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EVIDENCE FOR A PERIOD-LUMINOSITY-AMPLITUDE RELATION FOR RR LYRAE STARS

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ABSTRACT

It is demonstrated that the spread in apparent magnitudes of RR Lyrae stars in M3 and in ω Cen is real; the observed scatter in their apparent luminosities correlates with residuals in their periods as $\Delta m_{\rm bol} = 3\Delta \log P$, where $\Delta \log P$ is determined from either the period-temperature or the period-amplitude relation. As there is no systematic deviation of the abnormally bright or faint variables from the ridge lines of the amplitude-temperature relation in either cluster, it follows that amplitude is a unique function of depth of penetration into the instability strip. Observations show that amplitude is largest near the blue edge. Combining this $A = g(T_e)$ relation with the correlation of magnitude and period residuals requires the existence of a period-luminosity-amplitude relation. This function, as it is known for M3, should permit absolute magnitudes of any RR Lyrae star, pulsating in the fundamental mode, to be determined relative to those in M3, once only its period and amplitude are known.

Subject headings: stars: RR Lyrae — stars: variables

I. INTRODUCTION

One explanation for the observed differences in the period distributions of RR Lyrae stars among globular clusters in the different Oosterhoff (1939, 1944) groups is that variables in clusters of the long-period group have brighter absolute magnitudes than those in the short (Sandage 1958). Evidence for this possibility is that (1) at any given temperature, the periods of RR Lyrae stars in M15 are longer than those in M3 by a well-determined period shift, (2) the same shift is required to make the period-amplitude relations of the two clusters coincide, and (3) no shift is required to make the amplitude-temperature correlations coincide. The data and conclusions are discussed by Sandage, Katem, and Sandage (1981, hereafter SKS), where it is shown that the observed correlations can be understood if (1) amplitude is a unique monotonic function of temperature within the instability strip, and (2) the variables in M15 are ~0.2 mag brighter than those in M3, provided that the stellar mass at a given period is the same in both clusters.

However, the argument for luminosity differences between the M3 and M15 horizontal branches (HB) is not a direct proof, because SKS reached their conclusion by inference, using $P\langle\rho\rangle^{1/2} = Q$ with an assumed constant mass. As no adequately reliable measurement of HB absolute luminosities of the clusters is directly available,[1] it is desirable to find a more straightforward way to demonstrate the case that longer periods signal brighter absolute magnitudes either at constant temperature or, equivalently, at constant amplitude. It is shown in this *Letter* that such a direct proof follows by noting that the observed spread in luminosity of variables in M3 and in ω Cen correlates with their observed spread in period, individually. The advantage of the present demonstration is that the model is now applied to variables in a *given* cluster where the differences in absolute magnitude among the variables are observed directly.

[1] The machinery (Eggen and Sandage 1959, 1962) of fitting the observed M3 and M15 dwarfs, corrected for blanketing, to a set of evolved fiducial main sequences with different zero points to account for different metallicities and helium abundances (Sandage and Eggen 1959, 1969) is simply too uncertain at the necessary level of ±0.2 mag to measure the HB luminosities of M3 and M15 directly. A recent variation of these methods by Carney (1980) has given results similar to those derived previously (Sandage 1970)—results that contradict the Oosterhoff period requirement and are therefore suspect.

II. THE MODEL

A schematic of the main features of the model, applied to stars that vibrate in the fundamental mode, is shown in Figure 1. The lower diagram is a segment of the H-R diagram with the boundaries of the *ab* instability strip (assumed to be vertical) and the loci of constant period shown. From $P\langle\rho\rangle^{1/2} = Q$ with sensibly constant mass, the periods stand as $P_i > P_j$ because brighter stars at a given temperature have larger radii and, hence, have the lower mean densities. A portion of the zero-age horizontal branch (ZAHB) of a particular cluster is shown in black, with a typical variable j drawn as an open circle on its period line P_j.

Suppose now that an intrinsic dispersion in luminosity exists among the variables in the cluster, presumably owing to evolution. One such brighter variable i is shown, assumed here to be at the same temperature as variable j.

The upper panel shows amplitude as a function of position in the strip, going to zero at both boundaries but peaking close to the blue boundary as observed (SKS, and Fig. 4 discussed later here). From these observations, amplitude is a function only of T_e, hence $A_i = A_j$.

Note other properties of the model. The bulk of the

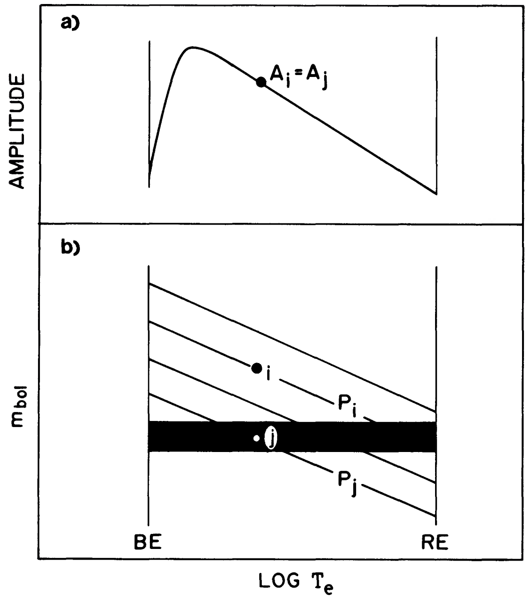

FIG. 1.—(*a*) Schematic representation of amplitude as a function of T_e. (*b*) Model for the instability strip of the H-R diagram showing lines of constant period and a typical zero-age horizontal branch. A nonevolved variable j and a brighter variable i, assumed to be at the same temperature $T_e{}^j$, are shown on their period lines P_i and P_j.

variables are assumed to lie within the narrow confines of the ZAHB. Hence, there will be a very narrow period-color relation, defined by the intersection of the period lines with the ZAHB at various temperatures. For an infinitely narrow HB, the dispersion in the period-color correlation would be zero (assuming constant mass). But consider variable i at the same temperature as j. This star, with its abnormally long period P_i, will stand off both the mean period-color and period-amplitude relations defined by the majority of the variables on the ZAHB, but because $A_i = A_j$ and $T_e{}^i = T_e{}^j$, will not deviate in the color-amplitude relation.

Consider now the model quantitatively. Calculations by van Albada and Baker (1971), Iben and Huchra (1972), and Stellingwerf (1975) give nearly identical results for the variation of the pulsation parameter Q with mass, luminosity, and temperature for a wide variety of compositions. The relation of van Albada and Baker (1971) is

$$\log P = -0.34\, M_{\rm bol} - 0.68 \log M/M_\odot - 3.48 \log T_e + \text{const.}, \quad (1)$$

for the fundamental period. The coefficients of the luminosity, mass, and temperature terms differ from -0.30, -0.50, and -3.0 that would follow from $P\langle\rho\rangle^{1/2} = Q$ if Q were strictly constant, using $\langle\rho\rangle \sim M/R^3$, and $L \sim R^2 T_e{}^4$. The reason is that Q is a slow function of L, M, and T_e, according to the cited calculations.

Consider first the case where the mass of all RR Lyrae stars is constant, independent of L, T_e, and composition. The lines of constant period, such as drawn in Figure 1 in the ($m_{\rm bol}$, log T_e)-plane, have the equation

$$M_{\rm bol} = -3.00 \log P - 10.24 \log T_e + \text{const.} \quad (2)$$

Hence variables in an assembly whose periods deviate from the mean period-color relation by $\Delta \log P$ should also deviate in absolute magnitude by $\Delta M_{\rm bol} = -3\Delta \log P$, in the sense that longer periods require brighter absolute magnitudes.

To what extent do mass variations cloud this conclusion? Calculations (cf., Iben and Rood 1970; Sweigart and Gross 1976, and references therein) require mass loss to be a free (variable) parameter for red giants at the He flash, so that the HB can be subsequently populated throughout its large observed temperature range. However, the models admit only a tiny mass range of at most $\Delta \log M \approx 0.03$ within the RR Lyrae instability strip (see Fig. 1 of Sweigart and Gross, at, say, $\log T_e = 3.83$). From equation (1), these variations give a nearly negligible contribution to $\Delta \log P$ compared with $\Delta \log P = 0.06$ that is often observed.

More serious is a possible variation of $M/M_\odot$ for ZAHB variables at a fixed T_e as [Fe/H] is varied; higher mass is predicted at fixed T_e for lower [Fe/H] (see Caloi, Castellani, and Tornambè 1978, Figs. 9 and 10), but the sense of the dependence requires an even greater $\Delta M_{\rm bol}$ difference between M3 and M15 than given by equation (2) (SKS, § VI). Furthermore, this predicted mass-metallicity effect is not detected using the ω Cen variables of different metallicity and, hence, may not in fact be a problem.

III. THE OBSERVATIONS

Detailed photometry exists for variables in M3 (Roberts and Sandage 1955; Sandage 1981*b*, hereafter Paper II) and in ω Cen (Butler, Dickens, and Epps 1978). The instability strip in $m_{\rm bol}$, log T_e is shown in Figure 2, based on a temperature scale of Bell, quoted by Butler *et al.*

The variables show a dispersion in $m_{\rm bol}$ with a range[2] of 0.85 mag in ω Cen and 0.30 mag in M3. Most of the dispersion is real, shown by noting that all the brighter stars have longer periods than fainter stars of the same temperature. The data and proof via the period-temperature relation are given elsewhere (Paper II). We give here a parallel proof using the period-amplitude relation which is equivalent to that using equation (2), since amplitude is a function of temperature alone.

The left panels of Figure 3 give the observed period-amplitude relations for ω Cen and M3, respectively. Note that the stars in Figure 2 which deviate in luminosity from the mean also deviate in period at a given amplitude. The sense is that required by the model; bright stars have longer periods.

[2] The large difference in the ranges is believed to be due to different directions of travel along evolutionary tracks from the ZAHB for ω Cen and M3 (see Sweigart and Gross 1976, Fig. 2, for an illustrative example). Such a difference is also suspected from the color overlap between the *c* and *ab* variables in ω Cen and its lack in M3 (SKS).

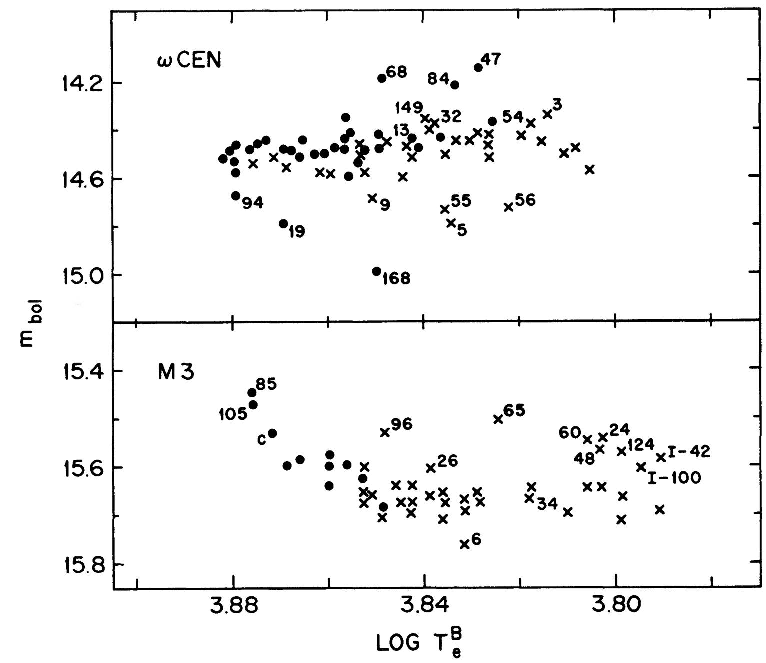

FIG. 2.—The observed ($m_{\rm bol}$, log T_e)-diagrams for ω Cen and M3. Particular bright and faint variables are identified. Type *c* RR Lyrae stars are filled circles; *ab* variables are crosses. Temperatures are on the scale by Roger Bell. Data for variables in ω Cen are by Butler, Dickens, and Epps (1978). Those for M3 are from Roberts and Sandage (1955) and Sandage (1981*b*).

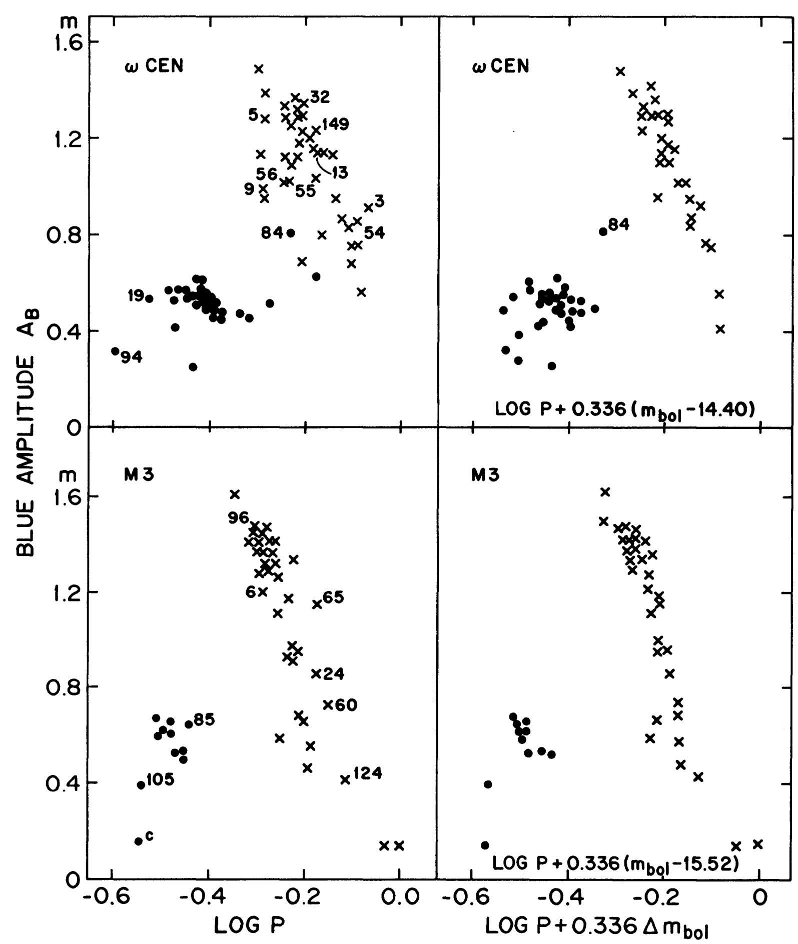

FIG. 3.—Observed period-amplitude relations for ω Cen and M3 in panels (*a*) and (*c*). The correlations using the reduced periods log $P + 0.336\Delta m_{\rm bol}$ are in panels (*b*) and (*d*). The specific bright and faint stars marked in Fig. 2 are also marked here.

The expected period-luminosity-amplitude (P-L-A) relation for constant mass now follows by substituting the supposed $T_e = f(A)$ relation into equation (1) to obtain

$$\log P + 0.34 M_{\rm bol} = g(A) + {\rm const.} \quad (3)$$

The test that such a relation exists is for the scatter in Figures 3*a* and 3*c* to decrease if the data are plotted with $\log P + 0.34 m_{\rm bol}$ as the abscissa. Figures 3*b* and 3*d*, so plotted, show a marked reduction in scatter; the remaining deviations are now of the order of the observational errors alone of ± 0.1 mag for A_B and $\sim \pm 0.1$ mag for $m_{\rm bol}$.

We now again test the requirement that amplitude and temperature are uniquely related by plotting the color-amplitude relation for M3 in Figure 4. The stars that deviate most in Figures 2 and 3, also marked in Figure 4, show no deviation from the mean, proving that A_B is not a function of L. Furthermore, as shown by SKS (their Fig. 11) for M3 and M15, and now for ω Cen itself (Fig. 3*b*), where stars of different metallicities are plotted), the amplitude-temperature relations coincide for all stars, despite their large differences in [Fe/H]. Hence, A_B is neither a function of L or of

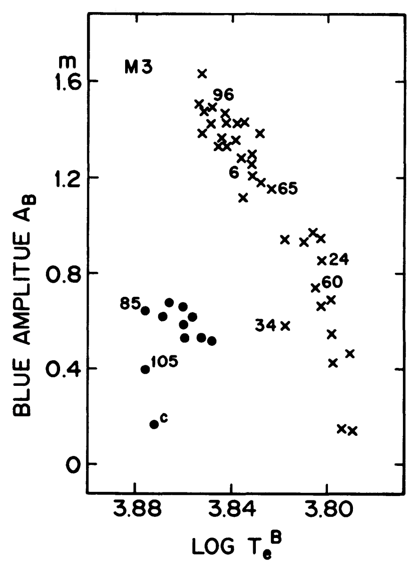

FIG. 4.—Amplitude-temperature relation for M3. Stars that deviate in $m_{\rm bol}$ in Fig. 2 are marked here. No deviation of these stars occurs in this diagram.

[Fe/H] at a given T_e but, to the accuracy of the present data, depends principally on T_e alone.

IV. APPLICATION AND USES OF THE P-L-A RELATION

There is no need to parametrize the $g(A)$ relation of equation (3) so as to give the P-L-A relation analytically. The graphical representation of Figure 3*d* for M3 contains the total information.

Equation (3) can be used to obtain the absolute magnitude of any RR Lyrae star with known P and A_B, as follows. (1) Adopt the M3 P, $m_{\rm bol}$, A_B relation in Figure 3*d* as fiducial. (2) For any given RR Lyrae, the deviation $\Delta \log P$ is read directly from this diagram relative to the mean M3 line, at a given A_B. (3) From equation (3), the difference in absolute magnitude of this star from those of the same temperature in M3 is $\Delta M_{\rm bol} = 3\Delta \log P$, in the sense that stars of longer period than the fiducial relation are brighter than the variables in M3.

Two uses of individual absolute magnitudes for field RR Lyrae stars can be mentioned. (1) Reduction of the proper motion of any given variable to a common distance can now be made with an accuracy of $\sim \pm 0.1$ mag. This should improve the homogeneity of the data for statistical parallax solutions and, hence, improve the zero point of $M_{\rm bol}$ for M3, leading most directly to the age of the globular cluster system. (2) It is known that $M_{\rm bol}$ of variables near the ZAHBs of a large sample of globular clusters is very tightly correlated with [Fe/H] (Fig. 4, Sandage 1981*a*, *c*, hereafter Paper III). Therefore, metallicities of most individual field RR Lyrae stars (except the few that are far evolved from their ZAHB) can perhaps be obtained knowing only their period and amplitude via equation (3). Hence, we may be able to study the chemical gradient in the high galactic halo directly from the RR Lyrae (P, A_B) pairs alone[3] for very faint variables.

[3] The correlation of $M_{\rm bol}{}^{\rm HB}$ with [Fe/H] for the *system* of galactic globular clusters has a different and more fundamental reason than the lack of such correlation in a given cluster, such as ω Cen (§ III). The cluster-to-cluster correlation is explained by a solution to the Oosterhoff problem developed in Paper III (which in outline is that: [1] all clusters are nearly the same age; [2] given different [Fe/H] values, $L_{\rm TO}$ must differ according to the stellar models and, because $\Delta M_{\rm bol}$ (TO − HB) = 3.5 for all clusters regardless of metallicity, $L_{\rm HB}$ must also differ in the way that makes $M_{\rm bol}{}^{\rm HB}$ brighter for low-metallicity clusters; hence such clusters will have longer periods for their variables). The dispersion of $M_{\rm bol}$ *within* a given cluster is due to the much simpler cause of evolution itself from its particular ZAHB, and this $\Delta M_{\rm bol}$ is *not* correlated with [Fe/H].

REFERENCES

Butler, D., Dickens, R. J., and Epps, E. 1978, *Ap. J.*, **225**, 148.
Caloi, V., Castellani, V., and Tornambé, A. 1978, *Astr. Ap. Suppl.*, **33**, 169.
Carney, B. W. 1980, *Ap. J. Suppl.*, **42**, 481.
Eggen, O. J., and Sandage, A. 1959, *M.N.R.A.S.*, **119**, 255.
———. 1962, *Ap. J.*, **136**, 735.
Iben, I., and Huchra, J. 1972, *Astr. Ap.*, **14**, 293.
Iben, I., and Rood, R. T. 1970, *Ap. J.*, **161**, 587.
Oosterhoff, P. Th. 1939, *Observatory*, **62**, 104.
———. 1944, *Bull. Astr. Inst. Netherlands*, **10**, 55.
Roberts, M. S., and Sandage, A. 1955, *A.J.*, **60**, 185.
Sandage, A. 1958, *Stellar Populations*, ed. D. O'Connell, *Ricerche Astr. Specola Vaticona*, **5**, 41.
———. 1970, *Ap. J.*, **162**, 841.
———. 1981*a*, *Report of the Director*, *Carnegie Inst. Yrb.*, No. 81.
———. 1981*b*, *Ap. J.*, in press (Paper II).
———. 1981*c*, *Ap. J.*, in press (Paper III).
Sandage, A., and Eggen, O. J. 1959, *M.N.R.A.S.*, **119**, 278.
———. 1969, *Ap. J.*, **158**, 685.
Sandage, A., Katem, B. N., and Sandage, M. C. 1981, *Ap. J.*, in press (SKS).
Stellingwerf, R. F. 1975, *Ap. J.*, **195**, 441.
Sweigart, A. V., and Gross, P. G. 1976, *Ap. J. Suppl.*, **32**, 367.
van Albada, T. S., and Baker, N. 1971, *Ap. J.*, **169**, 311.