

The IAU (1976) System of Astronomical Constants

TRUDPERT LEDERLE

Astronomisches Rechen-Institut, Heidelberg

Summary. A system of astronomical constants can be defined as a set of parameters whose adopted numerical values are needed for the reduction of observations. It should be a consistent set, i. e. the theoretical relations known between the constants have to be exactly fulfilled. The IAU (1976) System of Astronomical Constants will be introduced in 1984 together with the FK5. Finally a detailed explanation for the change of the coefficients in the conventional formula for GMST at 0^h UT1 is given.

1. Introduction

The purpose of a fundamental catalogue is to provide the best approximation to an inertial system to which the locations of celestial bodies and their variations on the sky can be referred. The coordinates of that system at a certain instant of time are represented by the positions and proper motions of a number of stars as given in the fundamental catalogue. For the reduction from one epoch to another of observations which are made from the Earth moving with respect to the inertial system, a set of parameters is required; this set is called the system of astronomical constants. All astrometric work is in fact based on these two elements — a fundamental catalogue and a system of astronomical constants.

2. Definition

It is much easier to give a negative characterization rather than an exact definition of what a system of astronomical constants should be. From its special purpose, it follows that it does not include all the constants of importance in astronomy; it is restricted to astrometric parameters. Because they are to be used for reductions in general, constants which are merely related to the motion of a single body, e. g. the elements of the orbit of a planet, are not part of the system. It should be noted that the importance of a constant may change in the course of time, and thus some may be omitted and replaced by others in the system. Finally it has to be pointed out that several parameters of the system are

variable with time; in such cases the values of these constants for a certain epoch (which has to be specified) are included in the system but not their variations.

The designation “system” is not unimportant, it emphasizes the consistency: All relationships which are known between the constants have to be fulfilled by the adopted numerical values. In recent systems this requirement is met by distinguishing between “primary” and “derived” constants. The numerical values for the first category only are fixed independently; those for the latter are derived from them by the relationships. It must be admitted that this principle may lead to numerical values adopted for the derived constants which may not always be identical with the best values provided by observations.

The preceding remark gives the opportunity for clarifying a wide-spread misunderstanding: The adopted system of astronomical constants is *not* a list of the best values for ever. The required consistency prevents that, and there is another reason: Because most of the research in astrometry has to be based on results of observations which cover a long period of time, it would be extremely troublesome, if one would have to take into account that different values of constants were used in reducing them. Changes of the conventional values of the constants are therefore avoided as long as possible, and if changes have to be made, they are introduced for all constants simultaneously. It is wise to make a change of the whole system of astronomical constants only when the effects of erroneous values on the results can no longer be corrected differentially. Of course this strong principle merely concerns fundamental observations and the data given in the national and international almanacs. It is quite obvious that everybody may use the best known value for a constant in his research if he regards that necessary; but the deviation from the adopted value should then be explicitly stated.

3. Historical Review

It seems to be of some interest to give here a brief survey on the conventional systems of constants. There have been in fact not more than three which were adopted by international agreement:

(i) The first system consisted mainly of Newcomb's values and has been adopted by two conferences held at Paris in 1896 and 1911 (Bureau des Longitudes 1896 and 1911). This system of constants was valid for more than half a century.

(ii) In 1950 an international colloquium on fundamental constants was held in Paris again. The only change which was decided was the introduction of the concept of Ephemeris Time. In 1963 the IAU Symposium No. 21 (1965) prepared the adoption of the IAU System of Astronomical Constants at the General Assembly in Hamburg in 1964 (IAU 1964); this system was introduced in 1968.

(iii) As a consequence of the rapid development of new methods of observations and reductions and of the demands for higher accuracy the lifetime of this second system of constants will be fairly short. Prepared by the IAU Colloquium No. 9 at Heidelberg in 1970, the new IAU (1976) System of Astronomical Constants has been adopted at the IAU General Assembly at Grenoble in 1976 (IAU 1976). Some supplementary resolutions have been endorsed at Montreal in 1979 (IAU 1979), and the complete system will be introduced in 1984.

4. The IAU (1976) System of Astronomical Constants

This new system of astronomical constants which is to replace the system adopted in

1964, is given in the “Joint Report of the Working Groups of IAU Commission 4 on Precession, Planetary Ephemerides, Units and Time-Scales“, which has been adopted at the IAU General Assembly at Grenoble in 1976. The Report consists of the following parts:

I. The Introduction which describes the background of the Report.

II. The Resolution by which the Working Group recommends that the new system “shall be used in the preparation of the fundamental catalogue FK5 and of the national and international ephemerides for the years 1984 onward, and in all other relevant astronomical work“.

III. Recommendations (No. 1: IAU (1976) System of Astronomical Constants; No. 2: The new standard epoch and equinox; No. 3: The fundamental reference frame; No. 4: The procedures for the computation of apparent places and the reduction of observations; No. 5: Time-scales for dynamical theories and ephemerides; No. 6: Other quantities for use in the preparation of ephemerides).

IV. Notes on Recommendations.

No detailed description of this Report shall be given here. Some information which is in particular relevant to the fundamental reference system will be given in the next paragraph, together with the discussion of a Resolution adopted in 1979 at Montreal.

However it seems to be of general interest to give the complete list of the numerical values of the constants of the IAU (1976) System which are part of Resolution 1 of the Joint Report, in Table 1. The following remarks may be made: For the constant of nutation (No. 11) the value given is that adopted in 1979; the constant of gravitation has been changed by 2 units of the last decimal.

In the notes to the 1976 System, as well as to the 1964 System, limits are given between which each constant is believed to lie; to these limits correspond the estimated deviations ϵ_{76} and ϵ_{64} in Table 1. The “corrections“ in the sense 1976 minus 1964 are all smaller than the corresponding estimates ϵ_{64} . This shows that the estimations were fairly good; one may, therefore, expect that the corresponding quantities ϵ_{76} are also realistic.

5. The Consequences on the Fundamental Reference Frame

Some of the consequences on the fundamental system of the introduction of the IAU (1976) System of Astronomical Constants are already mentioned in the contribution by Fricke (1980). It may be useful to draw attention to two items: (a) Changes in the reduction from mean to apparent star places, (b) Clarification of the reasons for the change in the expressions of UT1 in Terms of GMST.

The requirement of higher precision in the calculation of the reductions to apparent places is a clear demand in view of the much improved accuracy of observations, in particular with modern methods. This is explicitly expressed in Recommendation 4(c) of the Joint Report as adopted in 1976. Moreover, the following changes will be made in the reductions (1976 and 1979 in the following enumeration refers to the respective Recommendation):

- (i) New expressions for precession (1976, Rec. 1; see also Lieske et al. 1976);
- (ii) new expressions for nutation (1976, Rec. 4 (b), and 1979, Res. 2, of Comm. 4 etc.) according to the recommendations of the Working Group on Nutation (1979 IAU Theory of Nutation; see Seidelmann 1980);

Table 1. The IAU (1976) System of Astronomical Constants

| | Diff. | | |
|--|---|-------------|-----------------|
| | ϵ_{76} | $(76)-(64)$ | ϵ_{64} |
| I. Defining Constants | (in units of the last decimal) | | |
| 1. Gaussian gravitational constant | k = 0.017 202 098 95 | 0 | 0 |
| II. Primary constants | | | |
| 2. Speed of light | c = 299 792 458 m s ⁻¹ | 1.2 | 42 |
| 3. Light-time for unit distance | τ_A = 499.004 782 s | 6 | 7 218 |
| 4. Equatorial radius for Earth | a_e = 6 378 140 m | 5 | 20 |
| 5. Dynamical form-factor for Earth | J_2 = 0.001 082 63 | 1 | 7 |
| 6. Geocentric gravitational constant | GE = 3.986 005 x 10 ¹⁴ m ³ s ⁻² | 3 | 25 |
| 7. Constant of gravitation | G = 6.672 x 10 ⁻¹¹ m ³ kg ⁻¹ s ⁻² | 4 | 2 |
| 8. Ratio of mass of Moon to that of Earth | μ = 0.012 300 02 | 5 | 10 |
| 9. General precession in longitude, per Julian century, at standard epoch 2000 | p = 5 029".096 6 | 1534 | +11 000 |
| 10. Obliquity of the ecliptic, at standard epoch 2000 | ϵ = 23°26'21".448 | 102 | 0 |
| 11. Constant of nutation, at standard epoch 2000 | N = 9".205 5 | 3 | - 65 |

III. Derived constants

| | | | | | |
|---|---|-----|---|-------|-------|
| 12. Unit Distance | $c\tau_A = A = 1.495\,978\,70 \times 10^{11} \text{ m}$ | 2 | - | 2 130 | 3 000 |
| 13. Solar parallax | $\text{arc sin}(a_e/A) = \pi_\odot = 8''.794\,148$ | 7 | + | 98 | 290 |
| 14. Constant of aberration, for standard epoch 2000 | $\kappa = 20''.495\,52$ | 0.2 | - | 28 | 40 |
| 15. Flattening factor for the Earth | $f = 0.003\,352\,81$ | 2 | - | 9 | 91 |
| 16. Heliocentric gravitational constant | $GS = 1.327\,124\,38 \times 10^{20} \text{ m}^3 \text{ s}^{-2}$ | 5 | - | 5 562 | 8 000 |
| 17. Ratio of mass of Sun to that of Earth | $S/E = 332\,946.0$ | 3 | - | 120 | 230 |
| 18. Ratio of mass of Sun to that of Earth + Moon | $(S/E)/(1+\mu) = 328\,900.5$ | 3 | - | 115 | 215 |
| 19. Mass of the Sun | $S = 1.989\,1 \times 10^{30} \text{ kg}$ | 12 | - | 7 | 22 |

IV. System of planetary masses

20. Ratios of mass of Sun to those of the planets and their satellites (in brackets: Differences (76)-(64) in % of the IAU (1964) values)

| | | | | | |
|--------------|-----------|---------|---------|-----------|---------|
| Mercury | 6 023 600 | (+0.39) | Saturn | 3 498.5 | (-0.09) |
| Venus | 408 523.5 | (+0.13) | Uranus | 22 869 | (0) |
| Earth + Moon | 328 900.5 | (-0.14) | Neptune | 19 314 | (0) |
| Mars | 3 098 710 | (+0.17) | Pluto | 3 000 000 | (- -) |
| Jupiter | 1 047.355 | (0) | | | |

(iii) stellar aberration computed from the total velocity of the Earth referred to the barycentre of the solar system (1979, Rec. 4 (a)); this means also the end of an anachronism: the so-called E-terms, i. e. the part of the aberration depending on the ellipticity of the Earth's orbit will be removed from the mean positions and included in the reduction from mean to apparent places.

Although not expressis verbis mentioned, the high-precision reduction will include relativistic effects.

One particular point should be discussed here in more detail because misunderstanding should be avoided. In Recommendation 3 (c) of the Joint Report 1976, it had been recommended that "the expression for Greenwich mean sidereal time at 0^h UT shall be amended by the same equinox correction and motion as adopted for the FK5 in order to avoid a discontinuity in UT". That recommendation was specified in 1979 by the following Resolution (The numerical values have been adjusted according to the recent provisional expression for E (T) given by Fricke, 1980):

"In considering that it is planned to introduce the IAU (1976) System of Astronomical Constants, the 1979 IAU Theory of Nutation, and the equinox of the FK5 on 1984 January 1, it is recommended that:

(a) the relationship between mean sidereal time und UT1 be modified so that there is no change in either value or rate of UT1, due to a correction to the zero point of right ascensions of the FK4 and a correction to the motion of the zero point, to be introduced in FK5;

(b) the new (provisional) expression for Greenwich mean sidereal time of 0^h UT1 be

$$\text{GMST of 0}^{\text{h}} \text{ UT1} = 6^{\text{h}}41^{\text{m}}45^{\text{s}}8245 + 8\,640\,184^{\text{s}}627 T_{\text{u}} + 0^{\text{s}}0929 T_{\text{u}}^2,$$

where T_{u} is the number of Julian centuries of 36525 days of Universal Time elapsed since 1900 January 0, 12^h UT1 (JD 2 415 020.0). This expression is rigorously equivalent to the following

$$\text{GMST of 0}^{\text{h}} \text{ UT1} = 6^{\text{h}}41^{\text{m}}50^{\text{s}}5414 + 8\,640\,184^{\text{s}}8128 T_{\text{u}} + 0^{\text{s}}0929 T_{\text{u}}^2,$$

where T_{u} is measured from 2000 January 1, 12^h UT1 (JD 2 451 545.0)."

The reason for the necessity of the change as adopted by that resolution may be best understood, if one refers to the derivation of UT1 from meridian observations. Let α be the right ascension of a star, whose transit through the meridian at a place with longitude λ (West of Greenwich) is observed, θ the Greenwich mean sidereal time of preceding 0^h UT1 and χ the ratio of mean solar to mean sidereal time. Then, after correction for polar motion, one obtains

$$\text{UT1} = (\alpha - \theta + \lambda)\chi \quad (1)$$

for the universal time at the moment of the transit; α and θ may be expressed as functions of time T:

$$\begin{aligned} \alpha &= \alpha_0 + \dot{\alpha}T + \ddot{\alpha}T^2 \\ \theta &= \theta_0 + \dot{\theta}T + \ddot{\theta}T^2 \end{aligned} \quad (2)$$

and thus

$$UT1 = [(\alpha_0 + \dot{\alpha}T + \ddot{\alpha}T^2) - (\theta_0 + \dot{\theta}T + \ddot{\theta}T^2) + \lambda]\chi \quad (3)$$

The introduction of a correction to the equinox, the zero point of right ascensions,

$$E = E_0 + \dot{E}T$$

has to be added to all right ascensions. Hence, we have first to replace (3) by

$$UT1 = \llbracket [(\alpha_0 + E_0) + (\dot{\alpha} + \dot{E})T + \ddot{\alpha}T^2] - (\theta_0 + \dot{\theta}T + \ddot{\theta}T^2) + \lambda \rrbracket \chi \quad (4)$$

Second, we have the choice between 3 possibilities:

- (a) a discontinuity in UT1 and its rate proportional to E ;
- (b) correcting either all longitudes λ , or
- (c) correcting the expression for θ by E in order to compensate the term E added to α .

It is obvious that (b) has to be excluded, because this would mean not only a constant correction to all longitudes but also a motion which is unacceptable. Since a discontinuity in UT1 is undesirable, because UT1 is practically the measure of the rotation of the Earth, there remains only the modification of the conventional expression for θ , GMST of 0^h UT1 as recommended:

$$\theta_{\text{new}} = \theta_{\text{old}} + E = (\theta_0 + E_0) + (\dot{\theta} + \dot{E})T + \ddot{\theta}T^2 \quad (5)$$

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