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THE OOSTERHOFF PERIOD GROUPS AND THE AGE OF GLOBULAR CLUSTERS. III. THE AGE OF THE GLOBULAR CLUSTER SYSTEM

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ABSTRACT

The Oosterhoff period shifts, determined from the (light curve shape)-period relation in 30 globular clusters, are found to form a continuum rather than to fit into two nearly distinct period bins. The shifts are correlated with metallicity; longer periods occur in clusters with lower metal abundance. Results from Papers I and II require the RR Lyrae absolute magnitudes to be brighter in clusters where the variables have longer periods. A composite color-magnitude diagram, made from eight clusters whose luminosity levels are normalized according to $\Delta M_{\rm bol}^{\rm RR}=3\Delta \log P$, shows that the main sequence turnoff luminosity varies with metallicity as $\partial M^{\rm TO}/\partial[{\rm Fe/H}]=0.29$. Lower metallicity clusters have brighter turnoffs. This observed gradient agrees with the models first calculated by Simoda and Iben as the requirement that clusters of different metallicity have the same age. Analysis of the cluster data in the metallicity range $0\geq[{\rm Fe/H}]\geq-2.3$ using the Simoda and Iben condition with the Yale models shows that the ages of all clusters studied here are the same to within the accuracy of the method, which is about $\pm10\%$. An independent demonstration that the clusters are coeval to within this limit uses the magnitude difference between the horizontal branch and main sequence turnoff levels. The data show that the metal-rich clusters 47 Tuc and NGC 6838 cannot be younger than the metal-poor clusters, again by more than $\Delta t/t\approx\pm10\%$. The condition for nearly equal age requires that the Oosterhoff period shifts are correlated with metallicity in the way that is observed. It can also be understood why the individual period-amplitude relations for variables in *different* clusters are ordered by metallicity, provided that amplitude is a unique function of position in the RR Lyrae instability strip. The absolute age of the globular cluster system is $t=(17\pm2)\times10^9$ years if the absolute magnitude of RR Lyrae variables in M3 is $M_V=+0.80$, justified from external data. This age agrees with a *global* value of the Hubble constant near $H_0=50$ km s^{-1} Mpc^{-1} when we add to the age of the clusters the formation time of galaxies from the "beginning," determined from the observed upper redshift limit of $\Delta\lambda/\lambda_0\approx+4$ for quasars. The data are, however, in clear violation with the larger values of the Hubble constant occasionally quoted.

Subject headings: clusters: globular — stars: abundances — stars: RR Lyrae

I. INTRODUCTION

Unsolved problems concerning RR Lyrae stars in globular clusters and the general field include (1) an understanding of the cause of the Oosterhoff-Sawyer period shifts for variables in different clusters; (2) the reason for the tight correlation of these shifts with metallicity when it is known that the luminosity level of the horizontal branch does not depend strongly enough on [Fe/H], of and by itself, to account for the observed $\Delta \log P$ (cf. eq. [8] put into eq. [13] of Sandage, Katem, and Sandage 1981, hereafter SKS); (3) the cause of the progressive shift in the period-amplitude relation toward longer periods in clusters of lower metallicity; and (4) an explanation for the same correlation of $\Delta \log P$ with metallicity for the field RR Lyrae variables.

In this report we continue the analysis of the Oosterhoff problem begun in earlier papers of the series (Paper I, SKS; Paper II, Sandage 1981*a*). It was shown there (*a*) that a difference in horizontal branch luminosity by $\Delta M_{\rm RR}^{\rm HB}=3\Delta \log P$ can explain item (1), (*b*) that such a correlation between $\Delta M_{\rm bol}^{\rm RR}$ and $\Delta \log P$ also holds for *individual* variables in M3 and ω Cen, and (*c*) that the only other explanation for the *star-by-star* period shift in different clusters at a given temperature is a mass difference; variables at fixed T_e in clusters of low metallicity would have to have lower mass than those at equal T_e in clusters of high metallicity. As this requirement is in the *opposite sense* and of a magnitude several times larger than given in all current zero-age horizontal-branch model calculations for stars of different metallicity and of different helium abundance,[1] we considered mass differences to be unlikely as an explanation for the period shifts.

It was also found in Paper II that the shifts in the *period-amplitude* (*P-A*) relations can be understood by the same difference in horizontal branch (HB) luminosities between clusters, provided that amplitude is a unique function of position in the instability strip. Further, the *equality* (to within the observational errors related to reddening and photometry, which are $\sim\pm 0.015$ mag or $\Delta \log T = 0.005$) of the period shifts between the *P-A* and the P-T_e relations require the existence of a period-luminosity-amplitude relation for RR Lyrae stars: analysis of data for variables in M3 and ω Cen (Sandage 1981*b* and Paper II, Figs. 10–12) appear to provide a confirmation. (Note, however, that the slight inequality of the M3 and M15 *c* type variables in the amplitude-temperature relation of Fig. 12, Paper I, may still not be understood.)

In the present paper we extend the discussion to items (2) and (3). Analysis of the field RR Lyrae variables is given in Paper IV that follows.

Data are given in § II to show that the Oosterhoff-Sawyer period groups form a continuum in the distribution of $\Delta \log P$, [Fe/H], $(B-V)_{0,g}$, and $\Delta V_{1.4}$. The correlation of the period shifts with [Fe/H] and/or $(B-V)_{0,g}$ and height of the giant branch $\Delta V_{1.4}$ is given for 30 clusters in § III. New composite *C-M* diagrams of clusters, normalized in luminosity using the observed $\Delta \log P$(RR) values, are set out in § IV. It is shown in § V that this HB normalization gives main sequence turnoff (TO) luminosities that become progressively brighter with decreasing metallicity. The amount is that required by the available age-dating models if globular clusters of all metallicities have closely the same age. A limit of $\Delta t/t \approx \pm 10\%$ can be put from the data, indicating a relatively rapid collapse of the early Galaxy (Eggen, Lynden-Bell, and Sandage 1962).

An independent proof of nearly equal age is given in § VI using the observed luminosity ratio of the horizontal branch and turnoff levels for globular clusters of all metallicities. Comparison with age determinations by others is made in § VII, where it is shown that claims of very young ages for 47 Tuc and NGC 6838 violate the requirements set out in § VI for the ratio of the HB and TO levels. Finally, a summary is given in § VIII to show how the condition of equal age can explain the correlation of period shifts with [Fe/H] for the RR Lyrae variables, and why their period-amplitude relations are nearly uniquely ordered with metallicity.

II. THE OOSTERHOFF GROUPS FORM A CONTINUUM

When the Oosterhoff period groups for RR Lyrae variables in clusters were first isolated (Grosse 1932; Hachenberg 1939; Oosterhoff 1939*a*, 1944; van Agt and Oosterhoff 1959), the two period groups centered at $\langle P_{ab}\rangle \approx 0.55$ and 0.65 days appeared to be almost discrete. This narrowness, with each of bin-width only $\sigma(P) \approx 0.01$ days, was a principal difficulty in explaining the groups because it would have required a sort of quantization.

However, in recent years, accumulating evidence shows that the period distributions of the variables are, in fact, wide. Wehlau and Demers (1977, their Table 4) found that variables in Oosterhoff group I clusters have values of $\langle P_{ab}\rangle$ that range from 0.526 to 0.592 days, with a 1σ spread of ± 0.017 days. The distribution approaches a continuum over the stated range. We show here that a similar continuum exists in the periods of Oosterhoff group II cluster variables, and in other parameters of the cluster *C-M* diagrams that are related to $\langle P\rangle$ and to metallicity.

[1] It was shown in Paper I that mass differences between M3 and M15 which are related to changes in metal abundance Z are small enough to be neglected and, as stated, are of the wrong sign to explain $\Delta \log P$ between the clusters without changing $L_{\rm RR}$. However, no discussion was given there of the mass dependence on helium abundance Y as well because, when the ideas of this series were in embryo and the mass dependence on Z for the argument in Paper I had been completed, no thought of varying Y between M3 and M15 had yet occurred, and the matter was never taken up subsequently.

Both R. P. Kraft and the referee have together pointed out that this neglect leaves the argument incomplete that we can, in fact, neglect the mass variations between variables in different clusters in attempts to explain the period shifts. The point is important because the fundamental premise throughout this series is that $\Delta M_{\rm bol}^{\rm RR} = 3\Delta \log P$. That this relation is *approximately* true is shown in Figure 10 of Paper II for variables in M3 and in ω Cen individually, but as the referee remarks, the scatter there is large enough to admit variations in the coefficient between 2 and 4 (however, it cannot be zero).

We now show that the neglect of the mass dependence on ΔY, tacitly assumed heretofore, *is* justified and, in a manner similar to the $\partial \log \text{mass}/\partial Z$ effect of equation (6) (Paper I), is also of the wrong sign to admit a $\Delta \log L_{\rm RR} = 0$ solution to the Oosterhoff period shift problem. The grids of horizontal branch models of Sweigart and Gross (1976) embrace a wide range of Z, Y, core mass M_c, and total mass. From them we can obtain the required $\partial \log \text{mass (RR)}/\partial Y$ ratio at $\log T_e = 3.85$ for fixed Z and M_c. Interpolation in their tables gives $\partial \log \text{mass}/\partial Y = +0.4$ for $M_c = 0.525$ and [Fe/H] $= -2.3$ for $0.2 < Y < 0.3$. The ratio is $\partial \log \text{mass}/\partial Y = +0.1$ between the same Y values for [Fe/H] $= -1.3$. If the core mass is $M_c = 0.475$, the partial ratio is $+0.30$ for [Fe/H] $= -2.3$ and is $+0.07$ for [Fe/H] $= -1.3$. The sense is that higher mass goes with higher Y. In Paper I we require ΔY (M15 minus M3) $\approx +0.05$. This then requires the mass of M15 RR Lyrae stars due to the Y difference alone to be $\Delta \log \text{mass} \approx (0.05)(0.2) \approx +0.01$ larger for M15. Putting this into the pulsation equation (eq. [5] of Paper I) at fixed L would produce $\Delta \log P = -0.007$ between M15 and M3. This is eight times smaller and of the wrong sign to explain the observed $\Delta \log P = +0.055$ between the clusters. The smallness of the $\partial \log \text{mass}/\partial Y$ coefficient using the Sweigart-Gross models is the justification for the neglect of mass variations with Y as well as Z.

A literature survey, complete to roughly the beginning of 1980, has been made of globular cluster color-magnitude diagrams (CMD) and RR Lyrae light curves, with the results given in Table 1. Values of $E(B-V)$, $(B-V)_{0,g}$, and $\Delta V_{1.4}$ listed in columns (3)–(5) are averages of the literature data, weighted by a judgment of reliability concerning the original photometry. The major studies of reddening include those of Kron and Mayall (1960), Racine (1973), Harris and van den Bergh (1974), Burstein and McDonald (1975), Harris (1976), and Zinn (1980). The literature sources for the CMDs and the RR Lyrae data are noted in columns (10) and (11), and are listed in the footnote. The reddening-free colors of the subgiant branch and the height of the giant branch at $(B-V)_0 = +1.4$ in columns (4) and (5) have been determined from the published CMDs using the reddenings in column (3).

Mean periods of the variables in column (6) are calculated from the *Third Catalog of Variable Stars in Globular Clusters* (Hogg 1973), based on the number of stars in column (7). In discussing the Oosterhoff-Sawyer period problem, it has been customary to use these mean $\langle P_{ab}\rangle$ and $\langle P_c\rangle$ values, determined simply by averaging the periods of whatever *ab* or *c* variables exist in the cluster. Such averages measure some ensemble property, depending on the color distribution of the variables in the instability strip. Because population gradients exist along the HB depending on metallicity (metal-poor clusters are usually richer in blue HB stars than in red; the opposite is usually true for metal-rich clusters) the ensemble averages are not pure indicators of the period properties of *individual* variables at say constant T_e. As the test between the model in Papers I and II (used here) and the hysteresis model of van Albada and Baker (1973) depends on properties of the individual period shifts, star by star, we have sought a different measure of the period shifts. In Paper II it was found that the period shift of one cluster relative to another using individual stars can be determined by plotting $\Delta\phi_{\rm rise}$ (which measures light curve shapes) versus $\log P$, and shifting the plot for any cluster relative to that for M3. Period shifts determined in this way are listed as $(\Delta \log P)_s$ in column (8) of Table 1, where s denotes shifts by light curve *shape*. The sense in column (8) is M3 *minus* cluster. A representative error is $\varepsilon(\Delta \log P)_s \approx \pm 0.01$ (it, of course, has no meaning to put an error on $\langle P_{ab}\rangle$ because of its hybrid nature).

The period shifts in column (8) are similar to those determined from the period-amplitude relations (cf. Table 7 of Paper II), but they are less subject to errors in the photometry. The listed $\Delta \log P$ values have, however, been checked by using the *P-A* relations plotted by Cacciari and Renzini (1976) from Hogg's (1973) catalog. The agreement is good, but, as expected, the individual *P-A* relations often show larger scatter than the *P*-$\Delta\phi$ correlations, presumably due to occasionally unsatisfactory photometry. The literature sources for the cluster RR Lyrae data are listed in column (11). The adopted [Fe/H] values in column (9) are taken from Zinn (1980). It should be noted that Zinn's zero point is 0.2 smaller than the direct calibration of [Fe/H] by Butler (1975*a*).

The data in Table 1 are plotted in Figure 1 and show the following features: (1) The $\langle P_{ab}\rangle$ values do divide into two groups, one with periods in the range $0.620 < \langle P_{ab}\rangle < 0.698$, and the other with $0.525 < \langle P_{ab}\rangle < 0.573$ days. This is the Oosterhoff-Sawyer dichotomy. (2) The distribution of the period shifts determined from the light curve shapes (LCS), shown in Figure 1*b*, again divide into two groups with a break between. However, within each group the distribution of $(\Delta \log P)_{\rm LCS}$ is not discrete, but wide. (3) The distributions of $(B-V)_{0,g}$, $[\mathrm{Fe/H}]_{\rm Zinn}$, and $\Delta V_{1.4}$ are shown in panels (*c*), (*d*), (*e*). Note the break at $(B-V)_{0,g} \approx 0.98$ in panel (*c*). Clusters with reliable $(B-V)_{0,g} > 0.98$ have few or no RR Lyrae stars, and hence are not plotted in panels (*a*) and (*b*). They are the metal-rich clusters with $[\mathrm{Fe/H}] > -0.6$ that include 47 Tuc, NGC 5927, 6304, 6352, 6356, 6624, 6637, and 6838. The same clusters also occur in panels (*d*) and (*e*) on the extreme left-hand side of the distribution.

The conclusion from Figure 1 is that the distributions of all parameters form a bimodal continuum rather than two very narrow bins.

III. CORRELATION OF Δ LOG P AND [Fe/H] FOR GLOBULAR CLUSTERS

a) *The Observations*

Arp (1955) was the first to show that clusters of Oosterhoff group II have lower metal abundance and steeper giant branches than those of group I. The correlations, extended by Kinman (1959), are now known to be general.

Another fact, known since the early 1950s, is that the giant branch color $(B-V)_{0,g}$ at the HB level varies with [Fe/H]. A recent calibration by Butler (1975*b*) gives $\partial(B-V)_{0,g}/\partial[\mathrm{Fe/H}] \approx 0.14$, which agrees approximately with the calculations of Bell and Gustafsson (1975) that give $\partial(B-V)_{0,g}/\partial[\mathrm{Fe/H}] \approx 0.23$, based on the giant branch models of Rood (1972).

These correlations can be seen again in the data of Table 1. In Figure 2, $[\mathrm{Fe/H}]_Z$ is plotted versus $(B-V)_{0,g}$ and $\Delta V_{1.4}$. The dependences are well determined and show very little scatter. A fit by eye to Figure 2*b* has the equation

$$(B-V)_{0,g} = 0.180[\mathrm{Fe/H}] + 1.10, \qquad (1)$$

which agrees well with Butler's (1975*b*) calibration of $(B-V)_{0,g} = 0.139[\mathrm{Fe/H}] + 0.97$ if the previously mentioned difference of $+0.2$ in the zero point of $\Delta[\mathrm{Fe/H}]$ (Butler minus Zinn) is taken into account.

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TABLE 1

CHARACTERISTIC PARAMETERS FOR GLOBULAR CLUSTERS WITH ADEQUATE DATA

Name (1)	Messier (2)	E(B−V) (3)	$(B-V)_{o,g}$ (4)	$\Delta V_{1.4}$ (5)	$\langle P_{ab} \rangle$ (6)	N_{ab} (7)	$(\Delta \log P)_s$ (8)	$[Fe/H]_z$ (9)	Source CMD[a] (10)	Source RR Lyrae[b] (11)
47 Tuc		0.04	0.99 ± 0.01	1.92	...	...	...	−0.64	1	...
NGC 288		0.02	0.84 ± 0.03	2.46	...	...	...	−1.61	2	...
362		0.04	0.83 ± 0.03	2.25	0.542	7	+0.017	−1.13	3	a
1261		0.00	0.88 ± 0.03	2.45	0.563	10	0.000	−1.28	4	b
1851		0.02	0.88 ± 0.03	2.58	0.573	14	0.000	−1.33	5	c
1904	79	0.01	0.81 ± 0.03	2.80	...	...	...	−1.76	6	...
2419		0.03	0.70 ± 0.02	3.14	0.650	19	−0.085	...	7	d
2808		0.24	0.75 ± 0.04	2.43	...	...	...	−1.22	8	...
3201		0.21	0.79 ± 0.03	2.90	0.558	69	−0.010	−1.40	9	e
4147		0.00	0.79 ± 0.02	>2.85	0.525	6	...	−2.07	10	f
4590	68	0.03	0.74 ± 0.03	3.10	0.625	14	−0.050	−2.19	8	g
4833		0.36	0.65 ± 0.04	3.07	0.684	6	−0.090	−1.91	11	h
5024	53	0.01	0.71 ± 0.03	2.94	0.633	18	−0.066	−1.99	12	i
5053		0.02	0.68 ± 0.02	2.8:	0.672	5	−0.080	−2.29	13	j
5272	3	0.00	0.85 ± 0.03	2.70	0.551	148	0.000	−1.69	14	k
5286		0.24	0.76 ± 0.03	...	...	...	...	−1.72	15	...
5466		0.00	0.70 ± 0.05	2.94	0.637	11	−0.065	−2.17	16	ℓ
5634		...	...	...	...	...	−0.050	−1.92	...	m
5897		0.11	0.80 ± 0.03	2.84	...	...	...	...	17	...
5904	M5	0.03	0.79 ± 0.02	2.59	0.547	67	0.000	−1.58	18	n
5927		0.42	1.04 ± 0.05	1.45	...	...	...	−0.16	19	...
5986		0.28	0.77 ± 0.04	2.69	...	...	...	−1.80	15	...
6093	80	0.16	0.76 ± 0.04	2.74	...	...	...	−1.76	20	...
6121	4	0.36	0.92 ± 0.02	2.28	0.538	30	+0.035	−1.46	21	o,p
6171		0.28	1.02 ± 0.03	2.12	0.527	14	+0.065	−0.89	22	q
6205	13	0.03	0.79 ± 0.03	2.80	...	...	...	−1.73	14	...
6218	12	0.17	0.75 ± 0.03	2.78	...	...	...	−1.56	23	...
6229		...	...	...	0.527	11	+0.010	−1.49	...	r,s
6254	10	0.26	0.80 ± 0.06	2.95:	...	...	...	−1.70	15	...
6266		...	...	...	0.544	62	+0.020	−1.26	...	g
6304		0.54	1.06 ± 0.07	1.82	...	...	...	−0.27	24	...
6333	9	...	...	...	0.614	7	−0.040	−1.81	...	t
6341	92	0.02	0.70 ± 0.02	3.05	0.626	9	−0.080	−2.19	14	u,v
6352		0.25	1.05 ± 0.03	1.52	...	...	...	−0.36	25	...
6356		0.24	0.99 ± 0.04	1.83	...	...	...	−0.29	26	...
6362		0.11	(0.97 ± 0.10)	(2.35)	0.535	7	+0.020	−0.96	27	w
6397		0.17	0.71 ± 0.03	2.90	...	...	...	−2.24	2	...
6402	14	...	...	...	0.564	31	−0.007	−1.49	Need	x,y
6426		...	...	...	0.665	4	−0.085	−2.28	Need	z
6624		0.31	(1.11 ± 0.10)	(1.53)	...	...	...	−0.23	28	...
6626	28	...	...	...	0.565	6	...	−1.44	...	...
6637	69	0.17	1.05 ± 0.04	1.38	...	...	...	−0.39	29	...
6656	22	0.36	0.72 ± 0.04	2.87	0.651	8	−0.040	−1.86	30	c
6712		0.48	0.89 ± 0.05	2.24	0.557	7	+0.025	−1.17	31	aa
6715	54	...	...	...	0.551	31	0.000	−1.41	Need	bb
6723		0.01	0.91 ± 0.02	2.35	0.540	24	+0.045	−1.17	32	cc
6752		0.02	0.81 ± 0.03	2.78	...	...	...	−1.52	33, 34	...
6809	55	0.08	0.80 ± 0.02	3.10	(0.620)	2	...	−1.85	35	...
6838	71	0.21	1.01 ± 0.05	1.54	...	...	...	−0.40	36	...
6864		0.17	0.79 ± 0.03	2.36	...	...	...	−1.39	8	...
6934		0.11	0.86 ± 0.05	2.69	0.545	30	...	−1.65	37	...
6981	72	0.04	0.77 ± 0.03	2.78	0.552	26	−0.010	−1.58	38	dd
7006		0.05	0.77 ± 0.05	2.66	0.567	54	0.000	−1.53	39	ee
7078	15	0.08	0.76 ± 0.02	>2.50	0.640	27	−0.055	−2.15	14	ff
7089	2	0.04	0.72 ± 0.04	2.73	0.636	13	−0.060	−1.79	8	gg,hh
7099	30	0.02	0.70 ± 0.02	3.10	0.698	3	−0.110	−2.26	40	ii
Pal 5		0.03	0.83 ± 0.03	...	...	...	...	−1.48	13	...

[a]SOURCE (CMD).—(1) Lee 1977*b*. (2) Cannon 1974. (3) Menzies 1967. (4) Alcaino 1979. (5) Stetson 1981. (6) Stetson and Harris 1977. (7) Racine and Harris 1975. (8) Harris 1975. (9) Lee 1977*c*. (10) Sandage and Walker 1955. (11) Menzies 1972. (12) Cuffey 1965. (13) Sandage and Hartwick 1977. (14) Sandage 1970. (15) Harris, Racine, and De Roux 1976. (16) Cuffey 1961. (17) Sandage and Katem 1968. (18) Arp 1962. (19) Menzies 1974*b*. (20) Harris and Racine 1974. (21) Lee 1977*a*. (22) Dickens and Rolland 1972. (23) Racine 1971. (24) Hesser and Hartwick 1976. (25) Hartwick and Hesser 1972. (26) Sandage and Wallerstein 1960. (27) Alcaino 1979. (28) Liller and Liller 1976. (29) Hartwick and Sandage 1968. (30) Hesser, Hartwick, and McClure 1977. (31) Sandage and Smith 1966. (32) Menzies 1974*a*. (33) Wesselink 1974. (34) Carney 1979*b*. (35) Lee 1977*d*. (36) Arp and Hartwick 1971. (37) Harris and Racine 1973. (38) Dickens 1972*a*. (39) Sandage and Wildey 1967. (40) Dickens 1972*b*.

[b]SOURCE (RR Lyrae).—(a) Hogg 1931. (b) Wehlau and Demers 1977. (c) Wehlau and Hogg 1978. (d) Pinto and Rosino 1977. (e) Wright 1941*a*. (f) Newburn 1957. (g) Van Agt and Oosterhoff 1959. (h) Wright 1941. (i) Margoni 1964, 1965, 1967. (j) Mannino 1963. (k) Roberts and Sandage 1955. (l) Bartolini, Biolchini, and Mannino 1961. (m) Liller and Hogg 1976. (n) Oosterhoff 1941. (o) Sturch 1977. (p) Cacciari 1979. (q) Dickens 1971. (r) Mannino 1960. (s) Mayer 1961. (t) Hogg 1951. (u) Hackenburg 1939. (v) Oosterhoff 1939*b*, 1944. (w) Van Hoff 1961. (x) Hogg and Wehlau 1966. (y) Hogg and Wehlau 1968. (z) Grubissich 1958. (aa) Sandage, Smith, and Norton 1966. (bb) Rosino and Nobili 1959. (cc) Menzies 1974*a*. (dd) Dickens and Flinn 1972. (ee) Rosino and Ciatti 1967. (ff) Sandage, Katem, and Sandage 1981. (gg) Hogg 1935. (hh) Mantegazza 1961. (ii) Rosino 1949, 1961.

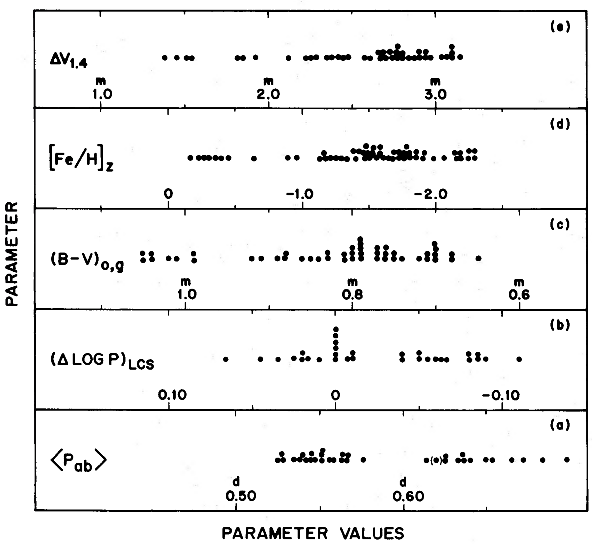

FIG. 1.—Histograms of the five parameters listed in columns (4), (5), (6), (8), and (9) of Table 1. The Oosterhoff dichotomy into two period groups is seen in panels (*a*) and (*b*). It may be masked in panels (*c*), (*d*), and (*e*) by the observational errors. The separations of the Oosterhoff groups should occur near $(B-V)_{0,g} \approx 0.75$ in panel (*c*), near [Fe/H] ≈ -1.7 in panel (*d*), and near $\Delta V_{1.4} = 2.7$ mag in panel (*e*). The estimated observational errors are $\varepsilon\Delta\log P_{\rm LCS} = \pm 0.01$, $\varepsilon(B-V)_{0,g} = \pm 0.04$ mag, ε[Fe/H]$=\pm 0.2$, and $\varepsilon\Delta V_{1.4} = \pm 0.15$.

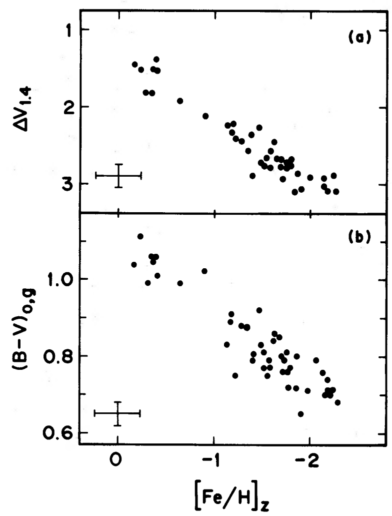

FIG. 2.—Correlation of metallicity with $(B-V)_{0,g}$ and $\Delta V_{1.4}$ from the data in Table 1.

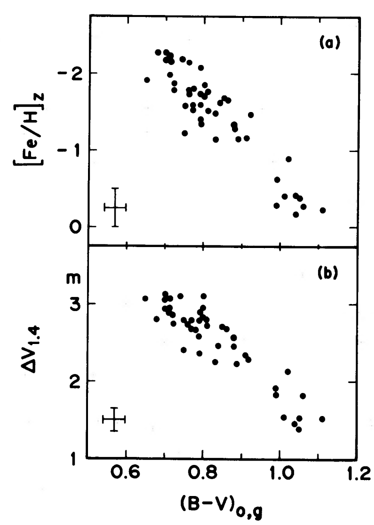

FIG. 3.—Correlation of [Fe/H] and $\Delta V_{1.4}$ with $(B-V)_{0,g}$ from the data in Table 1.

The correlations are shown differently in Figure 3, where $(B-V)_{0,g}$ and $\Delta V_{1.4}$ are compared in panel (*b*). The correlation of Figure 2*b* is repeated in Figure 3*a*, but with the independent variables switched.

The Oosterhoff period dichotomy is not clearly visible in either Figure 2 or Figure 3. However, this is undoubtedly an artifact of the errors, which are $\varepsilon(B-V)_{0,g} \approx \pm 0.04$ mag, $\varepsilon\Delta V_{1.4} \approx \pm 0.15$ mag, and ε[Fe/H]$\approx \pm 0.2$. Inspection of Figure 1 shows that such errors are large enough to smear the distribution appreciably in the three mentioned parameters, but the dichotomy is not smeared away in $\Delta \log P_s$ because the error $\varepsilon\Delta \log P$

$= \pm 0.01$ is small compared with the observed range of $\Delta \log P_s$ in Figure 1.

What *is* visible in Figures 2 and 3 is the gap between the metal-rich clusters with [Fe/H]> -0.8 and those of lower metallicity. The same gap occurs at $\Delta V_{1.4} \approx 2$ mag and $(B-V)_{0,g} \approx 0.9$. Evidently in this sample, the metal-rich clusters are part of the still unknown formation history of *three* separate Oosterhoff groups, broadly sorted into metallicities centered at [Fe/H]~ -2.2, -1.5, and -0.3.

The purpose of showing Figures 2 and 3 is to emphasize that the photospheric parameter [Fe/H] measured by Butler (1975*a*), and independently by Cohen (1978, 1979), is related to the metal abundance used by model builders for their envelope opacity calculations. Evolved models calculated by Demarque and Geisler (1963), Rood (1972), Sweigart and Gross (1978), and others show that $(B-V)_{0,g}$ and $\Delta V_{1.4}$ are sensitive functions of the mix of heavy elements that determines the envelope opacity. Because Figures 2 and 3 show that $(B-V)_{0,g}$ and $\Delta V_{1.4}$ vary with the *photospheric* [Fe/H] value, it follows that this parameter measures in some gross way the required Z (*interior*) value.

With that connection made, we return to the correlation of metallicity with period shift. The correlation is shown in Figure 4; $(\Delta \log P)_s$ is plotted versus [Fe/H] in panel (*a*) and $(B-V)_{0,g}$ in (*b*). The deviations, read as abscissa, are $\sigma(\Delta \log P) \approx \pm 0.018$, which are hardly more than the errors. Read as ordinate, they have σ([Fe/H])$\approx \pm 0.155$, also nearly equal to the errors.

The correlation is

$$\Delta \log P = 0.116\,[\mathrm{Fe/H}] + 0.173, \qquad (2)$$

put through the points in Figure 4*a* by eye. The correlation is the generalization of Arp's (1955) discovery that the Oosterhoff groups divide the clusters by metallicity. Figure 4 shows that the parameters are correlated in a continuum. *This is the principal result of the present paper*. From it, and from the requirements that $\Delta M_{\mathrm{bol}}^{\mathrm{RR}} = 3\Delta \log P$, it follows that the luminosity level of cluster horizontal branches must, for some presently unknown reason, be related to metallicity by $\Delta M_{\mathrm{bol}} = 0.34\Delta$[Fe/H].

It should be emphasized that we need *not* understand the reason in order to use the result in setting the relative luminosities of the horizontal branches of any cluster whose [Fe/H] is known. This is because $\Delta M_{\mathrm{bol}}^{\mathrm{RR}} = 3\Delta \log P$ has been established from other considerations (for example, Fig. 10 of Paper II), and that conclusion is independent of Figure 4. Once having set the relative HB levels for those clusters whose main sequence photometry is available, the relative ages follow directly from L_{TO}, [Fe/H], and some assumption of Y, using the available main sequence termination models.

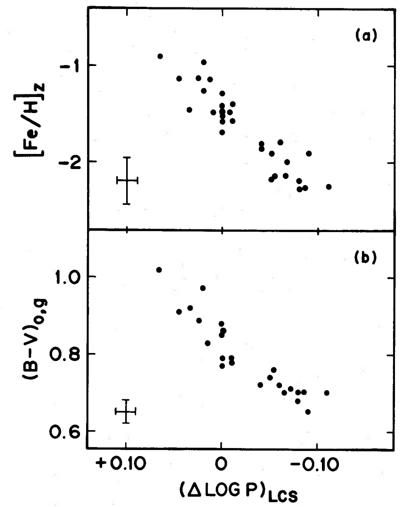

FIG. 4.—The period shifts determined from the light curve shapes (LCS) are correlated with metallicity in panel (*a*) and with $(B-V)_{0,g}$ in panel (*b*). The two Oosterhoff groups are separated near $\Delta \log P = -0.04$ in each panel.

Because age is only a *slow* function of Y, we could proceed directly from equation (2), using $\Delta M_{\mathrm{bol}} = 3\Delta \log P$, to the color-magnitude fittings in § IV and then to the ages in § V without inquiring further into the cause of the correlation in Figure 4. Hence the next subsection, where the possibilities of an explanation of Figure 4 are discussed, is not crucial to the conclusions of § V on ages. The result to be derived in § III*b* enters § V only as a detail.

b) *How Can the Correlation of* $\Delta \log P$ *and* [Fe/H] *Be Produced*?

The simplest explanation of Figure 4a would be that $M_{\mathrm{bol}}^{\mathrm{RR}}$ changes directly with [Fe/H], being brighter for low [Fe/H]. The point is discussed in Paper I (cf. n. 2), where neither equation (14) nor Iben's (1974) equation (8) with its stronger dependence of L_{RR} on [Fe/H] can produce the correlation of Figure 4*a* directly. Iben's dependence is about 3 times too weak, while our result (eq. [3] below) is yet a factor of 2 weaker still.

To show the problem, recall that equation (14) of Paper I is

$$M_{\mathrm{bol}}^{\mathrm{RR}} = \mathrm{const} - 4.20(Y-0.3) - 0.126(\log t - 10.2) + 0.06\,([\mathrm{Fe/H}] + 2.3), \qquad (3)$$

derived also in the Appendix here. Note that the dependence on age is small; for a factor even as large as 2 in age, $\Delta M_{\mathrm{bol}}^{\mathrm{RR}} \approx 0.04$ mag, which is negligible compared with $\Delta M \approx +0.2$ mag between M3 and M15 that is required from the observed period shifts. Equation (2) also shows that a change of Δ[Fe/H]$=1$ gives only

$\Delta M_{\rm bol}^{\rm RR}=0.06$ mag; hence if Δ[Fe/H]=0.5 as between M3 and M15, then $\Delta M_{\rm bol}^{\rm RR}=0.03$ mag, which is 7 times too small. Therefore, if equation (3), or even Iben's stronger dependence on [Fe/H], is correct, the observed strong dependence of $\Delta \log P$ on [Fe/H] in Figure 4*a* requires that another parameter must vary. Because age in equation (3) is so weakly coupled, the only other candidate is Y. Varying Y will change $M_{\rm bol}^{\rm RR}$ drastically; a variation of $\Delta Y=0.05$ produces $\Delta M_{\rm bol}^{\rm RR}=0.21$ mag in the sense that larger Y gives brighter luminosities.

We now inquire how Y must vary with [Fe/H] so as to produce the observed correlation of $\Delta \log P$ (and hence $\Delta M_{\rm bol}^{\rm RR}$) with [Fe/H]. Differentiating equation (3) gives

$$\Delta M_{\rm bol}^{\rm RR}=-4.20\Delta Y-0.126\Delta \log t+0.06\Delta[{\rm Fe/H}]. \quad (4)$$

Adopting the Oosterhoff requirement that $\Delta M_{\rm bol}^{\rm RR}=3\Delta \log P$ and using equation (2) gives

$$\Delta M_{\rm bol}^{\rm RR}=0.348\Delta[{\rm Fe/H}], \quad (5)$$

which, when put into equation (4) neglecting the age term requires that Y and metallicity be correlated as

$$\Delta Y=-0.069\Delta[{\rm Fe/H}]. \quad (6)$$

The sense is against intuition, but it is the same as required by the vertical rather than the sloping boundaries of the H-R diagram (Paper I, eqs. [9] and [10]).

We are now faced with a choice. The arguments of Papers I and II seem persuasive enough to require that a magnitude difference of $\sim 3\Delta \log P$ exists between RR Lyrae stars whose period-amplitude relations differ by $\Delta \log P$, read at constant amplitude. This, together with the correlation of Figure 4*a*, requires that either (1) the dependence of $M_{\rm bol}^{\rm RR}$ on [Fe/H] must be as high as $\partial M^{\rm RR}/\partial[{\rm Fe/H}]=0.34$ so as to produce equation (5) directly if only [Fe/H] is to be varied between the clusters (in which case eq. [3] here and even Iben's eq. [8], both with their much weaker dependence of $M_{\rm bol}^{\rm RR}$ on [Fe/H], would be wrong), or (2) Y must vary as [Fe/H] is changed, coupled as in equation (6).

To determine which of these possibilities is more likely, we seek an independent test of the $\partial M^{\rm RR}/\partial[{\rm Fe/H}]$ term in equation (3). The cluster variables in ω Cen appear to provide such a test because the variation of their $M_{\rm bol}$ values with [Fe/H] can be found from the observations, and hence the $\partial M^{\rm RR}/\partial[{\rm Fe/H}]$ coefficient can be determined directly.

c) *Dependence of* $M_{\rm bol}^{\rm RR}$ *on* [Fe/H] *for* ω *Centauri Variables*

Figure 13 of Butler, Dickens, and Epps (1978) shows that there is virtually no correlation between $M_V^{\rm RR}$ and [Fe/H] for the ω Cen variables studied by them. The [Fe/H] values for these stars, which are averages of their data and those of Freeman and Rodgers (1975), range between -0.5 and -2.2. The result is shown more explicitly in Figure 5, where the magnitude deviations from the ridge line of Figure 10*a* of Paper II are plotted versus [Fe/H] for each star listed by Butler *et al.* (their Table 4). We use these residuals rather than $M_{\rm bol}$ directly so as to correct for the effect on the observed magnitudes of evolution away from the zero-age horizontal branch (cf. Paper II).

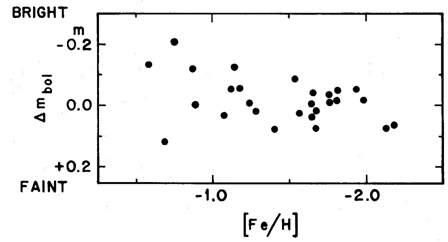

FIG. 5.—Magnitude residuals from the ridge line of Fig. 10, Paper II, for variables in ω Cen as a function of [Fe/H]. There is no obvious correlation. In no case is the sense that which would be required for a direct variation of $L_{\rm RR}$ with [Fe/H] of the size required to explain Fig. 4*a* if no other parameter but [Fe/H] varies. This is taken as justification for the near independence of $L_{\rm RR}$ on [Fe/H] directly, given by eq. (3).

Figure 5 shows no large systematic trend in $M_{\rm bol}$ with metallicity. The trend that is present is of opposite sense to even equation (3): higher metallicity variables are observed to be slightly brighter than stars of lower metallicity. The conclusion from Figure 5 (and Fig. 13 of Butler *et al.*) is that $\partial M_{\rm bol}^{\rm RR}/\partial[{\rm Fe/H}]\approx 0$. In agreement with equation (3), a value as high as $+0.34$ is excluded by Figure 5, both in size and, even more importantly, in sign.

The reason why ω Cen variables do not show the equation (5) dependence of $M_{\rm bol}^{\rm RR}$ on [Fe/H] would then have to be that Y is sensibly the same for all the ω Cen variables. In a *given* cluster, Y apparently is *not* coupled with [Fe/H]. With this proviso, the ω Cen variables would seem to provide a confirmation of the weak dependence of $L_{\rm RR}$ on [Fe/H] in equation (3).

IV. COMPOSITE COLOR-MAGNITUDE DIAGRAMS NORMALIZED BY THE RR LYRAE PERIOD SHIFTS

Eight globular clusters have accurate enough *C-M* diagrams to determine ages using main-sequence turnoff luminosities. Because their *C-M* diagrams have been observed to faint levels, turnoff levels can be fixed once the luminosity levels of the horizontal branches are known. The *relative* HB levels can be set from the period shifts via $\Delta M_{\rm bol}^{\rm RR}=3\Delta \log P$. Their absolute values follow by adopting $M_V({\rm RR})=+0.8$ for the variables in M3. This luminosity differs slightly from that used in

Paper I; it is based on the available absolute calibrations discussed in § VIII*b*.

The photoelectric scale for each of the eight calibrating clusters extends at least 1 mag below the main-sequence turnoff using direct measurements, with no Racine-Pickering wedge extrapolations to this level. The photographic photometry also has high weight.

The calibrating clusters are listed in Table 2 in order of metallicity. Data to be used later are also set out. The adopted metallicity from Zinn (1980) is in column (2). The reddening from Table 1 is in column (3). Columns (4) and (5) give the absorption-corrected magnitudes for the horizontal branch and main-sequence turnoff point, taken from the published *C-M* diagrams in the sources listed in column (13). The ridge-line sequence levels are adopted from the given sources when available, otherwise the mean points have been read from the published *C-M* diagrams. The visual absorption is assumed to be $3E(B-V)$.

Column (6) lists the V magnitude difference between the HB and main-sequence turnoff, obtained from columns (4) and (5). The near constancy of this difference, whose value is $\langle \Delta M_V(\mathrm{TO}-\mathrm{RR})\rangle = 3.40 \pm 0.12$ mag independent of [Fe/H], is also a clue to the ages (§ VI).

The period shifts, multiplied by 3, are listed in column (7). These are *calculated* from equation (2) using the adopted metallicities rather than from column (8) of Table 1, so as to smooth the data. The values in column (7) are renormalized to be zero for M3. The $M_V(\mathrm{RR})$ for each cluster in column (8) follows from column (7) by adopting $M_V(\mathrm{RR}) = +0.8$ for M3. These luminosities, together with $V^0(\mathrm{HB})$ in column (4), give the cluster modulus in column (9).[2]

[2] We have assumed that the $\Delta \log P$ values for 47 Tuc and NGC 6838 can be obtained in this way, and *would* be the period shifts for their RR Lyrae stars, had they any. The assumption seems reasonable because (1) the extrapolation of Fig. 4*a* into the [Fe/H] > -0.7 range is very small, and (2) the continuity of the correlations in Figs. 2 and 3 into this high metallicity range is complete. Furthermore, the consequences of this assumption for ages (§ V) agrees with the independent argument of § VI that uses the observed constancy of the magnitude difference between HB and turnoff for clusters of different metallicity.

Column (10) lists the main-sequence termination luminosity, obtained by applying column (9) to column (5). The bolometric correction at the temperature of the turnoff, listed in column (11), is from Carney (1980, Table 4), based on the calibration of Carney and Aaronson (1979). Applying these to column (10) gives the bolometric magnitude of the turnoff, taken here to be the bluest part of the main sequence region of the *C-M* diagram.

The results of this HB normalization are shown in Figures 6 and 7. Only four clusters are plotted in Figure 6 so as to show the main features of the distribution without the crowding in Figure 7, where all clusters but M13 are shown.

The most important feature of these diagrams is the *progressively fainter main-sequence termination luminosities as the metallicity increases*. The TO in M92 is $\Delta M_V = 0.59$ mag brighter than in 47 Tuc. It is $\Delta M_{\mathrm{bol}} = 0.67$ mag brighter in bolometric magnitude. The mean gradient of TO luminosity with [Fe/H] is $\partial M_{\mathrm{bol}} / \partial [\mathrm{Fe/H}] \approx 0.30$.

These main-sequence data for the eight clusters are plotted in Figure 8. Error bars of ± 0.2 mag are used, which is the approximate accuracy with which the apparent magnitudes at main-sequence termination can be determined. The lines in the diagram discussed in the next section are loci of constant fractional age differences $\Delta t/t$. The line marked zero is the required correlation between M_{TO} and [Fe/H] for equal age.

V. AGE OF THE GLOBULAR CLUSTER SYSTEM

a) *Ages of the Eight Calibrating Clusters*

The isochrones of Ciardullo and Demarque (1977, hereafter CD) have the equation (cf. Fig. 14 and eq. [18]

TABLE 2

CHARACTERISTICS OF GLOBULAR CLUSTERS USED FOR AGE DATING

Name (1)	$[\mathrm{Fe/H}]_z$ (2)	$E(B-V)$ (3)	V^0(HB) (4)	V^0(TO) (5)	$\Delta M_{\mathrm{TO}}^{\mathrm{RR}}(V)$ (6)	3Δ Log P (7)	M_V^{RR} (8)	$(m-M)^0$ (9)	$M_0^{\mathrm{TO}}(V)$ (10)	BC(TO) (11)	M_0^{TO}(bol) (12)	Source (13)
NGC 6838..	−0.40	0.21	13.91	17.15	3.24	0.45	1.25	12.66	4.49	−0.15	4.34	1
47 Tuc..........	−0.64	0.04	13.98	17.38	3.40	0.37	1.17	12.81	4.57	−0.14	4.43	2
NGC 6752..	−1.52	0.02	13.84	17.15	3.31	0.06	0.86	12.98	4.17	−0.22	3.95	3
M5	−1.58	0.03	15.02	18.55	3.53	0.04	0.84	14.18	4.37	−0.18	4.19	4
M3	−1.69	0.00	15.65	19.10	3.45	0.00	0.80	14.85	4.25	−0.20	4.05	5
M13	−1.73	0.03	14.81	18.42	3.61	−0.01	0.79	14.02	4.40	−0.20	4.20	5
M15	−2.15	0.08	15.61	18.92	3.31	−0.16	0.64	14.97	3.95	−0.21	3.74	5
M92	−2.19	0.02	14.99	18.34	3.35	−0.17	0.63	14.36	3.98	−0.22	3.76	5
					Mean: $3.40 \pm 0.12\sigma$							

SOURCE.—(1) Arp and Hartwick 1971. (2) Hesser and Hartwick 1977. (3) Carney 1979. (4) Arp 1962. (5) Sandage 1970.

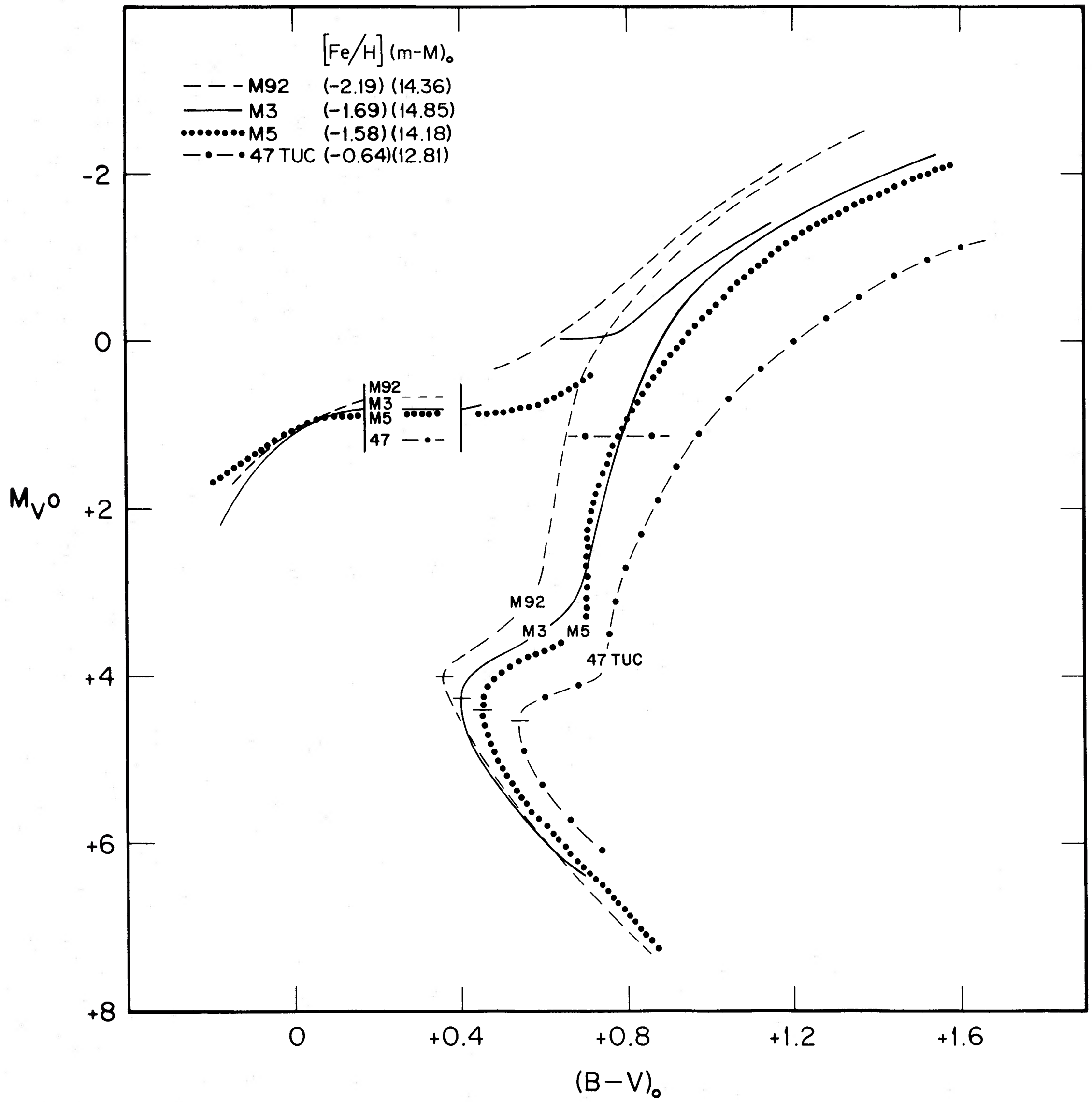

FIG. 6.—Composite *C-M* diagram for four clusters of different metallicity. Normalization of the horizontal branches is from Table 2, column (8), based on the Oosterhoff-Sawyer RR Lyrae period shifts. Note that the main sequence turnoff luminosities are ordered by metallicity.

of Paper I)

$$\log t = 10.2 + 0.41\left(M_{\rm bol}^{\rm TO} - 3.81\right) - 0.15([{\rm Fe/H}] + 2.3) - 0.43(Y - 0.3), \quad (7)$$

hence the age difference between two clusters with different $M_{\rm bol}^{\rm TO}$, Y, and [Fe/H] is

$$\Delta t/t = 0.944\,\Delta M_{\rm bol}^{\rm TO} - 0.345\Delta[{\rm Fe/H}] - 0.99\Delta Y. \quad (8)$$

From the argument in § III we require Y and [Fe/H] to be anticorrelated, with the dependence of equation (6). This, when substituted into equation (8), gives

$$\Delta t/t = 0.944\,\Delta M_{\rm bol}^{\rm TO} - 0.277\Delta[{\rm Fe/H}], \quad (9)$$

which shows that if two clusters have the same age but different metallicities, the cluster with lower metallicity must have a brighter main sequence turnoff by

$$\Delta M_{\rm bol}^{\rm TO} \approx 0.293\Delta[{\rm Fe/H}]. \quad (10)$$

This strong theoretical dependence of turnoff luminosity on metallicity was discovered by Simoda and

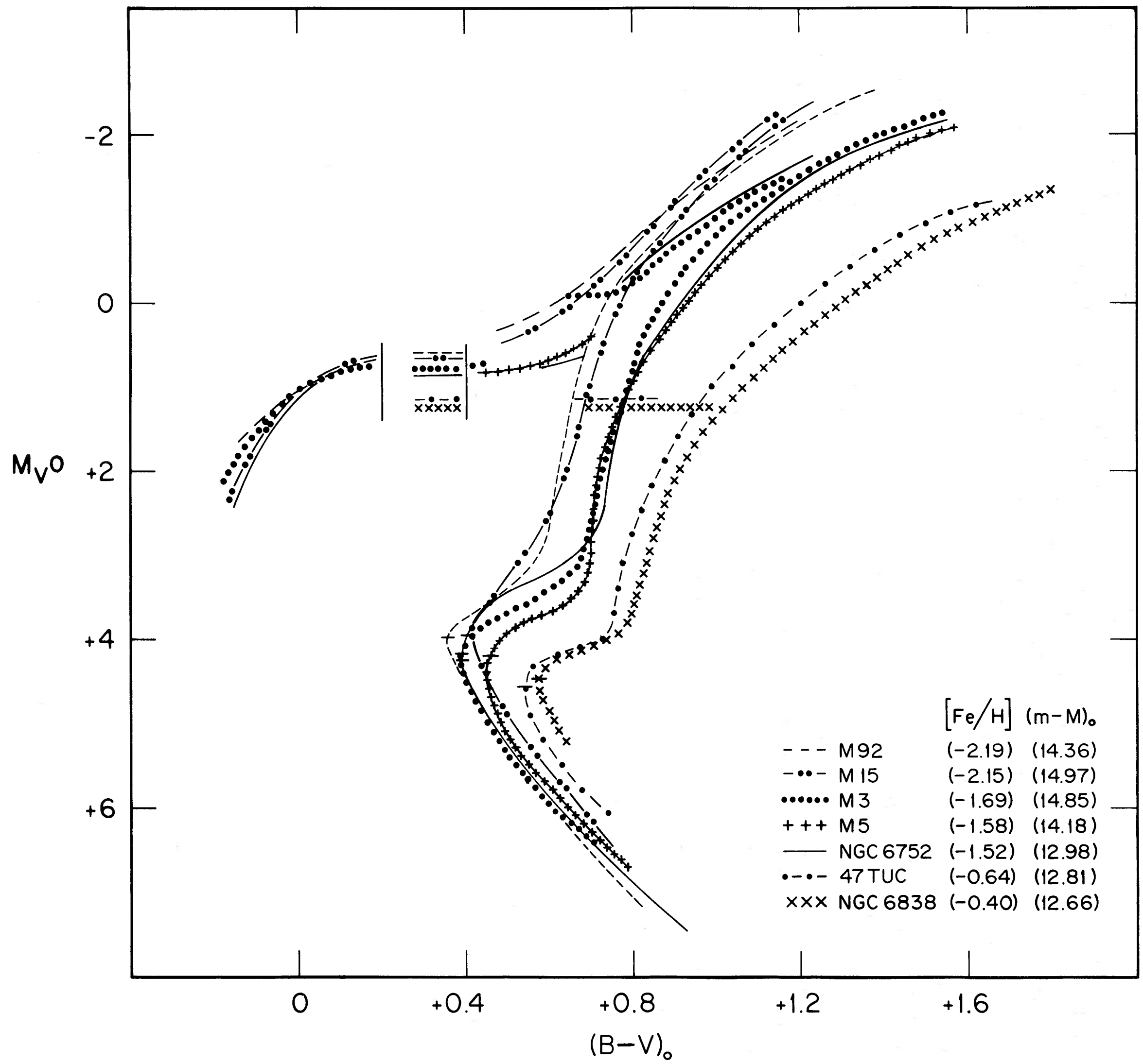

FIG. 7.—Same as Fig. 6 for all clusters in Table 2 except M13. The [Fe/H] and $(m-M)_0$ values are given in the code.

Iben (1968, 1970), who calculated $\partial M/\partial[\mathrm{Fe/H}]\approx 0.32$ as the requirement for equal age. This is nearly identical to equation (10), showing that the Yale models, which are the parents of equations (7) and (10), are isomorphic with those of Simoda and Iben in their $t=t(L^{\mathrm{TO}}, Z)$ dependence.

We now contend that since the *observed* dependence of $M_{\mathrm{bol}}^{\mathrm{TO}}$ on [Fe/H] displayed in Figures 6 and 7 is closely that of equation (10), the eight clusters must be of nearly equal age, and further (cf. § VIII) that this, of and by itself, leads to an explanation of why the Oosterhoff-Sawyer period shifts are correlated with metallicity.

How closely are the ages the same? The lines in Figure 8 have been drawn for various values of $\partial M/\partial[\mathrm{Fe/H}]$. The line marked zero has the equal-age slope of $\partial M/\partial[\mathrm{Fe/H}]=0.293$. The lines marked $\Delta t/t=\pm 0.10$ bracket the two possibilities that metal-poor clusters are either older or younger than more metal-rich systems by $\pm 10\%$.

A different representation of the same result is shown in Figure 9, where absolute ages have been calculated from equation (7) under three assumptions concerning helium abundances. Ages in panels (a) and (b) are based on variations of Y between the clusters, calculated from equation (6), normalized to a He value of Y(M15) $=0.30$ and 0.25, respectively. Panel (c) shows ages as if $Y=0.23$ for all clusters. The data are set out in Table 3 for the three cases. Error bars in Figure 9 are arbitrarily put on each cluster at $\pm 20\%$.

Figures 8 and 9 show no dependence of age on [Fe/H], contrary to the results of Saio (1977) and

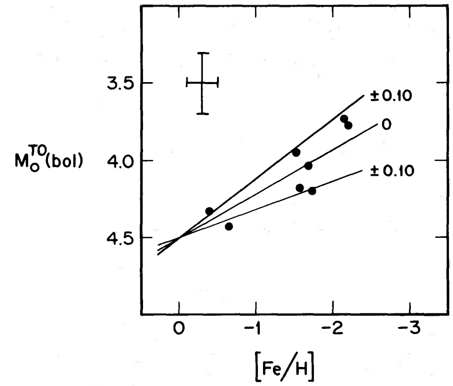

FIG. 8.—Correlation of main sequence termination luminosities with metallicity for the eight calibrating clusters. The line marked zero is the required correlation (eq. [9]) if the clusters have the same age. Lines marked ±0.10 show the required ridge line correlations if the age spread among the clusters is 10%.

Carney (1980). This is due to our different treatment of the absolute magnitudes, resulting in fainter horizontal branches for 47 Tuc and NGC 6828, and hence fainter main-sequence turnoffs and older ages here. We return to this comparison in § VII.

b) *Age of the Globular Cluster System*

With one additional assumption, the ages of the clusters in Table 1 which have RR Lyrae stars can be determined by the same argument. Assume that the nearly constant magnitude difference which is observed between the horizontal branch and the main-sequence turnoff, independent of cluster metallicity for the eight calibrating clusters (cf. col. [6] of Table 2), can be generalized to the Galactic globular cluster *system*. If all clusters have this constant difference, those with brighter main-sequence turnoffs must also have brighter horizontal branches. The condition for equal age in equation

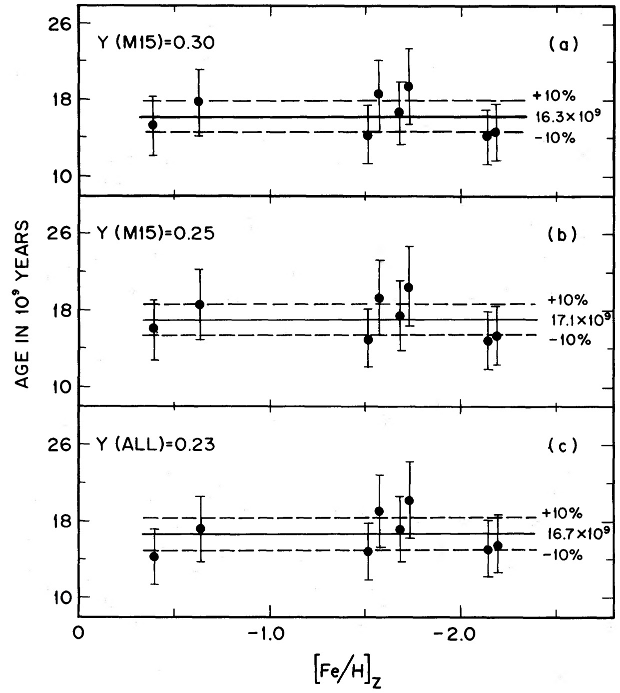

FIG. 9.—Different representation of the results of Fig. 8. The absolute ages, based on data in Table 2 put into eq. (7), are plotted vs. [Fe/H] for three assumptions concerning the He abundance. In panels (*a*) and (*b*), *Y* and [Fe/H] are assumed to be anticorrelated via eq. (6), normalized to $Y=0.30$ and 0.25 for M15, respectively. In panel (*c*), *Y* is assumed constant at 0.23 for all clusters. Error bars of ±20% are put on individual cluster ages.

(10) is then replaced by a corresponding relation that involves *horizontal branch* luminosity as

$$\Delta M_{\rm bol}^{\rm RR} = 0.293\Delta[\mathrm{Fe/H}]. \quad (11)$$

This requires a period shift to occur among the RR Lyrae stars of amount $\Delta \log P = 0.33\Delta M_{\rm bol}^{\rm RR}$, to be correlated

TABLE 3

AGES FOR THE CALIBRATING CLUSTERS WITH VARIOUS ASSUMPTIONS CONCERNING HELIUM ABUNDANCE

		VARIABLE HELIUM					CONSTANT *Y*	
CLUSTER (1)	Fe/H (2)	$M_{\rm bol}^{\rm TO}$ (3)	*Y* (4)	$(t\pm\varepsilon)_9$ (5)	*Y* (6)	$(t\pm\varepsilon)_9$ (7)	*Y* (8)	$(t\pm\varepsilon)_9$ (9)
NGC 6838	−0.40	4.34	0.18	15.3±3.0	0.13	16.1±3.2	0.23	14.3±2.9
47 Tuc	−0.64	4.43	0.20	17.7±3.5	0.15	18.6±3.7	0.23	17.2±3.4
NGC 6752	−1.52	3.95	0.26	14.4±2.9	0.21	15.1±3.0	0.23	14.8±3.0
M5	−1.58	4.19	0.26	18.4±3.7	0.21	19.3±3.9	0.23	19.0±3.8
M3	−1.69	4.05	0.27	16.6±3.3	0.22	17.4±3.5	0.23	17.2±3.4
M13	−1.73	4.20	0.27	19.4±3.9	0.22	20.4±4.1	0.23	20.2±4.0
M15	−2.15	3.74	0.30	14.1±2.8	0.25	14.8±3.0	0.23	15.1±3.0
M92	−2.19	3.76	0.30	14.6±2.9	0.25	15.3±3.1	0.23	15.6±3.1
Mean	...	...	...	16.3±2.0σ	...	17.1±2.0σ	...	16.7±2.1

with metallicity as

$$\Delta \log P = 0.097\Delta [\mathrm{Fe/H}]. \tag{12}$$

In deriving equation (9) for the relation between age, turnoff luminosity, and [Fe/H] we have assumed that Y and [Fe/H] are anticorrelated via equation (6). On the other hand, if Y is constant (and the HB absolute magnitude differences would then have some as yet unknown cause other than a variation of Y), then equation (8) requires that the expected difference in HB absolute magnitudes for constant age should be $\Delta M_{\mathrm{HB}} = 0.360\Delta[\mathrm{Fe/H}]$, or

$$\Delta \log P = 0.120\Delta [\mathrm{Fe/H}], \tag{13}$$

rather than the slightly weaker dependence of equation (12). Either argument is, then, the prediction of a correlation between period shift and metallicity for RR Lyrae stars in *any* given globular cluster (differences relative to any fiducial cluster), provided that (1) all clusters have nearly the same age, and (2) the magnitude difference between the HB and main-sequence turnoff is nearly constant.

A comparison of equations (12) and (13) with the observations for 30 clusters is given in Figure 10, which is Figure 4a with the two theoretical lines drawn. The solid line is the predicted $\partial\Delta \log P/\partial[\mathrm{Fe/H}]$ relation of equation (12). The dotted is equation (13). Both fit the data almost equally well, and no decision between them can be made.

Figure 11 shows the accuracy with which the condition of equal age can be claimed. Plotted again are the data from Figure 4a with superposed lines calculated for various values of the percentage difference $\Delta t/t$. Suppose the true correlation between period shift and metallicity is $\Delta \log P = a\Delta[\mathrm{Fe/H}]$. The coefficient a is the slope in Figure 11. Substitution into the age dating equation (9), using $\Delta M_{\mathrm{bol}}^{\mathrm{TO}} = 3\Delta \log P$, gives $\Delta t/t = \Delta[\mathrm{Fe/H}](2.83a - 0.28)$. Hence, for any value of a over a stated range of [Fe/H], the percentage age difference can be found. The lines in Figure 11 are calculated for the range $\Delta[\mathrm{Fe/H}] = \pm 0.5$. The values that label the lines should be doubled if the range is doubled to $\Delta[\mathrm{Fe/H}] = \pm 1.0$.

Figure 11 shows that the tightness of the observed correlation between period shifts and metallicity permits limits to be placed on equality of ages for clusters of all metallicities to within $\Delta t/t \approx \pm 10\%$. The actual limit is, of course, smaller because the scatter of the points in Figures 4, 10, and 11 can be almost entirely accounted for by the observational errors in both coordinates, hence the true correlation of $\Delta \log P$ with [Fe/H] is even tighter than shown in these diagrams.

VI. THE NEARLY CONSTANT ΔM(RR$-$TO) MAGNITUDE DIFFERENCE ALSO REQUIRES $\Delta t/t$ TO BE SMALL

a) *Theoretical Expectations*

The nearly constant difference between the horizontal branch and the turnoff magnitudes for the eight calibrating clusters over the metallicity range of $-2.3 <$ [Fe/H] < -0.4 requires explanation. An equation for this difference can be obtained from available main sequence and HB models with the following steps.

1. From tables of isochrones by Ciardullo and Demarque (CD) (1977) one can read $M_{\mathrm{bol}}^{\mathrm{TO}}$ and total mass of stars at main-sequence turnoff for various ages, using Y and Z as independent variables. These listings are not published in the cited reference, but a convenient table was supplied by Demarque.

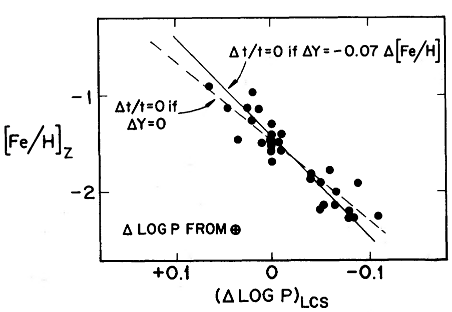

FIG. 10.—Observed correlation of RR Lyrae period shifts with metallicity for the clusters in Table 1. The condition for equal age via eqs. (12) and (13) is superposed. The solid line assumes that anticorrelation of Y with metallicity via eq. (6) is correct. The dashed line is for constant Y for all clusters.

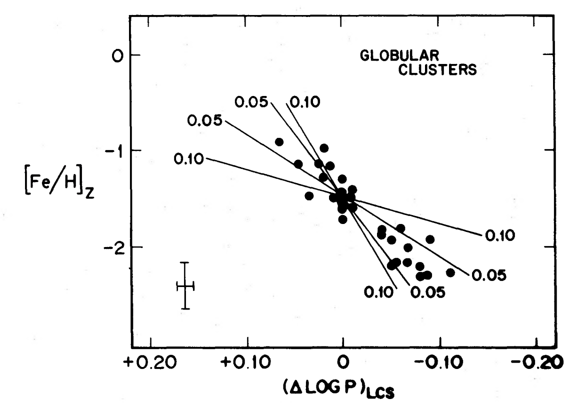

FIG. 11.—Same as Figs. 4a and 10 with lines for various values of $\Delta t/t$ superposed. The anticorrelation of Y with Fe/H via eq. (6) is imposed. The assumed range of [Fe/H] is ± 0.5. The numbers on the labels should be increased by a factor of 1.5 if $\Delta[\mathrm{Fe/H}] = \pm 0.75$.

2. From this listing the total mass, M_T, for given t, Y, and Z values can be found. The helium core mass, M_c, at the tip of the giant branch can then be calculated from the models of Rood (1972), and Sweigart and Gross (1978); cf. equation (A2) of the Appendix here.

3. Using these parameters, the luminosity of the zero-age horizontal branches at $\log T_e=3.85$ can be found from models by Iben and Rood (1970*b*), and Sweigart and Gross (1976). The results reduce to equation (A6) of the Appendix.

4. For each value of the age, Y, and Z, we now know $M_{\text{bol}}^{\text{RR}}$ from equation (A6) and also $M_{\text{bol}}^{\text{TO}}$ from step 1. Hence, the magnitude difference $\Delta M_{\text{RR}}^{\text{TO}}(\text{bol})$ can be found. The resulting table of values can be linearly approximated by

$$\Delta M_{\text{RR}}^{\text{TO}}(\text{bol})=3.46+5.2(Y-0.3)+2.56(\log t-10.2)$$
$$+0.29([\text{Fe/H}]+2.3). \qquad (14)$$

This equation satisfies the calculations to within 0.08 mag everywhere in the interval $0.2 \leq Y \leq 0.3$; $-2.3 \leq [\text{Fe/H}] \leq -0.7$; $10 \exp 9 \leq t \leq 25 \exp 9$. It is better than 0.04 mag for $Y=0.4$ between [Fe/H] of -2.3 and -1.3 if $t > 12$ billion years, and is within 0.1 mag for [Fe/H] as large as -0.3 in the same age interval.

To better fix the steps and to illustrate the agreement of equation (14) with the directly calculated values, a sample set of tables, and a more detailed explanation of the method, is set out in the Appendix.

Before this procedure had been completed, the problem was posed to Icko Iben, who supplied an algebraic solution based on linear interpolations of his choice of the relevant equations. His final result, after reduction, is

$$\Delta M_{\text{RR}}^{\text{TO}}(\text{bol})=3.68+4.42(Y-0.3)+2.23(\log t-10.2)$$
$$+0.15([\text{Fe/H}]+2.3), \qquad (15)$$

where t is in years, and $\log Z=[\text{Fe/H}]-1.7$ is adopted using $Z=0.02$.

Iben's route to equation (15) is the same as described above, but his approximations are different. He adopts the explicit age dating equation with its Y, Z, and L_{TO} dependences from Iben and Rood (1970*a*) rather than from the Yale isochrones. His L_{RR} is taken from Sweigart and Gross (1976), and the core He mass is from Rood (1972). He uses slightly different assumptions such as neglecting the mass term in L_{RR}, but not its dependence on Z (we adopt the opposite; see n. 2 of Paper I and also §§ III*b* and III*c* here for a justification). He applies a small correction of ~ 0.1 mag for RR Lyrae evolution which we have neglected because we read the RR Lyrae apparent magnitudes as a lower envelope to the observed distribution, which presumably refers to the zero-age horizontal branch.

Nevertheless, equations (14) and (15) are closely the same over the range of application between $Y=0.2$ and 0.3, [Fe/H] between -2.3 and -0.2, and t between 10×10^9 and 20×10^9 years. The maximum difference is ~ 0.35 mag, being larger at low [Fe/H] due to the different Z dependences. The difference goes to zero in particular regions of the parameter space. The zero point difference would be reduced by ~ 0.1 mag if Iben had applied no RR Lyrae evolution.

What conclusions can be drawn from equations (14) and (15)? We again adopt the condition that Y and [Fe/H] are coupled via equation (6). Substitution of this dependence into equations (14) and (15) gives

$$\delta\Delta M_{\text{RR}}^{\text{TO}}(\text{bol})=-0.069\Delta[\text{Fe/H}]+2.56\Delta\log t, \qquad (14a)$$

and

$$\delta\Delta M_{\text{RR}}^{\text{TO}}(\text{bol})=-0.153\Delta[\text{Fe/H}]+2.23\Delta\log t, \qquad (15a)$$

respectively, for the variation of the magnitude difference (TO$-$RR) as [Fe/H] and age are varied.

b) *Observational Comparison*

Limits on possible variations in the ages can be found by comparing these equations with the observations of the eight calibrating clusters. The data in column (6) of Table 2 must be changed to bolometric magnitude differences. The bolometric correction (BC) at main-sequence turnoff is in column (11) of that table. The BC for RR Lyrae variables at $\log T_e=3.85$ is adopted to be zero, following Morton and Adams (1968, Table 2), Newell (1970, Table 3), and Bell (quoted by Butler, Dickens, and Epps 1978). Hence, column (11) added to column (6) of Table 2 gives the required $\Delta M_{\text{RR}}^{\text{TO}}(\text{bol})$.

We do not consider the *zero point* of the (TO$-$RR) magnitude difference to be well determined from theory, but examine here only the trend of $\Delta M_{\text{RR}}^{\text{TO}}(\text{bol})$ with [Fe/H].

The data are set out in Figure 12, where ΔM_{bol} is plotted versus [Fe/H]. Each observed point is assigned an observational error of $\varepsilon\Delta M=\pm0.2$ mag. The solid line in the top panel is the mean value $\langle\Delta M(\text{TO}-\text{RR})\rangle = +3.21\pm0.13\sigma$. The $\pm1\sigma$ lines are dashed. This line, with no dependence on [Fe/H], adequately fits the data.

Equation (14a) or (15a) can be made to give the expected variation of $\Delta M_{\text{RR}}^{\text{TO}}(\text{bol})$ with [Fe/H] for various age differences. Suppose that age *is* a function of metallicity in the form

$$\log t=a[\text{Fe/H}]+\text{const}. \qquad (16)$$

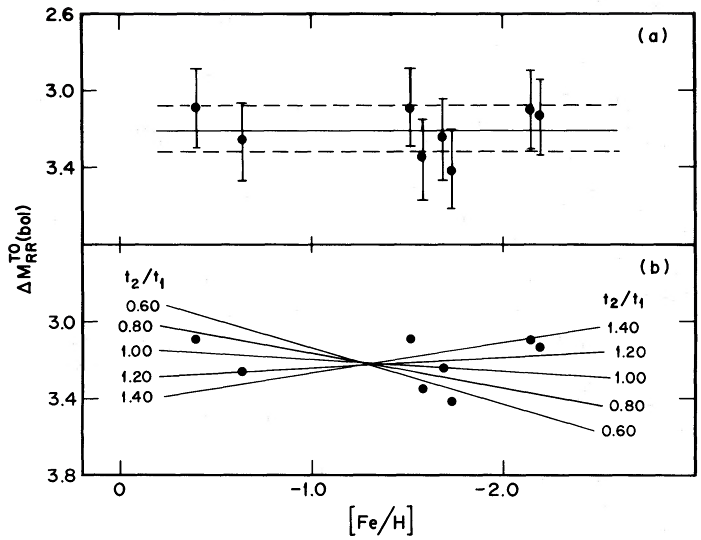

FIG. 12.—Correlation of metallicity with the observed difference in bolometric magnitude between the main sequence turnoff and the zero age horizontal branch. Panel (a) shows the observations with their assumed errors of ± 0.2 mag. The mean line is put at $\langle \Delta M(\text{bol}) \rangle = +3.21 \pm 0.13\sigma$. One σ lines are dashed. Panel (b) shows the same observations with age ratios using eq. (14a). If $t_2/t_1 < 1$, the sense is that older clusters have lower metallicity. From these data the maximum permissible age differences are of order $\Delta t/t \approx 20\%$. The best fit is for $\Delta t/t \approx 0 \pm 10\%$.

Over the stated range of [Fe/H], the variation of log t is then fixed once a is known, and this, substituted into equation (14a) or (15a), gives the prediction that

$$\delta \Delta M_{RR}^{TO} = (2.56a - 0.069)\Delta[\text{Fe/H}].$$

This result is shown in Figure 12b as a family of curves labeled as age ratios t_2/t_1 for clusters that span Δ[Fe/H]=2. The relation between the slope a in equation (16) and the age ratios is $a = 0.217(t_2/t_1 - 1)$; hence if a is negative (i.e., smaller metal abundance for older age), then $t_2/t_1 < 1$. This sense is also clear from equations (14a) and (15a).

Inspection of Figure 12b shows that the data can be fitted with age ratios in the range $0.8 \lesssim t_2/t_1 \lesssim 1.4$, or $-0.04 < a < 0.08$. The best fit is close to the horizontal mean line of Figure 12a, which is $t_2/t_1 \approx 1.1$, or $a \approx 0.02$, but $t_2/t_1 = 1.00$ (or $a = 0$) is within the errors. Carney's (1980) claim that $a = -0.168$ is inconsistent with the observations by about 4σ.

VII. COMPARISON WITH PREVIOUS STUDIES OF GLOBULAR CLUSTER AGES

Discussion of globular cluster ages became possible when their observed main sequence turnoffs were identified with the Schönberg-Chandrasekhar (1942) mass limit for hydrogen-exhausted convective cores. The first estimate gave $t = 3.5 \times 10^9$ years (Sandage and Schwarzschild 1952). The value increased steadily in later years, passing through 6×10^9 years (Hoyle and Schwarzschild 1955), $\sim 10 \times 10^9$ years (Hoyle 1959), finally reaching $\gtrsim 20 \times 10^9$ years (Sandage 1962), but subsequently reduced by half (Woolf 1962).

These early estimates were based primarily on various calculations of the core mass at main-sequence turnoff. However, until the late 1960s no systematic exploration had been made of the dependence of age on metal abundance. Simoda and Iben's crucial discovery of the (age, Z)-dependence was made in 1968.

In recent age determinations, summarized by Carney (1980, his Table 9), most authors have used the same models, due now principally to Iben and Rood (1970a) and Ciardullo and Demarque (1977, CD), supplemented with special models such as those of Hartwick and van den Berg (1973) and Saio (1977); hence the principal difference in the calculated ages in these discussions is due more to different adopted turnoff luminosities than to differences in the models.

The methods used to obtain turnoff luminosities from the observations all suffer from various uncertainties, and each method exploits particular advantages peculiar to itself. There are two principal disadvantages to the straightforward, semiempirical procedure of main sequence fitting (Sandage 1970). The method relied on a group of trigonometric parallax stars (groups A, B, C, and D in Table 12 of the cited reference) to define the position of the fiducial main sequence as a function of ultraviolet excess. The globular clusters of known excess were then fitted to the relation. The disadvantages are: (a) The fiducial relation is poorly determined. There are few trigonometric parallax stars; their parallaxes are small; a bias (Wallerstein 1967; Lutz and Kelker 1973) in their M_V values must be corrected. The most serious objection by Eggen (1973, his § VIII) is that new data change my adopted relation between $\delta(U-B)$ and main sequence displacement fundamentally. (*b*) The color data for the globular clusters must be of very high accuracy. The reddening must be known to $\varepsilon[E(B-V)] \approx \pm 0.02$ mag to avoid errors as large as $\Delta M_V^{\mathrm{TO}} \approx \pm 0.2$ mag; the color system and the observed colors must be known as accurately; the $U-B$ colors needed to obtain $\delta(U-B)$ must be known to the same accuracy. As the relevant globular cluster stars are generally fainter than $V \approx 17$, the entire procedure is generally unsatisfactory in obtaining main-sequence absolute accuracies to better than ~0.3 mag, and hence ages to within ~30%.

The method we have now adopted here avoids all this reliance on colors. It depends only on the apparent *magnitude* difference between the horizontal branch and turnoff as observed directly, normalized by the differences in HB absolute luminosities given by the *RR Lyrae period shifts*, and hence is very powerful, if indeed period shifts *are* related to $\Delta M_{\mathrm{bol}}^{\mathrm{RR}}$ values. To be sure, the setting of the absolute zero point at $M_V(\mathrm{RR})_{\mathrm{M3}} = +0.8$ determines the absolute age; however, the *relative* ages and hence the age spread depend only on the period shifts and the slight extrapolation of the period shift–[Fe/H] relation to 47 Tuc and NGC 6838.

The method of Carney (1980) has two disadvantages. (1) It relies again on the observed main sequence colors in the clusters where the effects of errors in the reddening and the photometry are the same as stated above. (2) More seriously, the cluster moduli are determined by fitting to the calculated *temperatures* of the Ciardullo-Demarque isochrones rather than to any observed fiducial sequence. Reliance is, then, put entirely on the model radii; these are known to be quite uncertain because no fundamental theory of convective energy transport exists.

The difference between the strong dependence of age on [Fe/H] claimed by Saio (1977) and by Carney (1980) and the counterclaim here rests with the input data, primarily for the metal-rich clusters NGC 6838 and 47 Tuc. Our modulus for 47 Tuc is $(m-M)_0 = 12.81$; Carney's is 13.37; Saio gives 13.03, which was also used by Demarque and McClure (1977); Hartwick and van den Berg (1973) obtain 13.49 using the observed main-sequence colors and the earlier fitting method (Sandage 1970). Carney does not explicitly state his modulus for NGC 6838. Our modulus is 12.66; Saio gives 13.04; Hartwick and van den Berg derive 13.22.

Rather than discuss in detail the reasons for these differences, which can be traced to differences in the uncertain main-sequence fittings, it is more instructive to show the consequences for the predicted *magnitude difference* between the HB and turnoff for clusters with different [Fe/H] using Carney's claim for a strong dependence of age on *Z*. Carney's (1980, Table 9, last row) adopted ages correlate with [Fe/H] as $\log t_c = -0.168[\mathrm{Fe/H}] + 9.924$, where t_c is in years. When put into equation (14a), one predicts the variation of HB to TO magnitude difference to vary with [Fe/H] as $\delta\Delta M_{\mathrm{HB}}^{\mathrm{TO}}(\mathrm{bol}) = -0.50\Delta[\mathrm{Fe/H}]$, a very strong dependence that is not observed. The observations of the eight clusters plotted in Figure 12*a* span the range $\Delta[\mathrm{Fe/H}] = 1.8$ which would require $\Delta M_{\mathrm{HB}}^{\mathrm{TO}}(\mathrm{bol})$ to vary by 0.9 mag between NGC 6838 and M92. Table 2 (col. [11] added to col. [6]) shows a negligible difference of only 0.04 mag.

The comparison between the observations and Carney's age-metallicity relation is shown more explicitly in Figure 13. Plotted again are the observations from Table 2, found by combining columns (6) and (11). The line has a slope of $\partial\Delta M/\partial[\mathrm{Fe/H}] = -0.5$ which does not fit the data.

The size of the discrepancy between the prediction and the observations can be eased somewhat if Y does not vary inversely with Z, for then equation (14) with constant Y (but again using Carney's age-Z relation) yields $\partial\Delta M/\partial[\mathrm{Fe/H}] = -0.14$; however, as previously stated, constant Y has no justification if we must vary the HB luminosities to explain the period shifts. Note also that Carney's equation (9) gives brighter luminosities for higher metallicities, which is the opposite sense to that required to understand the Oosterhoff period shifts and their correlation with metallicity.

We close the section by noting that high ages for many of the individual low-metallicity clusters have also been obtained earlier by others. Both Carney and Saio obtain ages $\gtrsim 15 \times 10^9$ years for clusters of low metallicity. Saio, Shibata, and Simoda (1977) obtained an age between 15 and 20×10^9 years for M92. Demarque and McClure (1977) found ages of $13-16 \times 10^9$ years for M3,

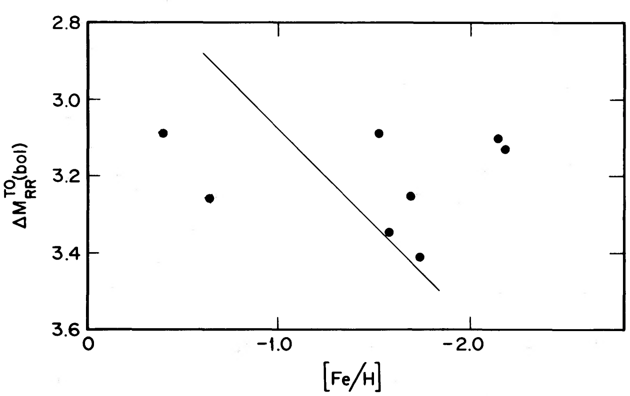

FIG. 13.—Same as in Fig. 12 but with the expected variation of $\Delta M_{\mathrm{RR}}^{\mathrm{TO}}(\mathrm{bol})$ with [Fe/H] if Carney's claim of age variation with [Fe/H] would have been correct. The solid line is computed as if Y is anticorrelated with [Fe/H] via eq. (6). The prediction violates the observations. The equal age hypothesis of Fig. 12 more nearly fits the data.

M13, M15, and M92; Bertelli *et al.* (1979) found even larger values. Böhm-Vitense and Szkody (1973) found 23×10^9 years for M15 and M92.

VIII. DISCUSSION AND SUMMARY

This paper contains two principal results. (a) An explanation of the Oosterhoff RR Lyrae period shifts, their correlation with metallicity, and the progression of the period-amplitude relations toward longer periods in clusters of lower metallicity can be given from the fact that all clusters have nearly equal ages, despite the large variations in [Fe/H] among the clusters. The consequences of this condition of equal age in the presence of Δ[Fe/H] variations of ~ 2 (i.e., a factor of 100 in Z) have been worked out in §§ III, IV, and V. (b) The absolute age of the cluster system in the galaxy is near 17×10^9 years (§ V).

Discussion of these two results follows.

a) *The Oosterhoff Period Shifts*

The principal point made in § III is that the metallicity and the period shifts are tightly correlated; the dependence is given by equation (2). The results of Papers I and II show that these period shifts are almost certainly caused by luminosity differences of the variables; longer period RR Lyrae stars are brighter than others of equal amplitude. And because the period shifts are determined star by star at fixed T_e (or amplitude), the van Albada-Baker (1973) hysteresis mechanism which they invoke to explain part of the Oosterhoff period shifts (they also propose small changes in L and mass as well) can only change the *mean* period $\langle P_{ab} \rangle$ over a group of variables without changing L_{RR}, and therefore is not germane. None of the arguments in Papers I and II rely on the shift $\Delta \log \langle P_{ab} \rangle$ of the *mean* period but rather on the actual period shifts star by star read either at constant T_e, constant amplitude, or constant light curve shape. No hysteresis mechanism can change these values of $\Delta \log P_{\mathrm{LCS}}$ for the individual stars.

Adopting the required luminosity differences for the horizontal branches determines the main sequence turnoff levels for the eight clusters in Table 2. These correlate with [Fe/H] (Fig. 8), in such a way that all clusters have nearly the same age, because the Simoda-Iben theoretical requirement that $\partial M^{\mathrm{TO}}/\partial[\mathrm{Fe/H}] \approx 0.3$ if $\Delta t = 0$ is satisfied by the observations.

It was further shown (§ VI) that the (Y, [Fe/H])-dependence that is needed to produce the required $\Delta \log L_{\mathrm{RR}}$ between clusters, when applied to clusters of nearly equal age, produces a nearly constant absolute magnitude difference between the horizontal branch and the turnoff point (Fig. 12 and eqs. [14a] and [15a]), as observed. This is the justification for believing that, as the TO luminosity is brighter for lower metallicity, so then is the luminosity of the HB, and this is the reason why the HB luminosity itself is correlated with [Fe/H].

Given the above, it is a single step more to understand why the period-amplitude relations for RR Lyrae variables *depend on metallicity*. Amplitude is a unique function of depth of penetration into the instability strip (Paper II). Hence, in clusters with different HB luminosities, and therefore different metallicities, variables at a given temperature (i.e., amplitude) have different periods. Therefore, the period-amplitude relations are ordered by metallicity, given that the period shifts *are* correlated with [Fe/H] because clusters of different metallicity have equal ages. An important test of the model satisfied by the observations is that the period shift determined from the period-temperature relation be the same as from the period-amplitude relation (Paper II, Table 7).

Two deeper questions remain unanswered. (1) What is the cause of the large variations in [Fe/H] throughout the cluster system, given that the age difference of its members is so small? (2) Why is the helium abundance anticorrelated with [Fe/H]?

The answer to the first question may be that the enrichment near the beginning of the collapse was very spotty, caused by many small, self-contained separate enrichment events, each of mass $\sim 10^7 M_{\odot}$. These events presumably occurred early in the initial stages of the galactic collapse, caused perhaps by unstable supermassive objects with lifetimes of $\sim 10^6$ years in which the initial heavy elements were made and subsequently dispersed. If the products of these events were not well stirred throughout the halo during the further stages of the collapse, the large variation in [Fe/H] among the smaller units that became globular clusters could be produced within the first $\sim 5 \times 10^8$ years of the galactic evolution (Sandage and Hartwick 1977; Searle and Zinn 1978). No clue concerning the second question is evident.

How secure are these results? The single most critical point in the analysis of Papers I–II is that period shifts, read at constant T_e, or A_B, or $\Delta\phi_{\mathrm{rise}}$, signal luminosity differences of size $\Delta M_{\mathrm{bol}}^{\mathrm{RR}} = 3\Delta \log P$. Once this is granted, the normalization of the color-magnitude diagrams in Figures 6 and 7, and thence the ages, follow automatically. The data from Papers I and II suggest that luminosity differences provide, in fact, a reasonable explanation for the star-by-star period shifts. Mass variations due to a mass dependence on [Fe/H] at fixed T_e along the zero-age horizontal branch, discussed in Paper I (§ VI), were shown to be of the wrong sign to explain the period shifts. Mass variations due to evolution from the zero-age horizontal branch could also be ruled out. It is also shown in n. 1 here that mass variations that accompany a variable Y value at fixed T_e are also negligible. The results, generalized in Paper II and applied there (Fig. 10) to individual stars in M3 and ω Cen, showed that $\Delta M_{\mathrm{bol}}^{\mathrm{RR}} = 3\Delta \log P$ was also required from the data directly. It was emphasized in §§ III*a* and III*c* of the present paper that we need not understand the reason for the $\Delta \log L_{RR}$ between the clusters before we can use $\Delta M_{bol}^{RR} \approx 3\Delta \log P$ to normalize the *C-M* diagrams in § IV. Hence, we emphasize once again that the conclusion that Y and [Fe/H] may be anticorrelated could, in fact, be wrong without jeopardizing the principal conclusions (*a*) and (*b*) of this paper. Only rather minor details of the age dating would be changed (cf. Table 3 and Fig. 9) if Y need not be varied between the clusters so as to produce $\Delta \log L_{RR}$ as in equation (3).

b) *Absolute Ages*

The absolute ages given in Table 3 and Figure 8 depend mainly on the absolute magnitudes of the turnoffs, and to a much lesser extent on the helium abundance. In turn, M^{TO}(bol) depends on our assumption of $M_{RR} = +0.80$ for M3. Justification for this value is a central issue.

The direct method of main-sequence fittings using blanketing corrected colors and trigonometric parallax stars (Sandage 1970, § IV) gave M_V(M3)$=0.60$, M_V(M15)$=0.98$, M_V(M92)$=0.91$ where, as previously mentioned, the luminosity differences between the clusters are in the opposite sense to that required by the Oosterhoff problem. We rejected the individual values in 1970 for this reason. However, the mean value may have some weight, provided that the method itself is not fundamentally flawed. The mean of the three values is $\langle M \rangle_V = +0.83 \pm 0.15\sigma$. In the cited reference we corrected by 0.2 mag for bias in the parallaxes of the calibrating stars to obtain $\langle M \rangle_V +0.63 \pm 0.15\sigma$. The mean Oosterhoff type of the three clusters is 1.7 and hence is closest to M15.

A second method that avoids blanketing corrections altogether is to use only those nearby trigonometric parallax subdwarfs to define a fiducial main sequence for [Fe/H] values that are similar to those of the clusters under discussion. Such stars will have the same blanketing, hence their colors can be directly compared with the cluster data. If these field subdwarfs also have the same Y values as the clusters, their main-sequence positions in the (M_{bol}, $\log T_e$)-plane should also be the same. The method was used initially to obtain $(m-M)_0$ for M3 and M13 taking the Groombridge 1830 moving group as the fiducial sequence (Eggen and Sandage 1959). These authors obtained $M_V = +0.8$ for RR Lyrae itself, and $M_V = +0.6$ for four other field variables.

As a variation of the method, we can restrict the cluster sample to the metallicity range of $-1.3 <$[Fe/H]< -2.4 and use the eight subdwarfs with adequately large trigonometric parallaxes summarized by Carney (1979*a*). The mean metallicity of these stars is $\langle$[Fe/H]$\rangle = -1.7$, which is about one-third of the way between M3 and M15. Using these stars to form a fiducial sequence, the fit gives M_V(M3)$= +0.84$, M_V(M5)$= +1.02$, M_V(NGC 6752)$= +0.99$, M_V(M92)$= +0.64$, and M_V(M15)$= +1.00$ if $E(B-V)=0.08$ or $+0.75$ if $E(B-V)=0.12$ for M15. The mean of the values is $\langle M_V \rangle = +0.87 \pm 0.16\sigma$. Note that the ordering of the weakest-lined clusters (M15 and M92) relative to the M3 group is now in the sense that we require by the Oosterhoff period shifts.

As previously mentioned, the difficulty with the method is its great sensitivity to the observed colors, and to the reddening and blanketing corrections. As the M15 comparison shows, errors of only 0.04 mag in either the color *system* or the colors themselves give errors of ~ 0.3 mag in M_V(RR).

A third method is a modified Baade-Wesselink procedure using the Barnes-Evans surface brightness relation, first worked out by Wesselink (1969). A preliminary analysis of X Ari ($\Delta S = 10$) and RR Lyr ($\Delta S = 6$) by Manduca, Moffett, Barnes, and Evans (Manduca 1980, private communication) gives $M_V = +0.60 \pm 0.20$ for the two stars, whose mean ΔS is 8.

The mean of the three methods support the adopted values of M_V(RR)$=0.80$ for M3 and M_V(RR)$=0.64$ for M15 in Table 2, to within ± 0.05 mag, from which it follows that the system of globular clusters has the ages in Table 3. The preferred ages are those from column (7) where the He abundances generally agree with external data.

The age of $(17 \pm 2) \times 10^9$ years for the galactic halo, when added to $t = 0.2 H_0^{-1}$ for the formation time for galactic nuclei obtained from the observed upper redshift limit of $\Delta\lambda/\lambda_0 \approx 4$ for quasars, gives $H_0^{-1} = 17 \times 10^9 + 0.2 H_0^{-1}$ years, if $q_0 = 0$. This value of $H_0^{-1} = (21 \pm 2) \times 10^9$ years gives $H_0 = 46 \pm 4$ km s^{-1} Mpc^{-1}, which is in clear violation with appreciably larger values occasionally mentioned by some authors.

It is a pleasure to thank Icko Iben for his solution to the problem posed in § IV, Pierre Demarque for sending the convenient summary of the Ciardullo and Demarque isochrones at main sequence turnoff, Robert Kraft for a very helpful correspondence, and the referee for requesting wider explanations. It is also a pleasure to thank Pamela Gilman, Maria Anderson, and Nancy Newton for their work in preparing the many drafts of the manuscript and the diagrams for publication.

APPENDIX

DERIVATION OF EQUATIONS FOR L_{RR} AND ΔM_{TO}^{RR}(bol)

The absolute magnitude of globular cluster horizontal branches and the luminosity ratio of horizontal branch to main sequence levels as functions of age and chemical composition can be derived by combining several auxiliary equations.

a) *Horizontal Branch Luminosities*

The models of Iben and Rood (1970*b*) and Sweigart and Gross (1976) show how L_{RR} depends on core mass, Y, and Z. Relations between these parameters have been determined from Table 1 of the latter reference. Reading this table at $\log T_e=3.85$ along the zero age horizontal branch gives

$$\log L_{RR}=1.80+3.18(M_c-0.50)+2.27(Y-0.3), \tag{A1}$$

where the slight dependence on Z has been neglected. The consequences of this neglect are discussed in Paper I (n. 2). A justification for its neglect via its daughter equation (3) for L_{RR} is given in § III*c* and Figure 5 of the present paper. The helium core mass is related to the total mass M_T, to Y, and to Z by

$$M_c=0.474-0.24(Y-0.3)-0.007([Fe/H]+1.3)-0.038(M_T-0.8), \tag{A2}$$

as given directly by Rood (1972) and Sweigart and Gross (1978). The dependence of M_c on total mass in equation (A2) requires M_T to be the model mass of any star during its pre–helium flash evolutionary track from the main sequence to the tip of the giant branch; i.e., no mass loss is assumed to occur until the helium flash. Hence, M_T can be found from the main sequence models of the progenitors of any given giant star track. The tabulated data for the isochrones (Ciardullo and Demarque 1977, hereafter CD), computed from the Yale models, give M_T for any listed age for particular Y and Z values. Reading the CD tables at the bluest point of the isochrones (i.e., the main sequence turnoff) gives values of L_{TO} for given age, Y, and Z. A convenient summary table of these parameters, as read from the CD isochrones, was very kindly supplied by Demarque.

An extract of the needed values is shown in Table 4 here. For any given Y and [Fe/H] value, the other parameters are listed as follows. The age, taken as the independent tabular value, is given in column (1). Columns (2) and (3) give the bolometric turnoff luminosity and the total mass, taken directly from the CD tables. Column (4) gives the helium core mass (in solar units) calculated from equation (A2). The resulting RR Lyrae star luminosity (in solar units), listed in column (5), is calculated from equation (A1) using M_c from column (4). The resulting absolute bolometric magnitude is given in column (6), found by adopting $M(\mathrm{bol})=+4.72$ for the Sun.

The variation of $M_{\mathrm{bol}}^{\mathrm{RR}}$ with the parameters, as determined from the run of values in the table, is

$$\partial M^{\mathrm{RR}}/\partial Y=-4.20, \tag{A3}$$

in the sense that larger Y gives brighter RR Lyrae luminosities for constant age and [Fe/H];

$$\partial M^{\mathrm{RR}}/[\mathrm{Fe/H}]=+0.06, \tag{A4}$$

in the sense that lower metal abundance gives brighter luminosities for constant age and Y; and

$$\partial M^{\mathrm{RR}}/\log t=-0.125, \tag{A5}$$

in the sense that older ages give brighter luminosities for constant Y and [Fe/H]. The gradients in equations (A3), (A4), and (A5) are valid in the interval $10\lesssim t_9\lesssim 25$, $-2.3<[\mathrm{Fe/H}]<-0.7$, and $0.2\leq Y\leq 0.4$.

A linear approximation is

$$M_{\mathrm{bol}}^{\mathrm{RR}}=0.34-4.20(Y-0.3)-0.125(\log t-10.2)+0.06([\mathrm{Fe/H}]+2.3), \tag{A6}$$

which is equation (14) of Paper I.

b) *Magnitude Difference between the Horizontal Branch and the Main Sequence Turnoff*

Combining columns (2) and (5) of Table 4 gives the luminosity ratio $L(\mathrm{RR})/L(\mathrm{TO})$, from which the absolute magnitude difference in column (7) follows. The partial gradients, determined from the tabular entries, are

$$\partial \Delta M/\partial Y=+5.10, \tag{A7}$$

TABLE 4

REPRESENTATIVE MODEL PARAMETERS THAT LEAD TO L_{RR} AND ΔM_{TO}^{RR}

Age 10^9 (1)	Log L_{TO} (2)	M_T $\odot$ (3)	M_c $\odot$ (4)	Log L_{RR} (5)	M_{bol}^{RR} (6)	ΔM_{RR}^{TO} (7)	ΔM_{RR}^{TO} calc. Eq. 14
			Y = 0.2; [Fe/H] = -2.3				
10	0.620	0.940	0.500	1.573	0.79	2.38	2.43
14	0.465	0.856	0.503	1.583	0.76	2.80	2.80
18	0.352	0.800	0.505	1.589	0.75	3.09	3.08
22	0.266	0.758	0.507	1.595	0.73	3.32	3.30
			Y = 0.2; [Fe/H] = -1.3				
10	0.486	0.941	0.493	1.551	0.84	2.66	2.72
14	0.313	0.857	0.496	1.556	0.83	3.11	3.09
18	0.222	0.804	0.498	1.564	0.81	3.35	3.57
22	0.134	0.763	0.499	1.570	0.80	3.59	3.59
			Y = 0.2; [Fe/H] = -0.7				
10	0.378	0.964	0.488	1.535	0.88	3.89	2.89
14	0.209	0.880	0.490	1.541	0.87	3.33	3.26
18	0.117	0.826	0.492	1.548	0.85	3.58	3.54
22	0.052	0.787	0.494	1.554	0.84	3.76	3.76
			Y = 0.2; [Fe/H] = -0.3				
10	0.296	1.002	0.483	1.519	0.92	3.06	3.01
14	0.160	0.923	0.485	1.525	0.91	3.41	3.38
18	0.064	0.868	0.487	1.532	0.89	3.67	3.66
22	-0.006	0.826	0.489	1.539	0.87	3.86	3.88
			Y = 0.3; [Fe/H] = -2.3				
10	0.568	0.795	0.481	1.740	0.37	2.93	2.95
14	0.424	0.725	0.484	1.749	0.35	3.31	3.32
18	0.317	0.678	0.486	1.755	0.33	3.59	3.60
22	0.230	0.642	0.487	1.759	0.32	3.82	3.82
			Y = 0.3; [Fe/H] = -1.3				
10	0.420	0.795	0.474	1.717	0.43	3.24	3.24
14	0.284	0.728	0.477	1.727	0.40	3.61	3.61
18	0.164	0.680	0.479	1.733	0.39	3.92	3.89
22	0.092	0.646	0.480	1.736	0.38	4.11	4.11
			Y = 0.3; [Fe/H] = -0.7				
10	0.294	0.811	0.470	1.705	0.46	3.53	3.41
14	0.167	0.746	0.472	1.711	0.44	3.86	3.78
18	0.080	0.702	0.474	1.717	0.43	4.09	4.06
22	-0.004	0.667	0.475	1.721	0.42	4.29	4.28
			Y = 0.4; [Fe/H] = -2.3				
10	0.521	0.663	0.462	1.906	-0.04	3.46	3.47
14	0.383	0.606	0.464	1.912	-0.06	3.82	3.84
18	0.285	0.567	0.466	1.918	-0.08	4.08	4.12
22	0.197	0.537	0.467	1.923	-0.09	4.31	4.34
			Y = 0.4; [Fe/H] = -1.3				
10	0.358	0.662	0.455	1.884	0.01	3.82	3.76
14	0.228	0.608	0.457	1.886	0.01	4.15	4.13
18	0.130	0.569	0.459	1.889	0.00	4.40	4.41
22	0.052	0.541	0.460	1.892	-0.01	4.60	4.63
			Y = 0.4; [Fe/H] = -0.3				
10	0.122	0.697	0.447	1.858	0.08	4.34	4.05
14	0.016	0.642	0.448	1.862	0.07	4.62	4.42
18	-0.067	0.605	0.449	1.867	0.05	4.84	4.70
22	-0.136	0.576	0.451	1.872	0.04	5.02	4.92

at $\log t = 10.2$ and $[\mathrm{Fe/H}] = -2.3$ in the sense that the luminosity ratio becomes larger for greater Y, at constant age and [Fe/H];

$$\partial \Delta M / \partial [\mathrm{Fe/H}] = +0.29, \tag{A8}$$

at $\log t = 10.2$ and $Y = 0.2$ in the sense that the luminosity ratio becomes larger for higher metallicity, for constant age and Y;

$$\partial M / \partial \log t = 2.56,$$

at $Y = 0.3$ and $[\mathrm{Fe/H}] = -1.3$ in the sense that the luminosity ratio becomes larger for greater ages, at constant Y and [Fe/H].

Hence, a linear approximation to the Table 4 values is

$$\Delta M_{\mathrm{RR}}^{\mathrm{TO}}(\mathrm{bol}) = 3.46 + 5.2(Y - 0.3) + 2.56(\log t - 10.2) + 0.29([\mathrm{Fe/H}] + 2.3), \tag{A9}$$

which is equation (14) of the text. Values calculated from this equation are listed in column (8) of Table 4 for comparison with column (7).

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THE OOSTERHOFF PERIOD GROUPS AND THE AGE OF GLOBULAR CLUSTERS. IV. FIELD RR LYRAE STARS: AGE OF THE GALACTIC DISK

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ABSTRACT

Individual period shifts, $\Delta \log P$, determined by comparing the periods and amplitudes of ~ 125 Bailey type *ab* RR Lyrae field stars with the period-amplitude relation for M3, are well correlated with Preston's ΔS index. The distribution of $\Delta \log P$ forms a continuum over the interval $+0.15 > \Delta \log P > -0.12$ which embraces the metallicity range of $0 \leq \Delta S \leq 10$. The slope of the correlation is $\partial \Delta \log P / \partial [\mathrm{Fe/H}] = 0.116$ which, by the argument in Paper III, is the condition that all RR Lyrae stars that obey the correlation have the same age to within the accuracy of the method at $\sim \pm 10\%$.

Because the high-metallicity RR Lyrae variables with $\Delta S \leq 2$ are known to be old disk objects, it follows from their great age that this component of the disk began to form at closely the same time as the halo globular clusters. A similar conclusion follows by noting that older giants than those in NGC 188 also exist in the disk. The field giants studied by Wilson, by Helfer, and by Janes in the luminosity range $0.5 \leq M_v \leq 1.5$ became progressively bluer than those in NGC 188, as [Fe/H] is decreased. The observed color gradient with metallicity of $\partial (B-V)_{0,g} / \partial [\mathrm{Fe/H}] = +0.44$ agrees with the color difference between giants in NGC 188 and the 47 Tuc in the composite *C*-*M* diagram at $M_v = +1$.

A summary diagram is given which shows again that the variation of [Fe/H] with age for various components of the Galaxy was rapid during halo collapse, but that the enrichment of the disk was slower as this structure was built over a longer time.

Subject headings: clusters: globular — stars: evolution — stars: RR Lyrae

I. INTRODUCTION

Absolute magnitudes of the main sequence termination points for low-metallicity globular clusters are brighter than the turnoffs of more metal-rich clusters if the luminosities of their respective horizontal branches are placed at different levels according to their Oosterhoff-Sawyer period shifts (Sandage 1982, hereafter Paper III). If their horizontal branch levels are ordered by $\Delta M_{\mathrm{RR}} = 3 \Delta \log P$, expected from $P \langle \rho \rangle^{1/2} = Q$ in the sense that clusters with longer period variables have brighter horizontal branches, then the observed gradient of turnoff magnitudes with metallicity is $\partial M^{\mathrm{TO}} / \partial [\mathrm{Fe/H}] = +0.3$. This is also the required theoretical value if clusters with different metallicities have equal ages (Simoda and Iben 1968, 1970; Iben and Rood 1970; Ciardullo and Demarque 1977). From this it was concluded in Paper III that the sample of 30 clusters studied there have the same age to within $\sim \pm 10\%$, centered upon an absolute value of $t = (17 \pm 2) \times 10^9$ years.

Most of the clusters that could be age dated in this way are in the halo; hence because of their nearly equal age, it follows that the formation of these halo components was relatively rapid, taking less than $\sim 2 \times 10^9$ years, which is the accuracy of the method. Even the high-metallicity, old-disk globular clusters of NGC 6838 and 47 Tuc were found to have the same old age, and hence must also have formed during the initial rapid galactic collapse, not during the later more gradual building of the disk.

Gradual as the formation of the *complete* disk may have been, it must nevertheless have begun to form as soon as the first gas from the halo, not yet fragmented into stars (Hoyle 1953), met gas falling from other directions and, by dissipation, settled into the rotation plane (Eggen, Lynden-Bell, and Sandage 1962).

The minimum time for the beginning of suck disk formation is the free-fall period from the wider halo regions. As there is a hierarchy of free-fall times (Searle and Zinn 1978) depending on the mean density of the Galaxy inside a given spherical boundary centered on the nucleus (Sandage 1970, n.1), the disk has formed over an appreciable time; matter far out took longer to fall than that closer in. As the shortest fall time is of order $\lesssim 5 \times 10^8$ years, it is of considerable interest to know if the *oldest* disk stars are, in fact, nearly as old as the halo, to within this difference.

To this end, we apply here the age dating method of Paper III to the field RR Lyrae variables. The high-metallicity variables with $\Delta S \lesssim 3$ belong to the disk; their W velocities are low, and the eccentricity of their orbits is small (Preston 1959; Spite 1960; Epstein 1969; Epstein and de Epstein 1973). And because these variables are undoubtedly among the oldest disk stars that exist, their age dating provides important information for this problem.

In this paper we continue the analysis of the Oosterhoff period shifts, applied here to the field RR Lyrae stars as a whole. It is shown in § II that the period shifts, determined by comparison with the period-amplitude relation for M3, form a *continuous* distribution over the large range $-0.12 < \Delta \log P < +0.15$, at constant amplitude. It is also shown that the correlation of [Fe/H] with these period shifts for field variables, discovered by Preston (1959, his Fig. 5), is the same correlation that applies to variables in the clusters. Furthermore, the correlation extends linearly to the field variables of highest metallicity ($\Delta S \leq 2$) not represented in the clusters themselves. It is this property that permits age dating of the disk variables.

It is shown in § III that much older disk stars exist than are present in NGC 188. The observed lower metallicities of the older disk giants place them blueward of the NGC 188 giant-star envelope; hence this envelope itself does *not* age date the oldest component of the disk.

Finally a new summary is given in § IV of the rapid variation of [Fe/H] with age in the early disk, and its much slower subsequent enhancement. The data are ages of globular clusters of different [Fe/H] determined in Paper III, together with the ages and [Fe/H] values of NGC 188, M67, and the Sun, and disk stars in the solar neighborhood.

II. CORRELATION OF PERIOD SHIFTS WITH METALLICITY FOR FIELD RR LYRAE STARS

Preston (1959, Fig. 5) showed that field variables with different ΔS values populate different regions of the period-amplitude diagram. He divided his sample into three categories of metallicity and found that the period-amplitude relations were different for each. RR Lyrae stars of lowest metallicity have the longest period at a given amplitude, and vice versa. This discovery was the extension to the field variables of the same effect found by Arp (1955) for globular clusters; the two broad Oosterhoff period groups are also separated by metallicity. However, Preston's discovery in the field extended Arp's result to the highest metallicity variables with $\Delta S \leq 2$ that are not well represented, if at all, in the clusters. These variables have the shortest periods for any given amplitude, reaching values as small as $P_{ab} \approx 0.33$ days.

From the form of these results it had been widely believed that the period-amplitude-metallicity relations, shown as three broad groups in Preston's Figure 5 and as the two broad Oosterhoff groups in Arp's dichotomy, gave only a general indication of a correlation of period shifts with metallicity, but did not hold precisely for any individual star. It then came as a surprise that the ($\Delta \log P$, [Fe/H])-correlation was, in fact, good for the globular clusters (Paper III, Fig. 4). The scatter is scarcely larger than the observational errors in [Fe/H] and $\Delta \log P$ considered together.

From Paper I (Sandage, Katem, and Sandage 1981) and Paper II (Sandage 1981) it is, however, possible to understand this if (1) amplitude is a unique and very precise function of horizontal position in the RR Lyrae instability strip and (2) the absolute luminosity level of the horizontal branch is tightly correlated with [Fe/H] for reasons stemming from the equality of age of globular clusters of different metallicity (Paper III, § V).

We now show that a *continuous* correlation exists between metallicity and period shift at fixed amplitude for the field RR Lyrae stars, and that the relation has the same form as for the globular clusters. If the cause is the same as described above, it then follows by the argument of Paper III that all field RR Lyrae variables which obey this ($\Delta \log P$, [Fe/H])-relation have the same age to within limits set there. Recall that this will be the case if the gradient of $\Delta \log P$ on [Fe/H] is close to

$$\partial \Delta \log P / \partial [\mathrm{Fe/H}] \approx +0.1, \qquad (1)$$

(Paper III, § V*b*). We now show that the field RR Lyrae stars satisfy equation (1).

The period-amplitude relation for M3 is adopted as fiducial, and $\Delta \log P$ is calculated for any given field variable from it. Using the linearized approximation to this M3 relation gives

$$\begin{aligned} \Delta \log P &\equiv \log P(M3) - \log P(\text{observed}) \\ &= -[\log P + 0.129\, A_B + 0.088]. \end{aligned} \qquad (2)$$

This equation has been applied to the sample of field RR Lyrae stars that have (1) known ΔS values either from Preston (1959) or from other observers on the Preston system, and (2) photoelectric light curves from which the A_B amplitudes can be found. Only variables with photoelectric photometry were considered. The sources for light curves are Spinrad (1959), Sandage (1960), Kinman (1961), Jones (1965), Harding and Penston (1966), Fitch, Wisniewski, and Johnson (1966), Clube, Evans, and Jones (1967), and Lub (1977).

The correlation of the calculated period shifts with metallicity for ~125 field variables is shown in Figure 1. The top panel shows the correlation with ΔS, the bottom with [Fe/H] using Butler's (1975) calibration of

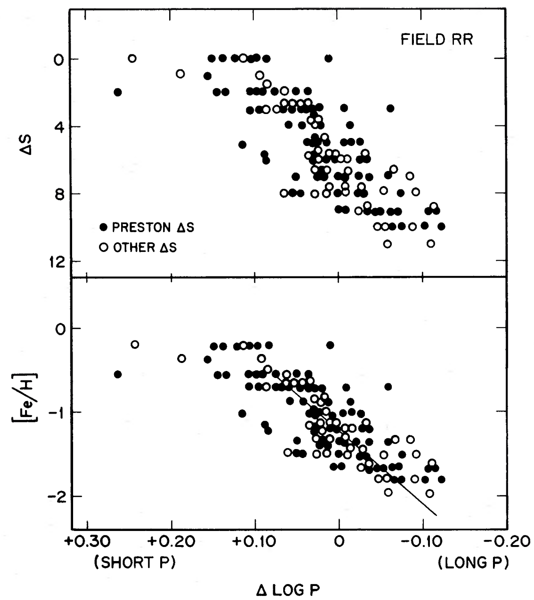

FIG. 1.—*Top panel*, correlation of ΔS with period shifts, determined using the M3 period-amplitude relation as fiducial, for field RR Lyrae stars that have photoelectric light curves. *Bottom panel*, same as the top using [Fe/H] from Butler's (1975) calibration of ΔS and metallicity. The line is the globular cluster correlation of Paper III, shifted by 0.2 in [Fe/H] for the difference between Butler and Zinn's calibrations.

$[\mathrm{Fe/H}] = -0.16\Delta S - 0.23$. The errors on the coordinates are $\varepsilon\Delta S \approx \pm 2$ and $\varepsilon\Delta \log P \approx \pm 0.02$, both of which are larger by a factor of ~ 2 than the corresponding errors for the equivalent parameters in the clusters (Paper III, Fig. 4). The reason is that only single determinations exist of A_B and ΔS for individual field stars, whereas the period shifts and [Fe/H] for the clusters represent averages over many stars. Given the observed errors, the observed scatter in Figure 1 is of the order expected from them alone; hence the true $(\Delta \log P, [\mathrm{Fe/H}])$-correlation must also be nearly as tight for these field variables as for the clusters.

The line drawn in the lower panel of Figure 1 is the cluster correlation (Paper III, Fig. 4 and eqs. [2] and [13]), but shifted by $\Delta[\mathrm{Fe/H}] = 0.2$ for the known difference between the metallicity scales of Butler (1975) and of Zinn (1980). The slope of this line is $\partial\Delta \log P / \partial[\mathrm{Fe/H}] = 0.116$, which fits the data well enough.

The line has been drawn only to $\Delta \log P \approx +0.10$ and $[\mathrm{Fe/H}] \approx -0.5$ which is the limit of the globular cluster correlation. The next most important feature of Figure 1, after noting that the slope is the same as for the clusters, is that the correlation extends into the $\Delta S \leq 2$ high metallicity domain with the same slope. These RR Lyrae stars are clearly part of the same distribution, forming its high-metallicity end. It therefore seems plausible that the explanation developed in Paper III for the $(\Delta \log P, [\mathrm{Fe/H}])$-correlation that applies in the $-2.3 < [\mathrm{Fe/H}] < -0.5$ domain should also apply to them. Hence, rather than repeat the argument given there, we assert that the slope of the correlation in Figure 1 requires the ages of field RR Lyrae variables of all metallicities including those with $\Delta S \leq 2$ to be the same to the accuracy of the method, which is $\Delta t/t \approx \pm 10\%$.

III. AGE OF THE OLDEST COMPONENT OF THE GALACTIC DISK

a) *The High Metallicity RR Lyrae Stars*

The kinematic properties of the short-period $\Delta S \leq 2$ Bailey *ab* variables show them to be disk objects (Pavlovskaya 1953; Preston 1959; Spite 1960; Epstein 1969; Saio and Yoshii 1979). To illustrate this again, the $|W|$ velocity (perpendicular to the galactic plane) and the eccentricity of the galactic orbits for RR Lyrae stars are plotted versus ΔS in Figure 2, taken from the calculations of Saio and Yoshii (their Table 4). In the lower panel only RR Lyrae stars with proper motions marked A in that reference have been plotted. In the upper panel such variables have been separated into those whose $|W| \leq 60$ km s^{-1}.

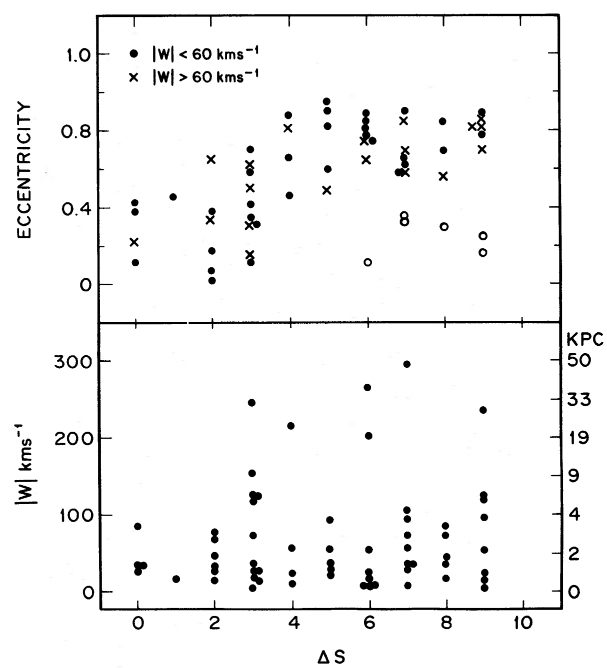

FIG. 2.—Correlation between ΔS and W velocity perpendicular to the plane and eccentricity of the galactic orbit for RR Lyrae stars used by Saio and Yoshii (1979). The maximum height reached by stars with given W values are shown along the right hand ordinate of the lower panel, as calculated in the cited reference. Open circles are low metallicity variables in nearly circular orbits, and may form a separate group.

In both these diagrams it should be remembered that the proper motions of the RR Lyrae stars are very uncertain; hence, errors in the ordinates are large. Nevertheless it is clear from Figure 2 that variables with $\Delta S \leq 2$ have small $|W|$ velocities and small orbital eccentricities; hence they belong to the galactic disk. The conclusion is strengthened further by noting that Saio and Yoshii used $M_v = +0.6$ as the absolute magnitude for RR Lyrae variables of all ΔS values, whereas if absolute magnitudes are assigned by $\Delta M_{\rm bol} = 3\Delta \log P$ as required by the Oosterhoff shifts, then M_v for $\Delta S =$ 0–2 stars will be $\sim$0.8 mag fainter than for $\Delta S = 10$. Hence $|W|$ and e for them will be even smaller than shown in Figure 2.

Since these high-metallicity variables are disk objects, and since, by the arguments of § II, they are the same age as the lower metallicity variables and the globular clusters, it follows that the oldest component of the disk has an age comparable to that of the halo. The same conclusion can be reached using old disk field giants.

b) *The Old Field Giants*

It is well known that the subgiant and giant branches of the old open cluster NGC 188 form an adequate lower envelope to the *C-M* diagram of field giants in the solar neighborhood (Sandage 1962; Wilson 1976, Fig. 6). An earlier demonstration that the lower envelope of the field star distribution was fainter than the sequences in the somewhat younger cluster of M67 had been given by Wilson (1959). Because it was also known that the M67 and NGC 188 stars have nearly the solar metal abundance, it could be concluded already in 1962 that there has been no appreciable enrichment of the

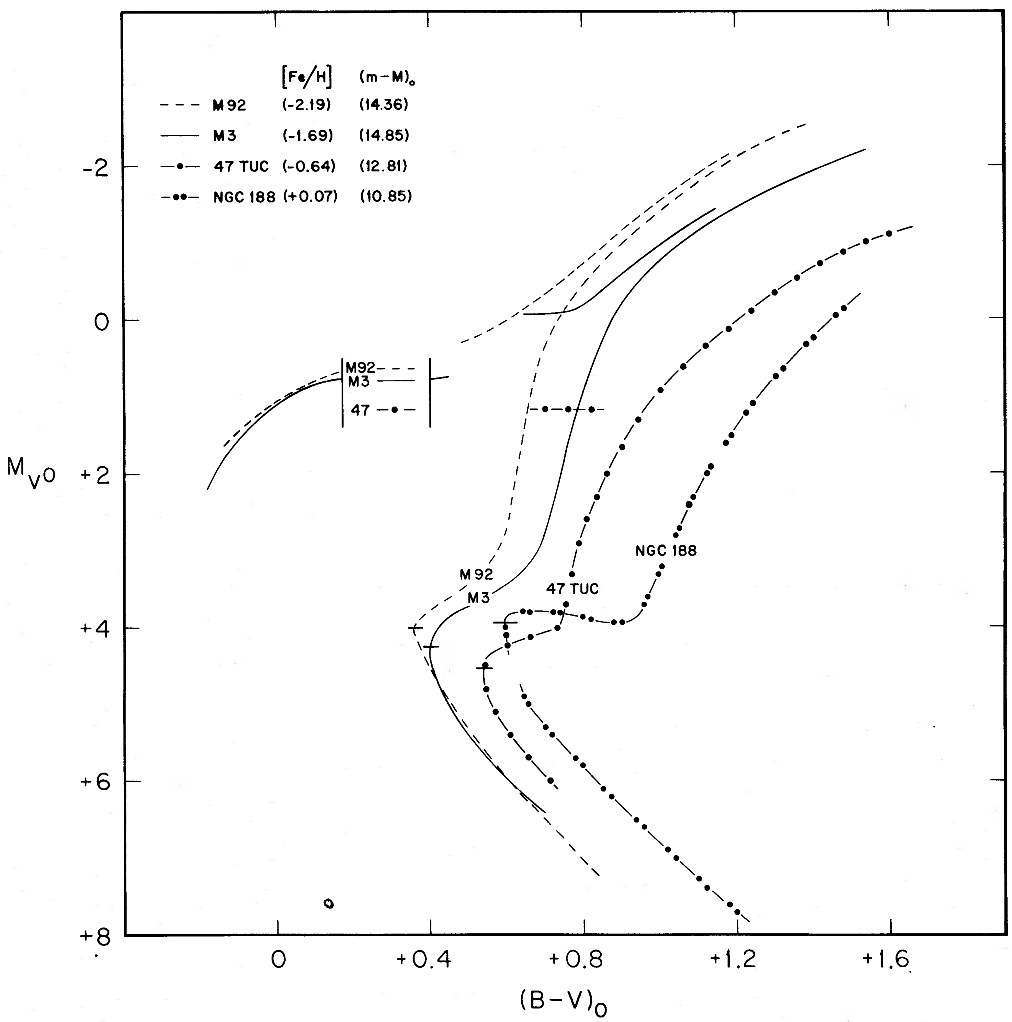

FIG. 3.—Composite *C-M* diagram for three globular clusters and the old open cluster NGC 188. Although NGC 188 is a factor of $\sim$3 younger than the globulars, its giant branch still forms the lower envelope of the diagram.

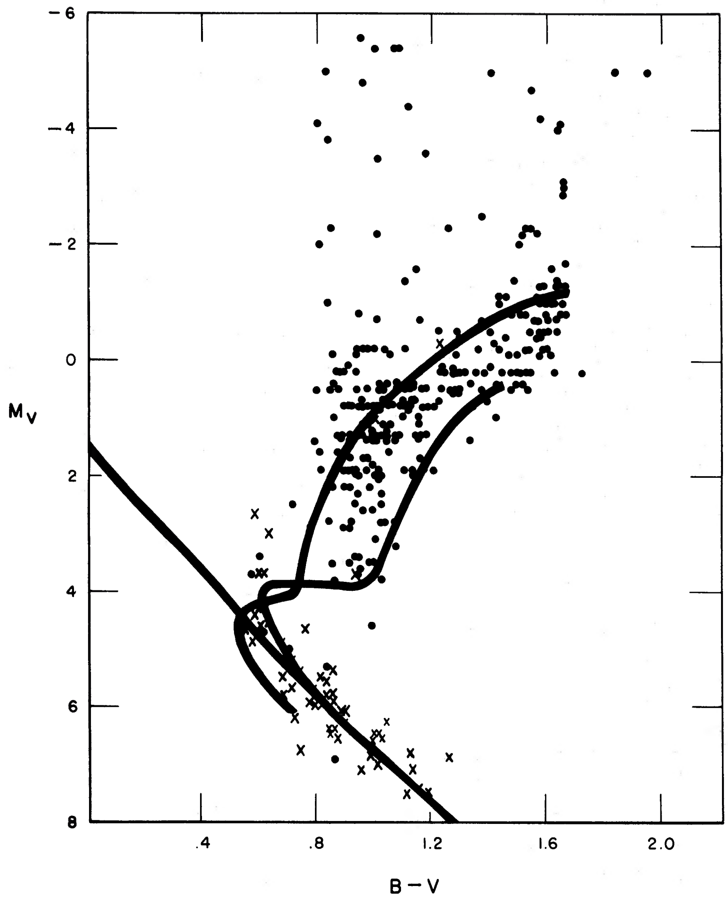

FIG. 4.—*C-M* diagram for field stars after Wilson (1959) with the 47 Tuc and NGC 188 sequences superposed. Crosses on the main sequence are trigonometric parallax stars.

galactic disk in the time since M67 and NGC 188 were formed.

However, simply because NGC 188 forms the lower envelope to the distribution it does not follow that there are no older stars in the galactic disk than represented by that cluster. The point made by Demarque and McClure (1977) is that "the position of the giant branch is sensitive to metal abundance, and stars more metal-poor than NGC 188 could be older and still lie on a Hayashi track above that for the NGC 188 giant branch." Carney's (1980) conclusion that no disk stars older than NGC 188 exist, based on the NGC 188 lower envelope, need not be correct.

The case is made in Figures 3–6. A schematic composite *C-M* diagram for four clusters of different metallicity is given in Figure 3. The data and luminosity normalization for three globular clusters are from Paper III; those for NGC 188 are from Eggen and Sandage (1969). Recall that NGC 188 is younger than the globular clusters, with an age somewhere in the range 5–8$\times 10^9$ years (Saio, Shibata, and Simoda 1977; Demarque and McClure 1977); yet, because of its higher metallicity than 47 Tuc, it *still* forms the lower envelope to the spread.

Plotted in Figure 4 are the field giants from Wilson's H-K absolute magnitudes used in an earlier diagram (Sandage 1962, Fig. 9), but with the 47 Tuc schematic from Figure 3 superposed.

Many of Wilson's giants have been studied by Helfer (1969) to determine their metallicities, with the result shown in Figure 5. The $B-V$ colors are from Johnson *et al.* (1966), and the absolute magnitudes are Wilson's, as corrected for an abundance effect by Helfer. The stars are binned into three metallicity groups, shown by different symbols. It is evident that there is a metallicity gradient in the diagram; stars of highest metallicity are closer to the NGC 188 envelope than those of lower abundance, which lie progressively blueward. (It should be noted, of course, that a few of these giants with $0 < M_v < 2$ will be young and hence will have *horizontal* evolutionary tracks like the Hyades. Therefore they cut across the more vertical tracks of the majority of the

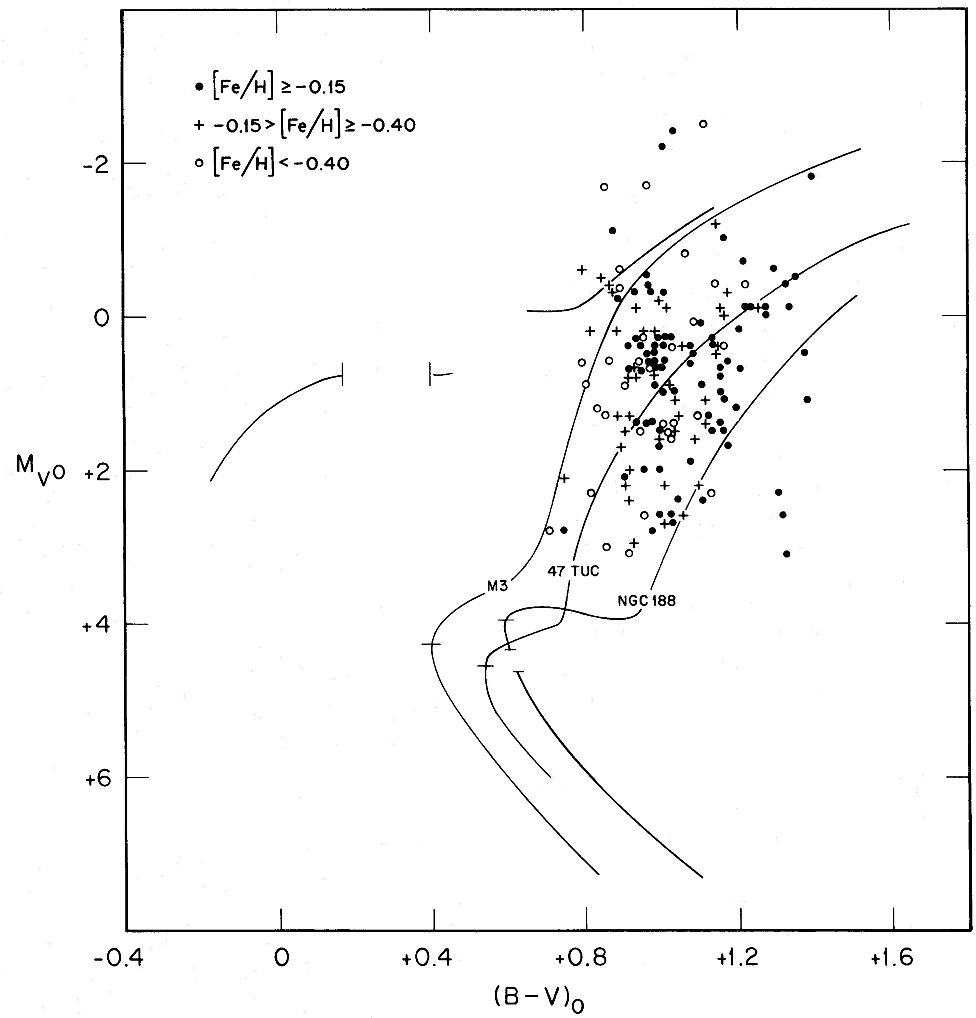

FIG. 5.—Similar to Figs. 3 and 4 with stars studied by Helfer (1969) binned into three metallicity groups. Note that as one proceeds blueward of the NGC 188 envelope, the metallicities generally decrease. It should be remembered that some of the giants will be young and will evolve on horizontal tracks from the main sequence as in the Hyades. The color-metallicity relation should not apply to them.

older giants where the color-metallicity correlation is expected, diluting the correlation for the old giants.)

The color-metallicity effect is shown quantitatively in Figure 6 where the observed colors are plotted versus Helfer's [Fe/H] for those giants whose luminosities are in the range $0.5 \leq M_V \leq 1.5$. The least squares line, averaged between two solutions made by exchanging the dependent and independent variables, shows the trend. It has a slope $\partial(B-V)/\partial[\mathrm{Fe/H}] = +0.44$. The expected slope, obtained from Figure 3 using the NGC 188 and 47 Tuc giant branch colors at $\langle M_v \rangle = +1.0$, with $[\mathrm{Fe/H}]_{188} = 0.0$ and $[\mathrm{Fe/H}]_{47} = -0.64$, is $+0.42$. The same conclusion follows from more recent data on the field giants by Janes (1975, Fig. 10).

This close agreement between the observed variation of metallicity blueward of the NGC 188 envelope and

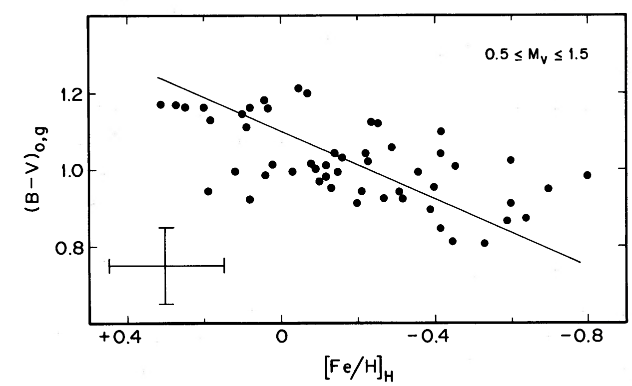

FIG. 6.—Correlation of observed $B-V$ color with Helfer's measured [Fe/H] for field giants in Fig. 5 between absolute magnitudes $+0.5$ and $+1.5$. The line is the impartial least squares fit.

that expected from the *schematic cluster diagrams* supports the assertion that the oldest disk stars began to form long before NGC 188 was born, and, in fact are as old as the halo globular clusters. The evidence in § III*a* suggests that such disk stars formed within $\sim 2\times 10^9$ years (the accuracy of the method) from the beginning of halo cluster formation.

IV. VARIATION OF METALLICITY WITH AGE

Beginning with the evidence from M67 and NGC 188 in 1962, it has been known that the enrichment of the galactic disk with time has been very gradual over the past $\sim 5\times 10^9$ years. Furthermore, given the large variation of [Fe/H] among globular clusters of nearly the same age, it follows that earlier enrichment must either have been very rapid or very spotty, or both.

The results of Paper III together with those known earlier for M67, NGC 188, Hyades, and the Sun have been combined in Figure 7 to again show the gross features of the enrichment history. The adopted metallicities of M67 and NGC 188 are [Fe/H]$= +0.07$ (relative to the Sun), based on their measured ultraviolet excess values of $\delta(U-B)=0.05$ relative to the Hyades (Eggen and Sandage 1964, 1969), using the calibration of photometric excess versus [Fe/H] due to Eggen (1964, Fig. 19).

Many studies have been made of the distribution of [Fe/H] for field stars in the solar neighborhood. Results of the latest of these studies by Twarog (1980), quoted by Pagel (1980), are shown as crosses in Figure 7.

The line, showing the general trend, is close to that obtained in an earlier analysis of the distribution of [Fe/H] for stars in the solar neighborhood by Clegg and Bell (1973). They also gave a theoretical calculation of this line from a simple model of the enrichment history. Later models such as Larson's (1976) show that a wide variety of enrichment gradients, including those observed in Figure 7, can be produced by reasonable collapse models. The very rapid enrichment during the halo collapse, shown in Figure 7, is reproduced in all of Larson's models (his Fig. 14).

This most striking historical feature of very rapid initial enrichment had already been suggested by the earlier data, when NGC 188 was found to have nearly solar abundance, although, to be sure, it was then assumed that its age was nearly the same as that of the globular clusters (Sandage 1968, Fig. 7; Eggen and Sandage 1969, Fig. 7). However, this rapid phase *still* remains in the present diagram because the ages of the high metallicity disk globulars, NGC 6838 and 47 Tuc, have been taken here, following Paper III, to be the same as the halo clusters. The principal change from the 1968 picture is that the enrichment increase of the disk *subsequent* to the halo collapse is more gradual than in the two previously cited diagrams. This is because the age of NGC 188 has been reduced from $\sim 11\times 10^9$ years used there to $\sim 5\times 10^9$ years adopted here, as determined by Torres-Peimbert (1971), and by Demarque and McClure, and Saio *et al.* cited earlier. During this more gradual enrichment of the slowly forming disk, the mean metallicity gradient is of the order $\partial[\mathrm{Fe/H}]/\partial \log t \approx 0.07$.

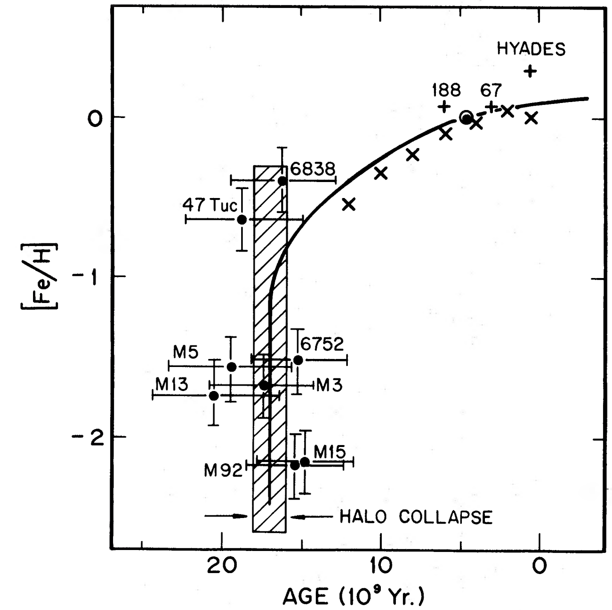

FIG. 7.—Schematic representation of the variation of [Fe/H] with age for selected galactic objects and regimes. The globular cluster data are from Paper III. The data from the solar neighborhood, shown as crosses, are due to Twarog (1980) quoted by Pagel (1980). The clusters of NGC 188, M67, and the Hyades, together with the Sun, are also shown.

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