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R ANDROMEDAE AND THE METHOD OF WAVELENGTH COINCIDENCE STATISTICS

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ABSTRACT

The wavelength lists of Merrill and of Merrill and Greenstein for R Andromedae (λλ3342–6892) have been analyzed by the method of wavelength coincidence statistics (WCS).

The atomic species identified at high confidence levels by WCS are in excellent agreement with the results of the classical methods. Lack of agreement can almost always be attributed to small numbers of expected atomic lines (e.g., Ca II, Na I, etc.), where WCS, like any statistical procedure, is at a disadvantage. WCS, without further spectroscopic considerations, finds Tc I at the 99.5% confidence level.

A theoretical model of the WCS procedures is developed which makes it possible to give estimates of the expected number of marginal results (95–99.5% confidence). The *number* of marginal identifications of WCS estimated from the model to be real—not due to chance coincidences—is in excellent agreement with the number of species identified by Merrill and Greenstein on the basis of small numbers of lines or with question marks concerning their presence. The most practical aspects of WCS appear to be well-understood, and there is no indication that confidence levels are being overestimated as can happen when the Russell-Bowen formulae are applied.

Attention is called to the weakness (or absence) of La II and Ce II, which appears to contradict the predictions of the *s*-process. The proposed explanation is depletion of the atomic species by the formation of molecules.

Subject headings: line identifications — stars: abundances — stars: individual — stars: long-period variables

I. INTRODUCTION

The method of wavelength coincidence statistics (WCS) is a powerful tool for the investigation of stellar spectra with large numbers of lines. Its usefulness is not limited to qualitative analyses. In some cases a good indication of the relative strength of certain atomic spectra is given by the parameters of WCS. Cowley and Aikman (1980) have shown that abundance estimates of certain elements can be made by calibrating the WCS parameters against published analytical results. The calibration reproduces the adopted abundances with a typical uncertainty of 0.5 dex.

Automated WCS procedures were first introduced by Hartoog, Cowley, and Cowley (1973) in connection with the proposed identification of promethium in HR 465 (Aller and Cowley 1970). Because the technique itself has never escaped association with what was a highly controversial matter, we have felt it would be useful to present a comparison of the results of WCS with one of the classical studies of a stellar spectrum. We have chosen for this purpose the work of Merrill (1947, 1948) and Merrill and Greenstein (1956) on the spectrum of the long-period variable R And. The high quality and thoroughness of these authors' work is well known. Significant disagreements between their results and WCS would have been a strong indication of weaknesses in the method.

The comparison will also enable us to present a theoretical discussion of marginal statistical events which is of practical importance to the optimum use of WCS. Marginal events are at once exciting and invidious. While an individual marginal identification may be the subject of debate and uncertainty, this need no longer be true when we consider a *number* of marginal events. Our current lists of laboratory wavelengths include nearly 400 species including not only various stages of ionization, but also several subgroups, by intensity category, within a given stage.

In a typical analysis of a stellar spectrum, between 10 and 20 marginally significant ($>95\%$ confidence) results may occur. A certain fraction of these results are simply due to chance. Another fraction will be due to the real presence of atomic spectra which are too weak or involve too few lines to give more than marginal confidence.

In our past work, we have treated all of these marginal cases by the traditional techniques. Many marginal results may be accepted (e.g. H and K) or dismissed immediately, but in other cases a decision is not simple. Partial, highly relevant information is available from the theory of statistics which enables one to calculate the probability distribution of chance events. It is then possible to assign a *confidence level* at which a certain number of marginally significant results may be rejected. Similarly, one can say, with a calculable confidence level, that another fraction of the marginal results are not due to chance.

II. COMPARISON OF THE ANALYSES OF R ANDROMEDAE

We present the results of Merrill (1947, 1948) and Merrill and Greenstein (1956) on the identification of *atomic* features. Their conclusions are compared with output from our WCS program. The spectroscopic material available to Merrill and Greenstein is amply documented in the papers cited and will not be repeated. Our WCS program used the wavelengths in Merrill and Greenstein's Table 7 for $\lambda\lambda$4000–6900, supplemented by Table 1 of Merrill (1948) for $\lambda < 4000$. The standard tolerance for a coincidence was ± 0.06Å, and 200 Monte Carlo trials were used for each atomic species.

The comparison of the two methods is made in Table 1 whose format is adapted from a similar tabulation by Merrill and Greenstein. The results of the WCS survey are presented symbolically. Usually, the symbols entered are from the list for a given atomic species giving the *most positive* results. The symbols under WCS have the following meaning:

- $-$: indicates a result for which the confidence level is less than 95%.
- 0: indicates a "marginal" result: $95 \leq$ confidence $<$ 99.5%.
- $+$: indicates a highly confident result, $\geq 99.5\%$ confidence but with the S parameter less than 4.0.
- S: for results with the S parameter >4.0, the parameter itself is listed.

If Merrill or Merrill and Greenstein said lines of a certain spectrum *may* be present, we entered a question mark in the appropriate column, usually, the one headed "few lines." When these authors said lines *were* present, an X is entered. Their "no evidence" column applies primarily to the rare earth elements. Unsuccessful searches for other spectra were generally not indicated. The column headed H/N gives the number of coincidences or hits, H, out of N laboratory lines sought. Agreement of the two techniques is indicated in the penultimate column; a question mark in this column means that a one-word description of the situation is simplistic.

In a comparison of this type we *expect* good agreement of WCS and classical methods whenever the S parameter is entered, i.e., $S>4.0$ *and* the number of hits H upon which S is based is not small. For the present, the reader may note that the instances of disagreement may be almost exclusively attributed to small numbers; usually it is palpably obvious that the numbers are small.

One case of disagreement that might be discussed in some detail is that of neutral praseodymium, Pr I. Merrill and Greenstein entered an X in their "no evidence" column. This must have been an overall assessment, based in part upon Bidelman's (1953) identifications which were made using Merrill's (1947) list. Bidelman has concluded that neutral lines of praseodymium were absent. However, since the later Merrill-Greenstein list was more extensive it is entirely possible that different conclusions might have been based on it. Indeed, five out of our six coincidences do not occur in the older list used by Bidelman. The fact is that in Merrill and Greenstein's Table 1, two lines ($\lambda\lambda$* 4951.37 and 5045.52) are identified as "?Pr" and "Ti ? Pr." King (1928) had assigned these lines correctly to the first spectrum, but it is relevant to note this classification could quite properly have been regarded as tentative at the time of Merrill and Greenstein's work in the mid-1950's.

Gaussian statistics cannot be assumed for the S parameter for so small a number of coincidences as six; the only valid method of establishing the probability that the coincidences are spurious is to run a prohibitively large number of trials. We can say that the identification is *at least* 99.95% confident (based on 2000 trials). Moreover, even if we remove two of the six coincidences, the result is still 99% confident.

We conclude that Pr I is probably present, and that Merrill and Greenstein concur. Since $\lambda\lambda$4951.37 and 5045.52 are now classified *within* the Pr I spectrum (i.e., we know the energy levels from which they arise; Meggers, Corliss, and Scribner 1975), there can be little doubt that they belong to the neutral atom. The spectrum of Pr I is surely weak, and as long as the statistical results are based on only six coincidences, some lingering doubts are not out of place.

Further detailed discussion of Table 1 will not be given. The agreement between the two techniques is entirely satisfactory.

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TABLE 1 A

COMPARISON OF MERRILL-GREENSTEIN ID WITH RESULTS FROM WAVELENGTH COINCIDENCE STATISTICS

Spectrum	Merrill-Greenstein: Numerous Lines	Merrill-Greenstein: Few Lines	Merrill-Greenstein: No Evidence	WCS: ID	WCS: H/N	Agreement	Remarks
Li I		x		–	0/8	No	Small numbers
Na I		x		–	1/2	No	Small numbers
Mg I		x		6.5	5/9	Yes	
Al I		x		–	1/5	No	Small numbers
Si I			?	–	1/21	?	
K I		x		–	2/5	No	Small numbers
Ca I	x			14.3	28/51	Yes	
Ca II		x		–	0/2	No	Small numbers
Sc I	x			8.6	8/9	Yes	
Sc II	x			4.8	18/71	Yes	
Ti I	x			19.6	68/92	Yes	
Ti II	x			0	2/5	?	
V I	x			11.3	31/34	Yes	
Cr I	x			11.9	28/48	Yes	
Mn I		x		4.9	17/42	Yes	
Fe I	x			11.5	52/135	Yes	
Co I	x			6.7	19/58	Yes	
Ni I	x			6.1	14/37	Yes	
Ga I		x		–	2/6	No	Small numbers
Sr I		x		–	1/1	No	Small numbers
Y I	x			8.1	14/25	Yes	
Y II	x			7.7	20/41	Yes	
Zr I	x			8.1	16/24	Yes	
Zr II	x			4.5	21/79	Yes	
Nb I	x			+	7/11	Yes	
Ru I	x			5.1	18/53	Yes	
Rh I		x		+	12/14	Yes	
Ag I		?		–	1/7	?	Small numbers
In I		?		–	1/3	?	Small numbers
Ba II	x			6.2	5/7	Yes	
La (I)			?	–	6/28	Yes	La II, $p=0.055$
Ce			x	–	19/98	Yes	
Pr I			x	5.4	6/14	?	See text
Pr II		x		5.4	16/33	Yes	
Nd I			x	–	1/16	Yes	
Nd II	x			7.2	28/67	Yes	
Pm			x	–	2/38	Yes	
Sm I		?		–	3/21	?	Small numbers
Sm II	x			6.8	26/63	Yes	
Eu I			?	–	2/12	Yes	
Eu II		x		+	5/19	Yes	
Gd I		x		0	3/5	Yes	Small numbers
Gd II		x		+	19/69	Yes	
Tb II		?		–	6/24	?	
Dy I		x		–	3/8	No	Small numbers
Dy II		x		+	17/69	Yes	
Ho			?	0	7/16	?	Ho I, $p=0.025$
Er		x		0	7/21	Yes	Er I, $p=0.025$
Tm I		x		–	3/14	No	Small numbers
Tm II			x	–	3/22	Yes	
Yb I		x		–	1/1	No	Small numbers
Yb II			x	–	1/25	Yes	
Lu			x	–	8/75	Yes	
Hf I		?		–	5/19	?	Small numbers, $p=0.09$
Re I		?		0	3/14	Yes	Small numbers
Os I		?		0	5/11	Yes	Small numbers
Pb I		?		–	1/8	No	Small numbers

TABLE 1B

SUMMARY OF TABLE 1A

Agreement	Remarks	Number of Cases
Yes	...	37
No	Small numbers	11
?	(various)	9

III. THE TECHNETIUM IDENTIFICATION

According to our standard Tc I list, number of trials (200), and tolerance (± 0.06 Å), Tc I is identified at the 99.5% confidence level, based on coincidences with 10 out of 29 lines sought. Using 2000 random trials, the confidence level is 99.7%, hardly changed.

These confidence levels are quite high, though they are not in the category we describe as "ineluctable." Merrill, and later Merrill and Greenstein based their arguments in favor of the identification on considerations other than WCS. It is difficult to evaluate their arguments quantitatively, but they are generally accepted and are surely sound. WCS simply reinforces the Tc I identification.

IV. SPURIOUS AND MARGINAL RESULTS OF WCS

In Table 2, we list a number of marginal statistical results on R And. It will be the problem of the present section to try to understand these results by building up a theory of WCS. Tables 1 and 2 are, with a few exceptions, exclusive. All marginal results have been included in one or the other of the two tables.

There are nine entries in Table 2; most of them are highly unlikely identifications. How many may be written off as due to chance? A rigorous answer to this question turns out to be extremely difficult and quite impractical to give for an arbitrary stellar line list. The reason is that the elementary probability of a coincidence with a given laboratory line varies somewhat for each line because the *density* of lines in the stellar list is variable. This is the reason the Monte Carlo trials were originally introduced.

We may, however, attempt to *model* the true situation by introducing a "characteristic" elementary probability "ω" which is the same for all lines in our idealized model. Then for an atomic species with n laboratory lines within the stellar list, the probability of k coincidences is simply given by the binomial formula

$$p_{\rm model} = \binom{n}{k} \omega^k (1-\omega)^{n-k}. \tag{1}$$

Table 2 shows values of $p_{\rm model}$ calculated by summing equation (1) from k to n, using estimates of ω from the Monte Carlo runs, i.e.,

$$\omega = \frac{\langle H_N \rangle}{N}. \tag{2}$$

These $p_{\rm model}$'s should be compared with the entries in the fifth column, which give the direct Monte Carlo probabilities that the k coincidences are due to chance. Also shown are values of $p_{\rm RB}$ using ω calculated as in a Russell-Bowen (1929) procedure

$$\omega_{\rm RB} = \frac{\mathfrak{N}}{\lambda_{\max} - \lambda_{\min}} 2\Delta\lambda = 0.071, \tag{3}$$

where $\mathfrak{N} = 2101$, the total number of lines in the list, and $\Delta\lambda = 0.06$ Å for our work with R And.

Let us now apply the concept of modeling to the problem of understanding the statistics of the spurious marginal events which occur in a search for several hundred atomic species.

TABLE 2

MARGINAL COINCIDENCES

Sp	H/N	$\langle H_N \rangle$	$\sigma(\langle H_N \rangle)$	p(WCS)	S	ω	$p_{\rm MODEL}$	$p_{\rm RB}$	$N_{\rm LIST}$[a]	$N_{\rm SIG}$[b]
He I ...	6/19	2.21	1.45	0.025	2.61	0.1163	0.018	0.002	3	1
Ne I ...	4/15	0.97	0.96	0.010	3.15	0.0647	0.013	0.018	2	1
Al II ...	5/34	2.18	1.34	0.045	2.10	0.0641	0.064[c]	0.090	2	1
Si III ...	3/9	0.92	0.91	0.040	2.27	0.1022	0.056[d]	0.022	1	1
Mn II ..	21/110	13.31	3.36	0.015	2.29	0.1210	0.023	<0.001	3	2
Ce III ..	16/113	8.60	2.72	0.005	2.72	0.0761	0.012	0.006	2	2
Gd III ..	22/132	13.75	3.39	0.020	2.43	0.1042	0.018	<0.001	2	1
Er III...	13/77	7.49	2.35	0.020	2.34	0.0973	0.034	0.003	3	1
Os II ...	5/18	2.03	1.34	0.050	2.22	0.1128	0.044	0.007	2	1

[a] There are, for example, 3 He I lists.
[b] Of these 3 He I lists only 1 gave a "significant" result.
[c] $p_{\rm MODEL} = 0.020$ for $H=6$.
[d] $p_{\rm MODEL} = 0.009$ for $H=4$.

This problem could also be attacked by the Monte Carlo procedure. We need only make up a large number of nonsense *stellar* wavelength lists, run the standard identification program against them, and tabulate the results. Since this procedure would have to be done for each star, the cost of such trials are prohibitive, and we therefore turn to a model from which useful analytical results may be obtained. Let us use capital letters to distinguish quantities in the present model from the analogous ones developed above.

The probability of obtaining a "significant" result by chance varies from one atomic species to another because the number of laboratory wavelengths vary and the positions of these wavelengths vary. It varies also from star to star because of variable wavelength dependence on the line density. However, let us assume that one mean probability, Ω, can be applied to all of the atomic species. More precisely, let Ω be the probability that a 95% confidence result or greater will arise by chance for an individual atomic species. We can then write down the expected distribution of chance results in a search for N atomic species with the help of the binomial formula. The following sums are of particular interest:

$$S_{\max}=\sum_{K=K_{\max}}^{N} P(k)=\sum_{K=K_{\max}}^{N}\binom{N}{K}\Omega^{K}(1-\Omega)^{N-K}, \quad (4)$$

where $K_{\max}$ is the smallest number such that $S_{\max}<.05$, and

$$S_{\min}=\sum_{K=0}^{K_{\min}} P(K), \quad (5)$$

where $K_{\min}$ is the largest number such that $S_{\min}<0.05$.

With these definitions we can specify at the $>95\%$ confidence level that at least $K_{\min}$ and no more than $K_{\max}$ atomic species will be *spuriously* identified. As we shall see later, the *expected* number of spurious marginal results is approximately nine for R And if we use our current list containing some 400 laboratory wavelength sets. This means that we would be justified in rejecting most or even all of the entries of Table 2. The credible identifications appear in Table 1 and the probably spurious cases in Table 2 because of the prior identification work of Merrill and Greenstein. WCS alone would not tell us which of the marginal cases (e.g., He I, Ne I,...) were bogus, although in many instances rather casual spectroscopic considerations would rule out a number of atomic spectra.

V. FIXING THE PARAMETER Ω

The expected number of spurious 95% confident results for N uncorrelated laboratory wavelength lists is $N\Omega$. One might guess that since our cutoff for a marginal event is 95% confidence, Ω would be close to 0.05. In fact, it is closer to half of that value. To show this, we avoid momentarily the problem of correlation among our laboratory line lists and suppose them to be uncorrelated.

Let us now use equation (1) to investigate the mean value of $p(k)$ for which $p(k)<0.05$. Here, we need a value of the parameter ω about which we have extensive knowledge from the Monte Carlo trials. We performed a complete analysis on the R And wavelengths shifted by -2 Å, and determined the ω parameter for each species. The results are summarized as follows:

mean value of ω for 393 atomic species	...	0.101
standard deviation	...	0.036
maximum ω	...	0.215
minimum ω	...	0.015.

Using these figures, it is not difficult to show that for most cases, a $>95\%$ significant result will be typically 97 or 98% confident. It is most difficult to generalize precisely, because, even within the context of the model, the problem varies with ω, n, and k. Let us consider a specific example: $\omega=0.10$, $n=12$. An equal to or greater than 95% confident result means that $\geq k^*$ coincidences were found such that

$$p_{\leq .05}=\sum_{k=k^*}^{n=12}\binom{n}{k}\omega^{k}(1-\omega)^{n-k}\leq 0.05. \quad (6)$$

This "summed binomial distribution function" is tabulated in various sources (see Burlington and May 1970). We find from the tables that k^* must be 4 for a significant result. The probability of three or more coincidences is 0.1109, not significant. For $k^*=4$, the sum in equation (6) is 0.0256, considerably below the threshold of 0.05. This may be regarded as a simple consequence of the discrete nature of the coincidences.

In deciding upon a value of Ω to use in equation (4) we must consider the fact that n varies considerably among the atomic species. We have attempted to investigate the sensitivity of the Ω parameter to the length of the line lists by dividing our laboratory list into 112 species containing less than or equal to (.LE.) 10 lines and 90 species with equal to or greater than (.GE.) 40 lines. On the experiment with the R And list shifted by -2 Å, the results were as follows:

.GE. 40 lines: 3 of 90 cases were
$>95\%$ confident $\Rightarrow \Omega \approx 0.0333$

.LE. 10 lines: 2 out of 112 cases were
$>95\%$ confident $\Rightarrow \Omega = 0.018$

All lab lists: 11 out of 393 cases were
$>95\%$ confident $\Rightarrow \Omega = 0.028$.

It appears, then, that an appropriate value of Ω is close to 0.03. We must, to be prudent, consider a reasonable range of values in the neighborhood of 0.03.

VI. THE PARAMETER N: PRESENT AND CORRELATED ATOMIC SPECIES

A number of the species in our laboratory line list are *surely* present in the spectrum of R And. These species should be excluded from the number N used in equations (4) and (5), since our null hypothesis will be that the stellar and laboratory line lists are uncorrelated. We have subtracted 54 line lists as belonging to species which are surely present. The precise number which ought to be subtracted involves some subjectivity, as will be discussed below.

A difficult problem concerns the correlation of the laboratory line lists with each other. Such correlations occur in several ways:

1. For many atomic species we purposefully included short lists of strong lines and weak lines as well as inclusive lists containing both categories of lines.
2. Among the actinides, especially uranium, we have attempted to make separate lists for individual isotopes by applying isotope shifts. Many of these shifts are small, so that the lists are correlated with one another.
3. In some of our newer laboratory lists, a few lines from impurities may not have been deleted. We have become sensitive to this problem and have made special efforts to eliminate such lines. However, we cannot pretend to have had complete success in every case.
4. Certain laboratory line lists are, by accident, weakly correlated with one another. Such (weak) correlation may however, in the case of a short laboratory list, lead to an appreciable underestimate for the probability of a chance event when the correlation is not taken into account properly.

These correlations may be treated by the theory of probability provided they can be established accurately. Using our present methods, this is cost prohibitive; we must therefore postpone a rigorous discussion of these correlations to a future paper. From a practical point of view we note that the lists which are highly correlated are easy to recognize in some cases. In a number of instances, these highly correlated lists are for well-identified atomic species, Fe I, Nd II, etc., which have already been eliminated from possible consideration vis-à-vis the null hypothesis that *none* of the species are present.

If we exclude the lists which are correlated or well identified, we find some 226 such lists remain out of an original 418. The number N of species sought should therefore be between $418-54=364$ and 226. We will surely overestimate the number of chance coincidences if we work with the first figure, while we will *underestimate* it if we use the second. In fact, the overlap among marginal coincidences on lists which contain some of the same lines is not large. Of the $364-226=138$ partially correlated lists some number should be added to obtain a workable value of N. We shall use $N=325$ in what follows. If $\Omega=0.03$, then the expected number of chance coincidences is 11, 10, or 7 if $N=364$, 325, or 226. This will give some insight into the uncertainties introduced by these correlations.

VII. MARGINAL COINCIDENCES IN R AND

There are nine marginal entries in Table 2, and an additional six described by the "0" symbol, and five described by the "+" in Table 1. While WCS and the statistical model do not discriminate among species with similar confidence levels, Table 3 gives a very strong indication that all 20 of the above cases cannot be due to chance. Our best estimates at the parameters $N=325$ and $\Omega=0.028$ tell us that most probably nine of these marginal results are due to chance, while as many as 15 or 17 may be. The fact that nine is precisely the number of entries that we find in Table 2 is a good indication that our statistical model is functioning well. Again, the five "+" signs comport well with $K_{max}=15$ for our best estimates of N and Ω. From the WCS alone, we could estimate that another ~six marginal identifications are credible, but we can only be 95% confident of five out of the 20 cases.

For R And, the work of Merrill and Greenstein identifies the individual cases. It is worth pointing out that a great deal of sophistication is not necessary to enable one to choose He I, Ne I, Al II, etc., as unlikely identifications in a star as cool as R And. One should always examine the *particular* coincidences, their strengths and multiplet membership, etc., in attempting to reach conclusions about marginal identifications. However, even before this very tedious work has been carried out, WCS plus a minimal knowledge of stellar spectroscopy would have led us to make virtually the same identifications as Merrill and Greenstein.

The power of this technique is apparent.

TABLE 3

MAXIMUM, MINIMUM, AND EXPECTED SPURIOUS COINCIDENCES

N, Ω	(364, 0.025)	(364, 0.030)	(325, 0.028)	(226, 0.030)
K_{MAX} ...	15	17	15	12
$\langle K \rangle$	9	10	9	6
K_{MIN} ...	4	5	4	2

TABLE 4

DISSOCIATION ENERGIES OF RARE EARTH OXIDES

Molecule	D(eV)
Yo	7.3
Lao	8.2
Ceo	8.2
Pro	7.7
Ndo	7.3
Smo	5.9
Euo	4.8
Gdo	7.4
Hoo	6.4

VIII. ON THE ABSENCE (WEAKNESS) OF La II AND Ce II

The standard interpretation of the anomalous abundances in R And is that *s*-processed material has been mixed, recently, to the surface of the star. If this is true, then cerium must be comparable or in somewhat greater abundance than the even-Z lanthanides neodymium and samarium, which are well identified in the stellar spectrum. Lanthanum, though odd-Z, should also be detectable (as La II) when Nd II and Sm II are present at high confidence levels unless it is genuinely underabundant. In fact, neither La nor Ce is well identified.

A reasonable interpretation of these observations that is consistent with the traditional picture of R And is that cerium and lanthanum are locked up in oxides while the intermediate and heavier lanthanides are not. Table 4 gives the dissociation energies of yttrium and lanthanide oxides from the recent compilation of Huber and Herzberg (1979). We see that there is a $\sim$0.5 eV difference between the dissociation energies of cerium and lanthanum oxides and the next highest dissociation energies. This small difference can be significant at the (low) temperatures extant in the atmosphere of R And. At 2000 K, the difference in the exponential term $(\exp - D/kT)$ is more than a factor of 10.

IX. CONCLUSIONS

The method of wavelength coincidence statistics (WCS) has been applied to the line lists of Merrill and Merrill and Greenstein. It has been shown that WCS gives results in excellent agreement with the conclusions of these authors in their classical study. *We emphasize that a complete identification study should use* both *WCS and the traditional techniques*. No flaws in the Monte Carlo WCS techniques have been revealed by this comparative study apart from the obvious disadvantages the technique has in dealing with atomic spectra with small numbers of lines.

The statistics of the marginal results of WCS have been discussed in terms of a simple analytical model. The model allows realistic estimates to be made of the *number* of marginal coincidences to be expected. While this knowledge does not allow one to predict *which* of the marginal results are likely to be real, it does provide a sound basis attacking these difficult cases. The conservative procedure is that of writing off *all* marginal results. In some cases this is much too severe and discards useful information. Simple spectroscopic considerations can often give a strong indication of a spurious result among the marginal cases.

The identification of lines in complex stellar spectra can be a long and tedious job. The spectroscopist must use all of the tools at his disposal. Among these tools one of the more powerful and rapid is the method of wavelength coincidence statistics.

This study would have been impossible unless we had some assurance that the R And wavelengths were unbiased, i.e. measured without foreknowledge of identifications. We wish to thank Professor Greenstein for a letter discussing this matter which provided the basis for us to proceed with the present investigation. Dr. D. Davis Locanthi was kind enough to read a preliminary version of the manuscript and make a number of useful comments. We thank Mr. Richard Leighton for assisting with some of the programming, and the referee of this paper, who corrected a number of errors. C.R.C. wishes to thank the Director and staff of the Dominion Astrophysical Observatory, where part of the work was carried out. This research was supported in part by the U.S. National Science Foundation.

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