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GIANT GLITCHES AND PINNED VORTICITY IN THE VELA AND OTHER PULSARS

M. A. ALPAR[1]
Physics Department, Bogazici University, Istanbul, Turkey
P. W. ANDERSON[2]
Bell Laboratories, Murray Hill, New Jersey
D. PINES[1]
Physics Department, University of Illinois, Urbana
AND
J. SHAHAM[1]
Racah Institute of Physics, Hebrew University, Jerusalem, Israel
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ABSTRACT

We call attention to a number of hitherto unexamined regularities in the Vela pulsar timing data and show how these find a natural explanation in a theory of giant glitches in the Vela and other pulsars as the dynamic consequence of catastrophic unpinning events in the pinned crustal neutron superfluid, with postglitch behavior resulting from glitch-induced vortex creep.

Subject headings: pulsars — stars: neutron

I. INTRODUCTORY REMARKS

New facts and new ideas have led us to reexamine our earlier ideas (Anderson and Itoh 1975; Pines 1980; Shaham 1980; Alpar *et al.* 1980) on the interpretation of the "superglitches" which occur in Vela as well as several other pulsars. We have recently examined (Pines *et al.* 1981) the dynamic consequences of pinned vorticity in a rotating superfluid using a simple model: the behavior of a rotating cylinder which contains a uniform region of either weakly or strongly pinned vorticity and which is being spun up or spun down by an external torque. Moreover, Downs (1981) has shown that the postglitch behavior of the Vela pulsar is considerably more complex than is obtained from the simple two-component model of a weakly coupled crust and superfluid core (Baym *et al.* 1969).

In this *Letter* we discuss several quantitative regularities in the Vela pulsar timing data and show how these strengthen our recent proposal (Pines *et al.* 1981) that the giant glitches in this and other pulsars represent vorticity jumps, while the postglitch behavior results from vorticity creep. A feature of our new interpretation is the likelihood that the core superfluid can equilibrate rapidly enough with the crust of the star (Alpar, Anderson, and Sauls 1981; Sauls and Stein 1981) so that the only superfluid uncoupled on the time scale of the experimental results is the pinned superfluid in the crust. We show in some detail elsewhere that the crust contains two superfluid regions: a "pinning" region where the vortex lines pin to the crust nuclei rather strongly; and a "threading" region where the vortex lines avoid the nuclei and can probably flow relatively easily (Alpar *et al.* 1981*a*). The result is a wide variety of possible relaxation rates and of modes of slowdown of the crust superfluid, from essentially viscous outward flow of vorticity through a highly nonlinear "vortex creep" process to catastrophic "unpinning" events in the region of strong and rapidly varying pinning forces. We will ascribe the glitch phenomenon to these catastrophic events.

II. THE VELA PULSAR GIANT GLITCHES

The detailed analysis by Downs (1981) of timing data which spans 11 years provides valuable information concerning not only the magnitude of the four observed giant glitches ($[(\Delta\Omega_c)_0/\Omega_c] \sim 2 \times 10^{-6}$), but also concerning immediate postglitch behavior and long-term frequency variations. In this section, we interpret the observed frequency variations in terms of a new two-component model consisting of pinned crustal superfluid and the remainder of the star (the crust plus the interior liquid core).

We assume the giant glitches represent events in which the unpinning of vortices in a weakly pinned transition layer leads to a temporary decoupling from the crust of an extensive region of vortex creep. This

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[2]Also at Princeton University, Princeton, New Jersey. The work at Princeton University was supported in part by National Science Foundation grant DMR 78-03015 and U.S. Office of Naval Research grant N00014-77-C-0711.

happens because the fast outward motion of unpinned vortices through the creep region during the glitch decreases the "driving force" for creep. Before the glitch, to the extent that a quasi-steady-state may be assumed to have been reached, the crust plus core of moment of inertia I_c and angular frequency Ω_c obey the equation of motion

$$I_c\dot{\Omega}_c = N_{\text{ext}} - I_p\dot{\Omega}_c, \tag{1}$$

where N_{ext} is the torque on the crust resulting from pulsar spin-down, while $I_p\dot{\Omega}_c$ describes the internal torque produced by the creeping crustal superfluid of inertial moment $I_p \ll I_c$. If, as a result of the glitch, a fraction $\delta I_p/I_c$ of the moment of inertia associated with the region of the creeping vortices is decoupled temporarily from the crust, it follows from equation (1) that

$$\frac{\delta I_p}{I_c} = \frac{(\Delta\dot{\Omega}_c)_0}{\dot{\Omega}_c}, \tag{2}$$

where $(\Delta\dot{\Omega}_c)_0$ is the observed jump in $\dot{\Omega}_c$ immediately after the glitch. A second quantity which characterizes the nature of the glitch is the average frequency jump of the decoupled vortices, given by

$$\frac{\delta\Omega_p}{\Omega_c} = \left[(\Delta\Omega_c)_0/\Omega_c\right]/\left(\delta I_p/I_c\right), \tag{3}$$

where $(\Delta\Omega_c)_0$ is the magnitude of the initial frequency glitch. The quantities $(\Delta\Omega_c)_0$, $(\delta I_p/I_c)$, and $\delta\Omega_p/\Omega_c$ are tabulated for the four giant glitches in Table 1.

A glance at Table 1 reveals a high degree of correlation between δI_p and $\delta\Omega_p$ for a given glitch; for glitches of similar magnitude (cf. glitches 2 and 3 of Table 1), if δI_p is larger than some characteristic value, $\delta\Omega_p$ will be smaller. There is, moreover, a glitch-to-glitch correlation. If δI_p is smaller than some characteristic value for one glitch, it will be larger for the subsequent one, with a corresponding (reverse) correlation for $\delta\Omega_p$. Two still more striking correlations emerge if one examines the table closely: a quantitative ($\sim 5\%$) correlation between the observed value of $\delta I_p/I_c$ for a given glitch and the time t_g to the next glitch; and a semi-quantitative ($\sim 20\%$) correlation between the observed time to that glitch and the *subsequent* average frequency jump $\delta\Omega_p$.

Thus, if we write $t_g^{\text{th}} = 100(\delta I_p/I_c)t'$, assume that t' is a constant of the glitch process (i.e., does not vary from glitch to glitch), and fix t' to be 882 days on the basis of the 900 day interval between glitches 1 and 2, we obtain intervals of 1440 days and 960 days, respectively, between glitches 2 and 3 and between 3 and 4. These compare quite favorably with the observed values of 1500 days and 1000 days. Despite this measure of agreement, from a theoretical point of view (see § III) a 20% variation over time of t' would not be surprising. On this basis, we would conclude that the next giant glitch will not occur before 1984. If, however, t' continues to display no more than a 5% variation, then the next giant glitch in the Vela pulsar should take place some 6.8 ± 0.3 years after the 1978 July giant glitch (i.e., between 1985 February and 1985 August).

The average frequency jump of the next glitch is likewise set to a considerable extent by δI_p or the time t_g since the previous glitch, according to

$$\frac{(\delta\Omega_p)^{\text{th}}}{\Omega_c} = \frac{t_g}{T}, \tag{4}$$

where $T = \Omega_c/\dot{\Omega}_c$ is the pulsar spin-down time. As may

TABLE 1

OBSERVED[a] AND DEDUCED PARAMETERS FOR THE VELA PULSAR GLITCHES

Glitch	$\frac{(\Delta\Omega_c)_0}{\Omega_c} \times 10^6$	$\left[\frac{\delta I_p}{I_c} \equiv \frac{(\Delta\dot{\Omega}_c)_0}{\dot{\Omega}_c}\right] \times 10^2$	$\frac{\delta\Omega_p}{\Omega_c} \times 10^4$	t_g (days)	t_g^{th} (days)	$\frac{(\delta\Omega_p)^{\text{th}}}{\Omega_c} \times 10^4$	$\ddot{\Omega}_c \times 10^{22}(\text{s}^{-3})$
1	2.34	1.02	2.29	900	900	...	6.6 ± 1.2
2	1.97	1.63	1.21	1500	1440	0.99	6.0 ± 1.2
3	2.02	1.09	1.85	1000	960	1.64	11.1 ± 1.2
4	3.06	2.81	1.09	[2480 ± 100]	2480	1.10	6.5 ± 1.2
[5]	[2.5 ± 0.5]	[0.9 ± 0.2]	[2.7 ± 0.5]	[800 ± 160]	800 ± 160	2.74	...

NOTES.—$(\Delta\Omega_c)_0$ and $(\Delta\dot{\Omega}_c)_0$ are the observed initial jumps in Ω_c and $\dot{\Omega}_c$, respectively; $(\delta I_p/I_c) \equiv (\Delta\dot{\Omega}_c)_0/\dot{\Omega}_c$. The derived quantity, $\delta\Omega_p/\Omega_c$, is $\equiv [(\Delta\Omega_c)_0/\Omega_c]/[(\Delta\dot{\Omega}_c)_0/\dot{\Omega}_c]$. t_g is the observed time to the next glitch, t_g^{th} the time deduced from $t_g^{\text{th}} = 8.82 \times 10^4\ (\delta I_p/I_c)$ days. $\delta\Omega_p^{\text{th}}$ is the deduced average jump in superfluid velocity, using eq. (4). The quoted limits of $\ddot{\Omega}_c$ reflect variations which result from analyzing the observations using somewhat different baselines to determine $\delta\ddot{\Omega}_c$. All quantities listed in square brackets represent our *deduced* values based on a no more than 5% variation in t' and, thus, represent predictions to be tested against future observations. We have estimated the size of glitch [5] by arguing that it will probably fall in the range of previous glitches; the quoted "errors" in our other deduced quantities for this fifth glitch follow from this uncertainty in $(\Delta\Omega_c)_0$.

[a]Downs 1981.

be seen in Table 1, the theoretical values $(\delta\Omega_p)_{\rm th}$ agree with those observed to within 20%.

Finally, we interpret the observed postglitch behavior [comparatively rapid (~ 200 days) relaxation of a small fraction ($\lesssim 5\%$) of $(\Delta\Omega_c)_0$ toward its preglitch value, followed by a quadratic increase corresponding to an approximately constant value of $\ddot{\Omega}_c$] as the recoupling of the entire region of pinned vorticity which participated in the glitch, followed by the much slower recoupling of the vortices in the transition region which acted to trigger the glitch. The state is set for the next glitch once both kinds of recoupling have occurred. There appears to be some correlation as well between the observed values of $\ddot{\Omega}_c$ and the size $(\Delta\Omega_c)_0$ of the next glitch, in that when $\ddot{\Omega}_c$ is substantially larger (smaller) than some average value, the resulting value of $(\Delta\Omega_c)_0$ will likewise be larger (smaller) than its corresponding average value.

By combining these correlations with those discussed above, we obtain an estimate not only of the time of the next giant glitch in the Vela pulsar, but also of the corresponding $\delta\Omega_p$ and, more approximately of its magnitude and, hence, the time to the subsequent glitch. These estimates are also listed in Table 1.

III. VORTEX CREEP AND VORTICITY JUMPS

We now consider to what extent the striking glitch-to-glitch correlations exhibited by the Vela pulsar may be qualitatively explained in terms of a microscopic model for vortex creep and vorticity jumps in the crust of a neutron star. We note first that, since the Magnus forces are largest near the equator, it is likely that vorticity motion will tend to be concentrated in this region. Under these circumstances, the cylindrical model calculations we have carried out recently (Pines *et al.* 1981) should provide a useful first cut. We briefly summarize these here and then consider their application to the Vela glitch phenomena.

Vortices in the interior of a cylindrical container obey two equations of motion:

$$\frac{\partial n(r,t)}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}(rnv_r) = 0, \tag{5}$$

$$\frac{\partial\Omega(r)}{\partial r} = \frac{nK - 2\Omega}{r}, \tag{6}$$

where $n(r,t)$ is the vortex density per unit area, v_r their radial drift velocity, $K \equiv h/2m_n$ is the quantum of vorticity, and $\Omega(r)$ is the superfluid angular velocity at r. v_r is related to the angular velocity of the superfluid relative to the crust $\omega(r)$ by

$$v_r = \alpha r(\Omega - \Omega_c) = \alpha r\omega, \tag{7}$$

where α is a coupling coefficient. In the absence of pinning and creep, α is determined by the normal fluid-vortex core coupling and is probably independent of $\omega(r)$. In a pinning region, α determines the rate of creep and is dependent on some critical value of $\omega - \omega_{\rm cr}(r)$, which is itself proportional to the pinning force F. Thus, for $\omega \sim \omega_{\rm cr}$, $v_r \approx rF[\omega - \omega_{\rm cr}]$, where F is an exponential function of $\omega - \omega_{\rm cr}$.

On combining equations (5), (6), and (7), one finds

$$\dot{\omega} + \alpha\omega\left[r\frac{\partial\omega}{\partial r} + 2\omega + 2\Omega_c\right] = -\dot{\Omega}_c, \tag{8}$$

which in quasi-steady-state reduces to

$$\frac{d}{dr}\left[r^2\Omega(r)\right] = -\frac{r\dot{\Omega}_c}{\alpha\omega(r)}, \tag{9a}$$

which for a pinning region becomes

$$\frac{d}{dr}\left[r^2\Omega(r)\right] = -\frac{r\dot{\Omega}_c}{F(\omega - \omega_{\rm cr})}. \tag{9b}$$

Thus, a pattern of differential rotation builds up in the star.

It is clear from equation (9) that a sharp rise in $\Omega(r)$ comes about where $\omega(r)$ rises steeply (F becoming very small). One then finds a boundary layer marking a region of transition from weak to strong pinning (cf. Fig. 1) in which vortices pile up (cf. eq. [5]). The thickness of the boundary layer, $d \approx \delta\Omega/(d\omega_{\rm cr}/dr)$, can be quite small (Alpar *et al.* 1981*a*, *b*). In the boundary

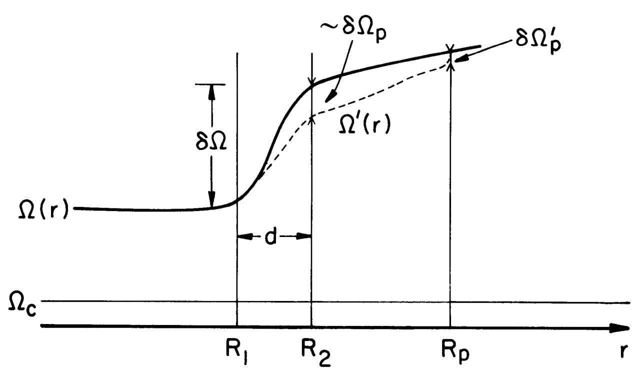

FIG. 1.—Changes in superfluid angular velocity $\Omega(r)$ due to an unpinning event in the boundary layer R_1R_2. $\delta\Omega$ is the height of the $\Omega(r)$ jump in the layer, and d is the boundary width. Unpinned vortices move in the direction of increasing radius r, some all the way to R_p. $\Omega'(r)$ is the new curve. $\delta\Omega_p$ is the average value of $\Omega - \Omega'$. $\delta\Omega'_p$ is the possible (small) discontinuity at R_p. Ω_c is the (spatially constant) crustal angular velocity.

layer, the random superfluid velocity fields, which add on to the existing macroscopic differential rotation, will generally be substantial ($\sim \delta\Omega$). Hence, the layer is a prime location for a coherent unpinning event.

We now focus on the vicinity of such a boundary layer. In a glitch, a bunch of vortices become unpinned and move out through d to an average distance R_p, where they repin and start again to participate in the creep process. As a result, the $\Omega(r)$ curve for the superfluid in the R_1R_p region drops to $\Omega'(r)$, as shown in Figure 1; the pinned superfluid has lost angular momentum, $\delta L \sim \delta I_p \delta\Omega_p$, where δI_p is the moment of inertia of the R_1R_p region; and the observed crust frequency Ω_c has undergone a glitch of magnitude, $\delta L/I_c$. One major dynamical consequence of the event is that in the R_1R_p region $\Omega(r) - \Omega_c$ drops, and the rate of vortex creep also drops substantially. Hence, the region becomes temporarily decoupled from I_c, and a net increase in the deceleration of the rigid component, given by equation (2), ensues. As Ω_c continues to decrease, $\Omega(r) - \Omega_c$ outside the boundary region returns toward its steady-state value, "burning" its way from R_p inward. Thus, a larger and larger fraction of I_p joins I_c, and the $\Delta\dot{\Omega}_c$ jump is damped.

The time scale t_0 for the "glitch" depends on the scattering—by the pinning sites—of vortices unpinned in R_1R_2. If each scattering moves the vortex out by a fraction η of the tangential distance, then $t_0 \approx [(R_p - R_1)/R]\,[1/\eta\omega(r)]$. The values of η and $\Omega(r) - \Omega_c$ have been estimated in calculations of vortex scattering (Henis and Shaham 1981) and pinning forces (Alpar *et al.* 1981*a*, *b*). For $[(R_p - R_1)/R] \sim (\delta I_p/I) \sim 10^{-2}$, $\eta \sim 10^{-3}$, $\omega(r) \sim 10^{-1}\ \mathrm{s}^{-1}$, t_0 is of the order of 100 s. Note that $R_p - R_1$ will generally increase as $\Omega'(r) - \Omega_c$ increases because of more effective scattering, so that $\delta\Omega_p$ is inversely correlated with $R_p - R_1$, hence δI_p. We thus expect that $\delta I_p \delta\Omega_p$ will show less glitch-to-glitch variability than $\delta\Omega_p$ or δI_p separately.

The unpinning event itself commences inside R_1R_2, where the density $n(r)$ and the random velocities are highest. It is carried through R_1R_2 by the rapidly varying random velocities introduced by the moving, unpinning vortices.

Following the glitch, the buildup of vortex density inside R_1R_2 will continue at a rate depending on the constant inward flow from the left and the creep rate to the right of R_2. The latter is smaller for higher $\delta\Omega_p$ values. Hence the vortex density for the next glitch will build up faster, so that the time t_g to the next glitch inversely correlates with $\delta\Omega_p$, and hence correlates with δI_p. The larger $(\Delta\dot{\Omega}_c)_0/\dot{\Omega}_c$ is for a given glitch, the longer is t_g to the next glitch.

On the other hand, the random velocities of vortices deposited throughout R_1R_2 must depend on t_g. Hence, the size of the subsequent change in $\delta\Omega_p$ (proportional to the number of excess vortices leaving R_1R_2 in an unpinning event in the following glitch) must also correlate with t_g.

Such are the qualitative features of this unpinning glitch scenario. Although its details require far more detailed knowledge of neutron stellar interiors, it provides a theoretical framework for understanding giant glitches in neutron stars and predicting their future behavior.

IV. CONCLUDING REMARKS

Although we have not developed a completely unique quantitative description of the giant glitch phenomenon, we have set it within very restrictive boundaries. We comment in passing on a few additional implications of our theory.

First, the energy release in this glitch scenario does not necessarily appear instantaneously on the surface as heat. The energy release during the unpinning, on the glitch rise time of ~ 100 s, is initially given to oscillations of charged nuclei. These oscillations have such small amplitude ($\sim 10^{-15}$ cm; Alpar *et al.* 1981*b*) that their coupling time to the electron gas is of the order of 1000 years. The subsequent creep (a thermal process) will have thermal time scales comparable to the dynamical times observed, i.e., months (the time scale of the observed fast relaxation) or longer. However, the surface layers of the neutron star have high opacity and, again, long thermal transport times. For steady-state conditions, we therefore conclude that the energy, $\Delta E \sim \delta I_p(\delta\Omega_p)^2 \lesssim 10^{39}$ ergs, which is released continuously at a rate $L = \Delta E/t_g$, is consistent with the observed upper limits on the surface temperature of the Vela pulsar (Helfand 1980), and no luminosity events are expected.

Second, the deduced values of $\delta I_p/I_c \sim 10^{-2}$ are reasonable from the point of view of neutron star structure calculations. The fractional moment of inertia in the pinned superfluid layers is 0.35 and 0.1, respectively (Alpar 1977), for the 1.33 $M_\odot$ TI equation of state and the 1.4 $M_\odot$ BJ equation of state models of Pandharipande, Pines, and Smith (1976). Our derived values of $\delta\Omega_p/\Omega_c$ are in turn consistent with calculated values of critical velocities for weak pinning (Alpar *et al.* 1981*a*, *b*).

Third, as we show elsewhere (Alpar *et al.* 1981*c*), the glitches observed in the Crab pulsar may be interpreted as smaller vorticity jumps expected from weakly pinned regions in a hot young neutron star, so that the differences in glitch behavior of the Crab, Vela, and older pulsars may be explained on evolutionary grounds.

Finally, giant glitches have been observed in other pulsars. For the glitch in PSR 1641−45 (Manchester *et al.* 1978) we have enough data to "predict" a previous glitch 90 years ago ($T = 7.5 \times 10^5$ yr; $\delta\Omega_p/\Omega_c = 1.2 \times 10^{-4}$), but not the next glitch. As previously stated

(Alpar *et al.* 1980), the number of such observed glitches in "old" pulsars such as PSR 1641−45 and PSR 1325−43 (Newton, Manchester, and Cooke 1981) is about right statistically. We note that pulsars with larger values of $(\dot{\Omega}_c/\Omega_c)$ should exhibit more frequent giant glitches. Finally, the several spin-down glitches which have been observed in the spinning-up pulsating X-ray sources, Her X-1 and Vela X-1 (Boynton 1981; Nagase *et al.* 1981), may likewise be of internal origin, corresponding to spin-up induced vorticity jumps. We see every reason to believe the pinned superfluid mechanism to be universal, at least in older pulsars.

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REFERENCES

Alpar, M. A. 1977, Ph.D. thesis, University of Cambridge.
Alpar, M. A., Anderson, P. W., Pines, D., and Shaham, J. 1980, in *IAU Symposium 95, Pulsars*, ed. R. Wielebinski and W. Sieber (Dordrecht: Reidel), in press.
______. 1981*a*, *Phil. Mag.*, in press.
______. 1981*b*, in preparation.
______. 1981*c*, *Proc. Nat. Acad. Sci.*, in press.
Alpar, M. A., Anderson, P. W., and Sauls, J. 1981, in preparation.
Anderson, P. W., and Itoh, N. 1975, *Nature*, **256**, 25.
Baym, G., Pethick, C., Pines, D., and Ruderman, M. 1969, *Nature*, **224**, 872.
Boynton, P. 1981, private communication.
Downs, G. 1981, *Ap. J.*, **249**, in press.
Helfand, D. 1980, in *IAU Symposium 95, Pulsars*, ed. R. Wielebinski and W. Sieber (Dordrecht: Reidel), in press.
Henis, Z., and Shaham, J. 1981, in preparation.
Manchester, R. N., Newton, L. M., Goss, W. M., and Hamilton, P. A. 1978, *M.N.R.A.S.*, **184**, 35P.
Nagase, S. *et al.* 1981, *Nature*, submitted.
Newton, L. N., Manchester, R. N., and Cooke, D. J. 1981, *M.N.R.A.S.*, submitted.
Pandharipande, V. R., Pines, D., and Smith, R. A. 1976, *Ap. J.*, **208**, 550.
Pines, D. 1980, *J. de Physique*, **41**, C2-111.
Pines, D., Shaham, J., Alpar, M. A., and Anderson, P. W. 1981, *Progr. Theor. Phys.*, in press.
Sauls, J. A., Stein, D. L., and Serene, J. W. 1981, preprint.
Shaham, J. 1980, *J. de Physique*, **41**, C2-9.

M. A. Alpar: Physics Department, Bogazici University, Istanbul, Turkey

P. W. Anderson: Bell Laboratories, Murray Hill, NJ 07974

D. Pines: Physics Department, University of Illinois, Urbana, IL 61801

J. Shaham: Racah Institute of Physics, Hebrew University, Jerusalem, Israel