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# AN ANALYSIS OF THE PROPER MOTIONS OF SS 433 RADIO JETS

R. M. HJELLMING
National Radio Astronomy Observatory,[1] Socorro, New Mexico

AND

K. J. JOHNSTON
E. O. Hulburt Center for Space Research, Naval Research Laboratory, Washington, DC
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## ABSTRACT

Four epochs of 4885 MHz VLA maps of SS 433 are used to show that the structure and changes in structure of the 0″.1–5″ size scale radio emission fit a kinematic model for the proper motion of ejected radio emitting material. This model is an extension of the kinematic twin-jet model for the optical jets of SS 433. The following model parameters give the best fit to all the data: the axis about which the jets rotate is inclined 80° to the line of sight in position angle 100°; the angle between the jets and the jet rotation axis is 20°; the jets rotate in a clockwise (left-handed) sense about the jet axis with a period of 164 days; the oppositely directed eastern and western jet rotation axes are on the near and far sides, respectively, of the central object; the ratio of constant jet velocity and the distance to SS 433 is 3″.0±0″.2 yr$^{-1}$; and the present capability of the radio data to determine an absolute jet velocity indicates a value of 0.26 times the speed of light and thus a distance of 5.5 kpc.

*Subject headings:* radio sources: variable — stars: binaries — stars: individual — stars: radio radiation — X-rays: binaries

## I. INTRODUCTION

The success of the twin-jet kinematic model of Abell and Margon (1979), Milgrom (1979), and Fabian and Rees (1979) in explaining the unusual radial velocity behavior of SS 433 has provided a solid basis for understanding much of the behavior of this object. The radio emission from SS 433, initially discovered by Ryle *et al.* (1978) and Seaquist *et al.* (1979), has been shown to be extended on arc second size scales by Gilmore and Seaquist (1980) and Hjellming and Johnston (1981, hereafter Paper I). In the latter paper, we reported proper motions of linearly polarized, radio-emitting plasma corresponding to 3″.2 yr$^{-1}$.

In this *Letter* we report on a more extensive analysis of the four highest-quality 4885 MHz VLA maps of Paper I in terms of a complete three-dimensional model of the kinematics of twin jets moving outward from the central object with a constant velocity. These radio data show proper motions of features moving along lines of constant position angle which give the appearance of a distorted, rotating "corkscrew." The parameters of the radio jets agree with the parameters of the kinematic model for the optical jets (Margon, Grandi, and Downes 1980) and supplement the optical data by determining all of the remaining parameters for a unified kinematic model of material moving outward from the central regions of SS 433.

## II. THE THREE-DIMENSIONAL KINEMATIC MODEL

The proper motion information contained in high-resolution VLA maps showing outward motion of radio jets is complementary to the radial velocity data about the optical jets. Before we can make full use of all of these data we must describe the necessary parameters and equations which relate the jet behavior in the reference frame of the jets to its appearance in the reference frame of the observations.

Let $z'$ be the axis about which the ejection velocity vector $\boldsymbol{v}$ rotates at an angle $\psi$ with an angular velocity $\Omega$. Let $y'$ be the axis of a right-handed coordinate system which is located in the plane of the sky, and let $x'$ be the axis perpendicular to $y'$ and $z'$. The origin of this and all other coordinate systems will be coincident with the position of the central object of SS 433. Since we must avoid confusion between two different sign parameters, let us define a rotation sign parameter $s_{\rm rot}$ and a jet sign parameter $s_{\rm jet}$. We then have $\Omega = s_{\rm rot} 2\pi/P$, so $s_{\rm rot} = +1$ means counterclockwise (right-handed) rotation with a period $P$. A value of $s_{\rm jet} = +1$ corresponds to the jet moving mostly toward the observer (blueshift). The geometry relating the $x'$-$y'$-$z'$ reference frame and the velocity vector $\boldsymbol{v}$ is shown in Figure 1$a$. Also shown in Figure 1$a$ is an $x$-axis pointing toward the observer, with

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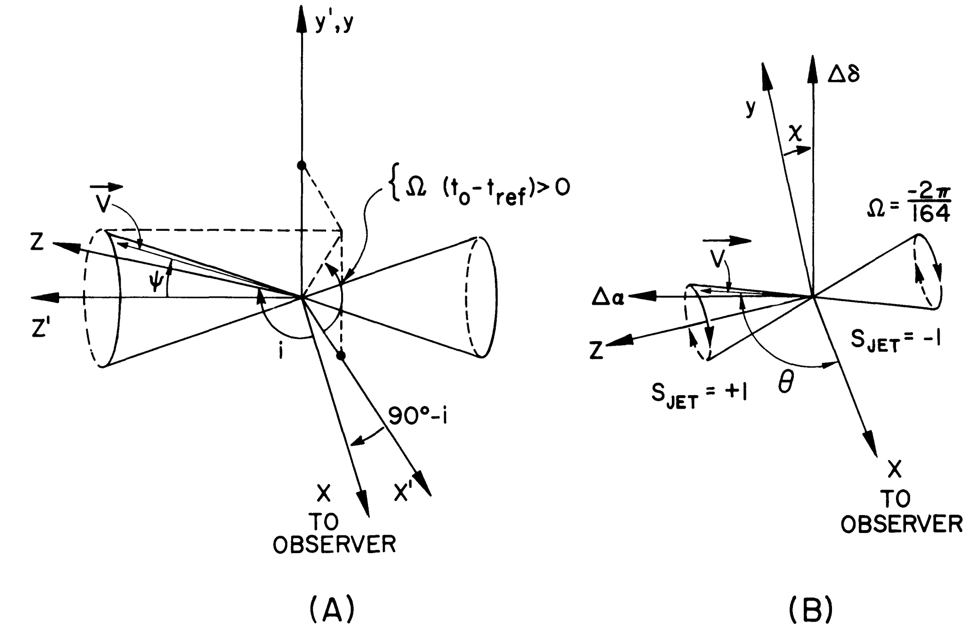


FIG. 1.—($a$) Schematic diagram relating the jet velocity $\boldsymbol{v}$ rotating at an angle $\psi$ about the $z'$-axis of the primed coordinate system. The jet rotation axis has an inclination $i$ with respect to the line of sight to the observer. The unprimed coordinate system in which the $x$-axis points to the observer is obtained by a $90°-i$ rotation about the $y'$-axis. The angular velocity, $\Omega = s_{\rm rot} 2\pi/P$, is positive for counterclockwise rotation (right-handed) of $\boldsymbol{v}$ about the $z'$-axis. ($b$) Rotation by an angle $\chi$ about the $x$-axis turns the $x$-$y$-$z$ coordinate system into one with the new $z$-axis corresponding to displacement in right ascension ($\Delta\alpha$), and the new $y$-axis corresponds to displacement in declination ($\Delta\delta$). The approximately correct geometry of the rotating twin jets of SS 433 is also shown.

an inclination $i$ with respect to the jet or $z'$-axis. The $x$-$y$-$z$ reference frame is obtained by a $90°-i$ rotation about the $y'$-axis.

Since $\psi$ corresponds to one of the angular coordinates of a spherical polar system and the angle $\Omega(t_0 - t_{\rm ref})$ describes the other angular coordinate of the vector $\boldsymbol{v}$ for a time $t_0$ after a reference time $t_{\rm ref}$ when the ejection vector lies in the $x'$-$z'$ plane, we can write

$$\boldsymbol{v} = s_{\rm jet} v\left[\sin\psi\cos\Omega(t_0 - t_{\rm ref}),\ \sin\psi\sin\Omega(t_0 - t_{\rm ref}),\ \cos\psi\right]. \quad (1)$$

The projections of the vector $\boldsymbol{v}$ on the $x$, $y$, and $z$ coordinate axes are

$$\boldsymbol{v}\cdot\hat{x} = s_{\rm jet} v\left[\sin\psi\sin i\cos\Omega(t_0 - t_{\rm ref}) + \cos\psi\cos i\right], \quad (2a)$$

$$\boldsymbol{v}\cdot\hat{y} = s_{\rm jet} v\left[\sin\psi\sin\Omega(t_0 - t_{\rm ref})\right], \quad (2b)$$

and

$$\boldsymbol{v}\cdot\hat{z} = s_{\rm jet} v\left[\sin i\cos\psi - \cos i\sin\psi\cos\Omega(t_0 - t_{\rm ref})\right], \quad (2c)$$

respectively. In order to describe proper motions, we still need to rotate the $y$- and $z$-axes by an angle $\chi$ so the new $y$-axis will point north and the new $z$-axis will point east, as shown in Figure 1$b$, where the new $z$-axis is called $\Delta\alpha$ and the new $y$-axis is called $\Delta\delta$. The specific identification of foreground and background jets, the sense of jet rotation, and the other geometric parameters which we will obtain from a fit to the radio data are incorporated in Figure 1$b$.

The projection of the vector $\boldsymbol{v}$ on the $\Delta\alpha$ and $\Delta\delta$ axes gives

$$v_\alpha = (\sin\chi)\boldsymbol{v}\cdot\hat{y} + (\cos\chi)\boldsymbol{v}\cdot\hat{z}, \quad (3a)$$

and

$$v_\delta = (\cos\chi)\boldsymbol{v}\cdot\hat{y} - (\sin\chi)\boldsymbol{v}\cdot\hat{z}. \quad (3b)$$

Let $t_{\rm eject}$ be the time when a particular pair of segments of the SS 433 jets are "ejected" from the central object; then the observed proper motions of these segments of jets at a later time $t_0$ are described by

$$\mu_\alpha = \frac{v_\alpha(t_0 - t_{\rm eject})/\cos\delta}{d(1 - \boldsymbol{v}\cdot\hat{x}/c)}, \quad (4a)$$

and

$$\mu_\delta = \frac{v_\delta(t_0 - t_{\rm eject})}{d(1 - \boldsymbol{v}\cdot\hat{x}/c)}, \tag{4b}$$

where $c$ is the speed of light, $d$ is the distance to the object, and the other factor in each denominator compensates for the finite travel time of observed radiation crossing the source. Because of this effect of special relativity, the near side of a twin jet is seen as it was at a time later than $t_0$ and the far side is seen as it was at a time earlier than $t_0$. The time of a particular observation $t$ is related to $t_0$ by the light travel time between SS 433 and the observer.

Examination of equations (1)–(4) shows that we have the parameters $s_{\rm jet}$, $s_{\rm rot}$, $\psi$, $i$, $\chi$, $P$, $t_{\rm ref}$, $v/d$, and $v$ which are common to all epochs of the object in a constant velocity model. It should be noted that these equations describe the apparent motion of any material "ejected" from a particular point, and they are correct for even relativistic velocities; therefore they apply not only to SS 433, but also to the jets of quasars and radio galaxies. In equation (4) the apparent proper motion can become arbitrarily large when the inclination is near 0° and $v$ becomes a significant fraction of $c$, as discussed by Ozernoi and Sazanov (1969) and Blandford, Reis, and McKee (1977).

In our analysis of the proper motions of the SS 433 radio jets, we will assume, based upon the results of Margon *et al.*, that either $i=80°$ and $\psi=20°$ or $i=20°$ and $\psi=80°$. In practice, one can show from the radio data alone that values of $i$ near 80° and values of $\psi$ near 20° are required, but since the values determined from the optical data are more accurate, they should be adopted. The same is true for the period $P$. One can show from the radio data that a period of the order of 164 days is required to fit all epochs of VLA maps with the same parameters; therefore we adopt the Margon *et al.* period of 164.0 days. We also adopt the Margon *et al.* value of $t_{\rm ref}$ = JD 2,443,501.48. We therefore treat the sign choices of $s_{\rm jet}$ and $s_{\rm rot}$, the two options for $i$ and $\psi$, and the values of $v/d$, $\chi$, and $v$ as the parameters to be adjusted to fit the proper motion data for the radio jets.

## III. FIT TO JET PROPER MOTIONS

The equations described in § II were used to generate plots of the observed effects of proper motions of SS 433 radio jets for a range of the possible values for each parameter. These plots were compared with the VLA 4885 MHz maps of Paper I for the following epochs ($t$): JD 2,444,215 (1979 December 7), JD 2,444,306 (1980 March 7), JD 2,444,335 (1980 April 5), and JD 2,444,411 (1980 June 20). In Figure 2 we display contour maps of these 4885 MHz data in a form where the core radio source (marked with a small +) is removed so the jets can be more easily seen. Superposed upon the contour maps of Figure 2 are the apparent proper motion paths of SS 433 radio jets as computed from equations (1)–(4) using the parameters of Table 1. The determination of the kinematic parameters in Table 1 was based only upon fits to the maps for JD 2,444,335 and 2,444,411; so the fits for the other days are independent checks on the model parameters. The filled circles along each proper motion curve in Figure 2 represent values of $t_{\rm eject}$ at intervals of 20 days in the range 20–320 days before $t_0$. Considering that the proper motion paths should be convolved with a synthesized beam which is typically $0\farcs7$ by $0\farcs4$, with a major axis along position angle $-35°$, these paths match details of structure in all four radio maps to an extraordinary degree. Even though there is variation in the strengths of particular portions of the radio jets, Figure 2 clearly shows that the dominant effect determining the extended structure of the radio jets of SS 433 is proper motion of material ejected from the central regions of the object.

All of the parameters listed in Table 1 are consistent with the optical jet parameters of Margon *et al.* All parameters could, in principle, be determined solely from radio data; however, as discussed previously, once we realized that parameters known from optical determinations were going to come out near the optically determined values, we adopted most of them on the basis of their greater accuracy. The choice of $i=80°$ and $\chi=20°$ corresponds to the slightly preferred narrow cone model as discussed by Margon *et al.* The $\chi=10°$ rotation angle in the plane of the sky corresponds to the mean position angle of 100° reported in Paper I and also indicated from X-ray (Seward *et al.* 1981) and VLBI (Spencer 1979; Walker *et al.* 1981) observations. Clockwise rotation ($s_{\rm rot}=-1$) with a period of 164 days is essential in allowing the same set of parameters to correctly describe the major structural features seen at all epochs. The time travel effects in equation (4) are most noticeable in the proper motion paths of Figure 2, establishing through differences in proper motion of a twin-jet "pair" that the eastern jet axis ($s_{\rm jet}=+1$) is on the near side of the central object and the western jet axis is on the far side, since equation (4) predicts that

$$\mu_W/\mu_E = [1-(v/c)\cos\theta]/[1+(v/c)\cos\theta], \tag{5}$$

where $\cos\theta = \boldsymbol{v}\cdot\hat{x}/v$, and $\mu_W$ and $\mu_E$ are the western and eastern proper motions of a particular twin-jet pair ejected at the same time.

The ratio $v/d = 3\farcs0 \pm 0\farcs2$ yr$^{-1}$ is directly determined by the observed size scale of each proper motion corkscrew. This is close to the value of $3\farcs2$ yr$^{-1}$ determined in Paper I from apparent motions of a few linearly polarized features in the jets. The values of $v/c = 0.26 \pm 0.05$ and $d = (5.5 \pm 1.1)$ kpc are less well

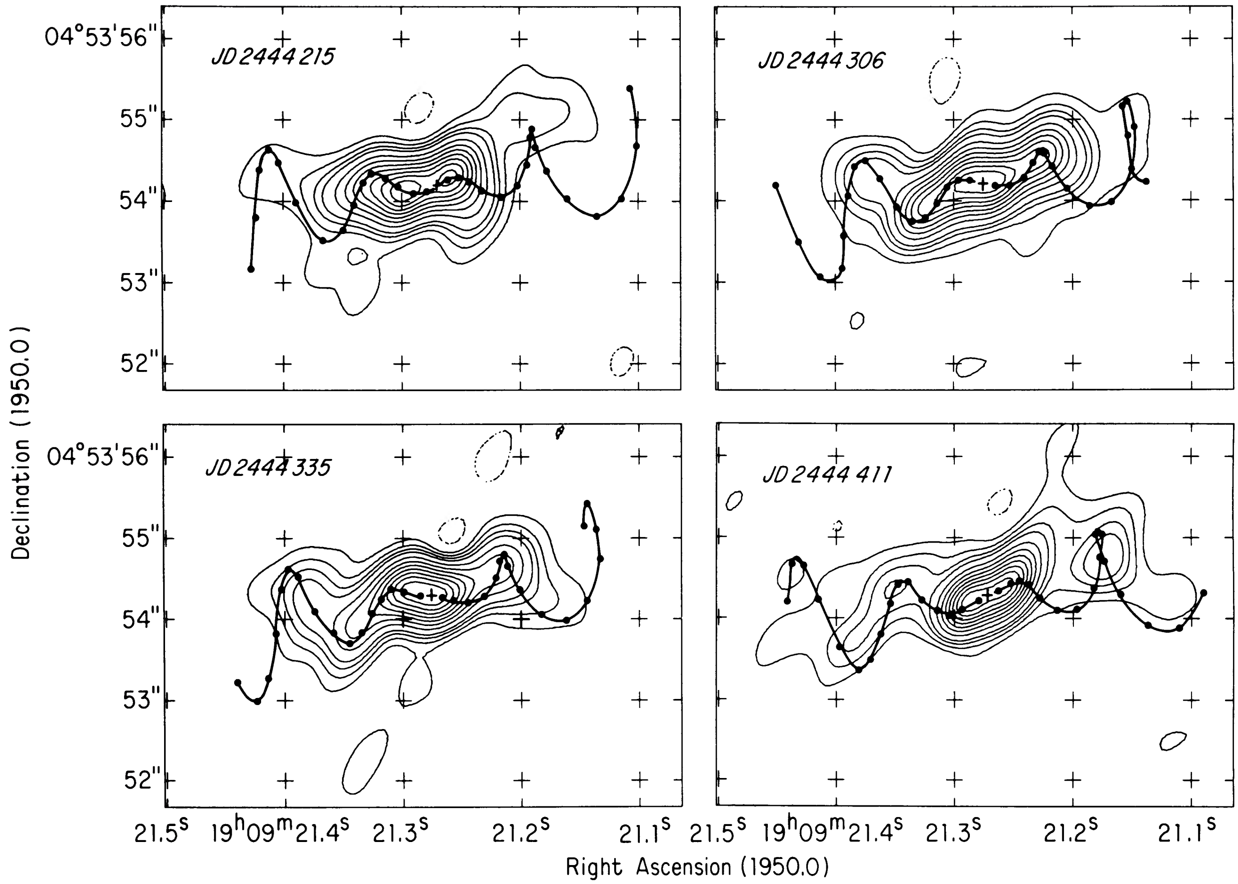


FIG. 2.—VLA radio contour maps of SS 433 at 4885 MHz for $t$=JD 2,444,215, 2,444,306, 2,444,335, and 2,444,111 are displayed in a form where the unresolved core radio source (small +) is removed, and the proper motion paths of material ejected at 20 day intervals with the parameters of Table 1 are drawn with filled circles. The contour levels correspond to 90, 80, 70, 60, 50, 40, 30, 20, 15, 10, 5, and −5% of the peak flux density values of 0.070, 0.030, 0.029, and 0.032 Jy per beam area for JD 2,444,215, 2,444,306, 2,444,306, and 2,444,411 maps, respectively.

determined and require special discussion. However, as can be seen from Figure 2, the proper motion corkscrews predicted from the parameters in Table 1 are in very good agreement with essentially all of the major structural features for all four epochs of VLA radio maps. The variations in intensity along the proper motion curves are caused by variations in the physical parameters of the ejected synchrotron radio-emitting plasma, a topic whose discussion is beyond the scope of this *Letter*.

TABLE 1

PARAMETERS OF THE TWIN-JET KINEMATIC MODEL FOR SS 433 RADIO JETS

| |
|---|
| $\psi=20°$ |
| $i=80°$ |
| $\chi=10°\pm2°$ (position angle 100°) |
| $s_{\rm jet}=+1$ for $\mu_\alpha>0$ (eastern jet) |
| $=-1$ for $\mu_\alpha<0$ (western jet) |
| $s_{\rm rot}=-1$ (clockwise or left-handed rotation) |
| $P=164$ days |
| $v/d=3''.0\pm0''.2$ yr$^{-1}$ |
| $v/c=0.26\pm0.05$ |
| $d=(5.5\pm1.1)$ kpc |

In principle, the differential effects of finite travel time across the source permit the absolute determination of $v$, and $d$ is then uniquely determined from the known ratio $v/d$. The data shown in Figure 2 indicate that $v=0.26c$ gives a slightly better fit than $v=0.17c$ or $v=0.31c$. The distinctions between these velocities are limited by the synthesized beam size of $0''.7$ by $0''.4$. This resolution size scale corresponds roughly to the changes in proper motion when $v$ changes by 20%. Fortunately, maps made with the full VLA will have symmetric beams $0''.35$ in size at 4885 MHz and will also have improvements in sensitivity of a factor of 2 or more. More accurate determinations of $v$ and $d$ from VLA maps of SS 433 will then be possible. In the meantime, since we find the radio jet velocity to be near $0.26c$ for all epochs, it is very likely that radio and optical jet velocities are both exactly $0.26c$, and the distance is therefore 5.5 kpc.

The measured $v/d$ ratio of 3″ yr$^{-1}$, and the nature of the models which fit most of the structural details out to 2″–3″ from the central object, indicate that the observed structure is an accumulation of about 300 days of ejection of radio-emitting material from the inner regions of SS 433. Specific structures can be assigned kinematic ages on the basis of the twin-jet model, and all jet features outside the roughly 0.″1 core radio source are greater than about 7 days in age (1″ = 127 light days = $8.2 \times 10^{16}$ cm with $d = 5.5$ kpc). An equipartition analysis, as discussed in Paper I, indicates jet magnetic fields of $10^{-3}$ to $10^{-2}$ gauss and $10^{43}$ to $10^{44}$ ergs in relativistic electrons and magnetic fields. Since this is the result of about 300 days of ejection, one can conservatively estimate the energy output in the observed radio jets to be at least $10^{35}$–$10^{36}$ ergs s$^{-1}$.

## IV. CONCLUSIONS

The kinematic, twin-jet model for SS 433 explains not only the extreme radial velocity behavior of the optical jets, but also the details of proper motions of radio jets with size scales from 0.″1 to a few arc seconds. The combination of radio and optical information provides a complete picture of the kinematics of these jets in which both radio and optical data are fitted with the same constant velocity model using the same geometric parameters.

Because the structure in the radio maps is a result of proper motions of radio-emitting material, both the radio and optical jets must be interpreted in terms of real ejection from a central, unresolved region. Models for the optical jets which involve "flashlight" effects or infall of matter can be excluded unless one can establish that the commonality of radio and optical jet parameters is a pure coincidence.

The effects of special relativity on proper motions of material moving at high velocities will, under certain circumstances, permit the absolute determination of velocity and distance. Such effects may have a much wider applicability for jets in quasars and radio galaxies. In any case, the type and variety of radio and optical data for SS 433 may make this object a "Rosetta stone" for understanding the general problem of relativistic jets.

## REFERENCES

Abell, G. O., and Margon, B. 1979, *Nature*, **279**, 701.
Blandford, R., Rees, M. J., and McKee, C. 1977, *Nature*, **267**, 211.
Fabian, A. C., and Rees, M. J. 1979, *M.N.R.A.S.*, **187**, 138.
Gilmore, W., and Seaquist, E. R. 1980, *A.J.*, **85**, 1486.
Hjellming, R. M., and Johnston, K. J. 1981, *Nature*, **290**, 100 (Paper I).
Margon, B., Grandi, S. A., and Downes, R. A. 1980, *Ap. J.*, **241**, 306.
Milgrom, M. 1979, *Astr. Ap.*, **76**, L3.
Ozernoi, L. M., and Sazanov, V. N. 1969, *Ap. Space Sci.*, **3**, 395.
Ryle, M., Caswell, J. L., Hine, G., and Shakeshaft, J. 1978, *Nature*, **276**, 571.
Seaquist, E. R., Garrison, R. E., Gregory, P. C., Taylor, A. R., and Crane, P. C. 1979, *A.J.*, **84**, 1037.
Seward, F., Grindlay, J., Seaquist, E. R., and Gilmore, W. 1981, *Nature*, in press.
Spencer, R. E. 1979, *Nature*, **282**, 483.
Walker, R. C. *et al.* 1981, *Ap. J.*, **243**, 589.

R. M. Hjellming: National Radio Astronomy Observatory, P.O. Box 0, Socorro, NM 87801

K. J. Johnston: E. O. Hulburt Center for Space Research, Naval Research Laboratory, Washington, DC 20375