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EFFICIENCY OF THE BOWEN FLUORESCENCE MECHANISM IN STATIC NEBULAE

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ABSTRACT

A simplified theory of the Bowen resonance fluorescence mechanism is presented for both the pumping of the O III Bowen lines by He II $\lambda304$ and for the pumping of the N III Bowen lines by O III $\lambda374$. Theoretical results, calculated on the basis of an escape probability approximation for resonance line transfer, obey simple scaling laws. The O III Bowen fluorescence yields calculated for a model planetary nebula agree well with those obtained in more rigorous calculations by Weymann and Williams and by Harrington. Intensities of O III and N III Bowen lines observed in the spectra of planetary nebulae and compact X-ray sources are discussed.

Subject headings: atomic processes — nebulae: planetary

I. INTRODUCTION

The Bowen resonance fluorescence mechanism was first suggested (Bowen 1934, 1935) to explain the anomalous strengths of certain emission lines of O III and N III observed in the spectra of planetary nebulae. The mechanism works because of an uncanny coincidence in wavelength which permits the He II $\lambda303.783$ resonance line (hereafter called Lα) to pump the resonance line O III $\lambda303.799$ (hereafter called $O1$). The resonantly excited O III may subsequently emit one or two of the Bowen lines, the strongest of which are $\lambda\lambda3444$, 3133, 3341, 3312, 3047, 3760 (Osterbrock 1974), and other EUV lines. One of these, O III $\lambda374.436$ (hereafter denoted $O4$), may in turn pump N III through a resonance doublet, N III $\lambda\lambda374.434$, 374.441 (hereafter denoted $N1$ and $N2$, respectively), followed by the emission of one of the N III $\lambda\lambda4634$, 4641, or 4642 Bowen lines. Grotrian diagrams indicating the relevant levels and transitions of O III and N III and notation are shown in Figures 1 and 2.

Early attempts to model the Bowen fluorescence process in planetary nebulae (Menzel and Aller 1941; Hatanaka 1947; Unno 1955) were hampered by a lack of knowledge of atomic processes and of the ionization structure of the nebula. Weymann and Williams (1969; hereafter WW) constructed the first accurate models for the O III resonance pumping. They computed the O III Bowen yield, y_B, defined as the total number of O III ($3p$–$3d$) Bowen photons produced per Lα photon, and found good agreement with observed line intensities in a model for the planetary nebula NGC 7027. They also noticed a discrepancy between the observed and predicted ratio of O III $\lambda3341$/O III $\lambda3444$ line strengths, and suggested that it could be explained if the $2s^22p3p$ levels are depleted by transitions to the $2s2p^3$ levels. Their results were confirmed by Harrington (1972). Recently, Saraph and Seaton (1980) have made improved calculations of the radiative transitions of O III and have shown that the theoretically predicted O III line ratios agree well with the observed ratios.

The advent of X-ray astronomy has given renewed impetus to the study of the Bowen mechanism. The brightest emission lines observed in the spectra of the optical counterparts of galactic X-ray sources are often N III $\lambda4641$ and He II $\lambda4686$ (McClintock, Canizares, and Tarter 1975). The suggestion that the strength of the N III lines is due to the Bowen mechanism was supported by model calculations (McClintock *et al.*; Hatchett, Buff, and McCray 1976) of the ionization structure of a nebula surrounding an X-ray source, in which He II, O III, and N III are coincident throughout a large region. This suggestion was controversial, because the N III $\lambda\lambda4634$–4642 emission can also be enhanced by dielectronic recombination in the atmosphere of an Of star (Mihalas and Hummer 1973). However, the observation by Margon and Cohen (1978) of the O III $\lambda\lambda3444$, 3760 Bowen lines in the spectrum of the X-ray binary HZ Her–Her X-1 suggests that the Bowen mechanism is the cause of the N III $\lambda\lambda4634$–4642 emission as well. Recently, strong O III Bowen emission lines have also been seen in the spectra of the transient X-ray sources A0620-00 (Oke and Greenstein 1977) and Cen X-4 (Canizares, McClintock, and Grindlay 1980) and in the optical counterparts of three X-ray bursters (Canizares, McClintock, and Grindlay 1979).

What is the significance of the Bowen mechanism in cosmic X-ray sources? We are forced to address this question in constructing detailed models for gaseous nebulae surrounding X-ray sources (Kallman and McCray 1980), including the effects of line emission and line transfer. Our calculations show that a significant fraction of the soft X-ray luminosity absorbed by the nebula is converted into He II Lα emission, which controls the ionization and temperature structure of the nebula until it is converted into optical and UV radiation that can escape from the nebula. Because the Bowen mechanism has a major effect on the fate of the Lα photons, we need to understand the Bowen mechanism in order to

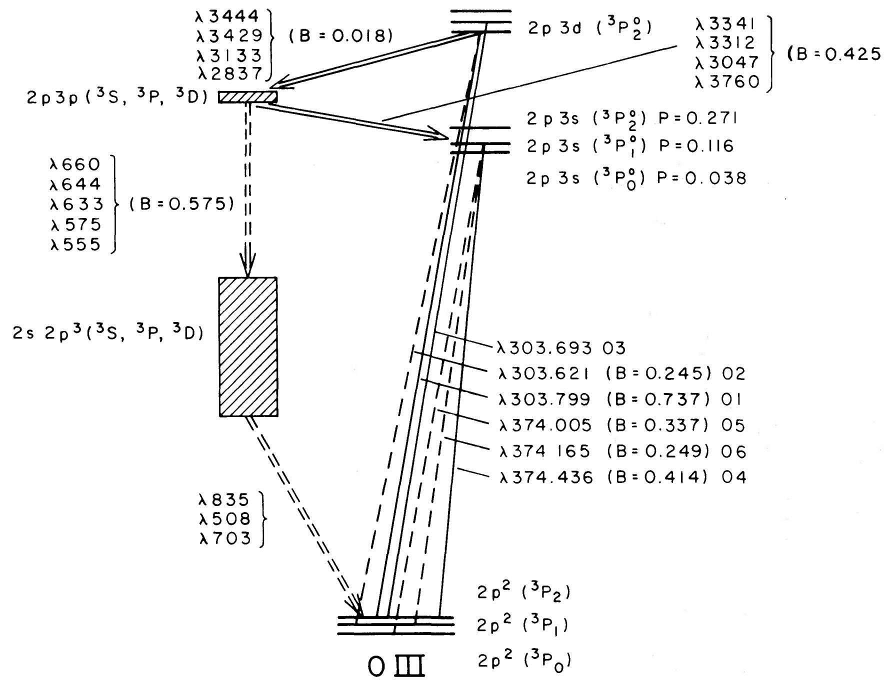

FIG. 1.—Grotrian diagrams for the O III transitions involved in the Bowen mechanism. Transitions subject to resonance absorption by He II Lα or N III λ374 are shown as solid lines. Other important transitions are shown as dashed lines. Wavelengths, branching ratios B per excitation to the upper level, and notation for the lines are specified. The probabilities of transitions to the various $2p3s(^3P^o)$ levels following emission of one of the O III upper Bowen photons are denoted by P (Saraph and Seaton 1980).

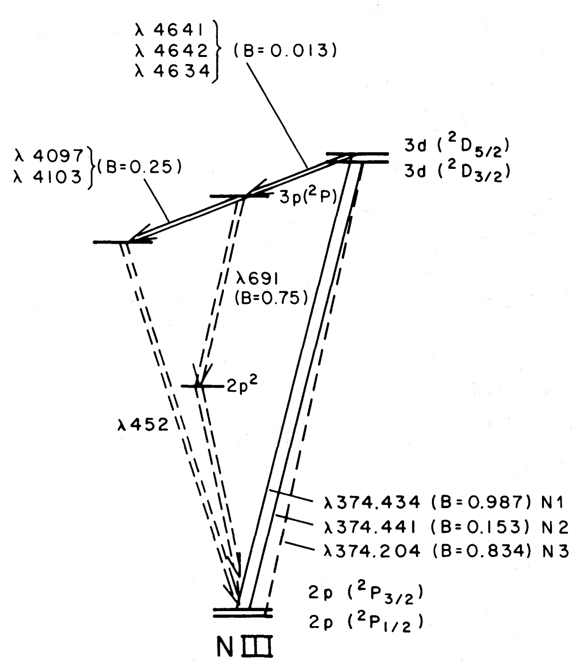

FIG. 2.—Grotrian diagrams for the N III transitions involved in the Bowen mechanism. Notation is the same as in Fig. 1.

interpret the emergent optical and UV spectra. In addition, the O III and N III Bowen lines themselves may be sensitive indicators of the physical conditions in the nebula.

Although the theory of the Bowen mechanism appears to be satisfactory for interpreting the O III lines in planetary nebulae, we encountered two difficulties in applying it to X-ray nebulae. First, the detailed theory of WW for the O III pumping in planetary nebulae is too elaborate and cumbersome to use in an exploratory investigation of X-ray nebulae, characterized by a broad range of uncertain parameters. Second, we were unable to find any detailed theory for the efficiency of the second Bowen resonance, whereby O III pumps the N III Bowen lines. In this paper, we present such a theory for both the O III and the N III Bowen lines. Our essential simplification is to approximate the spatial transfer of resonance lines by an "escape probability" formalism, which is known to give results in remarkably good agreement with more exact solutions of the transfer equation (see Hummer and Rybicki 1971). This approximation permits us to express the efficiency of each Bowen mechanism as a function of a few parameters characterizing a nebular model. One can convolve our results with the ionization structure of a complex nebular model to obtain the integrated yields of the Bowen lines. In § II we discuss the model, the approximate line transfer equations, and their formal solutions. In § III we present and interpret our results. As specific examples, we compare our results with those obtained by WW for their model of NGC 7027, and we present an illustrative calculation of the Bowen yield of a model nebula surrounding a compact X-ray source.

We emphasize that, although we were motivated to study this problem by our interest in X-ray sources, the results presented here are general and may be applied to any kind of nebula in which the Bowen mechanism occurs. However, as WW have pointed out, velocity gradients in the nebula may increase line escape probabilities, substantially suppressing the efficiency of the Bowen mechanism if turbulent or flow velocities become comparable to thermal velocities. These effects will be discussed further in § IV. Therefore, the present theory applies only to quasi-static nebulae. We shall examine the effects of velocity gradients in a subsequent paper.

II. THE MODEL

a) *Parameters*

We first consider the simple model of a semi-infinite, uniform slab of gas characterized by the following parameters: gas temperature T, atomic densities of relevant ions, $n_{\rm He\,II}$, $n_{\rm O\,III}$, $n_{\rm N\,III}$, and density of continuum absorbers n_A, defined as the equivalent density of H I atoms required to produce the actual continuum absorption coefficient at $\lambda 304$ (for example, if some He I is present, $n_A = n_{\rm H\,I} + 9.3 n_{\rm He\,I}$). As we shall show, the results at a depth L from the surface can be characterized by ratios of these parameters and by the central optical depths of the resonance lines. For purposes of illustration, we choose the following nominal values of the defining parameters: $T = 10^4$ K, $n_A/n_{\rm He\,II} = 0.10$, $n_{\rm O\,III}/n_{\rm He\,II} = 6.8 \times 10^{-3}$, $n_{\rm N\,III}/n_{\rm O\,III} = 8.5 \times 10^{-2}$, He II line-center optical depth $\tau_{\rm L\alpha}(0) = 10^5$, and $O4$ central optical depth $\tau_{O4}(0) = 45$. Given standard cosmic abundances, these values are typical of those found in the region of a planetary nebula in which the Bowen lines are produced, where the elements are predominantly in the ionization stages H II, He II, O III, and N III (Williams 1968).

The opacity in a resonance line of frequency $\nu_{i\alpha}$ due to an ion of mass M_i and density n_i is given by

$$\tau_{i\alpha}(x) = \kappa_{i\alpha}(x) L \,, \tag{1}$$

where

$$x = (\nu - \nu_{i\alpha})/\Delta\nu_{i\alpha} \tag{2}$$

is the frequency shift from line center in Doppler units, $\Delta\nu_{i\alpha} = (2kT/M_i c^2)^{1/2} \nu_{i\alpha}$. The line absorption coefficient is given by

$$\kappa_{i\alpha}(x) = \frac{\pi e^2}{mc} \frac{f_{i\alpha} n_i}{\Delta\nu_{i\alpha}} \phi_{i\alpha}(x) \,, \tag{3}$$

where $f_{i\alpha}$ is the oscillator strength and $\phi_i(x)$ is the normalized line-profile function. Figure 3 shows, for the stated nominal parameters, the normalized line absorption coefficients, $\alpha_{i\alpha}(x) \equiv \kappa_{i\alpha}(x)/\kappa_{\rm L\alpha}(0)$, of the resonance lines involved in the Bowen pumping of O III. (The fine-structure levels of the O III ^3P ground state are assumed to be populated according to statistical weights, which should be true if $n_e \gtrsim 10^3$ cm^{-3}.) Note that the overlap is not perfect and that $\alpha_{\rm L\alpha}$ is much greater than α_{O1} at line center, owing to the greater abundance of He II. The opacity in Lα is so great that the damping wings, evident for $|x| > 3$, are important. The dashed vertical lines indicate the frequency shifts at which Lα becomes transparent, $\tau_{\rm L\alpha}(x) < 1$. Also shown in this figure is the absorption profile due to the O III $\lambda 303.693$ ($O3$) line, which can absorb Lα photons and pump the Bowen lines. However, as shown by Harrington (1972), absorption by this line makes only a small contribution to the Bowen efficiency, and so we neglect it in the rest of this discussion. Figure 4 shows the line absorption coefficients responsible for the Bowen pumping of N III by $O4$, with the same normalization. In this case N III has two resonance lines, $N1$ and $N2$, that overlap the $O4$ line. Again, the dashed vertical lines indicate the frequency at which the lines become transparent. In this case, the resonance opacities are sufficiently small that the damping wings of the lines (which lie below the graph) are irrelevant.

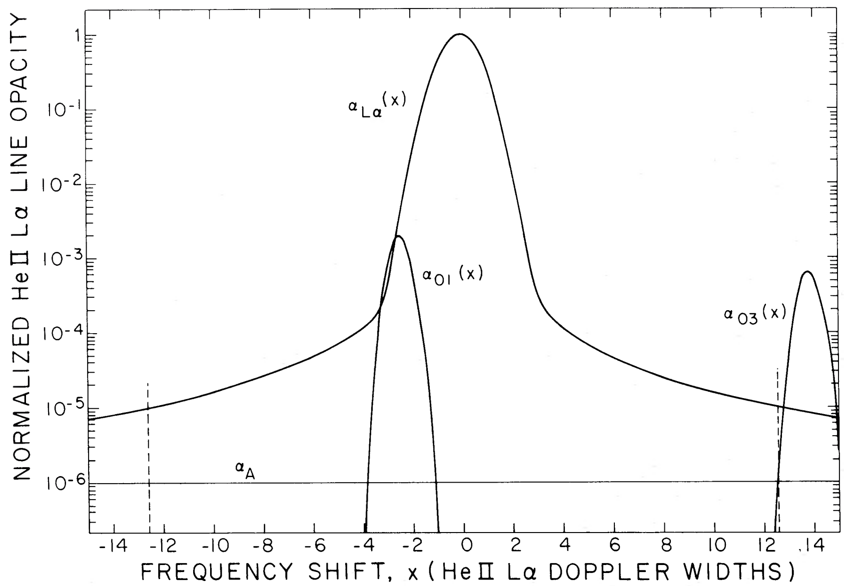

FIG. 3.—Normalized line absorption coefficients, $\alpha_{L\alpha}$, α_{O1}, α_{O2}, and continuum absorption α_A, as functions of x, the frequency shift from the Lα line-center measured in He II Doppler widths. Assumed abundances are given in the text, and the temperature is 10^4 K. The dashed vertical lines correspond to $\tau = 1$, assuming $\tau_{L\alpha}(0) = 10^5$.

b) *Spectroscopy*

Consider first the pumping of O III by He II Lα. Figure 1 gives the relevant spectroscopic information: transitions responsible for resonant pumping and the Bowen lines are indicated by solid lines; other important transitions are indicated by broken lines. The resonance transition O III λλ304 is a multiplet, only one member of which ($O1$) overlaps significantly with Lα. A trapped Lα photon has the following possible fates: first, it may escape from the nebula by diffusing to the wings of the He II line, with fractional probability (per Lα resonance scattering) $\beta_{L\alpha}$; second, it may be absorbed by H I atoms or other continuum absorbers, with fractional probability β_A; third, it may pump the $2p3d$ ($^3P_2{}^o$) level of O III, with fractional probability β_{O1}. The excited O III can decay in several ways, with branching ratios B_α indicated on Figure 1. First, it can simply reemit the resonance photon that pumped it. Second, it can emit the other O III resonance photon, denoted $O2$, which may also be subject to resonant trapping, absorption, escape, or Bowen fluorescence. Third, it can decay to one of the six $2p3p$ levels, indicated by the upper crosshatched zone in Figure 1, emitting one of the upper O III Bowen lines, λλ3444, 3133, 2837, etc., with a net branching ratio $B = 0.018$. (Grotrian diagrams for O III indicating more details of these transitions may be found in Harrington 1972 and Osterbrock 1974.)

The $2s^22p3p$ levels of O III may decay to one of the $2s^22p3s$ levels, emitting one of the lower O III Bowen lines, λλ3341, 3312, 3347, 3760, or to one of the $2s2p^3$ levels, indicated by the lower crosshatched zone, emitting an XUV line photon. (These latter transitions account for the discrepancy mentioned in § I.) The $2s^22p3s$ levels are populated with relative

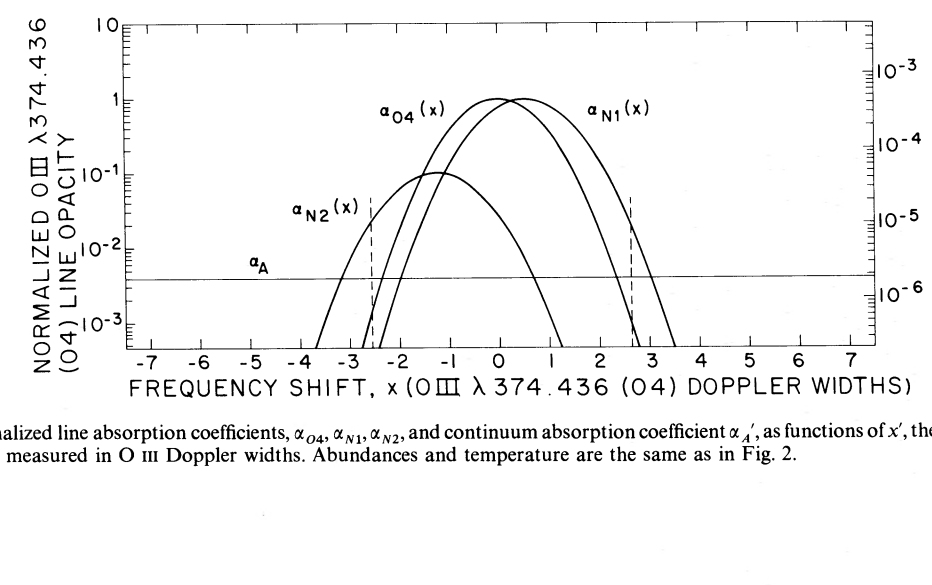

FIG. 4.—Normalized line absorption coefficients, α_{O4}, α_{N1}, α_{N2}, and continuum absorption coefficient α_A', as functions of x', the frequency shift from the $O4$ line center measured in O III Doppler widths. Abundances and temperature are the same as in Fig. 2.

probabilities p_i (per emission of an upper Bowen line), which have been calculated by Saraph and Seaton (1980). One of these levels, the $^3P_1$ level, decays via the $O4$ line that can resonantly pump N III, causing the secondary Bowen fluorescence of N III $\lambda\lambda$4634, 4641, and 4642.

Figure 2 indicates the relevant spectroscopy of N III. In this case, the resonant pumping can occur in two of the three resonance lines connecting the ground state to one of the $3d\,(^2D)$ levels, either of which can decay with the emission of one of the N III Bowen lines. (More details of these transitions may be found in Nussbaumer 1971.)

c) *Line Transfer*

In order to determine the Bowen yields, we need to know the probabilities of the various fates of photons created in the Lα and O III λ374 lines. These probabilities are found by solving the transfer equations for the respective line profiles and then convolving these profiles with appropriate absorption coefficients.

Consider first the Lα photons. The transfer equation for the specific intensity can be written (WW):

$$\frac{dI(x)}{d\tau_0} = -[\alpha_A + \alpha_{L\alpha}(x) + \alpha_{O1}(x)]I(x) + \alpha_{L\alpha}(x)/\phi_{L\alpha}(x)\int_{-\infty}^{\infty} R_{L\alpha}(x', x)J(x')dx' + (1 - \beta_{O1})\alpha_{O1}(x)/\phi_{O1}(x)\int_{-\infty}^{\infty} R_{O1}(x', x)J(x')dx' + \alpha_{L\alpha}(x)S(\tau_0)\,, \quad (4)$$

where τ_0 is the line-center optical depth, $J(x)$ is the mean intensity, $S(\tau_0)$ is the normalized source function of Lα photons, and α_A, $\alpha_{L\alpha}$, and α_{O1} are the normalized absorption coefficients due to continuum absorption, Lα resonance absorption, and $O1$ resonance absorption, respectively. The quantity β_{O1} is the probability per O III line scattering that the photon is lost and is given by $\beta_{O1} = \beta_{O2} + \beta_B$. The probability β_{O2} for conversion to the $O2$ (λ303.621) resonance line followed by escape of that line is given by (see Appendix):

$$\beta_{O2} = \frac{B_{O2}\epsilon_{O2}}{1 - B_{O2}(1 - \epsilon_{O2})}\,, \quad (5)$$

where B_{O2} is the branching ratio for emission of the line and ϵ_{O2} is the probability of escape per scattering of the line (to be discussed below). The probability of Bowen conversion per O III absorption is given by (see Appendix):

$$\beta_B = \frac{B_B}{1 - B_{O2}(1 - \epsilon_{O2})}\,, \quad (6)$$

where $B_B = 0.018$ is the branching ratio for emission of one of the upper Bowen lines of O III. The $O1$ redistribution function is well approximated by a Gaussian:

$$R_{O1}(x', x) = a^2/\pi \exp[-(ax + b)^2 - (ax' + b)^2]\,, \quad (7)$$

where $a = \Delta\nu_{L\alpha}/\Delta\nu_{O1}$, and $b = (\nu_{L\alpha} - \nu_{O1})/\Delta\nu_{O1}$. However, the L$\alpha$ opacity (cf. Fig. 3) is great enough that scattering in the damping wings is important, requiring the use of a more complicated function for $R_{L\alpha}(x', x)$ which accounts for partial redistribution (WW).

WW and Harrington (1972) obtained accurate numerical solutions of equation (4) using the Feautrier method for specific models of planetary nebulae. However, as we have remarked, the uncertainties in the ionization structure of most nebulae are so great that it makes sense to sacrifice some precision in the solution of the transfer equation in order to obtain a solution in a simple form that yields insight into the physics of the relevant processes. In that spirit, we make two major simplifications of equation (4). The first simplification is to make use of the fact that, when complete redistribution is assumed in the scattering, photons emitted in the core of Lα will scatter in a small spatial region near the emission point. Spatial transport will occur when a photon is scattered to a frequency where the nebula is optically thin, so the photon can escape in a single long flight (Osterbrock 1962). The probability per scattering for escape depends only on optical depth, and so escape is analogous to a destruction process. Accordingly, we replace equation (4) by:

$$0 = -[\alpha_c + \alpha_{L\alpha}(x) + \alpha_{O1}(x)]J(x) + (1 - \epsilon_{L\alpha})\alpha_{L\alpha}(x)/\phi_{L\alpha}(x)\int_{-\infty}^{\infty} R_{L\alpha}(x', x)J(x')dx' + (1 - \beta_{O1})\alpha_{O1}(x)/\phi_{O1}(x)\int_{-\infty}^{\infty} R_{O1}(x', x)J(x')dx' + \alpha_{L\alpha}(x)S(\tau_0)\,, \quad (8)$$

where $\epsilon_{L\alpha}$ is the escape probability per scattering. By comparing the two equations, we see that the term multiplied by $\epsilon_{L\alpha}$ acts as an effective destruction mechanism of Lα photons. In converting from equation (4) to equation (8), we have neglected the spatial transport that occurs in the wings of Lα at large optical depths. As will be shown, this does not introduce a significant error in the final answer, since most conversion occurs in the line core.

The second approximation is to replace the exact Lα redistribution function by an expression given by Jefferies and White (1960):

$$R_{\mathrm{L}\alpha}(x', x) = [1 - a(x)] \exp(-x'^2 - x^2)/\pi + a(x)\delta(x' - x) \exp(-x^2)/\pi^{1/2} , \tag{9}$$

where $a(x)$ is the function defined by Kneer (1975). This approximation simulates the effects of partial redistribution, but it permits a simple solution of equation (8).

It remains to define the escape probabilities of the resonance lines. For all resonance lines $\nu_{i\alpha}$, except the Lα line, the approximation of complete redistribution is valid, and the escape probabilities from a depth L in a semi-infinite slab are given approximately by (Mihalas 1978):

$$\epsilon_{i\alpha} = \frac{1}{2}\int_{-\infty}^{\infty} E_2[\tau_{i\alpha}\phi_{i\alpha}(x)]\phi_{i\alpha}(x)dx , \tag{10}$$

where $\tau_{i\alpha}$ is the line-center optical depth. But if $\tau_{i\alpha}(x) > 1$ for $x \gtrsim 3$, as can happen in the case of Lα, partial redistribution is important, and a more accurate expression for the escape probability is that calculated by Harrington (1973):

$$\epsilon_{\mathrm{L}\alpha} = (2.66\tau_{\mathrm{L}\alpha})^{-1} . \tag{11}$$

The inverse of this escape probability corresponds approximately to the mean number of scatterings required for a line photon to escape.

It is now a straightforward matter to solve the approximate equation (8) for $J(x)$. The redistribution terms are the products of unknown constants and known functions of x. Therefore, $J(x)$ can be written as an algebraic expression involving these constants. Equations defining the constants can then be obtained by evaluating the quadratures in equation (8). Given $J(x)$, the respective photon fates can be calculated in a straightforward manner, viz: O III Bowen yield,

$$y_{\mathrm{B}} = \beta_{\mathrm{B}} \int_{-\infty}^{\infty} J(x)\alpha_{O1}(x)dx ; \tag{12}$$

escape in $O2$,

$$y_{O2} = \beta_{O2} \int_{-\infty}^{\infty} J(x)\alpha_{O1}(x)dx ; \tag{13}$$

continuum absorption,

$$y_A = \alpha_A \int_{-\infty}^{\infty} J(x)dx ; \tag{14}$$

and escape in wings of Lα,

$$y_{\mathrm{L}\alpha} = \epsilon_{\mathrm{L}\alpha} \int J(x)\alpha_{\mathrm{L}\alpha}(x)dx . \tag{15}$$

Now consider the pumping of N III by the $O4$ photons. The equation for the $O3$ line profile analogous to equation (8) is

$$[\alpha_A' + \alpha_{O4}(x) + \alpha_{N1}(x) + \alpha_{N2}(x)]J(x) = \alpha_{O4}(x)S(\tau_0) + \sum_{i\alpha} (1 - \beta_{i\alpha})\alpha_{i\alpha}/\phi_{i\alpha}(x) \int_{-\infty}^{\infty} R_{i\alpha}(x', x)J(x')dx' , \tag{16}$$

where x is now the frequency shift from the O III line center measured in O III Doppler units, α_A' is the continuum absorption coefficient at $\lambda 374$, α_{O4}, α_{N1}, and α_{N2} are absorption coefficients due to the overlapping $O4$, $N1$, and $N2$ resonance lines, respectively, and the summation refers to these lines. In this case the approximation of complete redistribution is valid for all lines, so the redistribution functions are Gaussians as in equation (7), and the line escape probabilities are given by equation (10). The net losses per scattering in each of the resonance lines are given by (see Appendix):

$$\beta_{O4} = \frac{B_{O5}\epsilon_{O5} + B_{O6}\epsilon_{O6} + (1 - B_{O5} - B_{O6})\epsilon_{O4}}{1 - B_{O5}(1 - \epsilon_{O5}) - B_{O6}(1 - \epsilon_{O6})} , \tag{17}$$

$$\beta_{N1} = \frac{B_{N3}\epsilon_{N3} + (1 - B_{N3} - B_{\mathrm{B}}')\epsilon_{N1} + B_{\mathrm{B}}'}{1 - B_{N3}(1 - \epsilon_{N3})} , \tag{18}$$

$$\beta_{N2} = B_{\mathrm{B}}' + (1 - B_{\mathrm{B}}')\epsilon_{N2} , \tag{19}$$

where B_{O5} and B_{O6} are the branching ratios for production of the O III $\lambda 374.005$ and O III $\lambda 374.165$ resonance lines, respectively, B_{N3} and B_{B}' are the branching ratios for production of the N III $\lambda 374.204$ and N III Bowen lines, respectively, and the ϵ's are the corresponding line escape probabilities. The escape probability of the $O4$, $N1$, and $N2$ lines is equal to the probability that, given complete redistribution, a line photon will appear at a frequency where the net optical depth is less than or equal to 1.

Equation (16) is solved in the same way as equation (8). If the source function of O III pumping photons is $S(\tau_{O4})$, equations analogous to equations (12)–(16) give the respective yields of O III pumping photons converted to N III Bowen lines, absorbed by the continuum, or escaping in various line wings.

III. RESULTS

a) Uniform Slab Model

The general properties of the Bowen process are most easily understood by considering the simple model of a source of Lα photons at a depth L in a uniform, semi-infinite slab of gas. Consider first the pumping of O III by Lα. The fate of the Lα photons is determined by three parameters: the Lα line-center optical depth, $\tau_{L\alpha}$, the ratio $n_A/n_{\rm O\,III}$, and the ratio $n_{\rm O\,III}/n_{\rm He\,II}$.

Let us define τ_{cr} to be the optical depth where the yields for Bowen conversion and Lα escape are equal. Then Figure 5 shows the various yields as functions of the ratio $\tau_{L\alpha}/\tau_{cr}$ for fixed values of the ratios, $n_A/n_{\rm O\,III} = 16$, and $n_{\rm O\,III}/n_{\rm He\,II} = 6.8 \times 10^{-3}$. At small $\tau_{L\alpha}$, the photons scatter a few times before escaping to the wings of Lα (with yield $y_{L\alpha}$) or conversion to the $O2$ resonance line and escaping in the wings of that line. As $\tau_{L\alpha}$ is increased the probability of escape decreases; the photon remains trapped in the system of three resonance lines until it is either absorbed, with probability y_A, or converted to a Bowen photon, with probability y_B.

For $\tau_{L\alpha}/\tau_{cr} \gg 1$, escape in the wings of Lα is essentially precluded. We may estimate the critical optical depth τ_{cr} by equating the probability of Bowen conversion per Lα scattering to the probability of escape: $\epsilon_{L\alpha} = \beta_B \alpha_{O1} \Delta\nu_{O1}/\Delta\nu_{L\alpha}$, which yields

$$\tau_{cr} \approx 80\; n_{\rm He\,II}/n_{\rm O\,III}\,. \tag{20}$$

For Figure 5, $n_{\rm O\,III}/n_{\rm He\,II} = 6.8 \times 10^{-3}$, $\tau_{cr} = 1 \times 10^4$, and we see that for $\tau_{L\alpha}/\tau_{cr} \gg 1$ the Bowen yield becomes independent of $\tau_{L\alpha}$. Similarly, we may estimate the critical value, τ_{cr}'', of $\tau_{L\alpha}$ for which escape in the wings of the $O2$ line is precluded by equating the probability of escape in that line to the probability of Bowen conversion: $\beta_{O2} = \beta_B$, which yields $\tau_{cr}'' \approx 100\; n_{\rm He\,II}/n_{\rm O\,III}$. Since both τ_{cr} and τ_{cr}'' depend on $n_{\rm O\,III}/n_{\rm He\,II}$ in the same way, the curves in Figure 5 are universal functions of $\tau_{L\alpha}/\tau_{cr}$, provided that the ratio $n_A/n_{\rm O\,III}$ is fixed.

The dependences of the various yields on the ratio $n_A/n_{\rm O\,III}$, for fixed values of $\tau_{L\alpha}$, are shown in Figure 6. For $\tau_{L\alpha} \gg \tau_{cr}$, the competition is entirely between continuum absorption, which occurs primarily in the wings of Lα, and Bowen pumping, in the core of Lα (see Fig. 3). Since the line profile is essentially flat to a value $x \approx 10$, where escape can occur, we may estimate $y_B \approx \alpha_{O1}\beta_B/(\alpha_{O1}\beta_B + 10\alpha_A)$, which yields

$$y_B \approx \frac{1}{(1 + 2 \times 10^{-3}\; n_A/n_{\rm O\,III})}\,, \tag{21}$$

in agreement with the curve in Figure 6.

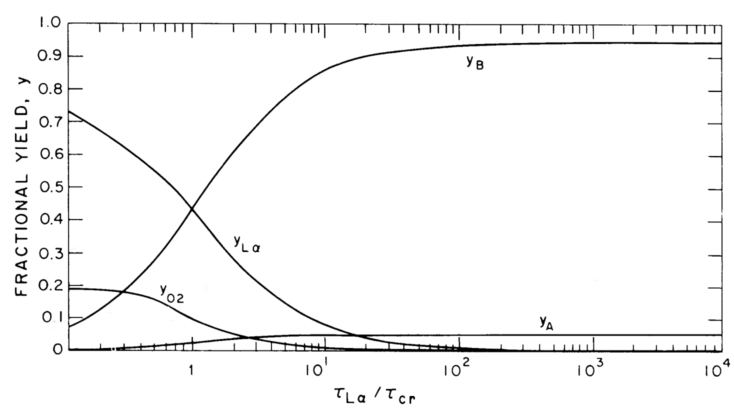

FIG. 5.—Fractional yields y for O III Bowen fluorescence, escape in the wings of the $O2$ line, and continuum absorption as functions of the ratio $\tau_{L\alpha}/\tau_{cr}$. Abundances are $n_{\rm O\,III}/n_{\rm He\,II} = 6.8 \times 10^{-3}$, $n_A/n_{\rm O\,III} = 16$, and the temperature is 10^4 K.

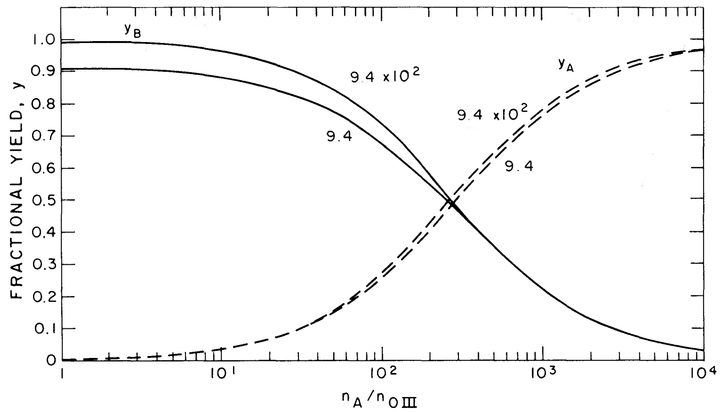

FIG. 6.—The yields, y_B, of O III Bowen fluorescence lines, and y_A, of continuum absorption as functions of the ratio $n_A/n_{O\,III}$, for fixed values of the ratio $\tau_{L\alpha}/\tau_{cr}$. The temperature is 10^4 K, and $n_{O\,III}/n_{He\,III} = 6.8 \times 10^{-3}$.

The behavior of the N III resonance fluorescence is entirely analogous, and the corresponding curves are shown in Figures 7 and 8. The critical $O4$ line-center optical depth to preclude line escape is given by

$$\tau_{cr}' = 3\, n_{O\,III}/n_{N\,III}\,, \tag{22}$$

and the curves of Figure 7 are universal functions of the ratio τ_{O4}/τ_{cr}'. For $\tau_{O4} \gg \tau_{cr}'$ the N III Bowen yield is given approximately by

$$y_B' \approx \frac{1}{(1 + 5 \times 10^{-3}\, n_A/n_{N\,III})} \tag{23}$$

in agreement with the curve in Figure 8.

b) *Planetary Nebula Model*

Now we address the problem of calculating the Bowen yields in models of actual nebulae, in which the defining parameters vary continuously through the source. To do this, we convolve our results of § III*a* with the distribution of parameters in a model for the source, according to a procedure described below. As a check on the validity of our theory, we compare our results with the results of more elaborate calculations by WW for a model planetary nebula.

In our procedure, we treat the results for the slab model as Green's functions, convolving the various local Bowen yields

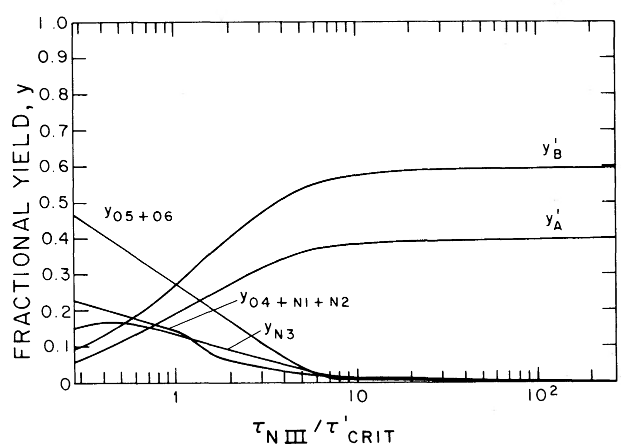

FIG. 7.—Fractional yields—N III Bowen fluorescence, y_B'; $O3$ resonance escape, y_{O4}; $O5$ plus $O6$ resonance escape, y_{O5+O6}; $N3$ resonance escape, y_{N3}; and continuum absorption, y_A'—as functions of the ratio τ_{O4}/τ_{cr}'. Abundances are $n_{N\,III}/n_{O\,III} = 2.5 \times 10^{-2}$, $n_A/n_{O\,III} = 1.9 \times 10^2$, and the temperature is 10^4 K.

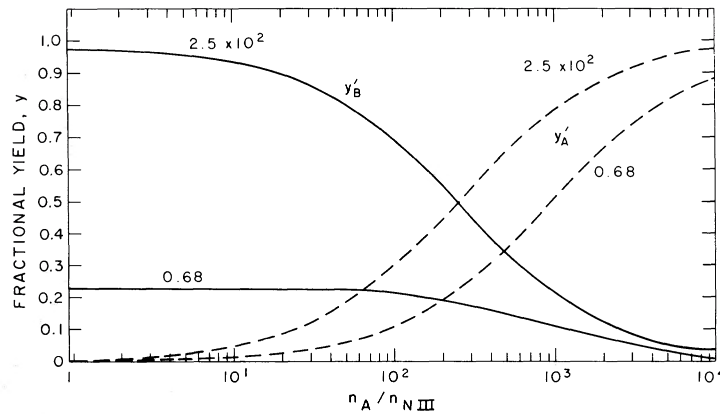

FIG. 8.—The yields, y_B', of N III Bowen fluorescence, and y_A', of continuum absorption, as functions of $n_A/n_{\rm N\,III}$, for fixed values of the ratio $\tau_{O4}/\tau_{\rm cr}'$. The temperature is 10^4 K and $n_{\rm N\,III}/n_{\rm O\,III} = 8.5 \times 10^{-2}$.

with the distribution of parameters through the source to calculate average yields for the nebula. For example,

$$\bar{y}_{\rm B} = \int y_{\rm B}(n_A, n_{\rm He\,II}, n_{\rm O\,III}, \tau_{\rm L\alpha}) S(\tau_{\rm L\alpha}) d\tau_{\rm L\alpha} \Big/ \int S(\tau_{\rm L\alpha}) d\tau_{\rm L\alpha}\,, \tag{24}$$

where the parameters, n_A, $n_{\rm He\,II}$, $n_{\rm O\,III}$, etc., are functions of $\tau_{\rm L\alpha}$. We have assumed that photons can escape both to the inside and outside of the nebula by using the sum of the slab escape probabilities measured inward and outward. We have made the further approximation that $\tau_{O1}/\tau_{\rm L\alpha}$ is constant, which seems adequate in view of the near constancy of $n_{\rm O\,III}/n_{\rm He\,II}$ in the model (cf. Fig. 1 of WW). This approximation is not essential and could be removed if necessary.

We have carried out this procedure for one of the planetary nebula models discussed by WW, in which a central star with effective temperature $T_* = 10^5$ K illuminates a nebula with gas temperature $T = 10^4$ K, using fits to the curves in their Figure 1 for the ion densities and optical depth needed to calculate $y_{\rm B}$, etc. Our results for the average yields are compared to those obtained by WW in Table 1.

Our theoretical results for $\bar{y}_{\rm B}$ and $\bar{y}_{O2}$ agree so well with those of WW that it is hardly worth commenting on the differences—they could be due to inaccuracies in our fit to their nebular model. However, the differences in $\bar{y}_{\rm L\alpha}$ and $\bar{y}_A$ are significant and may arise from our approximate description of partial redistribution in the wings of Lα. In contrast, the Bowen yield and the escape in O III result from scattering in the core of Lα, where partial redistribution is unimportant; therefore, $\bar{y}_{\rm B}$ and $\bar{y}_{O2}$ are insensitive to the approximation. But once a photon has diffused to the wings of Lα, its fate is determined primarily by a trade-off between continuum absorption and escape in Lα, which is a sensitive function of the details of the frequency redistribution. However, the sum $\bar{y}_{\rm L\alpha} + \bar{y}_A$ is not. To verify this statement, we have calculated the mean yield for Bowen fluorescence, escape, and absorption, assuming complete redistribution in the Lα scattering. The resulting yields differ by only a few percent from those presented in Table 1.

We have also calculated the mean yields for the formation of N III Bowen lines in planetary nebulae, using the same procedure as for the O III fluorescence. The uniform slab results of the previous subsection are again treated as Green's functions and are convolved with the photon source function according to an equation similar to (24). In this case, the source function at each depth is the product of the He II Lα source function and the O III Bowen yield, $y_{\rm B}$. Ionization fractions for the same nebular model are taken from Williams (1968). These ionization curves justify the assumption that τ_{N1}/τ_{O4}, τ_{N2}/τ_{O4} at each depth are proportional to the local ratio $n_{\rm N\,III}/n_{\rm O\,III}$.

The mean yields for this model are shown in Table 2. We see that the Bowen yield, $\bar{y}_{\rm B}'$, is suppressed mostly by the effects of escape in the resonance lines $O5$ and $O6$. These losses are due to the large branching ratios for production of

TABLE 1

EFFICIENCIES OF ESCAPE AND DESTRUCTION FOR He II Lα

	Average Yield (%)	
Conversion Process	Present Theory	Weymann and Williams
Bowen conversion, $\bar{y}_{\rm B}$	42	40
O III λ303.621 escape, $\bar{y}_{O2}$	7	8
He II Lα escape, $\bar{y}_{\rm L\alpha}$	40	33
Continuum absorption, $\bar{y}_A$	13	17

TABLE 2

EFFICIENCIES OF ESCAPE AND DESTRUCTION FOR $O4$ LINE PHOTONS

Conversion Process	Average Yield (%)
Bowen conversion, $\bar{y}_B'$	39
N III λ374.204 escape, $\bar{y}_{N3}$	11
O III λ374.436, N III λ374.434, λ374.441 escape, $\bar{y}_{O4+N1+N2}$	11
Continuous absorption, $\bar{y}_A'$	16
O III λ374.005, λ374.165 escape, $\bar{y}_{O5+O6}$	23

these lines following absorption in the $O4$ line and to the moderate optical depths in the nebula. Losses due to escape in other lines and to continuum absorption are also important. Unlike the case of O III fluorescence, the optical depths in the λ374 lines are small enough that the damping wings are optically thin. Therefore our results for the N III fluorescence yields do not suffer from errors due to the approximate treatment of partial redistribution in the line scattering.

IV. DISCUSSION

We now discuss the interpretation of observations of Bowen lines using the general theory developed here. Bowen line intensities have been observed for a large number of planetary nebulae, and intensity ratios are generally consistent with the theory. In the case of nebulae illuminated by X-rays, observed Bowen yields are potentially useful indicators of ion abundances in the fluorescing gas. However, interpretation of observed line intensities from these objects is complicated by the existence of competing emission mechanisms.

a) *Planetary Nebulae*

In the case of planetary nebulae, there is a relatively simple connection between theoretical Bowen yields and observed line intensities. When the He II emission is due to recombination, the emissivity in Lα is proportional to the emissivity in λ4686, with the proportionality constant depending weakly on the temperature in the nebula. Then the O III Bowen yield may be written

$$\bar{y}_B = \kappa I(\lambda 3444)/I(\lambda 4686) . \tag{25}$$

Harrington (1972) has estimated the $\kappa \approx 1$ for a nebular temperature of 15,000 K, and that κ ranges from 1.1 at 10,000 K to 0.92 to 20,000 K.

The N III Bowen yields can be estimated from observed line intensities in a manner similar to that used for O III, that is,

$$\bar{y}_B' = \kappa' I(\lambda\lambda 4634, 4641)/I(\lambda 3444) . \tag{26}$$

Using the transition probabilities given by Saraph and Seaton (1980), we find that the probability of emission of a λ3444 photon per O III Bowen decay is 0.21 and that the probability of a transition to $2p3s(^3p_1{}^o)$ per O III Bowen decay is 0.12. Taking the ratio of these two values and converting to intensity gives $\kappa' = 2.4$.

Table 3 lists the intensities of Bowen lines observed from a sample of planetary nebulae together with values of $\bar{y}_B$ and $\bar{y}_B'$ calculated from equations (25) and (26). The first entry in the table, NGC 7027, has $\bar{y}_B = 0.36$ and $\bar{y}_B' = 0.45$. These values are representative of the mean values of $\bar{y}_B$ and $\bar{y}_B'$ for this sample of nebulae, which are 0.37 and 0.61, respectively. The dispersions are $\langle \bar{y}_B{}^2 \rangle^{1/2} = 0.04$ and $\langle \bar{y}_B{}'^2 \rangle^{1/2} = 0.08$. These observed values are in agreement with our calculated values for a model of a typical planetary nebula such as NGC 7027 (cf. Tables 1 and 2) of $\bar{y}_B = 0.42$ and $\bar{y}_B' = 0.39$, given the uncertainties in the observations, reddening corrections, nebular models, and atomic transition probabilities. In particular, we note that the data summarized by Kaler (1976), shown in Table 3, are largely photographic; so the error in the line intensities are likely to be much larger than is indicated by the dispersion in the data. The larger dispersion in $\bar{y}_B'$ can be attributed to the sensitivity of the ratio $n_{\rm N\,III}/n_{\rm O\,III}$ to the ionizing radiation field in the spectral region between the N III and O III edges.

The agreement between observed and predicted values of $\bar{y}_B$ and $\bar{y}_B'$ can be interpreted as a verification of our theory for Bowen line formation in planetary nebulae. It can also be used to constrain the possible range of the ion abundances and optical depths in the sources. For example, the data on O III fluorescence yields in Table 3 indicate that a lower limit on the mean $\bar{y}_B$ for our sample of nebulae is about 0.37. Using this lower limit, together with the equation (21), and the assumption $n_{\rm O\,III}/n = 6.2 \times 10^{-4}$, the cosmic abundance of O, we can derive that

$$n_A/n < 10^{-2} , \tag{27}$$

where n is the total density. If dust grains with density n_g are present in the fluorescence region, we obtain

$$n_g \sigma_g/n < 10^{-20}\ \mathrm{cm}^2 , \tag{28}$$

TABLE 3

LINE INTENSITIES AND MEAN BOWEN YIELDS FOR PLANETARY NEBULAE

Nebula	$I(\lambda 3444)$	$I(\lambda 4634)$	$I(\lambda 4641 + \lambda 4642)$	$I(\lambda 4686)$	$\bar{y}_B$	$\bar{y}_B'$	Ref.
NGC 7027	22.53	1.40	3.17	46.13	0.36	0.45	K
NGC 1535	9.5[a]	0.78	1.7	18.0	0.50	0.62	LHA
NGC 2022	16.0	0.95	1.54	107.0	0.15	0.37	LHA
NGC 2392	5.3	0.3	0.62	34.5	0.15	0.41	LHA
NGC 6309	33.74	3.99	5.65	70.23	0.35	0.69	K
NGC 6741	25.39	2.13	4.25	47.50	0.39	0.61	K
NGC 6818	18.6	1.74	3.8	71.0	0.26	0.71	LHA
NGC 6886	47.52	1.0	2.0	46.93	0.74	0.15	K
NGC 7009[b]	6.65	1.4	3.92	2.4	0.26	1.56	LHA
NGC 7026	17.69	2.47	6.04	21.19	0.61	1.16	K
NGC 7662	22.09	2.16	7.66	49.57	0.33	0.62	K
IC 351	47.71	1.29	2.79	48.42	0.72	0.21	K
IC 1747	6.47	0.46	2.05	26.95	0.18	0.93	K
IC 2003	24.83	1.10	2.75	65.50	0.28	0.37	K
IC 2165	36.16	1.85	4.24	50.46	0.53	0.41	K
IC 5217	7.44	1.09	2.17	14.84	0.39	1.06	K
I900	22.96	...	...	47.31	0.36	...	K
86–8°1	14.0	0.56	2.0	89.0	0.16	0.44	LHA
J900	14.0	0.40	0.92	45.5	0.31	0.23	LHA

[a] $I(\lambda 3444)$ for NGC 1535 is from Aller and Walker 1965.

[b] Data for NGC 7009 are from Czyzak and Aller 1979.

REFERENCES.—K = Kaler 1976; KHA = Aller and Czyak 1979.

where σ_g is the grain photoabsorption cross section at 304 Å. This upper limit is greater than that required to produce the observed infrared emission for NGC 7027 (Krishna Swamy and O'Dell 1968) by a factor $\sim 10^2$.

Comparison of theory and observations of Bowen lines may be complicated by the contribution to line emission by other processes. One such process, discussed by Seaton (1968), is the excitation of O III and N III by absorption of stellar UV in the discrete spectrum of these ions. However, the equivalent width in the He II continuum is greater than the equivalent widths summed over the O III or N III discrete spectra by at least 10^3, so that resonant pumping by Lα will dominate the excitation of the relevant levels of O III and N III for a wide range of Bowen yields. As we shall discuss below, other emission mechanisms will affect the Bowen process in the nebulae surrounding X-ray sources, but they are unimportant in planetary nebulae.

b) X-Ray Sources

With respect to the formation of Bowen lines, the primary difference between the nebulae surrounding X-ray sources and planetary nebulae is that the former are likely to have much greater column densities of ionized gas, owing to the large luminosities and hard ionizing spectra characteristic of X-ray sources. As a result, the optical depths in resonance lines are so great that photon escape is negligible compared with other loss processes, and Bowen fluorescence competes only with continuum absorption.

In the absence of photon escape, theoretical Bowen yields can be calculated from H I, O III, and N III ion abundances, using equations (21) and (23). As an example, we have calculated mean yields for an X-ray ionized cloud, using equations (21) and (23), and the same Green's function assumption and convolution procedure as was used for the models of planetary nebulae. The ion abundances, temperatures, and source functions were taken from a model X-ray nebula consisting of a 10^{38} ergs s^{-1} X-ray source with a 10 keV thermal bremsstrahlung spectrum at the center of a static, spherical gas cloud with density 10^{11} cm^{-3}, calculated by Kallman and McCray (1980), according to the method of Hatchett, Buff, and McCray (1976). The He II Lα optical depth at the center of this cloud is found to be 10^8, and the mean Bowen yields are found to be $\bar{y}_B = 0.95$ and $\bar{y}_B' = 0.70$. As a check, we have calculated mean Bowen yields for this nebula using accurate Green's functions instead of equations (21), (23), and the results are found to be within a few percent of the simpler calculation.

The interpretation of Bowen line strengths in X-ray nebulae is subject to a number of uncertainties. One is that He II Lα may be produced by electron collisions rather than recombination. In the model described above, the temperature in the He II Lα emission region ranges from 16,000 K to 50,000 K, greater than the value $T \approx 15{,}000$ K characteristic of planetaries. At such temperatures the ratio of He II $\lambda 304$/He II $\lambda 4686$ is much greater than the pure recombination value, and equation (25) must be modified accordingly. For our model X-ray nebula, the ratio of the total number of $\lambda 4686$ photons to Lα photons produced in the cloud is 3.2×10^{-3}, so that $\kappa = 2.2 \times 10^{-2}$. For a cloud where the He II emission is solely due to collisional excitation, the value of κ is

$$\kappa = 0.079 \exp\left[-11.6/(T/10^4 \text{ K})\right] . \tag{29}$$

The strong temperature dependence of this quantity makes observational estimates of the O III Bowen yield, $\bar{y}_B$, highly uncertain.

However, the ratio of N III to O III line intensities is not affected by this mechanism, so that the mean N III Bowen yield, $\bar{y}_B'$, may be derived from observed line intensities and equation (26). This value of $\bar{y}_B'$ depends only on the relative abundances of continuum absorbers and N III ions in the fluorescence region, according to equation (23).

Other effects which may modify the Bowen line intensities in X-ray nebulae are dielectronic recombination and charge transfer. At the temperature characteristic of X-ray nebulae, dielectronic recombination of N IV can also produce the N III $\lambda\lambda$4634, 4641 lines (Mihalas and Hummer 1973), but its contribution to the total production rate is small (McClintock, Canizares, and Tarter 1975). Charge transfer can be an important mechanism in X-ray nebulae, because the X-rays can preferentially produce O IV and N IV through the Auger effect, while leaving a significant abundance of H I. The charge transfer of O IV with H I leaves the resulting O III ion in either the $2p3s$ or the $2p3p$ states, which then decay and pump the N III Bowen lines (Dalgarno and Butler 1978). Excitation of O III due to this reaction will exceed excitation due to the Bowen process when the hydrogen neutral fraction, $n_{\mathrm{H\,I}}/n_{\mathrm{H}} > 10^{-3}$. The Bowen process in X-ray sources and in planetary nebulae will also be affected by velocity gradients. These may have the effect of decreasing the optical depths in certain lines, such as $O2$ or $N3$, thus suppressing the Bowen efficiency. On the other hand, very large gradients might broaden He II Lα sufficiently that it could pump the $O3$ line (cf. Figs. 1 and 3), thus increasing the Bowen efficiency and changing the relative intensities of O III Bowen lines. These effects will be discussed in a subsequent paper.

Observations of Bowen lines from X-ray sources are consistent with our discussion so far. In particular, the N III $\lambda\lambda$4634, 4641 Bowen lines are always prominent, and He II λ4686 is also visible in the spectra of these objects (McClintock, Canizares, and Tarter 1975). These lines are accompanied by variable O III Bowen lines (Margon and Cohen 1978; Canizares, McClintock, and Grindlay 1979, 1980; Oke and Greenstein 1977; Whelan *et al.* 1977). The ratio of λ4686 to $\lambda\lambda$4634, 4641 intensity is much less than in planetary nebulae. Although these observations span a variety of different objects with possibly very different physical conditions in the emitting regions, the basic physical picture that we have presented seems to be correct. However, many of the X-ray sources show pronounced variability in their line emission. This, together with the anomalous O III Bowen line ratios (Canizares, McClintock, and Grindlay 1979) observed from some sources, suggests that bulk motions of the emitting gas are affecting the fluorescence.

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APPENDIX

DERIVATION OF EQUATIONS (5), (6), (17), (18), AND (19)

Consider first the O III lines. We want to know the probability that a photon introduced into line $O1$ will escape in $O2$ or will be converted to Bowen line photons. Since $O1$, $O2$, and the Bowen lines all share the same upper level, we can define B_{O2} as the probability of emission in $O2$ per scattering in $O1$ or $O2$. Then, if the probability of escape per scattering in $O2$ is ϵ_{O2}, the probability that a photon survives in $O2$ for N scatterings is

$$[1-(1-B_{O2})-B_{O2}\epsilon_{O2}]^N=[B_{O2}(1-\epsilon_{O2})]^N\,. \tag{A1}$$

The probability of escape in $O2$ is then the product of the probability of being in line $O2$ after $N-1$ scatterings with the probability of escape on the Nth scattering, i.e.,

$$\sum_{N=1}^{\infty}[B_{O2}(1-\epsilon_{O2})]^{N-1}B_{O2}\epsilon_{O2}=\frac{B_{O2}\epsilon_{O2}}{1-B_{O2}(1-\epsilon_{O2})}\,. \tag{A2}$$

Multiplying this by the probability of emission as $O2$, and adding the probability of escape in $O2$ without scattering, gives

$$\begin{aligned}\beta_{O2}&=\frac{B_{O2}(1-\epsilon_{O2})B_{O2}\epsilon_{O2}}{1-B_{O2}(1-\epsilon_{O2})}+B_{O2}\epsilon_{O2}\\&=\frac{B_{O2}\epsilon_{O2}}{1-B_{O2}(1-\epsilon_{O2})}\,.\end{aligned} \tag{A3}$$

The probability of Bowen conversion is obtained by replacing $B_{O2}\epsilon_{O2}$ in the numerator of (A3) by B_B, the Bowen conversion probability per scattering

$$\beta_B = \frac{B_B B_{O2}(1 - \epsilon_{O2})}{1 - B_{O2}(1 - \epsilon_{O2})} + B_B$$
$$= \frac{B_B}{1 - B_{O2}(1 - \epsilon_{O2})} . \tag{A4}$$

The derivations of equations (17)–(19) follow in an analogous way.

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