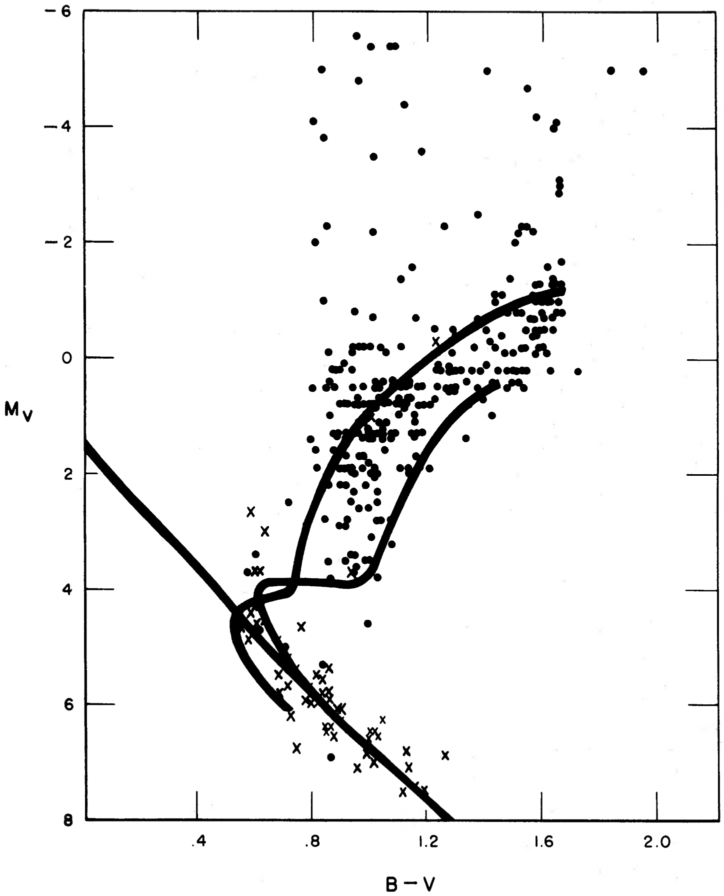

FIG. 4.—*C-M* diagram for field stars after Wilson (1959) with the 47 Tuc and NGC 188 sequences superposed. Crosses on the main sequence are trigonometric parallax stars.

galactic disk in the time since M67 and NGC 188 were formed.

However, simply because NGC 188 forms the lower envelope to the distribution it does not follow that there are no older stars in the galactic disk than represented by that cluster. The point made by Demarque and McClure (1977) is that "the position of the giant branch is sensitive to metal abundance, and stars more metal-poor than NGC 188 could be older and still lie on a Hayashi track above that for the NGC 188 giant branch." Carney's (1980) conclusion that no disk stars older than NGC 188 exist, based on the NGC 188 lower envelope, need not be correct.

The case is made in Figures 3–6. A schematic composite *C-M* diagram for four clusters of different metallicity is given in Figure 3. The data and luminosity normalization for three globular clusters are from Paper III; those for NGC 188 are from Eggen and Sandage (1969). Recall that NGC 188 is younger than the globular clusters, with an age somewhere in the range 5–8 × $10^9$ years (Saio, Shibata, and Simoda 1977; Demarque and McClure 1977); yet, because of its higher metallicity than 47 Tuc, it *still* forms the lower envelope to the spread.

Plotted in Figure 4 are the field giants from Wilson's H-K absolute magnitudes used in an earlier diagram (Sandage 1962, Fig. 9), but with the 47 Tuc schematic from Figure 3 superposed.

Many of Wilson's giants have been studied by Helfer (1969) to determine their metallicities, with the result shown in Figure 5. The $B-V$ colors are from Johnson *et al.* (1966), and the absolute magnitudes are Wilson's, as corrected for an abundance effect by Helfer. The stars are binned into three metallicity groups, shown by different symbols. It is evident that there is a metallicity gradient in the diagram; stars of highest metallicity are closer to the NGC 188 envelope than those of lower abundance, which lie progressively blueward. (It should be noted, of course, that a few of these giants with $0 < M_v < 2$ will be young and hence will have *horizontal* evolutionary tracks like the Hyades. Therefore they cut across the more vertical tracks of the majority of the

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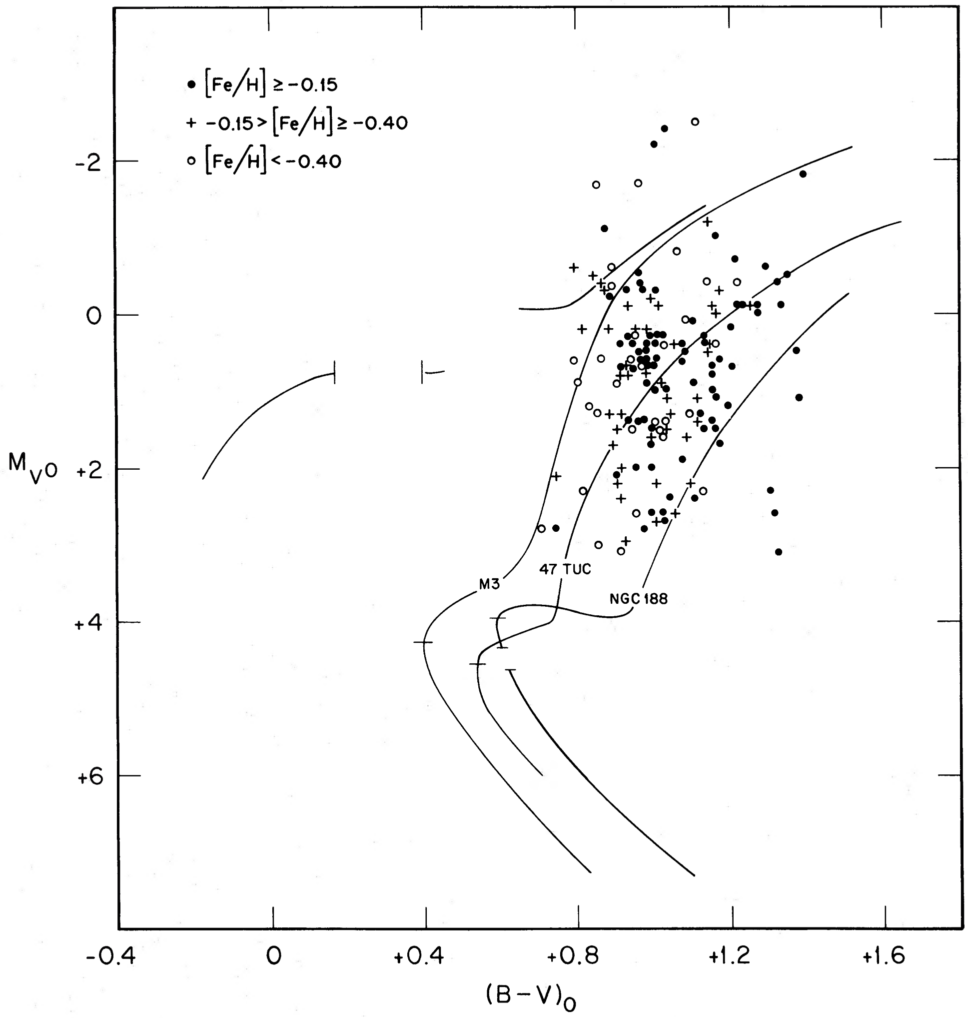

FIG. 5.—Similar to Figs. 3 and 4 with stars studied by Helfer (1969) binned into three metallicity groups. Note that as one proceeds blueward of the NGC 188 envelope, the metallicities generally decrease. It should be remembered that some of the giants will be young and will evolve on horizontal tracks from the main sequence as in the Hyades. The color-metallicity relation should not apply to them.

older giants where the color-metallicity correlation is expected, diluting the correlation for the old giants.)

The color-metallicity effect is shown quantitatively in Figure 6 where the observed colors are plotted versus Helfer's [Fe/H] for those giants whose luminosities are in the range $0.5 \leq M_V \leq 1.5$. The least squares line, averaged between two solutions made by exchanging the dependent and independent variables, shows the trend. It has a slope $\partial(B-V)/\partial[\mathrm{Fe/H}] = +0.44$. The expected slope, obtained from Figure 3 using the NGC 188 and 47 Tuc giant branch colors at $\langle M_v \rangle = +1.0$, with $[\mathrm{Fe/H}]_{188} = 0.0$ and $[\mathrm{Fe/H}]_{47} = -0.64$, is $+0.42$. The same conclusion follows from more recent data on the field giants by Janes (1975, Fig. 10).

This close agreement between the observed variation of metallicity blueward of the NGC 188 envelope and

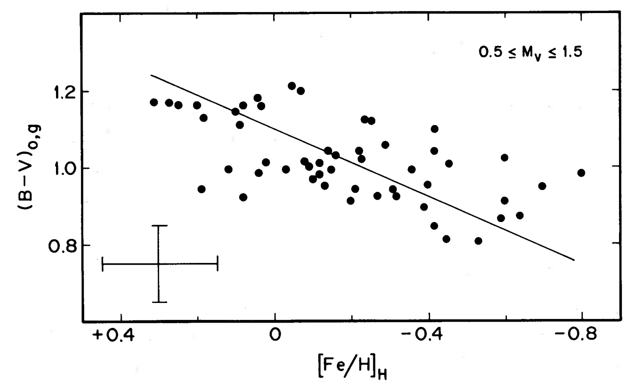

FIG. 6.—Correlation of observed $B-V$ color with Helfer's measured [Fe/H] for field giants in Fig. 5 between absolute magnitudes $+0.5$ and $+1.5$. The line is the impartial least squares fit.

that expected from the *schematic cluster diagrams* supports the assertion that the oldest disk stars began to form long before NGC 188 was born, and, in fact are as old as the halo globular clusters. The evidence in § III*a* suggests that such disk stars formed within $\sim 2\times 10^9$ years (the accuracy of the method) from the beginning of halo cluster formation.

## IV. VARIATION OF METALLICITY WITH AGE

Beginning with the evidence from M67 and NGC 188 in 1962, it has been known that the enrichment of the galactic disk with time has been very gradual over the past $\sim 5\times 10^9$ years. Furthermore, given the large variation of [Fe/H] among globular clusters of nearly the same age, it follows that earlier enrichment must either have been very rapid or very spotty, or both.

The results of Paper III together with those known earlier for M67, NGC 188, Hyades, and the Sun have been combined in Figure 7 to again show the gross features of the enrichment history. The adopted metallicities of M67 and NGC 188 are $[\mathrm{Fe/H}] = +0.07$ (relative to the Sun), based on their measured ultraviolet excess values of $\delta(U-B) = 0.05$ relative to the Hyades (Eggen and Sandage 1964, 1969), using the calibration of photometric excess versus [Fe/H] due to Eggen (1964, Fig. 19).

Many studies have been made of the distribution of [Fe/H] for field stars in the solar neighborhood. Results of the latest of these studies by Twarog (1980), quoted by Pagel (1980), are shown as crosses in Figure 7.

The line, showing the general trend, is close to that obtained in an earlier analysis of the distribution of [Fe/H] for stars in the solar neighborhood by Clegg and Bell (1973). They also gave a theoretical calculation of this line from a simple model of the enrichment history. Later models such as Larson's (1976) show that a wide variety of enrichment gradients, including those observed in Figure 7, can be produced by reasonable collapse models. The very rapid enrichment during the halo collapse, shown in Figure 7, is reproduced in all of Larson's models (his Fig. 14).

This most striking historical feature of very rapid initial enrichment had already been suggested by the earlier data, when NGC 188 was found to have nearly solar abundance, although, to be sure, it was then assumed that its age was nearly the same as that of the globular clusters (Sandage 1968, Fig. 7; Eggen and Sandage 1969, Fig. 7). However, this rapid phase *still* remains in the present diagram because the ages of the high metallicity disk globulars, NGC 6838 and 47 Tuc, have been taken here, following Paper III, to be the same as the halo clusters. The principal change from the 1968 picture is that the enrichment increase of the disk *subsequent* to the halo collapse is more gradual than in the two previously cited diagrams. This is because the age of NGC 188 has been reduced from $\sim 11\times 10^9$ years used there to $\sim 5\times 10^9$ years adopted here, as determined by Torres-Peimbert (1971), and by Demarque and McClure, and Saio *et al.* cited earlier. During this more gradual enrichment of the slowly forming disk, the mean metallicity gradient is of the order $\partial[\mathrm{Fe/H}]/\partial \log t \approx 0.07$.

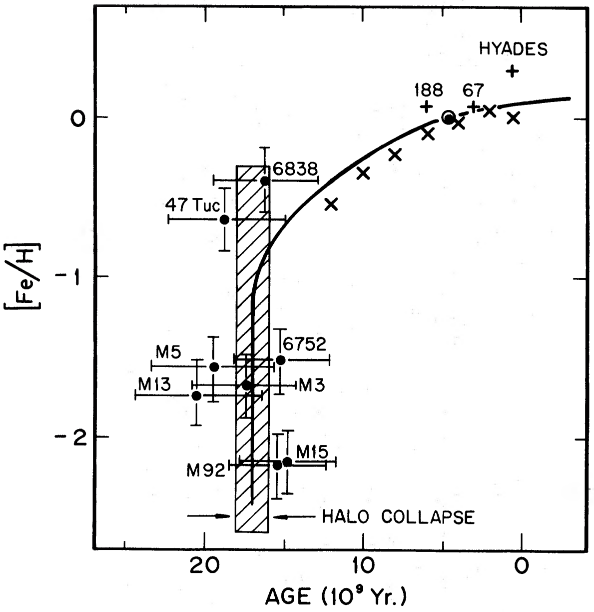

FIG. 7.—Schematic representation of the variation of [Fe/H] with age for selected galactic objects and regimes. The globular cluster data are from Paper III. The data from the solar neighborhood, shown as crosses, are due to Twarog (1980) quoted by Pagel (1980). The clusters of NGC 188, M67, and the Hyades, together with the Sun, are also shown.

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