

## INFRARED PHOTOMETRY OF RED GIANTS IN THE GLOBULAR CLUSTER 47 TUCANAE

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### ABSTRACT

Infrared broad-band *JHK* and intermediate-band CO and H<sub>2</sub>O photometry is presented for 64 stars in the globular cluster 47 Tucanae including the long period variables (LPV) VI–4. These data are combined with optical photometric data and compared with evolutionary tracks for giant branch (GB) stars, with models for asymptotic giant branch (AGB) stars, and with determinations of CN band strengths. The main results of this paper are as follows:

1. At a fixed luminosity, the 47 Tuc giants show negligible scatter in  $V-K$  or, equivalently, effective temperature. Except for the upper half-magnitude, the slope of the GB is similar to theoretical giant branch slopes of the appropriate metallicity. These two results set an upper limit of  $0.2 M_{\odot}$  to the mass lost by stars as they evolve from the level of the horizontal branch to within 0.5 mag of the GB tip.

2. Comparison of theoretical GB tracks with the observed giant branches of M92, M13, 47 Tuc, M71, and M67 show that shifts in  $T_{\text{eff}}$  are required to bring theory and observation into agreement. These shifts seem to be a function of  $[\text{Fe}/\text{H}]$  and are in the sense that the observed tracks lie progressively cooler than the theoretical tracks for higher metal abundance. This suggests that the ratio of the mixing length to the pressure scale height in the convective envelope may be a function of metallicity and/or stellar mass.

3. While an independent estimate of the metallicity of 47 Tuc cannot be made from the IR data alone, the close similarity in all observed parameters of the 47 Tuc stars and the M71 giants studied previously implies that the two clusters must have essentially the same metallicity.

4. At a given effective temperature, the CO absorption strengths of the 47 Tuc giants have a significant scatter. This scatter can be accounted for by an anticorrelation between the CO strengths and the CN strengths measured by Norris and Freeman. The origin of this anticorrelation is likely to be the effect of CN blanketing in one of the filter band passes used to measure the CO strengths.

5. At mean light the four LPVs lie sufficiently above the tip of the giant branch that they must be AGB stars. Their luminosities, temperatures, and periods are in qualitative agreement with model predictions for such stars. The periodic behavior of these variables is similar to that of Galactic LPVs.

6. Stellar H<sub>2</sub>O absorption in the band passes of the *H* and *K* filters is a strong influence on the  $J-H$  and  $H-K$  colors of some of the stars studied. Furthermore, the H<sub>2</sub>O strengths observed in the four LPVs are greater than those in the non-LPV variables which, in turn, are greater than those in the nonvariables. Again, this is similar to the situation for cool Galactic variable and nonvariable giants.

*Subject headings:* clusters: globular — photometry — stars: abundances — stars: late-type — stars: long period variables — stars: mass loss.

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## I. INTRODUCTION

The globular cluster 47 Tucanae has long been considered the prototype of a metal-rich halo cluster. Because it is one of the closest globulars to the Sun and lies at a high galactic latitude, problems due to reddening, crowding, and background contamination are minimal. Thus it is ideally suited for a study of several problems in stellar evolution.

The first extensive photometric studies of 47 Tuc which revealed metal-rich characteristics of the cluster were by Wildey (1961) and Tifft (1963).<sup>2</sup> Metal-rich characteristics were also shown by the integrated light spectroscopy of Kinman (1959), the finding of M type giants in the cluster by Feast and Thackeray (1960), and the DDO photometric study by McClure and Osborn (1974). The sequences in the H-R diagram are well defined with a negligible width to the giant branch (Hartwick and Hesser 1974; Hesser and Hartwick 1977; Lee 1977).

Recent photometric and spectroscopic work on 47 Tuc, however, shows that it may be neither as metal rich as previously thought, nor as homogeneous. For example, significant star-to-star variations in CN strengths have been reported by Hesser, Hartwick, and McClure (1976, 1977), Dickens, Bell, and Gustafsson (1979), Norris and Freeman (1979), Norris and Cottrell (1979), Mallia (1978), and Hesser (1978). The most surprising new result, though, is that of Pilachowski, Canterna, and Wallerstein (1980) who find from high resolution spectroscopy and from photometry that the logarithmic heavy metal abundance ratio with respect to the Sun is close to  $[\text{Fe}/\text{H}] = -1.2$ , lower than most previous estimates by more than 0.5 dex.

The purpose of this paper is to present new infrared data for 64 giants and variables in 47 Tuc (§ II). These data have the advantage of giving directly the bolometric luminosities, and, with a relatively well determined transformation, the effective temperatures of the stars (see the review by Persson and Frogel 1978). A comparison of a grid of theoretical giant branch (GB) tracks with empirical ( $\log L$ ,  $\log T_{\text{eff}}$ ) GB loci for 47 Tuc and other clusters with well determined abundances can be used to fix the values of several theoretical parameters that affect the stellar evolutionary calculations (§ III). Sections IV and V contain discussions of the stellar CO absorption data, a comparison with CN absorption data, and a discussion of the H<sub>2</sub>O indices and the  $[(J-H)_0, (H-K)_0]$ -diagram. Section VI examines the data we have obtained for the large amplitude variables and qualitatively compares them with the expected appearance of asymptotic giant branch (AGB) stars. A

<sup>2</sup>Subsequent photometry of limited samples of 47 Tuc stars by Eggen (1972), Menzies (1973), Cathey (1974), and Cannon (1974) showed systematic errors in the photometry of Wildey and Tifft, but did not alter the qualitative conclusions drawn from their work.

summary and conclusions are given in § VII. The Appendix contains a brief explanation of the procedures used to find  $M_{\text{bol}}$  and  $T_{\text{eff}}$  from the infrared data.

## II. OBSERVATIONS

The observations were made on the 4.0 m and 1.5 m telescopes of the Cerro Tololo Inter-American Observatory and the 2.5 m duPont telescope of the Las Campanas Observatory between 1978 September and 1980 January. Two InSb detector systems were used to make broad-band  $J$ ,  $H$ ,  $K$ , and  $L$  measurements (1.25, 1.65, 2.2, and 3.5  $\mu\text{m}$ , respectively) and narrow-band CO and H<sub>2</sub>O index measurements (2.36  $\mu\text{m}$ –2.20  $\mu\text{m}$  and 2.00  $\mu\text{m}$ –2.20  $\mu\text{m}$  colors, respectively) as described in Cohen, Frogel, and Persson (1978; hereafter CFP). The star/sky beam separations and aperture sizes were chosen to avoid stars which could contaminate one or the other of the beams. Work at the telescope was considerably facilitated by large scale finding charts supplied to us by K. Freeman.

The data from both observatories were transformed to the system of standard stars in Frogel *et al.* (1978) and Aaronson, Frogel, and Persson (1978).<sup>3</sup> A number of stars spread out in color and magnitude were observed at both observatories to check for possible systematic differences. As Table 1 shows, no systematic differences were found, and the dispersions of the pairs of measurements are consistent with the measurement uncertainties. The only other previously published infrared photometry of stars in 47 Tuc is that of Glass and Feast (1973). Nearly all of the stars in common are variables, and no meaningful comparison of the two data sets can be made.

Most of the 47 Tuc stars observed were selected from those for which Lee (1977) has published photoelectric  $UBV$  data. We also included a number of stars with DDO photometry from Hesser, Hartwick, and McClure (1977) and photoelectric  $BV$  photometry from Hesser and Hartwick (1977; identified with an HH number). Many of the well known red variables, including three suspected variables found by Lee (1977), were observed one or more times. Two stars identified as being particularly red in  $V-I$  by Lloyd Evans (1974) were also included.

Table 2 contains the observational data for all of the stars not specifically identified as variables by Hogg

<sup>3</sup>Since the publication of these two papers, a considerably enlarged body of observations has led to a slight revision of some of the magnitudes and colors of the standard stars. This revision will be discussed separately (Elias and Frogel, in preparation). Intercomparison of the cluster data which we have published to date is not affected in any systematic fashion.

Frogel *et al.* do not give standard values for  $K-L$  colors. We have established preliminary values of  $K-L$  for the standard stars on the assumption that HR 4689, an A2 V star, has  $K-L=0.01$ . Thus, in addition to the statistical errors, the  $K-L$  colors of the stars observed could have an additional zero point error of  $\pm 0.02$ .

TABLE 1  
COMPARISON OF THE LAS CAMPANAS AND CTIO OBSERVATIONS

Parameter	$K$	$J-K$	$H-K$	$H_2O$	CO
$\langle LC-CTIO \rangle \dots$	-0.01	+0.01	-0.01	0.00	0.00
Dispersion .....	0.02	0.03	0.01	0.02	0.02
$N$ .....	11	11	11	8	

(1973) and also the colors and magnitudes corrected for reddening and extinction corresponding to  $E(B-V) = 0.04$  (Lee 1977). No corrections were applied to the  $H_2O$  or CO indices. Throughout the rest of this paper corrected colors and magnitudes are noted by a subscript zero. The  $UBV$  data are from Lee (1977) unless otherwise noted. We have indicated which stars lie on the HB or AGB on the basis of a  $(V, B-V)$ -plot.

Table 3 gives the infrared data for the variables V1-4. For each star the phase has been arbitrarily set equal to zero on the first day of observation; the periods are taken from Hogg (1973).

Table 4 summarizes the photometry for all the stars identified as variables by Hogg (1973) or Lloyd Evans (1974). For V1-4 we have taken values representative of maximum and minimum  $K$  light from Table 3. Eggen (1975a) has given a photoelectric light curve for V3, and we have taken his maximum and minimum  $V$  values to form  $V-K$  colors for V3. Since V1 and V2 are similar in type to V3, we have scaled Eggen's  $V_{\max}$  and  $V_{\min}$  for V3 to the other two on the basis of the relative ranges indicated by Hogg (1973) in order to obtain representative maximum and minimum  $V-K$  colors. Estimates of the visual brightnesses of these stars at the time that the infrared observations were made indicated that the phase shifts between the  $V$  and  $K$  light curves were not large. Furthermore, the range in  $K$  is small compared to the range in  $V$  and the cycle to cycle changes in  $K$  are small as well. Thus we estimate that the errors in  $V-K$  for V1-3 are  $\pm 0.3$  mag. In any case, for the purposes of this paper more accurate  $V-K$  values are not required since, as discussed in the Appendix, neither the bolometric corrections nor the effective temperatures are particularly sensitive to such uncertainties for stars as red as these. For the other variables in Table 4, the expected range in  $K$  is on the order of 0.20 mag or less. Thus, as noted in the table, mean  $UBV$  colors and magnitudes from the literature were used to form the representative values listed.

### III. THE COLOR-MAGNITUDE DIAGRAM

#### a) The $[K_0, (V-K)_0]$ -Diagram

Figure 1 is an infrared color-magnitude diagram for the stars with data from Tables 2 and 4. The points

which represent variables V1-4 are straight averages of the maxima and minima. Since variations in  $V$  for the variables can be as much as 5 times greater than variations in  $K$ , we have given an indication of the size of these variations in Figure 1: Stars with a range in  $V$  exceeding 0.4 mag are given  $\pm 0.2$  mag error bars, while stars with smaller  $V$  variations are given  $\pm 0.1$  mag error bars. Fiducial giant branches for three clusters studied previously are also shown (Table 5).<sup>4</sup> For the distance modulus to 47 Tuc we take  $(m-M)_0 = 13.14$ , obtained from Lee's (1977) apparent magnitude of 14.06 for the horizontal branch,  $A_V = 0.12$ , and  $M_{V_0}(\text{HB}) = 0.8$ .

There is a striking difference between Figure 1 and conventional  $(V, B-V)$ -plots of metal rich and metal poor globular clusters. As was first noted by Sandage and Wallerstein (1960), in a  $[V_0, (B-V)_0]$ -plot, cluster giant branches generally do not extend redward of  $(B-V)_0 \approx 1.7$ , and the reddest giants in metal-rich clusters tend to be fainter in  $M_{V_0}$  than the reddest giants in metal-poor ones. The two physical effects which cause the  $[K_0, (V-K)_0]$ -diagram to look so different are first, the nearness of the  $K$  passband to the peak of the stellar energy distribution and its relative freedom from blanketing effects, and second, the continuing increasing sensitivity of  $(V-K)_0$  to  $T_{\text{eff}}$  as one goes to cooler stars. We particularly note that the reddest extent of the giant branch is a strong function of metallicity.

The stars that lie on the 47 Tuc giant branch display a scatter about a ridge line which is no larger than the observational uncertainties alone. This considerably extends Lee's finding because of the much higher sensitivity of  $V-K$  to effective temperature than that of  $B-V$ . With the exception of the variables V1-4 and R10, the

<sup>4</sup> Instead of using the values of  $(m-M)_0$  in Table 1 of CFP, we have put the distance moduli on the uniform scale based on the magnitude of the horizontal branch (HB) (Harris 1979). We differ from Harris, though, in taking  $M_{V_0}(\text{HB}) = +0.8$  instead of  $+0.9$  for G type clusters (e.g., M71 and 47 Tuc). Harris (1979) has discussed the reasons for his choices of values for  $M_{V_0}(\text{HB})$ . Our feeling is that the jump from 0.6 for F type to 0.9 for G type clusters is a bit too steep, and thus we chose to use  $+0.8$  for 47 Tuc and for M71 (Frogel, Persson, and Cohen 1979). None of the conclusions of the paper are affected by such small changes in  $(m-M)_0$  for metal rich clusters. We also changed  $E(B-V)$  for M67 from 0.06 to 0.03 (see the note added in proof at the end of CFP), and have taken  $(m-M)_0$  to be  $= 9.28$ . Thus the distance moduli are assumed to be 15.0, 14.33, 9.28, 12.90, and 14.44 for M3, M13, M67, M71 and M92, respectively.

TABLE 2  
47 TUCANAE PHOTOMETRY

Star <sup>b</sup>	Observed <sup>a</sup>			K <sub>0</sub>	(U-V) <sub>0</sub>	(B-V) <sub>0</sub>	Reddening Corrected <sup>c</sup>			H <sub>2</sub> O	C0	n	Notes
	K	(J-K)	(H-K)				(V-K) <sub>0</sub>	(J-K) <sub>0</sub>	(H-K) <sub>0</sub>				
5309	8.67 (3)	0.88 (3)	0.13 (2)	8.66	2.98	1.45	3.42	0.86	0.12	0.05	0.16	1	
5312	8.55 (3)	0.92 (3)	0.14	8.54	3.20	1.45	3.52	0.90	0.13	0.045	0.12	1	
7320	7.45	1.00	0.20	7.44	3.53	1.70	4.5	0.98	0.19	0.09	0.185	1	1, 3
1406	10.61	0.73	0.11	10.60	1.98	1.07	2.73	0.71	0.10	0.02 (3)	-0.02	1	
1407	11.13	0.66	0.10	11.12	1.78	1.03	2.54	0.64	0.09	...	0.05	1	
1414	11.92 (3)	0.60 (3)	0.08 (3)	11.91	1.66	1.02	2.21	0.58	0.07	...	0.045	1	
1421	6.84	1.10	0.21	6.83	3.28	1.65	5.1	1.08	0.20	0.11	0.14	2	2, 4 HH9065
1425	11.53	0.65	0.10	11.52	1.71	1.00	2.46	0.63	0.09	...	0.07	1	HH9062
2416	9.37	0.85 (4)	0.12	9.36	2.59	1.27	3.18	0.83	0.11	...	0.155	1	
2426	8.49	0.92	0.15	8.48	3.20	1.48	3.50	0.90	0.14	...	0.11	1	
3407	9.99	0.73	0.08	9.98	2.35	1.19	2.86	0.71	0.07	-0.005	0.08	1	HH9036
3410	10.30	0.75 (4)	0.11 (4)	10.29	2.15	1.13	2.77	0.73	0.10	...	0.13	2	
4411	11.54 (3)	0.64	0.07	11.53	...	1.00	2.45	0.62	0.06	...	...	1	
4415	9.86	0.61	0.09	9.85	1.85	1.02	2.37	0.59	0.08	0.06	0.09	3	AGB
4417	11.95 (3)	0.53	0.07	11.94	1.00	0.74	2.08	0.51	0.06	...	...	1	HB
4418	8.55 (3)	0.91	0.14	8.54	3.03	1.42	3.50	0.89	0.13	0.035	0.145	2	
5404	10.14 (3)	0.67	0.09	10.13	1.83	1.05	2.56	0.65	0.08	...	0.075	1	AGB
5406	9.69	0.81 (4)	0.10 (3)	9.68	2.49	1.26	3.01	0.79	0.09	...	0.13	1	
5422	9.13	0.85	0.14	9.12	2.80	1.36	3.23	0.83	0.13	...	0.11	1	
5427	9.85	0.83 (4)	0.12	9.84	2.35	1.18	2.97	0.81	0.11	...	0.10	1	
6407	9.98	0.73 (4)	0.08	9.97	2.20	1.12	2.71	0.71	0.07	0.02	0.04	1	AGB
6408	9.74	0.78 (4)	0.09	9.73	2.34	1.25	2.86	0.76	0.08	...	0.11	1	
8406	8.93	0.88	0.15	8.92	2.93	1.39	3.33	0.86	0.14	0.03	0.145	1	
8416	10.57 (6)	0.74	0.11	10.66	2.17	1.11	2.65	0.72	0.10	...	0.065 (3)	1	
1505	8.52	0.92	0.15	8.51	3.11	1.46	3.53	0.90	0.14	...	0.13	1	HH9083
1510	8.76	0.88	0.13	8.75	2.93	1.39	3.28	0.86	0.12	...	0.12	1	AGB, HH9085
1513	9.28	0.82	0.12	9.27	2.57	1.28	3.02	0.80	0.11	0.02	0.08	2	AGB
1518	10.33	0.68 (3)	0.09	10.32	1.76	1.04	2.62	0.66	0.08	...	...	1	AGB
2525	9.32	0.81 (3)	0.12	9.31	2.47	1.25	3.00	0.79	0.11	...	0.16	1	AGB
3501	8.57	0.91	0.15	8.56	3.07	1.43	3.45	0.89	0.14	0.03	0.12	3	HH9031
3512	7.43	1.04	0.19	7.42	3.51	1.58	4.25	1.02	0.18	0.05	0.13	3	2, 5, HH9006
4503	9.43 (3)	0.75 (3)	0.11	9.42	2.27	1.18	2.85	0.73	0.10	0.055	0.135	1	AGB
5527	10.88	0.68	0.10	10.87	1.84	1.04	2.61	0.66	0.09	...	...	1	HH9413
5529	7.92	0.97	0.16	7.91	3.36	1.55	3.86	0.95	0.15	0.04	0.16	2	HH9414
6502	12.02	0.46	0.06	12.01	1.12	0.80	1.91	0.44	0.05	...	...	1	HH9410, HB

TABLE 2—Continued

Star <sup>b</sup>	Observed <sup>a</sup>		$K_0$	(U-V) <sub>0</sub>	(B-V) <sub>0</sub>	Reddening Corrected <sup>c</sup>			n	Notes
	K	(J-K)				(H-K)	(V-K) <sub>0</sub>	(J-K) <sub>0</sub>		
6509	10.93	0.65	10.92	...	1.08	2.57	0.63	0.05	1	6, HH9409
7502	11.28	0.68	11.27	1.69	1.01	2.55	0.66	0.11	1	
7507	11.29 (6)	0.70	11.28	1.73	1.03	2.40	0.68	0.10	1	
7525	12.21 (6)	0.42	12.20	0.60	0.61	1.58	0.40	0.09	1	HB
8317	9.56 (3)	0.76 (3)	9.55	2.49	1.25	2.95	0.74	0.10	1	
8518	10.42 (6)	0.71	10.40	1.84	1.06	2.58	0.69	0.11	1	AGB
1602	10.64 (6)	0.69	10.63	1.69	1.00	2.49	0.67	0.10	1	AGB
1603	7.94	1.02	7.93	3.38	1.54	3.84	1.00	0.16	3	
1604	9.94	0.81	9.93	2.36	1.17	2.96	0.79	0.11	2	
2603	9.78 (3)	0.81 (3)	9.77	2.41	1.23	3.01	0.79	0.11	1	
2605	9.37 (5)	0.84 (5)	9.36	2.38	1.18	3.02	0.82	0.15	1	AGB
4603	8.74	0.85 (3)	8.73	2.93	1.37	3.28	0.83	0.12	1	AGB
5627	9.34	0.78	9.33	...	1.27	3.02	0.76	0.09	1	6, HH9411
5739	9.03	0.86	9.02	...	1.37	3.26	0.84	0.11	1	6, HH9405

<sup>a</sup> Observational uncertainties in K, H - K, CO, and H<sub>2</sub>O are ±0.02 mag or less unless otherwise noted (in units of hundredths of a magnitude in parentheses). The uncertainty in (V - K)<sub>0</sub> is ±0.04 unless the uncertainty in K is greater than 0.02.  
<sup>b</sup> All numbers are from Lee 1977. Hesser and Hartwick 1977 numbers are indicated in notes column by HH. Asymptotic giant branch and horizontal branch stars are indicated in notes column by AGB and HB, respectively.  
<sup>c</sup> All UBV photometry is from Lee 1977 unless otherwise noted.

Notes.—(1) Suspected red variable (Lee 1977). Observed only once in the IR. (2) Suspected red variable (Lee 1977). IR data are an average of two measurements whose K magnitudes differed by less than 0.05. (3) UBV photometry is mean from Lee (1977). Range in V is ~0.2 mag. (4) Same as note 3 except range in V is ~0.4 mag. (5) Same as note 3 with range in V of ~0.1 mag. (6) BV photoelectric photometry from Hesser and Hartwick 1977.

TABLE 3  
OBSERVATIONS<sup>a</sup> OF 47 TUCANAE VARIABLES

JD (2,440,000+)	Phase <sup>b</sup>	<i>K</i>	<i>J-K</i>	<i>H-K</i>	<i>K-L</i>	H <sub>2</sub> O	CO
A. V1							
3798 .....	0.0	5.75	1.11	0.34	0.51 (3)	0.35 (3)	0.115
3839 .....	0.19	6.19	1.21	0.46	0.51	0.74	0.15
3853 .....	0.26	6.36	1.16	0.46	...	0.775	0.13
3859 .....	0.29	6.44	1.16	0.49	...	0.82	0.11
3860 .....	0.29	6.44 (3)	1.14	0.50	0.55	0.815	0.10
3900 .....	0.48	6.64	1.20	0.55	0.66	0.78	0.04
4153 .....	1.67	6.14	1.14 (3)	0.35	0.54 (3)	0.335 (3)	0.00
4187 .....	1.84	5.75	1.16	0.34	...	0.245	0.075
4214 .....	1.96	5.74	1.11	0.34	...	0.28	0.10
4237 .....	2.07	5.90	1.09	0.37	0.52 (5)	0.41 (4)	0.11
4243 .....	2.10	5.99	1.07 (3)	0.38	...	0.48	0.12
4266 .....	2.21	6.25	1.11	0.43	0.49 (4)	0.74 (3)	0.15
B. V2							
3798 .....	0.0	5.96	1.15	0.34	0.49 (3)	0.30	0.09
3839 .....	0.20	5.87	1.12	0.36	0.45	0.44	0.135
3853 .....	0.27	6.04	1.08 (4)	0.36	...	0.50	0.135
3859 .....	0.30	6.11	1.07	0.40	...	0.52 (3)	0.10 (3)
3860 .....	0.31	6.13 (3)	1.05	0.40	0.51	0.56	0.125
3900 .....	0.50	6.68	1.11	0.53	0.62	0.86	0.12
4153 .....	1.75	6.64	1.12 (3)	0.38	0.51 (3)	0.83 (3)	0.19
4187 .....	1.92	6.44	1.04	0.34	...	0.66	0.18
4214 .....	2.05	6.15	0.97	0.28	...	0.56	0.205
4237 .....	2.16	6.06	0.96	0.27	0.51 (5)	0.53 (4)	0.225
4243 .....	2.19	6.08	0.95 (3)	0.28	...	0.51	0.24
4266 .....	2.31	6.07	1.06	0.33	0.36 (4)	0.62 (3)	0.26
C. V3							
3771 .....	0.0	6.01	1.05	0.29	...	0.32	0.16
3774 .....	0.01	6.01	1.06	0.31	...	0.32	0.16
3798 .....	0.14	6.03	1.10	0.34	0.39 (3)	0.34 (3)	0.14
3839 .....	0.35	6.35	1.23	0.43	0.44	0.61	0.14
3853 .....	0.43	6.48	1.23 (4)	0.41	...	0.57	0.17
3859 .....	0.46	6.51	1.23	0.43	...	0.55	0.08
3860 .....	0.46	6.50 (3)	1.21	0.45	0.49	0.54	0.08
3900 .....	0.67	6.45	1.09	0.35	0.50	0.29	0.08
4153 .....	1.99	6.08	1.02 (3)	0.25	0.33	0.39 (3)	0.22
4187 .....	2.17	6.07	1.15	0.35	...	0.51	0.23
4214 .....	2.31	6.43	1.12	0.40	...	0.725	0.22
4237 .....	2.43	6.62	1.11	0.41	...	0.71 (4)	0.135
4243 .....	2.46	6.60	1.10 (3)	0.43	...	0.67	0.15
4266 .....	2.58	6.53	1.08	0.38	0.55 (4)	0.475 (3)	0.10
D. V4							
3798 .....	0.0	6.48	1.19	0.35	0.49 (3)	0.365 (3)	0.08
3839 .....	0.25	6.58	1.17	0.35	0.43 (3)	0.49 (3)	0.11
3853 .....	0.33	6.47	1.09 (4)	0.30	...	0.38	0.15
3859 .....	0.37	6.41	1.09	0.31	...	0.35	0.15
3860 .....	0.38	6.41 (3)	1.10	0.31	0.42	0.37	0.13
3900 .....	0.62	6.47	1.22	0.36	0.46	0.34	0.08
4153 .....	2.15	6.64	1.14 (3)	0.32	0.49	0.44	0.12
4187 .....	2.36	6.42	1.13	0.32	...	0.35	0.16
4214 .....	2.52	6.40	1.12	0.33	...	0.24	0.11
4237 .....	2.66	6.47	1.15	0.33	...	0.26 (4)	0.08
4243 .....	2.70	6.49	1.15 (3)	0.34	...	0.27	0.075
4266 .....	2.84	6.69	1.18	0.36	0.45 (4)	0.43 (3)	0.07

<sup>a</sup>Values tabulated are uncorrected for reddening. Observational uncertainties when greater than  $\pm 0.02$  mag are given in parentheses in units of hundredths of a magnitude.

<sup>b</sup>Phase set equal to 0.0 on first day of observations.

TABLE 4  
PHOTOMETRY OF 47 TUCANAE VARIABLES<sup>a</sup>

Star	Observed				Reddening Corrected							Notes		
	K	(J-K)	(H-K)	(K-L)	K <sub>0</sub>	(U-V) <sub>0</sub>	(B-V) <sub>0</sub>	(V-K) <sub>0</sub>	(J-K) <sub>0</sub>	(H-K) <sub>0</sub>	(K-L) <sub>0</sub>		H <sub>2</sub> O	CO
V1	5.75	1.11	0.34	0.51	5.74	...	...	4.1	1.09	0.33	0.50	0.31	0.115	1, 2
V1	6.64	1.20	0.55	0.66	6.63	...	...	8.3	1.18	0.54	0.65	0.78	0.04	
V2	5.87	1.12	0.36	0.45	5.86	...	...	4.0	1.10	0.35	0.44	0.44	0.135	1, 2
V2	6.68	1.11	0.53	0.62	6.67	...	...	7.1	1.09	0.52	0.61	0.86	0.12	
V3	6.01	1.06	0.31	0.39	6.00	...	...	3.9	1.04	0.30	0.38	0.32	0.16	1, 2
V3	6.50	1.21	0.45	0.49	6.49	...	...	8.7	1.19	0.44	0.48	0.54	0.08	
V4	6.41	1.09	0.31	0.42	6.40	...	...	5.5	1.07	0.30	0.41	0.36	0.14	1, 3
V4	6.58	1.17	0.35	0.43	6.57	...	...	6.7	1.15	0.34	0.42	0.49	0.11	
V5	7.47	1.02	0.19	0.16	7.46	3.5	1.6	4.2	1.00	0.18	0.15	0.06	0.09	4
V6	7.44	1.03	0.21	0.18 (3)	7.43	...	...	4.3	1.01	0.20	0.17	0.09 (3)	0.105	5
V7	6.97	1.09	0.20	0.21 (3)	6.96	3.6	1.65	4.8	1.07	0.19	0.20	0.07	0.115	6
V8	6.67	1.14	0.31	0.34 (3)	6.66	...	...	5.3	1.12	0.30	0.33	0.225 (3)	0.13	7
V11	6.69	1.08	0.24	0.23 (3)	6.68	3.1	1.55	5.4	1.06	0.23	0.22	0.14 (3)	0.18	W12, 8
V13	7.66	0.99	0.20	0.14 (3)	7.65	...	...	4.25	0.97	0.19	0.13	0.09 (3)	0.12	9
V15	7.27	1.01 (3)	0.19	0.17 (3)	7.26	...	...	4.5	0.99	0.18	0.16	0.06 (3)	0.20	W300, 10
V18	7.45	1.02	0.22	0.19 (3)	7.44	3.65	1.6	4.45	1.00	0.21	0.18	0.06	0.16	L168, 11
V19	7.50 (5)	1.00	0.20	...	7.49	...	...	3.7	0.98	0.19	...	0.135	0.09	R10, 12, 13, 14
V21	6.75 (5)	1.23	0.28	...	6.74	...	...	5.7	1.21	0.27	...	0.22	0.16	A2, 12, 13, 15
A19	6.80 (5)	1.22	0.26	...	6.79	...	...	5.1	1.20	0.25	...	0.125	0.15	12, 13, 16

<sup>a</sup> This Table does not include the three suspected variables identified by Lee (1977). They are included in Table 2. Other commonly used designations for some of the stars are indicated in the notes column.

NOTES TO TABLE 4—(1) The infrared values for these stars are representative of time of maximum and minimum at *K* and are taken from Table 3. (2) The range in *V* magnitude has been estimated as discussed in the text. (3) The range in *V* is given by Lloyd Evans 1974 as 12.0–13.4. (4) Observed only once in the IR. Observed range in *V* is ~0.4 mag. The *UBV* colors are mean values (Lloyd Evans 1974; Eggen 1972, 1975b). (5) Observed only once in the IR. Lloyd Evans (1974) gives 0.2 mag for range in *V* with mean of 11.8. (6) Same as n. 4 except range in *V* is 0.9 mag with a mean of 11.9. (7) Observed only once in the IR; range in *V* is 0.6 mag with a mean of 12.1 (Lloyd Evans 1974). (8) Observed only once in the IR. Mean *UBV* magnitudes and colors from Lloyd Evans (1974) and Eggen

(1975a, b). The range in *V* is 0.5 mag. (9) Observed only once in the IR; range in *V* is 0.5 mag with a mean of 12.0 (Lloyd Evans 1974). (10) Observed only once in the IR; range in *V* is 0.3 mag with a mean of 11.9 (Lloyd 1974). (11) Same as n. 8. (12) Crowding problems make *K* magnitude uncertain. (13) Red variable; identification from Feast and Thackeray (1960) and/or Lloyd Evans (1974). (14) Observed only once in the IR; range in *V* is 0.4 mag with a mean of 11.3 (Lloyd Evans 1974). (15) Observed only once in the IR; range in *V* is 1.0 mag with a mean of 12.5 (Lloyd Evans 1974). (16) Observed only once in the IR; range in *V* is 0.3 mag with a mean of 12.0 (Lloyd Evans 1974).

TABLE 5  
RIDGE LINES FOR GIANT BRANCHES OF GLOBULAR CLUSTERS

$M_{K_0}$	$(V-K)_0$					$M_{K_0}$	$(V-K)_0$				
	M92	M13	M3	M71	47 Tuc		M92	M13	M3	M71	47 Tuc
-6.4	...	...	...	5.41	5.45	-3.8	2.50	2.66	2.68	3.12	3.14
-6.2	...	...	...	4.82	4.86	-3.6	2.46	2.61	2.62	3.04	3.06
-6.0	...	...	...	4.53	4.61	-3.4	2.41	2.57	2.57	2.96	2.99
-5.8	...	3.46	3.48	4.31	4.40	-3.2	2.36	2.51	2.51	2.88	2.92
-5.6	3.06	3.30	3.34	4.12	4.20	-3.0	2.32	2.46	2.46	2.81	2.85
-5.4	2.96	3.18	3.23	3.96	4.03	-2.8	2.28	2.40	2.40	2.74	2.77
-5.2	2.86	3.08	3.14	3.82	3.85	-2.6	2.24	2.35	2.35	2.67	2.71
-5.0	2.79	3.00	3.07	3.70	3.70	-2.4	2.19	2.30	...	2.60	2.65
-4.8	2.73	2.93	3.00	3.59	3.59	-2.2	2.15	2.24	...	2.53	2.58
-4.6	2.68	2.87	2.93	3.47	3.48	-2.0	2.11	2.19	...	2.47	2.53
-4.4	2.63	2.82	2.86	3.37	3.38	-1.8	2.07	2.13	...	2.42	2.47
-4.2	2.59	2.77	2.80	3.28	3.30	-1.6	...	2.07	...	2.37	2.41
-4.0	2.54	2.71	2.74	3.19	3.22	-1.4	...	2.02	...	2.31	2.36
						-1.2	...	1.98	...	2.27	2.31
						-1.0	...	...	...	2.24	...
$E(B-V)$	0.02	0.03	0.00	0.25	0.04						
$(m-M)_0$	14.44	14.33	15.0	12.9	13.14						

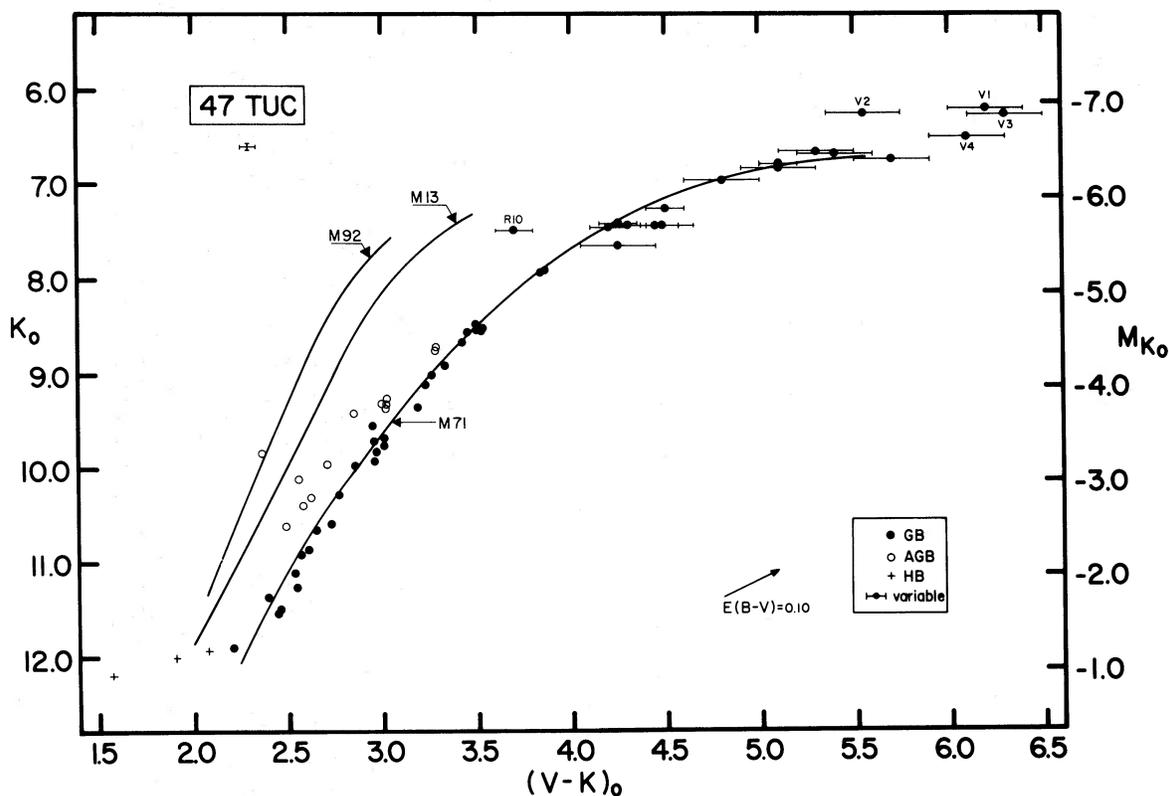


FIG. 1.— $K_0$  versus  $(V-K)_0$  for all stars observed in 47 Tuc. Giant branch (GB), asymptotic giant branch (AGB), and horizontal branch stars (HB) are distinguished based on their location in a  $(V, B-V)$ -plot (Lee 1977). The nominal error bar applies to the nonvariables. Variables are plotted at their mean values of  $K_0$  and  $(V-K)_0$  as discussed in the text. The lengths of the horizontal lines ( $\pm 0.1$  and  $\pm 0.2$ ) distinguish those stars which vary in  $V$  by less or more than 0.4, respectively. The fiducial line for the M71 giant branch is from the observations of Frogel *et al.* (1979) and assumed  $E(B-V)=0.25$  for M71 and  $(m-M)_0=12.90$  and 13.14 for M71 and 47 Tuc, respectively. The reddening vector displays the effect of extinction on  $M_{K_0}$ , not on  $K_0$ .

TABLE 6  
DERIVED PARAMETERS FOR 47 TUCANAE STARS IN TABLE 2

Star	BC <sub>V</sub>	M <sub>bol</sub>	log T <sub>eff</sub>	log g (M=0.8 M <sub>☉</sub> )	Notes	Star	BC <sub>V</sub>	M <sub>bol</sub>	log T <sub>eff</sub>	log g (M=0.8 M <sub>☉</sub> )	Notes
5309	-0.96	-2.02	3.601	1.0		1510	-0.86	1.97	3.610	1.0	AGB
5312	-1.02	-2.10	3.597	0.9		1513	-0.71	-1.56	3.625	1.3	AGB
7320	-1.74	-2.97	3.565	0.5	var.	1518	-0.51	-0.71	3.651	1.7	AGB
1406	-0.56	-0.37	3.642	1.8		2525	-0.70	-1.53	3.625	1.3	AGB
1407	-0.46	+0.06	3.653	2.0		3501	-0.98	-2.11	3.600	0.9	
1414	-0.28	+0.70	3.691	2.4		3512	-1.55	-3.02	3.573	0.5	var.
1421	-2.24	-3.45	3.551	0.2		4503	-0.61	-1.48	3.635	1.3	
1425	-0.42	+0.42	3.669	2.2	var.	5527	-0.50	-0.16	3.650	1.9	
2416	-0.81	-1.41	3.614	1.3		5529	-1.26	-2.63	3.586	0.7	
2426	-1.01	-2.17	3.598	0.9		6502	-0.19	+0.59	3.715	2.5	HB
3407	-0.63	-0.93	3.634	1.5		6509	-0.49	-0.14	3.652	1.9	
3410	-0.57	-0.65	3.640	1.7		7502	-0.46	+0.22	3.652	2.1	
4411	-0.42	+0.42	3.671	2.2		7507	-0.36	+0.18	3.675	2.1	
4415	-0.37	-1.29	3.676	1.6	AGB	7525	-0.08	+0.56	3.743	2.6	HB
4417	-0.27	+0.61	3.701	2.4	HB	8517	-0.68	-1.32	3.628	1.4	
4418	-1.01	-2.11	3.598	0.9		8518	-0.47	-0.62	3.652	1.7	AGB
5404	-0.47	-0.92	3.655	1.6	AGB	1602	-0.42	-0.44	3.666	1.9	AGB
5406	-0.71	-1.16	3.624	1.4		1603	-1.23	-2.60	3.587	0.7	
5422	-0.84	-1.63	3.612	1.2		1604	-0.67	-0.92	3.626	1.5	
5427	-0.68	-1.01	3.626	1.5		2603	-0.71	-1.07	3.624	1.4	
6407	-0.54	-1.00	3.645	1.6	AGB	2605	-0.70	-1.46	3.625	1.3	AGB
6408	-0.68	-1.13	3.627	1.4		4603	-0.87	-2.00	3.610	1.0	AGB
8406	-0.90	-1.79	3.606	1.1		5627	-0.72	-1.51	3.624	1.3	AGB
8416	-0.50	-0.33	3.646	1.8		5739	-0.86	-1.72	3.611	1.1	
1505	-1.03	-2.13	3.597	0.9							

TABLE 7  
DERIVED PARAMETERS FOR 47 TUCANAE VARIABLES IN TABLE 4

Star	BC <sub>V</sub>	M <sub>bol</sub>	log T <sub>eff</sub>	log g (M=0.8 M <sub>☉</sub> )	Notes <sup>a</sup>
V1	-1.41	-4.71	3.578	-0.2	max.
V1	-5.23	-3.44	3.458	-0.1	min.
V2	-1.32	-4.60	3.581	-0.1	max.
V2	-4.10	-3.47	3.504	0.0	min.
V3	-1.26	-4.50	3.584	-0.1	max.
V3	-5.60	-3.56	3.441	-0.3	min.
V4	-2.61	-3.85	3.543	0.0	max.
V4	-3.71	-3.58	3.516	0.0	min.
V5	-1.51	-2.99	3.575	+0.5	
V6	-1.62	-3.03	3.571	+0.5	
V7	-1.99	-3.37	3.556	+0.3	
V8	-2.42	-3.60	3.548	+0.1	
V11	-2.52	-3.58	3.545	+0.1	W12
V13	-1.59	-2.83	3.573	+0.5	
V15	-1.79	-3.17	3.564	+0.4	W300
V18	-1.71	-2.96	3.566	+0.5	L168
V19	-1.19	-3.14	3.592	+0.5	R10
V21	-2.75	-3.45	3.539	+0.2	A2
A19	-2.22	-3.47	3.551	+0.2	

<sup>a</sup> For V1-4 parameters are given at maximum and minimum K light.

red giants of 47 Tuc lie very close to the ridge line of the M71 giants even though the M71 GB redward of  $(V-K)_0 = 4.0$  is defined by only two stars (Frogel, Persson, and Cohen 1979). This agreement is consistent with their similarity in the optical (Lee 1977). (The M71 GB ridge line has been located in Figure 1 based on the data and parameters given in Frogel, Persson, and Cohen [1979]. No reference has been made to the present 47 Tuc data). An uncertainty of  $\pm 0.05$  in  $E(B-V)$  translates into an uncertainty of  $\pm 0.14$  in  $(V-K)_0$  and  $\pm 0.13$  in  $M_{K_0}$ . As shown by the arrow on Figure 1, such an uncertainty in the reddening has almost no effect on the relative giant branch locations of M71 and 47 Tuc.

Figure 1 also shows that all stars with  $(V-K)_0 > 4.0$  are variables. Of these 17 stars, 14 were specifically selected for observation because of their variability. However, we have observed all of the reddest stars from Lee, who has an unbiased sample. Also Lloyd Evans (1974) states that the reddest stars he has observed (whose V-I correspond to  $(V-K)_0 > 4.0$ ) are generally variable. Furthermore, of the two M71 stars which are redder than 4.0 in  $(V-K)_0$ , at least one of them, star 29, is a small-amplitude long period variable (Hogg, private communication quoted in Frogel, Persson, and Cohen 1979). We therefore believe that the abrupt transition between variables and nonvariables in Figure 1 is not due to the sample of stars selected.

How can we distinguish AGB stars from GB stars near the tip of the 47 Tuc giant branch? Calculations of the location of blue edge of the variability strip (e.g., Wood and Cahn 1977) show that it is vertical in  $T_{\text{eff}}$  with a minimum luminosity of  $M_{\text{bol}} = -4.0$ . The minimum luminosity deduced by Wood and Cahn has led most authors to believe that many types of luminous variables are AGB stars rather than GB stars. However, the existence of pulsations depend largely on the characteristics of the extensive convective envelopes of the red giants; differences in internal structure between AGB and GB stars, e.g., the existence of one or two energy generating shells in the deep interior, appear to be irrelevant to whether or not a star can pulsate (Stothers and Schwarzschild 1961; Iben, private communication). If there is a small error or metallicity dependence in the calculated minimum luminosity so that it is really  $M_{\text{bol}} = -3.0$  for metal poor systems, then it would naturally follow that all luminous stars cooler than some  $T_{\text{eff}}$ , observed to be  $(V-K)_0 = 4.0$  or  $T_{\text{eff}} = 3800$  K in 47 Tuc, M71, and probably  $\omega$  Cen (Persson *et al.* 1980) are variables. [In a similar vein, Lloyd Evans and Menzies 1973 suggested that at least in 47 Tuc, all stars with  $(V-I_K) > 1.8$  may be small amplitude variables.] This would also explain the change from nonvariables to variables in old disk stars occurring at similar  $T_{\text{eff}}$ , i.e., that corresponding to spectral type M3-M4 (Eggen 1975*a*). While, with the exceptions of V1-4 and R10 which we discuss later, we cannot be

certain that the variables in 47 Tuc are GB rather than AGB stars for the above reasons, we do not find that their variability compels us to ascribe them to the AGB. Rather, the variability is merely an expected result of the low  $T_{\text{eff}}$  for stars in these systems and populations.

A second point is that agreement between theoretical and observed luminosity functions for GB and AGB stars at lower luminosities where the two branches are clearly distinguished (Lee 1977) would lead us to believe that in an unbiased sample of bright red stars, the majority of stars lying within 1 mag (visual) or so of the apparent GB tip should be GB stars. Also, because of the simple relationship between core mass and stellar luminosity on the AGB, it is easy to show theoretically that the number of AGB stars per bolometric luminosity interval becomes equal to the number of GB stars only near the very tip of the giant branch (Wood 1974). At fainter luminosities, the number of GB stars per luminosity interval rapidly becomes larger than the number of AGB stars.

Henceforth, we explicitly assume that most of the 47 Tuc stars with luminosities less than that of the theoretical (e.g., Sweigart and Gross 1978; Rood 1972) GB tip, corresponding to  $M_{K_0} = -6.4$  in Figure 1, are in fact GB stars. The tip of the 47 Tuc GB then agrees to within  $\pm 0.1$  mag in  $M_{K_0}$  with that of M71. The mean magnitudes for V1-3 place them 0.5 mag brighter than this. At their brightest (Table 3) they are a full magnitude brighter than the tip, and at no time during their cycles have they been observed to be more than 0.1 mag fainter than the tip. For these reasons, we assume that they, and probably V4 as well, are AGB stars. These points will be discussed further later.

#### b) The $(\log L, \log T_{\text{eff}})$ -Diagram

The infrared colors and magnitudes can be used to calculate accurate effective temperatures and bolometric luminosities as described in the Appendix. Tables 6 and 7 list the resulting bolometric corrections, temperatures, luminosities, and surface gravities, based on a mass of  $0.8 M_{\odot}$ , for the 47 Tuc red giants in Tables 2 and 4.

The physical H-R diagram for the 47 Tuc variable and nonvariable giants is shown in Figure 2. The AGB and HB stars have been omitted for clarity. The fiducial lines for M92, M13, and M71 are taken from the data of CFP and Frogel, Persson, and Cohen (1979); they have been slightly revised as discussed in the Appendix and n. 4 and are given in Table 8. The fiducial giant branch for the old disk cluster M67 includes new unpublished data. Figure 2 emphasizes the conclusions drawn from Figure 1: At a given luminosity, the width of the 47 Tuc giant branch in  $T_{\text{eff}}$  is attributable solely to observational error and/or uncertainties in the transformation of  $(V-K)_0$  to  $T_{\text{eff}}$ . At a given  $M_{\text{bol}}$  the scatter in  $\log T_{\text{eff}}$  is no more than  $\pm 0.005$ , corresponding to  $\sim 50$  K. With the excep-

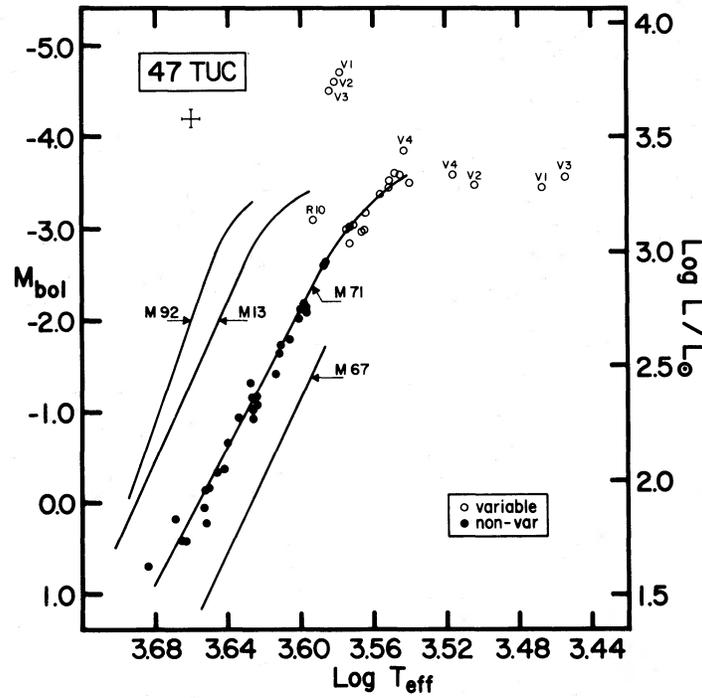


FIG. 2.—The H-R diagram in physical units for 47 Tuc giants. Luminosities and temperatures are computed as described in the Appendix. The single error bar includes uncertainties arising from the  $(V-K)$ ,  $T_{\text{eff}}$  calibration and from observational errors. Uncertainties in reddening and distance modulus are not included. The two values plotted for V1–4 correspond to the maximum and minimum observed  $K$  values. The fiducial giant branches for the other four clusters are from Table 8. The scatter about these fiducial lines for M92, M13, and M67 is comparable to or less than the scatter in the 47 Tuc stars. The scatter in the M71 stars is 2–3 times greater than that for 47 Tuc. AGB stars are not plotted.

TABLE 8  
FIDUCIAL ( $M_{\text{bol}}, \log T_{\text{eff}}$ )-LINES FOR GLOBULAR GIANT BRANCHES

$M_{\text{bol}}$	$\log T_{\text{eff}}$					$M_{\text{bol}}$	$\log T_{\text{eff}}$				
	M92	M13	M3	M71	47 Tuc		M92	M13	M3	M71	47 Tuc
-3.6	...	...	...	3.540	3.546	-0.8	3.681	3.672	3.668	3.635	3.633
-3.4	...	3.595	3.594	3.553	3.555	-0.6	3.684	3.677	3.673	3.640	3.638
-3.2	3.631	3.611	3.608	3.565	3.562	-0.4	3.688	3.681	...	3.646	3.643
-3.0	3.640	3.620	3.618	3.574	3.570	-0.2	3.692	3.686	...	3.651	3.649
-2.8	3.645	3.626	3.623	3.581	3.577	+0.0	3.695	3.690	...	3.656	3.654
-2.6	3.646	3.631	3.627	3.588	3.585	+0.2	...	3.695	...	3.662	3.659
-2.4	3.652	3.635	3.632	3.593	3.591	+0.4	...	3.700	...	3.667	3.664
-2.2	3.656	3.640	3.637	3.598	3.596	+0.6	...	...	...	3.673	3.670
-2.0	3.659	3.645	3.641	3.603	3.601	+0.8	...	...	...	3.677	3.675
-1.8	3.663	3.649	3.646	3.608	3.607	+1.0	...	...	...	3.682	...
-1.6	3.667	3.654	3.650	3.614	3.612						
-1.4	3.670	3.658	3.655	3.619	3.617						
-1.2	3.674	3.663	3.659	3.625	3.623						
-1.0	3.677	3.668	3.664	3.630	3.628						

The bolometric magnitudes are computed with  $BC_V(\text{sun}) = -0.08$ .

tion of the large amplitude variables V1-4 and R10, the mean locations of the other 47 Tuc variables define a smooth and tight extension of the giant branch defined by the nonvariables. The maximum luminosity of the 47 Tuc giant branch is  $M_{\text{bol}} = -3.6$ —slightly more luminous than the tips of the M3, M13, and M92 giant branches. The relatively low luminosity of the tip of M67's giant branch is almost certainly due to the sparseness of this cluster rather than to any physical mechanism.

We call attention to the variable R10. Its high luminosity and location blueward of the 47 Tuc giant branch in Figure 2 points to its being a member of a group of halo medium amplitude red variables discussed by Eggen (1972, 1977) which are AGB stars that are describing loops in the H-R diagram or else are evolving to higher temperature and into halo long-period Cepheids (Gingold 1974, 1976). Similar suggestions have been made by Lloyd Evans (1974). An effort should be made to determine the characteristics of its variability.

### c) Comparison with Theory

In Figure 3 we have replotted the fiducial giant branches from Figure 2 and superposed a sequence of theoretical giant branch tracks from Sweigart and Gross (1978). This particular sequence of models has  $M = 0.7 M_{\odot}$  and helium abundance  $Y = 0.30$ . The location of the theoretical tracks along the temperature axis is quite sensitive to the ratio,  $\alpha$ , of the mixing length to scale height in the convective stellar envelope. A doubling of  $\alpha$  for values in the range 0.5–2.0 changes  $\log T_{\text{eff}}$  of a given model by about +0.07 (Sweigart 1978). Thus to compare the theoretical and empirical tracks, we have assumed that the spectroscopically determined heavy metal abundances of M92 and M13 are reasonably certain. Cohen (1978, 1979) has determined abundances of  $[\text{Fe}/\text{H}] = -2.35$  and  $-1.6$  for M92 and M13, respectively. The entire  $M = 0.7 M_{\odot}$  model sequence was shifted by  $\Delta \log T_{\text{eff}} = +0.037$  in order to bring the  $[\text{Fe}/\text{H}] = -2.3$  and  $-1.7$  tracks into reasonable agreement with the M92 and M13 giant branches.

There are two important points of agreement between the models and the observations. First, the observed and predicted slopes of the giant branches are similar. The largest divergence between the two occurs only within 1 mag of the tip luminosity. This divergence is particularly noticeable for the metal-poor clusters M92, M3, and M13 and has been mentioned previously (CFP).<sup>5</sup> Second, we note that the models and the observations are in

<sup>5</sup>The present adopted values for the distance moduli to M3 and M13 (see n. 4) bring the giant branches of these clusters into very close agreement with each other, in contrast to Figure 5 of CFP and Figure 4 of Frogel, Persson, and Cohen (1979).

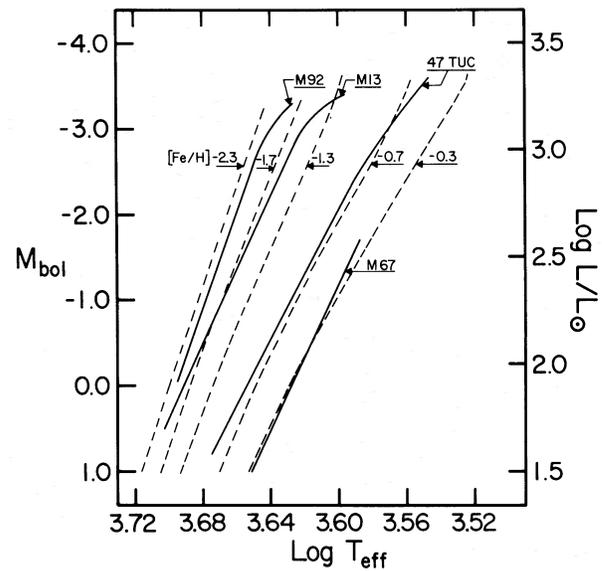


FIG. 3.—The fiducial giant branches for M92, M13, and M67 are repeated from Fig. 2. The fiducial line for 47 Tuc is drawn on the basis of the observations plotted in that figure. Five giant branch tracks from the models of Sweigart and Gross (1978) are indicated by dashed lines. They are all for stars with  $M = 0.7 M_{\odot}$  and  $Y = 0.30$ , and have been shifted by +0.037 dex on the temperature axis to make them correspond with metal poor clusters as described in the text. The most recent abundance determinations for the clusters, which are based on echelle spectrograms, are  $[\text{Fe}/\text{H}] = -0.3, -1.2, -1.6,$  and  $-2.35$  for M67, 47 Tuc, M13, and M92, respectively (Cohen 1978, 1979, 1980; Pilachowski *et al.* 1980).

agreement both in regard to the tip luminosity and the dependence, albeit slight, of this luminosity on metallicity. Some of the stars within 1 mag of the tip could be AGB stars, but it seems unlikely that they could *all* be such, based on theoretical estimates of their luminosity function relative to that of stars on the first ascent of the GB (Gingold 1974; also see discussion in § IIIa above).

The major point of disagreement between the models and the data is that, although the  $[\text{Fe}/\text{H}] = -0.7$  track lies close to the 47 Tuc giant branch, this is inconsistent with Pilachowski, Canterna, and Wallerstein's (1980) abundance of  $-1.2$  dex for the cluster. One interpretation of this misfit is that the shift applied to the metal-poor tracks is too large for the metal-rich tracks; i.e., the ratio of mixing length to scale height is a function of  $[\text{Fe}/\text{H}]$  (cf. Frogel, Persson, and Cohen 1980, Appendix B). On the other hand, Dickens, Bell, and Gustafsson (1979) derive a heavy metal abundance for 47 Tuc of  $[\text{Fe}/\text{H}] = -0.8$ , and they caution that a value as high as  $-0.5$  cannot be ruled out by their data. Thus if their value is adopted rather than that of Pilachowski *et al.*, the discrepancy between the tracks and the present 47 Tuc data vanishes. Cohen (1980), however, has derived a nearly identical value of  $[\text{Fe}/\text{H}] = -1.27$  for M71. As

discussed above, the giant branches of the two clusters define virtually the same locus in a  $(\log L, \log T_{\text{eff}})$ -plot. Theoretical calculations (Rood 1978) show that the relative locations of clusters' GBs in such a plot is sensitive only to *heavy* metal abundance and not to CNO abundances, all other things being equal; thus the H-R diagrams of the two clusters support the result of nearly identical metallicities. Any large error in the 47 Tuc abundance derived by Pilachowski, Canterna, and Wallerstein (1980) would have to be present in Cohen's independently derived value for M71 as well. While a systematic error in the model atmosphere codes or the effective temperatures cannot be absolutely ruled out, there is no evidence for this at present (see discussion by Cohen 1980). The abundances are derived for stars warmer than 4000 K. The temperature scale for these stars is not uncertain, as is the case for the cooler stars (see the discussion in the Appendix).

Further evidence that  $\alpha$  is a function of  $[\text{Fe}/\text{H}]$  comes from the location of the giant branch of the open cluster M67. Cohen (1980) has determined a spectroscopic abundance of  $[\text{Fe}/\text{H}] = -0.4$  for M67, in agreement with the earlier work of Griffin (1975) who determined an abundance for one M67 giant of twice that of Arcturus, or half solar. The  $[\text{Fe}/\text{H}] = -0.3$  track superposed on the M67 giant branch in Figure 3, however, is that for a model with  $M = 0.7 M_{\odot}$ , while most current estimates for the age of M67 require the cluster to have a turnoff mass of at least  $1 M_{\odot}$ . Such a track was obtained by interpolation between the  $0.9$  and  $1.1 M_{\odot}$  models of Sweigart and Gross (1978) and then shifted in  $T_{\text{eff}}$  in a manner identical to that for the other tracks. Like the case of 47 Tuc, this track lies hotter than the empirical GB. A shift in  $\log T_{\text{eff}}$  of  $0.02$ , half that applied previously for the metal-poor tracks, is required to fit the  $1.0 M_{\odot}$  track to the M67 GB. We thus conclude that if the echelle spectroscopic abundances are taken at face value, the theoretical giant branch loci of Sweigart and Gross (1978) can be brought into agreement with the observed giant branches of metal-poor and metal-rich clusters only if the loci are shifted in  $\log T_{\text{eff}}$  by amounts which depend on chemical composition, or mass, or both. Our present knowledge of the various uncertainties involved in computing these tracks suggests that the physical parameter which is varying is the ratio  $\alpha$  of mixing length to scale height. (Any reasonable cluster to cluster variation in the helium abundance would be too small to account for more than a minor fraction of the disagreement between observations and theory.) The sense of the variation is that  $\alpha$  decreases with increasing heavy metal abundance.<sup>6</sup>

<sup>6</sup>Comparisons can also be made between the empirical giant branches of Fig. 3 and the sequence of models computed by Rood (1972). His models RG1, RG3, and RG5 are relevant here. If they are adjusted to  $0.7 M_{\odot}$  instead of  $0.8 M_{\odot}$ , we find that they too are cooler than the cluster giant branches, but the shift required to match M92 and M13 is only about  $0.02$  in  $\log T_{\text{eff}}$ , or half of the

#### d) Limits on Mass Loss on the Upper Giant Branch

By comparing the observed giant branches with the theoretical tracks, we can place limits on the amount of mass lost by stars as they evolve up the giant branch. One of the current beliefs about the evolution of globular cluster stars is that somewhere between the subgiant branch and the horizontal branch such a star must lose about two tenths of a solar mass. Such mass loss is required both to account for the distribution of stars along the HB (Rood 1973) and the resulting integrated colors in the  $(U-B, B-V)$ -plane of globular clusters (e.g., Ciardullo and Demarque 1978). The detection of blueshifted  $\text{H}\alpha$  emission lines in the spectra of several GB tip stars in the clusters M13 and M92 by Cohen (1976) has been interpreted by her as being due to a mass loss rate sufficiently large to meet the above requirements. As discussed in CFP, we can investigate the possibility of random star-to-star *variations* in mass loss along the giant branch which would cause a finite width to the observed giant branch at a given  $M_{\text{bol}}$ . The small scatter in  $\log T_{\text{eff}}$  in Figure 2, all of which can be accounted for by observational and transformation uncertainties, translates into a dispersion of  $\pm 0.1 M_{\odot}$ , as found from the Sweigart and Gross (1978) models. Thus, the *difference* in mass loss from star-to-star in 47 Tuc cannot be greater than  $0.2 M_{\odot}$ . If there is even a small dispersion in heavy metal abundance, this estimate will decrease rapidly. We can also examine possible systematic mass loss as stars evolve up the giant branch, *provided* that  $\alpha$  is not a function of luminosity. Ignoring for the moment the upper half-magnitude of the giant branches in Figure 3, we conclude that the differences between the observed and calculated GB slopes can be accounted for by a systematic loss of mass of not more than  $0.2 M_{\odot}$  as the stars increase in luminosity from  $M_{\text{bol}} \approx -1$  to  $-3$ . These upper limits are just consistent with mass loss requirements derived from considerations of horizontal branch morphology (e.g., Iben and Rood 1970; Rood 1973; see also review by Renzini 1977).

Only on the upper half magnitude of all of the giant branches do differences between observations and theory become pronounced. We speculate that the change in slope of the observed GBs near their tips could result from the onset of mass loss. If this is true, it is interesting that the metal-poor clusters so far observed show this bending over of the GB more strongly than either 47 Tuc or M71, in apparent conflict with the theoretical expectation that the mass loss rate should increase with metal abundance (Reimers 1975) because the higher- $Z$  stars at the tip have lower surface gravities. If  $\alpha$  does turn out to be a function of one or more parameters of

value required for the Sweigart and Gross (1978) tracks. Rood's tracks have the same *relative* spacings as the Sweigart and Gross ones, so the problem of matching the tracks to *all* of the giant branches simultaneously remains. The tip luminosities of the two sets of giant branch models are in agreement.

the stellar envelope, then  $\alpha$  and the mass loss rate as calculated from arguments such as those just presented will be hopelessly entangled.

#### IV. CO ABSORPTION IN THE 47 TUCANAE GIANTS

The CO index data for 47 Tuc from Tables 2 and 3 are plotted in Figure 4. Only the mean values for V1–4 are shown as the variation in CO index through the cycles of these stars is large (Table 3 and Fig. 9). The dashed line encompasses all of the stars observed in M71 except for B and 29 which are plotted individually. Like the two M71 stars, the small-amplitude variables in 47 Tuc have CO strengths which are up to 0.1 mag weaker than field giants at the same color. There is apparently a large scatter in the amount of CO absorption at a given  $(V-K)_0$  in the 47 Tuc stars. This dispersion sets 47 Tuc apart from the other clusters we have studied so far, except for  $\omega$  Cen (Persson *et al.* 1980).

Let us examine the dispersion in CO index for the non-variable stars by looking at the well documented star-to-star variations in CN absorption (Norris and Freeman 1979, and references therein). Norris and Freeman's measurements of 152 stars on the 47 Tuc giant branch show that the distribution of CN absorption strengths, as given by the DDO CN index, is bimodal: over the brightest 2 mag the giants fall into either a CN strong or a CN weak group. Thirty-one of the stars for which CO measurements are available (Table 2) are in Norris and Freeman's sample. We have assigned stars to either the CN strong or CN weak group by locating a dividing line between the two groups 0.12 mag above the solid line on their Figure 1. Figure 5 shows that there is a strong correlation between the CO and CN distributions: almost without exception stars which belong to the CN weak group have CO indices larger than the stars in the CN strong group at the same

$V-K$ . Although only a few AGB stars are in our sample, their distribution in Figure 5 does not differ from that of the GB stars.<sup>7</sup>

One explanation for the observed CN enhancement in the 47 Tuc giants is that material which has been processed through the CN cycle has been mixed to the surface, resulting in an enhancement of up to a factor of 10 in the abundance of N compared with the solar N/Fe ratio. This nitrogen enhancement would be accompanied by a mild (not more than a factor of 3) depletion in the surface carbon abundance. These values result from spectral synthesis work by Dickens, Bell, and Gustafsson (1979) and Norris and Cottrell (1979). Qualitatively, then, one could explain the observed CO–CN anticorrelation in terms of this N enhancement and C depletion, and the bimodality of the CN distribution would then be evidence for the presence of a substantial fraction of unmixed stars.

The explanation for the observed CO–CN anticorrelation, however, is probably more mundane and involves an instrumental effect. The CO index is defined by a filter centered on the first overtone CO band at  $2.36 \mu\text{m}$  and a single continuum filter centered at  $2.20 \mu\text{m}$ . A perusal of the absorption due to CN in the  $2 \mu\text{m}$  region (see, for example, Johnson, Marenin, and Price 1970) shows that the continuum filter encompasses the 0–2 sequence of the red CN system, and that there is far less CN absorption at  $2.36 \mu\text{m}$ . Therefore, CN blanketing may be the sole cause of the spread in CO. To attempt to quantify this claim, we have studied high resolution FTS spectra of the cool giants  $\beta$  And and  $\alpha$  Boo obtained on the 4 m telescope at KPNO by Hall and Carbon (private communication). These spectra do show

<sup>7</sup>Note that Norris *et al.* (1980), who find the CN distribution of stars in NGC 6752 to be bimodal as well, did *not* find any AGB stars to belong to the CN strong group—all AGB stars in NGC 6752 belonged to the CN weak group.

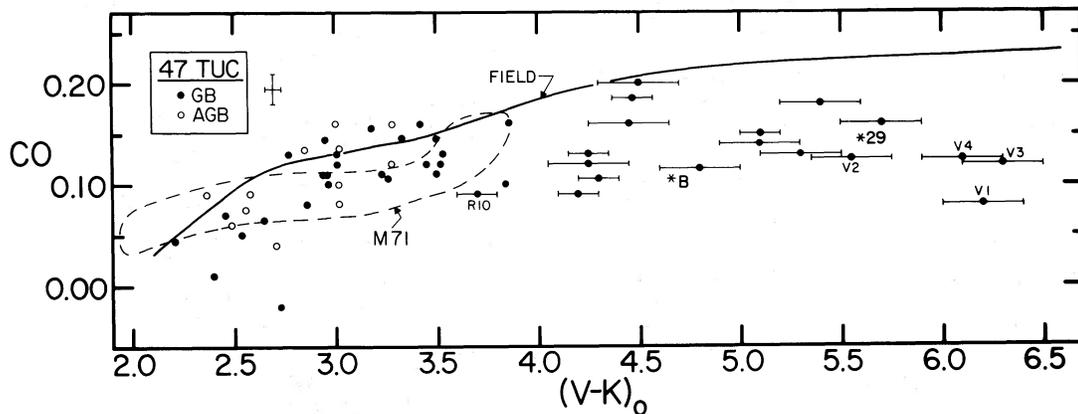


FIG. 4.—The CO versus  $(V-K)_0$  plot for stars in 47 Tuc. The notation and significance of error bars for variables are as in Fig. 1. No attempt is made to display the range in CO for the variables. The mean relationship for field giants and an envelope enclosing all but two of the observed stars in M71 are depicted (Frogel *et al.* 1978, 1979). The two remaining M71 stars, B and 29, are located by asterisks.

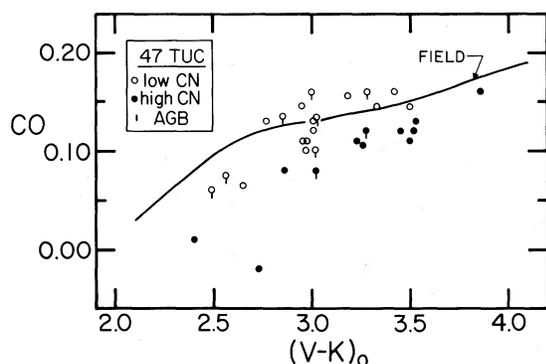


FIG. 5.—CO is shown as a function of  $(V-K)_0$  for those stars in common with Norris and Freeman (1979). We have qualitatively assigned each star to a high or low CN group on the basis of its location in Fig. 1 of Norris and Freeman. AGB stars are distinguished by tick marks.

many weak lines of CN throughout the region of our continuum filter (2.15–2.25  $\mu\text{m}$ ). Using a CN line list kindly provided by D. Carbon, we estimate that between 1.1 and 2.7% of the flux in the 2.2  $\mu\text{m}$  continuum filter is removed by CN lines in  $\alpha$  Boo and  $\beta$  And, respectively. Essentially no flux is removed in the 2.36  $\mu\text{m}$  filter due to the much lower CN opacity there. The DDO CN indices of  $\alpha$  Boo and  $\beta$  And (McClure 1976) would place them in the CN weak group of Norris and Freeman, and their temperatures of 4250 K (Mäckle *et al.* 1975) and 3900 K (from spectral type and the Ridgway *et al.* 1980 temperature scale) correspond to those of the more luminous 47 Tuc giants. In a CN strong star, however, there will be at least 4 times as much CN as calculated from the spectral synthesis results and from Deming's (1978) calibration applied to the observed DDO CN indices. Therefore, in a CN strong star the CN absorption in the continuum filter will be substantially larger than in a CN weak star—about 5% of the total flux there being removed by CN lines—while the flux in the CO filter will remain unaffected. Thus the CN strong stars will, by this blanketing effect due to CN, have artificially small CO indices, giving rise to the effect seen in Figure 5.

CFP and Pilachowski (1978) have pointed out a difference in the CO absorption between giants in M3 and M13, clusters which have similar values for  $[\text{Fe}/\text{H}]$ . Can the above CN blanketing effect be responsible for the M3/M13 CO difference? It seems highly unlikely because these clusters are so metal poor; they display essentially no CN absorption in the DDO system. On the other hand, it seems likely that no useful information on the relative abundances of metal rich clusters can be derived from low resolution CO data alone.

#### V. $\text{H}_2\text{O}$ ABSORPTION AND THE $[(J-H)_0, (H-K)_0]$ -DIAGRAM

Figure 6 shows the  $\text{H}_2\text{O}$  absorption indices for the stars in Table 2 and 4; the variables are plotted at their

mean colors and indices. Although the nonvariable 47 Tuc giants follow the mean relation of  $\text{H}_2\text{O}$  versus  $(V-K)_0$  for field giants quite well, it is noteworthy that the variables have  $\text{H}_2\text{O}$  indices considerably greater than the field giants at the same color. This is consistent with the findings of Johnson *et al.* (1968) and Frogel (1971) for variable and nonvariable Galactic stars. Only M supergiants have  $\text{H}_2\text{O}$  indices at least 0.15 mag larger than luminosity class III stars of similar spectral type (Aaronsen, Frogel, and Persson 1978), and we suggest that in late M giants the  $\text{H}_2\text{O}$  index is also quite sensitive to luminosity. Molecular equilibrium calculations in this temperature range by Tsuji (1964) and others show that the fractional abundance of  $\text{H}_2\text{O}$  increases sharply as  $T_{\text{eff}}$  decreases. Thus the stellar radius is important, as very extended atmospheres have cooler outer layers than those of smaller stars of the same  $T_{\text{eff}}$ . This effect produces a strong luminosity dependence in the observed  $\text{H}_2\text{O}$  index, and so the positions of the luminous, large-amplitude 47 Tuc variables far above the mean field giant line in Figure 6 is not surprising. Similar, but much less pronounced, behavior is seen for other globular cluster variables in M71 (Frogel, Persson, and Cohen 1979) and  $\omega$  Cen (Persson *et al.* 1980).

Perhaps the most interesting result to come out of the  $\text{H}_2\text{O}$  absorption data is simply that stellar  $\text{H}_2\text{O}$  indices are able to reach 0.5 or 0.6 mag in a system as metal poor as 47 Tuc. The large amount of  $\text{H}_2\text{O}$  absorption directly affects the broad band JHK colors of the variables. Figure 7 shows a  $[(J-H)_0, (H-K)_0]$ -plot for 47 Tuc, where for V1–3 the individual data points of Table 3 are shown connected by straight lines (only the mean value for V4 is plotted). Except for the extreme variables, the 47 Tuc stars are indistinguishable from the M71 stars which, in turn, intermix with the stars of the metal-poor clusters M3, M13, and M92 (CFP; Frogel, Persson, and Cohen 1979). Thus we confirm the preliminary result of Glass and Feast (1973) that the 47 Tuc stars are displaced from the mean relation for Population I field giants in the sense that  $(H-K)_0$  is consistently bluer than expected at a given  $(J-H)_0$ . The extreme variables V1–3, however, have  $(H-K)_0$  colors up to 0.4 mag redder than the field line. This was also noted by Glass and Feast. As these stars go through their cycles, they move roughly along a line perpendicular to the mean field line in Figure 7. At mean phase, their displacement from the mean field line is well correlated with their  $\text{H}_2\text{O}$  absorption as measured by their displacements upward from the mean field line in Figure 6. Figure 8 shows the correlation. We can conclude that  $\text{H}_2\text{O}$  molecule formation in the extended atmospheres of the stars strongly modifies their energy distributions. Although the wing of the 1.9  $\mu\text{m}$   $\text{H}_2\text{O}$  band overlaps both the  $H$  and  $K$  filter bands, the sense of the displacements in Figure 7 shows that most of the effect is in the  $H$  filter:  $J-H$  gets bluer and  $H-K$  redder as the  $\text{H}_2\text{O}$  opacity increases. Presumably this

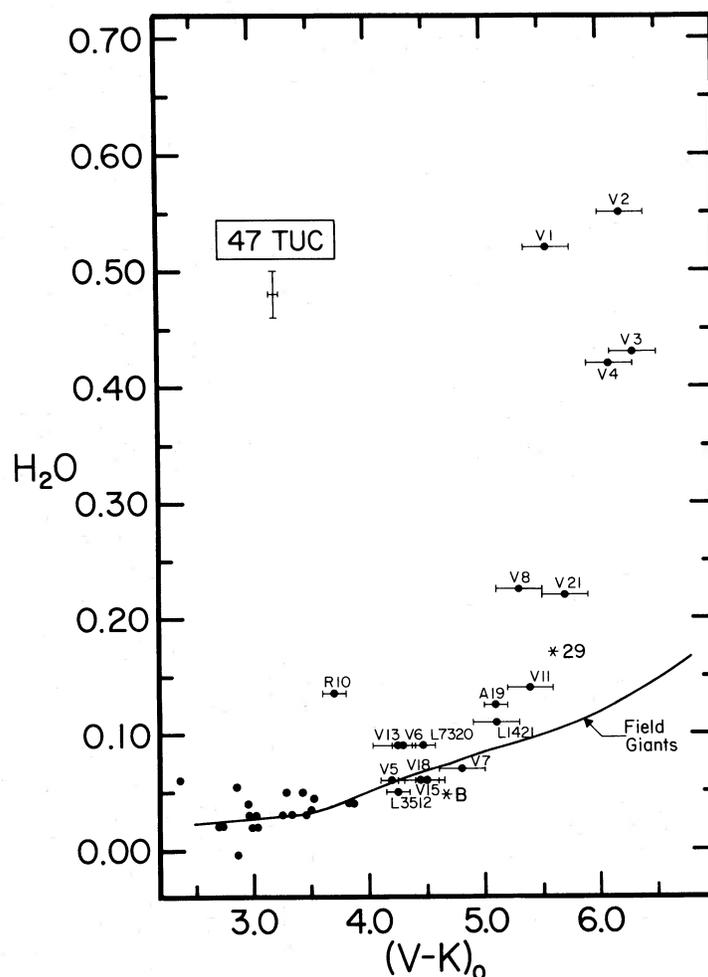


FIG. 6.—The  $H_2O$  index versus  $(V-K)_0$ . The notation and significance of the error bars used for variables are as in Fig. 1. No attempt is made to display the range in  $H_2O$  for the variable stars. Rather, the mean values from Tables 3 and 4 are plotted. The two reddest M71 stars from Frogel *et al.* (1979), B and 29, are plotted as asterisks at  $H_2O=0.05$  and  $0.17$  respectively. The mean relation for field giants from Aaronson *et al.* (1978) is shown.

arises from additional absorption in the  $H$  filter from the  $1.4 \mu\text{m}$   $H_2O$  band.

## VI. THE LARGE AMPLITUDE VARIABLES

### *a) Their Evolutionary State*

The evolution of a low-mass star up the first red giant branch terminates at a theoretically well-defined luminosity, that of the helium flash, which depends only weakly on initial mass (at least for  $1.4 M_\odot \geq M \geq 0.7 M_\odot$ ) or chemical composition (Rood 1972; Sweigart and Gross 1978). Red giants more luminous than this limit are thought to be on their second ascent of the giant branch. Calculations of AGB evolution have been presented by Rose and Smith (1970), Gingold (1974), and Wood (1974); see also the review by Renzini (1977).

If the value of  $(m-M)_0$  we use for 47 Tuc is correct, then the luminosities of V1–3, and probably V4 as well, indicate that these stars are AGB stars (Table 7).<sup>8</sup> These four variables are among the few long-period variables (LPVs) known with reliable individual distance determinations. Thus it is useful to compare their physical properties with those predicted by model calculations. The bolometric luminosities and amplitudes of V1–4 are qualitatively consistent with Wood's (1974) models having periods of  $\sim 200$  days. Differences in temperature between stars and models are not considered significant

<sup>8</sup>Our  $M_{\text{bol}}$  at maximum for V3 agrees to within 0.1 mag with Eggen's (1975a) value when allowance is made for the different distance modulus he used. His value for  $M_{\text{bol}}$  at minimum light is too faint because of nonlinearity in the technique Eggen uses to get bolometric corrections (Eggen 1975a; Dyck, Lockwood, and Capps 1974).

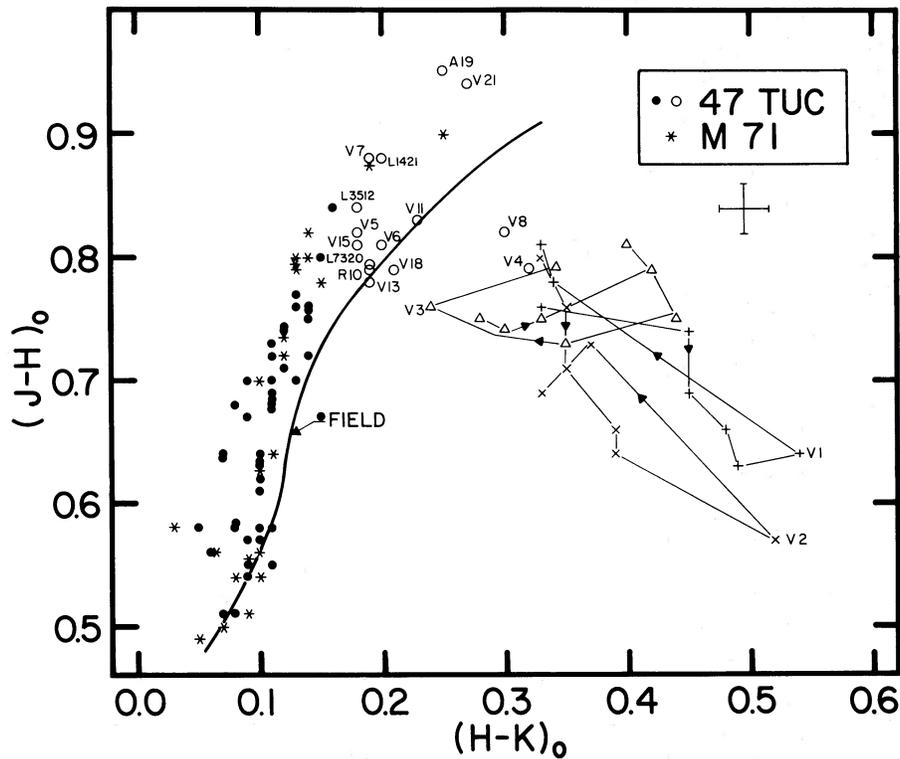


FIG. 7.— The  $[(J-H)_0, (H-K)_0]$ -plot for 47 Tuc stars. Individual observations corrected for reddening, are plotted for variables V1, 2, 3. The mean position of V4 from Table 3D is shown. All other variables from Figs. 1 and 2 are shown as open circles; nonvariables, as filled circles. The M71 stars are plotted as asterisks and the mean relation for field giants (Frogel *et al.* 1978) is shown.

in view of the discussion in § III above. Wood interprets the absence of variables more luminous than these in 47

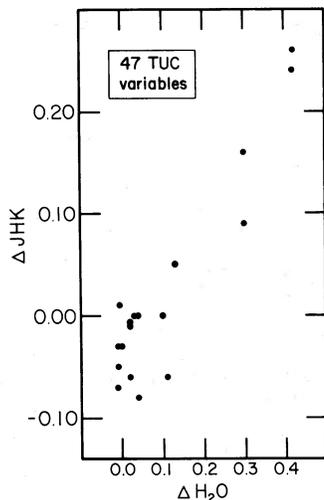


FIG. 8.— The quantity  $\Delta JHK$  measures the displacement of the 47 Tuc variables from the mean field line along a line of slope  $45^\circ$  with respect to the axes of Fig. 7.  $\Delta H_2O$  measures the displacement upwards from the mean field line in Fig. 6. In both cases the units are magnitudes. A negative  $\Delta JHK$  means that that star lies above and to the left of the mean field line.

Tuc and their complete absence in most other globular clusters as due to differing rates of mass loss. Sweigart, Mengel, and Demarque (1974) have shown that a horizontal branch star will never evolve into an AGB star if its mass is less than  $0.51 M_\odot$ . Using the core mass-luminosity relationship of Paczyński (1971), we calculate from the mean luminosities of V1–V4 core masses between  $0.56$  and  $0.58 M_\odot$ . Since the envelope mass is negligible, the core mass is essentially the total mass (Paczynski 1971). This suggests that the surface gravities tabulated for these stars in Table 7 (using a mass of  $0.8 M_\odot$ ) are  $0.15$  dex too high.

Recently, Mould and Aaronson (1980) have estimated ages for clusters in the Magellanic Clouds based on the maximum luminosities of the AGB stars in the clusters. They combined an age/turnoff mass relation with Paczyński's (1971) core mass–luminosity relation and the Reimers (1975) mass loss rate to derive an age/AGB luminosity relation. The amplitude changes of V1–V4 in 47 Tuc suggest that if this technique is to resolve age differences in a meaningful way, the brightest AGB stars must be observed for variability. The range in  $M_{bol}$  for V1–3 is  $\sim 1$  mag, corresponding to an age difference of  $7 \times 10^9$  yr according to Mould and Aaronson's Figure 7.

A further problem with their technique is a lack of knowledge of the dependence on metal abundance of

the mass loss process both on the GB and on the AGB. For clusters younger than  $6 \times 10^9$  yr this problem is not too serious, but the age dating technique is highly sensitive to  $\eta_R$ , the parametric efficiency of the mass loss rate, for clusters older than this. This point is easily appreciated by looking at Mould and Aaronson's Figure 7.

#### b) Their Contribution to the Integrated Cluster Light

In 47 Tuc the fractional contribution at  $K$  due to the four large amplitude variables is small. The total  $V$  magnitude of the cluster is 4.1 (Kron 1966), and we estimate  $V-K$  to be 2.9 on the basis of its similarity to M71 whose integrated  $V-K$  color has been measured (Aaronson *et al.* 1978). Thus the contribution of the four variables at mean light is 3.6% of the total  $K$  luminosity. (They contribute 0.2% at  $V$ .) However, the implication for the study of composite stellar systems is that in a younger cluster or galaxy which has had an episode of relatively recent star formation, the fractional contribution of the AGB to the integrated 2.2  $\mu\text{m}$  light is expected to be larger, and the integrated light  $\text{H}_2\text{O}$  index could conceivably be affected by upper AGB variables. Suppose there were 3 times as many as in 47 Tuc for a 10% AGB contribution at  $K$ . The effects on the integrated colors would be of the order of 0.035 mag in  $\text{H}_2\text{O}$  index, 0.015 mag in  $H-K$ , and 0.1 mag in  $V-K$ . Thus, although detailed modeling of the AGB contribution to the integrated light of stellar systems is not presently possible, it is qualitatively clear that the effects could be noticeable in  $V-K$  and  $\text{H}_2\text{O}$ , and that this would occur over a large range in metal abundance.

#### c) Their Temporal Variations

The temporal variations of the large amplitude variables through parts of three periods are displayed in Figures 9a-9d. V1 shows the best cycle to cycle consistency, while V2 and V3 show a considerable amount of variation from one cycle to the next, particularly in the CO and  $\text{H}_2\text{O}$  indices. The phase and amplitude relationships between the  $K$ , CO, and  $\text{H}_2\text{O}$  light curves are similar to those of 18 galactic variables studied by Frogel (1971). The correlation of the  $\text{H}_2\text{O}$  opacity changes through the cycle with those of  $H-K$  are seen clearly.<sup>9</sup> The accompanying variations in CO absorption

<sup>9</sup>Since strongest  $\text{H}_2\text{O}$  absorption occurs near minimum light and weakest near maximum, the effect of  $\text{H}_2\text{O}$  absorption on the  $K$  light curve will be to increase its apparent amplitude. A lower limit to this effect can be obtained by examining the difference in flux between the narrow band filter used to measure the CO and  $\text{H}_2\text{O}$  indices and the broad-band  $K$  filter as a function of  $\text{H}_2\text{O}$  index. We find from all of the data that went into making up Tables 3 and 4 that  $\Delta K = 0.20 \text{ H}_2\text{O}$  for  $0.0 \leq \text{H}_2\text{O} \leq 0.83$ , where  $\Delta K$  is in the sense that the  $K$  magnitude is too faint because of  $\text{H}_2\text{O}$  absorption. The scatter about this relationship is generally consistent with the errors of measurement.

and  $T_{\text{eff}}$  and the slight phase lags, probably due to propagation of a shock wave through various levels in these extended atmospheres (Frogel 1971), make the details of the light and color variations extremely difficult to model correctly. Further discussion of the physical origin of these variations is beyond the scope of this paper.

## VII. CONCLUSIONS AND SUMMARY

1. The giant branch of 47 Tuc lies very close to that of M71 in the infrared color-magnitude [ $K_0, (V-K)_0$ ]-diagram (Fig. 1) and equivalently in the ( $\log L, \log T_{\text{eff}}$ )-diagram (Fig. 2). There is very little scatter about the ridge line of the giant stars. Recent theoretical work, (e.g., Rood 1978) indicates that the location of the giant branch of a globular of given age and helium abundance is dependent on the abundance of only the heavy metals and is not at all sensitive to variations in the abundance of the CNO element group. On this basis we conclude that whatever the heavy metal abundances of M71 and 47 Tuc are, they must be essentially identical. Cohen (1980) and Pilachowski, Canterna, and Wallerstein (1980) in fact derived values of  $[\text{Fe}/\text{H}] = -1.27$  and  $-1.2$  for M71 and 47 Tuc, respectively.

2. A comparison of the 47 Tuc giant branch in the ( $\log L, \log T_{\text{eff}}$ )-plane with the Sweigart and Gross (1978) theoretical tracks, shifted in  $T_{\text{eff}}$  to fit the location of M92 and M13, indicates that the abundance of 47 Tuc may be close to  $[\text{Fe}/\text{H}] = -0.7$ , similar to the value derived by Dickens, Bell, and Gustafsson (1979). However, there is evidence that the relative spacings of these tracks may be a function of more than just the heavy metal abundance. The most likely additional parameter is the ratio of the mixing length to scale height,  $\alpha$ , in the stellar envelope. We conclude, along with Simoda and Iben (1970), that  $\alpha$  for the models may be a function of some combination of the metallicity (i.e., the stellar opacity), the stellar mass, and the effective temperature. The sense of the disagreement between observation and theory requires that  $\alpha$  decreases in the cooler, more metal-rich stars. We wish to emphasize, though, that this result is really based on three clusters, M67, M71, and 47 Tuc, and the low metal abundances derived for the latter two from high dispersion spectroscopic studies are at present controversial.

3. Aside from the problems associated with correctly locating the evolutionary tracks along the  $T_{\text{eff}}$  axis, there is reasonable agreement between the observed and predicted slopes, as well as between the observed and predicted level and trend of GB tip luminosity with cluster metallicity. The only stars which lie significantly above the theoretical tip luminosities in any of the clusters we have observed are V1-4, the four large amplitude variables in 47 Tuc.

4. The apparent lack of scatter of stars along the 47 Tuc giant branch and the slope of the branch itself have

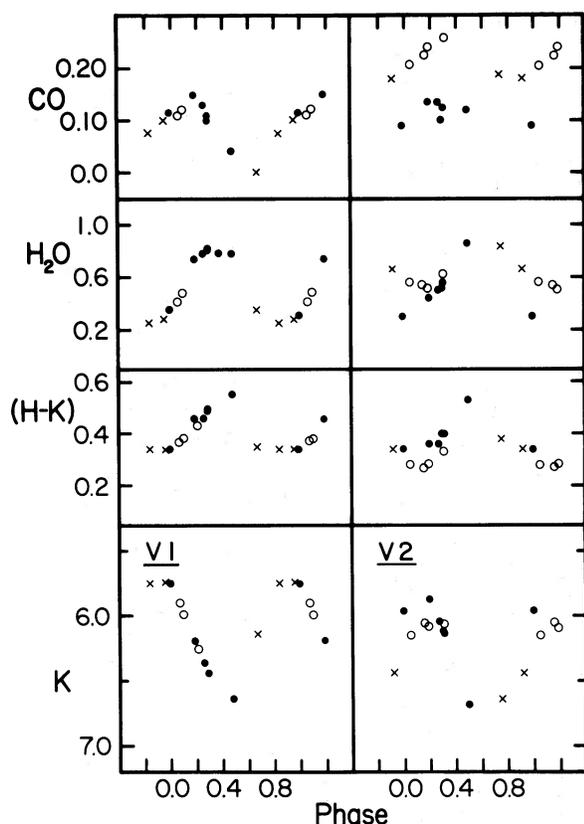


FIG. 9a 9b

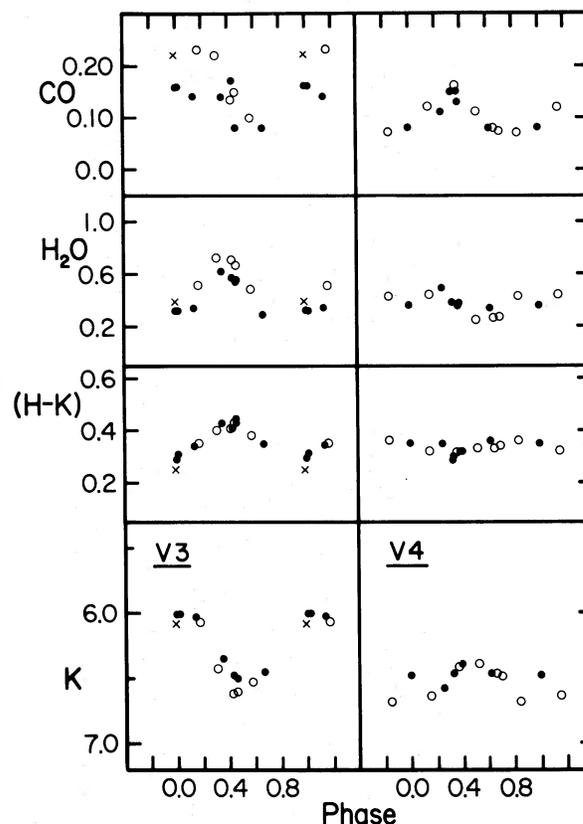


FIG. 9c 9d

FIG. 9(a-d).—Individual observed values from Table 3 for variables V1–4 are plotted as function of phase. Phase 0.0 is set arbitrarily at the date of first observation of each of the variables. The periods used are those from Hogg (1973). They are 212, 203, 192, and 165 days for V1–4, respectively. Observations during successive periods are differentiated by the symbols. Values within 0.2 of 0.0 and 1.0 are repeated at phase plus or minus 1.0, respectively.

been used to set limits on differential and systematic mass loss in the 47 Tuc giants of less than  $0.2 M_{\odot}$  for the region fainter than 0.5 mag below the GB tip; the limit on systematic mass loss depends critically on the validity of the slope of the models. If the theoretical slopes are reasonably correct, then the abrupt change in slope in the upper half-magnitude of the observed giant branches of all of the clusters observed to date, but particularly in the metal-poor ones, could be consistent with significant amounts of mass loss during this phase of a star's evolutionary history. The latter part of this conclusion depends critically on  $\alpha$  being independent of luminosity.

5. There is a significant spread in the amount of CO absorption at a given color among the 47 Tuc giants. There is a nearly one-to-one correspondence between our “CO strong” and Norris and Freeman’s “CN weak” stars and vice versa. The simplest interpretation of this result is that in a cluster as metal rich as 47 Tuc, the technique we use to measure the CO strength is affected by CN line blanketing.

6. Since the mean luminosities of variables V1–3 are 0.5 mag brighter than theoretical predictions of the maximum luminosity achievable in the first ascent of the giant branch, it is reasonable to conclude that these stars are on the AGB. V4 is probably in this category as well. Theoretical predictions of variable star characteristics have been shown to be qualitatively consistent with the observed and derived parameters of V1–4. A problem worth pursuing is why such stars are not found either as variables or as luminous giants in more metal-deficient clusters.

7. The  $H_2O$  indices of variables V1–3 attain large values and have mean values  $\sim 0.4$  mag greater than field giants at the same effective temperature. These large  $H_2O$  indices could be detected in integrated light if the contribution of the upper AGB were a few times stronger than in 47 Tuc, as might be the case in a younger stellar system.

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## APPENDIX

### BOLOMETRIC CORRECTIONS AND EFFECTIVE TEMPERATURES

In CFP and Persson *et al.* (1980), we described briefly our methods for obtaining bolometric corrections and effective temperatures for metal-poor globular cluster giants with  $5000 \geq T_{\text{eff}} \geq 3800$  K. The CFP model atmospheres upon which the  $T_{\text{eff}}$  scale was based do not include molecular absorption; this  $T_{\text{eff}}$  scale was not meant to be used for cool, metal rich stars whose energy distributions are affected by molecular absorption throughout the spectrum. Our survey of globular cluster giants now includes several cooler stars, and we are in need of a temperature scale for them. As discussed below, there does not seem to be a truly satisfactory solution at present, but we have adopted the Ridgway *et al.* (1980) scale which strictly applies only to field giants of a presumably solar or near solar metallicity. Mould and Aaronson (1980) have called into question the use of this latter scale for metal poor stars. They claim that  $J-K$  gives better values for  $T_{\text{eff}}$  than  $V-K$  because of blanketing effects at  $V$  that are systematically weaker in metal poor stars. The problem and solution are not quite this simple, as discussed below.

#### a) Bolometric Corrections

The bolometric corrections are based on a straightforward numerical integration of the energy distributions of the stars with the flux extrapolated below the wavelength of the  $U$  filter and beyond  $2.2 \mu\text{m}$ . For metal poor stars, no attempt need be made to compensate for molecular opacity between the windows in the Earth's atmosphere, as was done for field stars by Johnson (1966), but in any event, such corrections are not large (Johnson 1966). Our scale is based on the data given in Allen (1973) so that the bolometric correction for the Sun is  $-0.08$ . The absolute calibration for  $UBVRI$  is from Johnson (1966) and for  $JHKL$  from Wilson *et al.* (1972). We use the Wilson *et al.* data simply because our  $J$  filter differs in effective wavelength from that of Johnson and because Johnson did not measure at  $H$  ( $1.65 \mu\text{m}$ ).

Now that accurate near-infrared observations of faint globular cluster giants are routine, it makes sense to deal with bolometric corrections to the  $K$  magnitude, so that

$$m_{\text{bol}} = V + \text{BC}_V = K + \text{BC}_K$$

and

$$\text{BC}_K = \text{BC}_V + (V - K).$$

Because the  $K$  filter is near the peak of the stellar energy distribution for cool stars,  $\text{BC}_K$  will change slowly with spectral types compared to  $\text{BC}_V$ . Dyck, Lockwood, and Capps (1974) emphasized this point by plotting  $\log F_{\text{tot}}/F_{2.2 \mu\text{m}}$  versus spectral type for a number of ordinary giants, Mira variables, and red stars. Here  $F_{\text{tot}}$  is the total flux, and their quantity is obviously just a redefined  $\text{BC}_K$ . Between M0 and M10,  $\log F_{\text{tot}}/F_{2.2 \mu\text{m}}$  varies between  $-0.80$  and  $-0.50$ , with very little scatter.

Figure 10 plots  $\text{BC}_K$  versus  $V-K$  for cluster giants in M3, M13, M67, M71, M92, and 47 Tuc (CFP; Frogel, Persson, and Cohen 1979; this paper). The relationship agrees well with that of Johnson (1966), Dyck, Lockwood, and Capps (1974), and the unreddened stars measured by Ridgway *et al.* (1980). Johnson and Dyck *et al.* take  $\text{BC}_V(\odot) = 0.0$ , we take  $-0.08$ , and Ridgway *et al.* take  $-0.14$ . Figure 11 shows the relationship of Figure 10 together with that of Dyck, Lockwood, and Capps and the red M stars from Mendoza and Johnson (1965). The 0.08 mag shift in scale has been taken into account in plotting the data. Some of the stars of Dyck *et al.* and Mendoza and Johnson (1965) are probably reddened, and thus their  $V-K$  values should be reduced. The systematic errors in  $\text{BC}_K$  and hence  $m_{\text{bol}}$  for the late type giants under consideration here are probably of order 0.1 mag or less.

#### b) Effective Temperatures

As discussed by Ridgway *et al.* (1980), the new stellar diameters measured by the occultation technique have led to a substantial revision of the effective temperature scale for cool field giants (cf. Tsuji 1978). Several of the stars in the

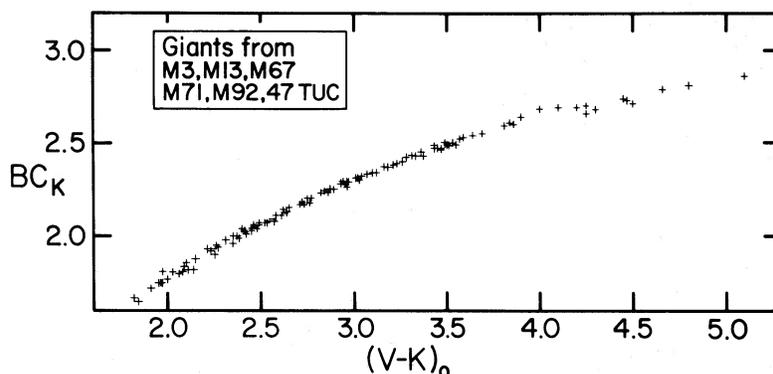


FIG. 10.—The bolometric corrections to the  $K$  magnitudes, calculated as described in the Appendix, are plotted as a function of  $(V-K)_0$  for cluster stars from CFP, Frogel *et al.* (1979), and this paper. The range of metal abundance for these stars is a factor of 100.

most metal-rich globular clusters have colors that are quite red, and in order to place these stars in the physical H-R diagram we need to know whether or not the new  $T_{\text{eff}}$  scale applies to these cluster stars. For stars warmer than  $T_{\text{eff}} > 4000$  K, molecular band blanketing is not important and there should not be any problems with the CFP scale so the abundance analyses by Pilachowski, Canerna, and Wallerstein (1980) and Cohen (1978, 1979, 1980) are unaffected. Figure 12 compares the CFP and the Ridgway *et al.* temperature scales and shows that the differences between the two are less than the uncertainties in the Ridgway *et al.* scale due to observational uncertainties in the diameters and the as yet small amount of data (see Fig. 1 of Ridgway *et al.* 1980). Mould and Aaronson (1980) criticized the use of our model atmosphere  $T_{\text{eff}}$  scale and prefer the Ridgway *et al.* scale. Figure 12 clearly shows that their criticism on this point is unfounded.

Turning to the question of the  $T_{\text{eff}}$  scale for the cooler, metal poor giants, we run into an inconsistency which was discussed by Mould and Aaronson (1980). Figure 13 shows the  $[(J-K)_0, (V-K)_0]$ -relationship for 47 Tuc together with the mean line for stars in M3, M13, M71, and M92, the mean line for  $\omega$  Cen (Persson *et al.* (1980)), and the mean line for field giants from Frogel *et al.* (1978), which agrees with that of Johnson (1966) when account is taken of the transformation between our systems at  $J$ . Mould and Aaronson's (1980) Figure 4 shows a similar displacement for giants in the LMC and SMC: there is a systematic shift between  $J-K$  and  $V-K$  which increases for cooler stars. The derivation of effective temperatures for such stars is clearly uncertain and depends on which observed color for metal poor stars is used to match the Ridgway *et al.* scale defined for metal-rich field giants. Mould and Aaronson dismiss

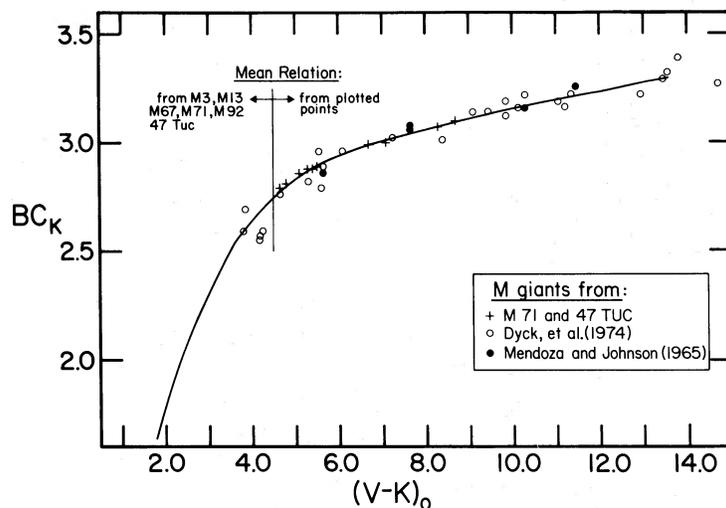


FIG. 11.—Bolometric corrections to the  $K$  magnitudes have been calculated, as discussed in the Appendix, for the M giants with photometry given by Mendoza and Johnson (1965). For stars observed by Dyck *et al.* (1974), we have taken their BCs and applied a correction for the different value of the solar BC which they used. These stars plus the red variables in 47 Tuc and M71 have been used to define the relation between  $BC_K$  and  $(V-K)_0$  for  $(V-K)_0 > 4.5$ . For  $(V-K)_0 < 4.5$  the mean relation is from Fig. 10.

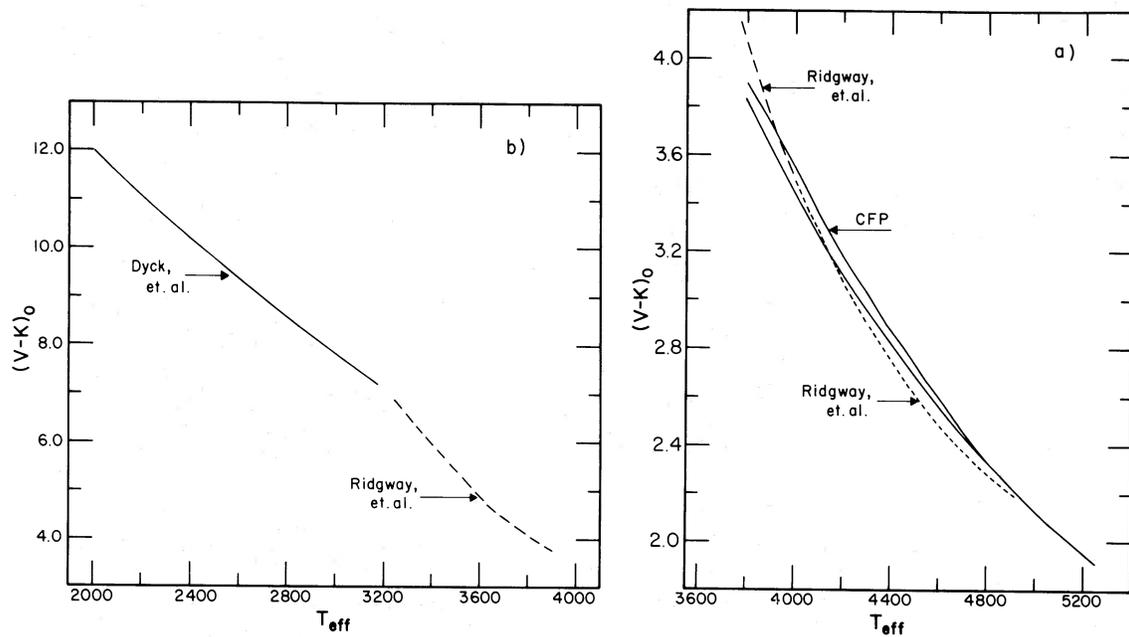


FIG. 12.— The  $[(V-K)_0, T_{\text{eff}}]$ -relationship from CFP is shown for the extreme sequences of models. Ridgway *et al.*'s (1980) temperature calibration based on lunar occultation data for approximately solar metallicity stars and the Dyck *et al.* (1974) temperature scale are also shown. Except for the coolest temperatures in the CFP scale, note the excellent agreement between CFP and Ridgway *et al.*

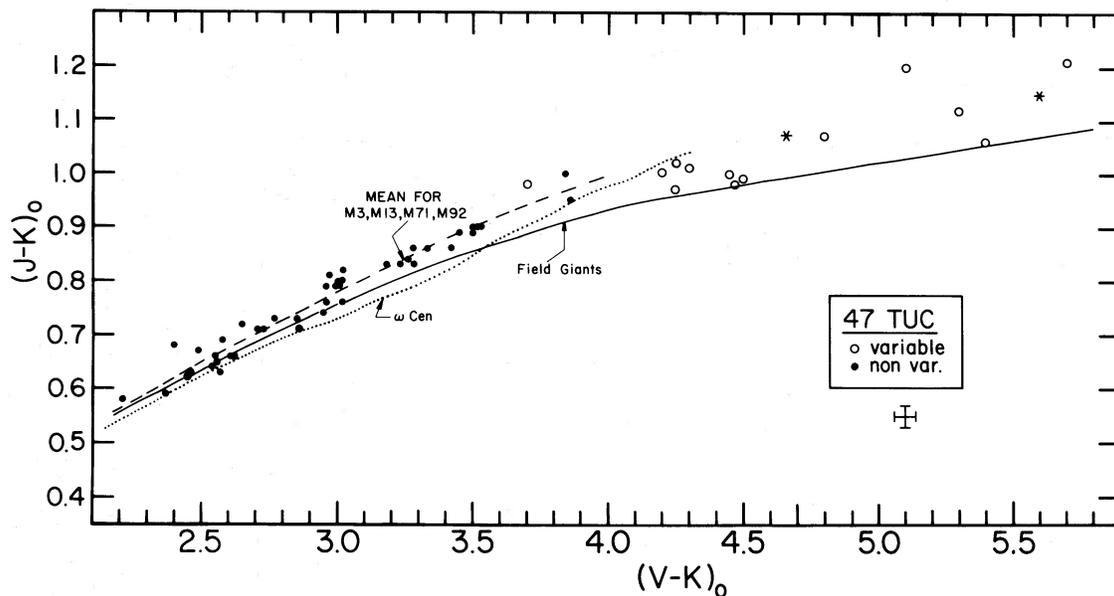


FIG. 13.—  $(J-K)_0$  versus  $(V-K)_0$  for 47 Tuc giants (solid circles) and variables (open circles). The dashed line is the ridge line drawn through the 96 stars of the clusters studied previously. The ridge line for giants in  $\omega$  Cen (Persson *et al.* 1980) is displaced downward from the ridge line for the other clusters by as much as 0.03 mag in  $(J-K)_0$ .

the temperature scale derived from  $V-K$  colors for cool metal poor stars claiming that most of the shift is due to TiO blanketing at  $V$  in metal rich stars. We have the following comments to make.

1. The size of the effect at  $V$  required by Mould and Aaronson is rather large. If  $(V-K)_0$  is 4.5, for example, then they want a 0.9 mag deblanketing change at  $V$  to move a metal rich star onto the metal-poor  $[(J-K)_0, (V-K)_0]$ -sequence. The required shift in  $(J-K)_0$  on the other hand, is 0.07 mag if only  $(J-K)_0$  is responsible. This difference in sensitivity of 0.9 mag versus 0.07 mag is a warning that opacity sources in the  $JHK$  region should at least be qualitatively explored. For a late  $K$  star at  $(V-K)_0 = 3.5$  (K5 III), the shift is 0.04 mag in  $J-K$  versus 0.25 mag in  $V-K$ . Inspection of spectral scans (discussed further below) shows that molecular blanketing at  $V$  at this spectral type cannot produce a 0.25 mag effect.

2. Mould and Aaronson base their argument in part on the appearance of a  $(J-H, H-K)$ -plot for stars in the LMC and SMC which they claim shows that the LMC and SMC giants lie roughly on the mean field line. Hence the  $JHK$  colors are supposed to be the same in all stars and only  $V$  is affected. First, there is a lot of scatter in their  $(J-K, V-K)$ -plot, with several stars lying as much as 0.2 mag off the mean line in  $(J-K)_0$  (this amounts to 1.8 in  $V-K$ ). Second, it was shown in CFP, in Frogel, Persson, and Cohen (1979), and in this paper, that globular cluster stars do *not* lie on the field line in a  $[(J-H)_0, (H-K)_0]$ -plot. Figure 7 shows the plot of 47 Tuc: there is a clear departure which is similar to that seen in more metal-poor clusters (CFP). Thus the  $JHK$  colors *are* affected by metal abundance/gravity effects.

3. Spinrad and Wing (1969) and Wing and Spinrad (1970) have discussed the various sources of infrared opacity in late type stars. They have pointed out the importance of CO and CN absorption at both  $H$  and  $K$ . Removal of these opacity sources in a late type star will to first order cancel out in  $H-K$  and cause  $J-H$  and  $J-K$  to redden. We showed in § IV of this paper that CN blanketing is probably responsible for reducing the *apparent* amount of CO absorption in the CN strong stars in 47 Tuc. As discussed there, the amount of CN blanketing in the 3900 K giant  $\beta$  And is 2.7% at 2.20  $\mu$ m. Together with the CO effect of 0.02 mag, this would account for much of the effect in  $J-K$ , which was ascribed by Mould and Aaronson to blanketing in  $(V-K)$ . (The effect of CO upon  $V-K$  is allowed for in the CFP temperature scale.)

4. We have obtained Reticon spectra of two of the M giants in 47 Tuc—viz., 1421 and 7320—and several somewhat hotter stars in the region of the  $V$  band, and have compared the amount of absorption due mainly to TiO with that seen in field M giants. Spectral scans at 20 Å resolution of a number of field giants from K5 to M6 were made available to us by J. Gunn. Both the Reticon spectra and the scanner data were integrated under the response function of the  $V$  filter and compared with a similar integration of an assumed continuum level. The continua are difficult to define for the later stars, but were located in a consistent way for the field and 47 Tuc stars. Therefore, the differential amount of TiO absorption between the field and cluster giants should be reliable. Table 9 shows the results. In both 1421 and 7320 the amount of TiO absorption is consistent with that seen in field giants of the same  $(V-K)_0$ . At M0 ( $V-K \sim 3.8$ ) and earlier the differential blanketing at  $V$  between the field and 47 Tuc giants is less than 0.1 mag. This result agrees with that of Wing (1973) who showed on the basis of eight-color photometry that several of the 47 Tuc variables obeyed the same  $\text{TiO}/T_{\text{eff}}$  relationship as do field giants. Mould and McElroy (1978) have found that the onset of TiO absorption at 7120 Å is a function of metal abundance for metal poor clusters. However, for M71 they found a differential TiO blanketing with respect to the field at  $(V-K)_0 = 3.8$  of 0.16 mag measured at 7120 Å, which corresponds to the head of a TiO band stronger than any in the bandpass of the  $V$  filter. This is completely consistent with the results given in Table 9 (integrated over the entire  $V$  bandpass) for star 5529 of 47 Tuc. It should be noted that giants significantly more metal poor than 47 Tuc do not reach  $T_{\text{eff}}$  less than 3800 K. Therefore the differential molecular blanketing between field and metal-poor cluster giants cannot be enhanced beyond the numbers cited above.

TABLE 9  
TiO ABSORPTION IN 47 TUCANAE STARS

Field Giant Spectral Type	47 Tuc Star No.	$(V-K)_0$	Apparent TiO Absorption at $V$ (mag)
K5 .....	...	3.67	0.10
M0 .....	...	3.79	0.15
	5529	3.9	0.05
M2 .....	...	4.11	0.30
	7320	4.5	0.29
M3 .....	...	4.58	0.48
	1421	5.1	0.60
M5 .....	...	6.06	0.55
M6 .....	...	7.01	1.05

For 47 Tuc then, the differential blanketing at  $V$  compared to field giants is not consistent with the 0.9–1.0 mag shifts in  $(V-K)_0$  for M3 and M4 stars advocated by Mould and Aaronson to explain the  $(J-K)_0/(V-K)_0$  discrepancy.

The true values of  $T_{\text{eff}}$  thus fall somewhere in between the values found from  $V-K$  alone and from  $J-K$  alone. At  $(V-K)=4.5$ ,  $T_{\text{eff}}(V-K)=3680$  and  $T_{\text{eff}}(J-K)\sim 3500$ . The best we can do presently is to assign errors of  $\pm 90$  K or  $\pm 0.011$  in  $\log T_{\text{eff}}$  to metal poor stars of this color.

A final point concerns the sense of the correction required and the effects upon the H-R diagram. Figure 3 shows a distinct bending over of the observed GBs for the 47 Tuc stars cooler than 3800 K ( $\log T_{\text{eff}}=3.580$ ). Adoption of a compromise between the  $J-K$  and  $V-K$  temperature scales will accentuate this effect because the  $J-K$  scale is cooler than the  $V-K$  scale. The location of the GBs for stars warmer than 4000 K is not affected.

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