THE ASTROPHYSICAL JOURNAL 241:858–863, 1980 November 1


# ON THE DETERMINATION OF THE DEGREE OF COSMOLOGICAL COMPTON DISTORTIONS AND THE TEMPERATURE OF THE COSMIC BLACKBODY RADIATION

YOEL REPHAELI
Center for Radiophysics and Space Research, Cornell University
*Received 1980 April 7; accepted 1980 June 10*

## ABSTRACT

We describe a method for a determination of the cosmic blackbody radiation temperature and its degree of cosmological Compton-distortions. *Differential* measurements at only three frequencies of the additional Compton distortions in a direction to a gas-rich cluster of galaxies are sufficient to determine both of these quantities. The method is essentially independent of the gas properties and, it is argued, can significantly improve on previous determinations of the radiation temperature. The method is implicitly based on the universality of the radiation and thus, if successfully applied, can give strong evidence for the universal nature of the radiation.

*Subject headings:* cosmic background radiation — cosmology — galaxies: clusters of

## I. INTRODUCTION

The temperature $T$ of the supposedly universal cosmic blackbody radiation (CBR) is not very accurately known. Since the discovery of the CBR (Penzias and Wilson 1965), many measurements of its intensity were made, mostly at the Rayleigh-Jeans (R-J) part of the spectrum. (For review of the observations and their analysis the reader is referred to Peebles 1971, and Danese and DeZotti 1977.) The microwave measurements only restrict $T$ to, roughly, the interval 2.4–2.9 K. Statistical analysis of measurements at various frequencies by various observing groups yields a mean temperature with smaller uncertainty. For example, analysis of 22 measurements made with ground-based detectors gives $T = 2.69 \pm 0.08$ K (Danese and De Zotti 1977). It is quite obvious, however, that the 1 $\sigma$ error quoted in this result has mostly formal significance. Woody and Richards (1979; see also Woody *et al.* 1975), performing high-altitude balloon measurements, have extended the observations to higher frequencies, measuring the CBR intensity in the region $7.5 \times 10^{10}$–$6 \times 10^{11}$ Hz. They find that the integrated flux of the radiation is equal to that from a 2.96 (+0.04, −0.06) K blackbody. These observations are quite significant, as they indicate the nearly thermal nature of the spectrum. However, since these are absolute measurements mostly in a frequency interval where the night-sky emission is as bright as or brighter than the CBR, their analysis and interpretation necessitate employing large (and not always accurately known) corrections.

In fact, the temperature may not be the only quantity needed to determine the radiation spectrum. Compton scattering of the photons by hotter electrons before and after recombination, as well as creation of bremsstrahlung photons, can significantly distort the spectrum (Zel'dovich and Sunyaev 1969; Chan and Jones 1975*a*). At frequencies $h\nu/kT \equiv x \gtrsim 1$, only Compton distortions are expected to be noticeable, with the relative distortion increasing with $x$. The observed spectrum may have to be characterized, therefore, by an additional quantity: the degree of Comptonization,

$$y = \int \frac{kT_e}{mc^2} d\tau , \tag{1}$$

where $T_e$ is the electron gas temperature and $\tau$ its optical depth to Compton scattering. Measurements at $x \gtrsim 1$ are thus essential not only for the determination of the radiation temperature but also for obtaining an estimate on the pressure of a universal distribution of hot electrons. Analysis of most of the microwave measurements yields $y \leq 0.05$ for $T \leq 3.0$ K (Chan and Jones 1975*b*). The same upper limit has also been deduced by Field and Perrenod (1977), who included the submillimeter measurements of Woody *et al.* (1975).

The recent measurements of Woody and Richards (1979) are consistent with those of Woody *et al.* in terms of the overall equivalent blackbody temperature. Significant differences between the two sets of measurements are particularly noticeable for $\nu > 3 \times 10^{11}$ Hz, in a frequency region in which large corrections for the night-sky emission need to be made. Looking at Woody and Richards spectrum for the night sky, we can obtain a very rough upper limit on the value of $y$, roughly equal to the above quoted value, by attributing all the night-sky emission at $\nu \sim 3 \times 10^{11}$ Hz to the CBR. In what follows we will assume that the value of $y$ is observationally restricted to be smaller than or equal to 0.05.

In view of the immense significance of the CBR in theoretical astrophysics and cosmology, it is quite desirable to have alternative methods for the determination of its temperature and degree of distortions. Here, we describe a method which involves *differential* measurements, at three different frequencies, of the change of the CBR intensity due to Compton scattering of the photons by hot electrons in clusters of galaxies. Extensive X-ray observations indicate the presence of large quantities of hot ($T_{e,c} \sim 10^8$ K) and relatively dense ($n_{e,c} \sim 10^{-3}$ cm$^{-3}$) gas in many clusters of galaxies (for a recent review, see Gursky and Schwartz 1977). Because of the differential nature of the measurements and since only ratios of measured quantities are used, the method can be regarded as a significant improvement on the presently employed method of absolute measurements at various frequencies. The method is essentially independent of the cluster gas properties as long as a gas-rich cluster is chosen so as to obtain higher signal-to-noise ratio.

A different approach for using cluster distortions to determine $T$ has already been suggested by Fabbri, Melchiorri, and Natale (1978). The latter authors have suggested to find the critical frequency $\nu_0$ at which the cluster Compton distortions vanish; comparison with the theoretical value, $x_0(y = 0) = 3.83$, can determine $T$. This alternate approach is observationally, we argue, less desirable, in addition to being less general, on theoretical grounds, since the possibility of nonnegligible cosmological distortions has to be entertained. We will discuss this further in the Discussion following a brief theoretical background and description of the method.

## II. THEORY

The interaction of a radiation field with a Maxwellian electron gas can be described in terms of the Kompaneets (1957) equation. When the radiation field photons are Compton scattered off a much hotter nonrelativistic electron gas, $T_e \gg T$, then, in the Thomson limit, the equation simplifies. If the initial photon occupation number is $\bar{n}(x)$, then (Zel'dovich and Sunyaev 1969),

$$\bar{n}(x, y) = \frac{1}{(4\pi y)^{1/2}} \int_{-\infty}^{\infty} \bar{n}(\zeta) \exp\left[-\frac{(\xi - \zeta)^2}{4y}\right] d\zeta \,,$$
$$\xi \equiv \ln x + 3y \,. \tag{2}$$

If the initial distribution is Planckian, $\bar{n}_0(x) = (e^x - 1)^{-1}$,

$$\bar{n}(x, y) = \frac{1}{(4\pi y)^{1/2}} \int_{-\infty}^{\infty} \frac{e^{-\eta^2/4y} d\eta}{\exp[xe^{\eta + 3y}] - 1} \,. \tag{3}$$

Suppose now that the above Compton-distorted radiation field further interacts with an additional distribution of hot electrons in some region of space. Then, along a line of sight to this region, the photon occupation number will be, practically independent of the region's redshift,

$$\bar{n}(x, y, y_c) = \frac{1}{4\pi(yy_c)^{1/2}} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{\exp(-\eta^2/4y - \rho^2/4y_c) d\eta d\rho}{\exp[xe^{\eta + \rho + 3(y + y_c)}] - 1}$$
$$y_c = \int \frac{kT_{e,c}}{mc^2} n_{e,c} \sigma_T dl \,. \tag{4}$$

$T_{e,c}$, $n_{e,c}$ are the region's electron temperature and density, and the integration is over the photon line-of-sight path length through the region ($\sigma_T$ is the Thomson cross section). For application to cluster-produced distortions, $y_c \ll 10^{-3}$, and so we can approximate the $\rho$-integral by expanding the integrand near its maximum retaining terms up to first order in $y_c$. Doing so, we find for the change of $\bar{n}$ for a line of sight through a cluster

$$\Delta\bar{n} = \bar{n}(x, y, y_c) - \bar{n}(x, y, 0)$$
$$= \frac{y_c}{(4\pi y)^{1/2}} \int_{-\infty}^{\infty} \frac{f \exp(f - \eta^2)}{(e^f - 1)^2} \left[f \coth\left(\frac{f}{2}\right) - 4\right] d\eta \tag{5}$$

with $f \equiv x \exp(3y + \eta)$. This approximation is valid for $x^2 y_c \ll 1$, or $x \lesssim 10$ here. For our purposes, no further substantial simplification of the expression in equation (5) is possible for $x \gtrsim 1$ without sacrificing essential accuracy. When $y = 0$, the expression for $\Delta\bar{n}$ reduces to the Zel'dovich and Sunyaev (1969) result

$$\Delta\bar{n} = \frac{xy_c e^x}{(e^x - 1)^2} \left[x \coth\left(\frac{x}{2}\right) - 4\right] , \tag{6}$$

which in the R-J part of the spectrum simplifies to $\Delta\bar{n}/\bar{n}_0 = -2y_c$. This portion of the spectrum can be characterized as Planckian with the reduced temperature $T_{\text{R-J}} = Te^{-2y_c}$. In this region, $\Delta\bar{n} = -2y_c e^{-2y}/x$ for small $y$.

The effect of the scattering of the photons off the much hotter electron gas is physically obvious. Since the number of photons is conserved, photons gain energy on the average, and thus the intensity decreases in the R-J side while it increases in the Wien side of the spectrum. The higher the value of $y$, the larger is the R-J diminution and the broader the distribution. The intensity change, $x^3\Delta\bar{n}/y_c$, is plotted in Figure 1 for various values of $y$. Note that for values of $x$ in the intervals (1, 2) and (2.5, 3.5) the dependence on $T$ is more sensitive than that on $y$, while the reverse is true in the interval (5, 6).

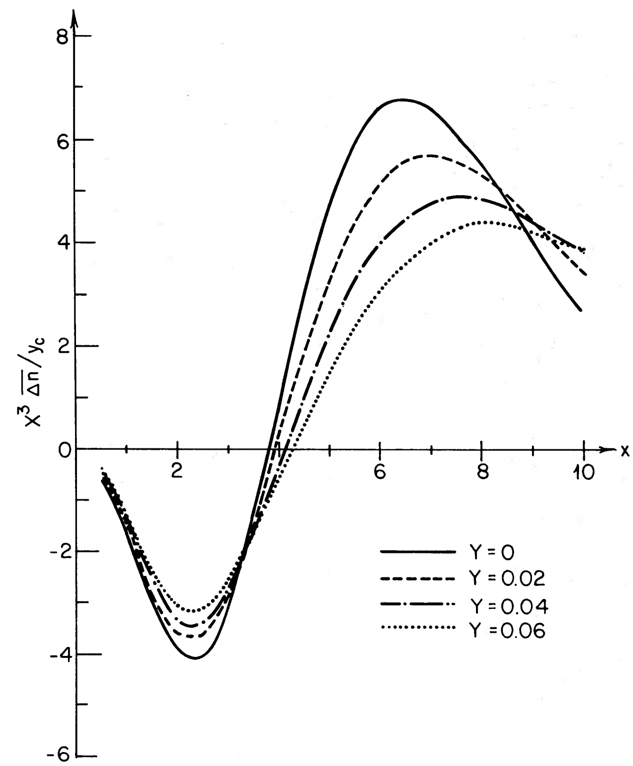


FIG. 1

## III. THE METHOD

The cluster Comptonization parameter, $y_c$, is very small, even for a gas-rich cluster. For example, $y_c \approx 2 \times 10^{-4}$ for $n_{e,c} = 4 \times 10^{-3}$ cm$^{-3}$, $T_{e,c} = 3 \times 10^8$ K in a cluster core region $\frac{1}{2}M_{\rm pc}$ in radius. Thus, the linear dependence on $y_c$ in equation (4) is certainly appropriate for the values of $x$ of interest here, $x < 10$. The ratio of intensity changes at two frequencies is therefore essentially independent of the cluster gas properties; it is only a function of the two frequencies, $y$, and $T$. A third measurement at one additional frequency can allow, in principle, a determination of $y$ as well. Obviously, it is desirable to choose the observing frequencies so as to achieve maximal separability in the dependence of $\Delta I$ on $y$ and $T$. A practical constraint on this choice is set by the spectral distribution of the night-sky emission. For ground based observations it is desirable to select frequencies in the atmospheric windows between the large absorption features of water vapor and oxygen. (Fabbri, Melchiorri, and Natale 1978 discuss the spectral shape of the atmospheric transmission function and other pertinent observational considerations.) In view of these requirements, we have chosen the three representative frequencies to be $\nu_1 = 8 \times 10^{10}$ Hz, $\nu_2 = 2\nu_1$, and $\nu_3 = 3 \times 10^{11}$ Hz in our discussion here. In Table 1 we give the values of $\Delta I_1/y_c$ and $\Delta I_1/\Delta I_2$ for $2.50 \leq T \leq 3.10$ K and $0 \leq y \leq 0.06$; in Table 2, values of $-\Delta I_3/\Delta I_1$ are listed for the same domain in the $(y, T)$-plane. (A superscript indicates the minimum value of $T$ allowed for a given value of $y$, the criterion being that $T_{\rm R\text{-}J} = Te^{-2y} > 2.45$ K.)

Looking at the tables, it is evident that in most of the specified $(y, T)$-domain the dependence of the ratios $\Delta I_1/\Delta I_2$ and $\Delta I_3/\Delta I_1$ on $y$, $T$ is as desired: the former ratio is much more sensitive to variation in $T$ than in $y$, while for the latter the reverse is true. Fabbri, Melchiorri, and Natale (1978) claim that experimental accuracy of $\sim 1\%$ can be achieved in the measurement of $\Delta I$ in the millimetric region. For a more realistic estimate of 5% (1 $\sigma$) accuracy in the measured value of $\Delta I$, $T$ can be determined to 0.1 K. For our particular choice of frequencies the accuracy is expected to be higher if $T < 2.85$ and somewhat lower otherwise. We emphasize that $\nu_1$, $\nu_2$ can be selected such as to steepen the dependence of the ratio $\Delta I_1/\Delta I_2$ on $T$. This can be readily achieved for $\nu_1$, $\nu_2$ in the interval $(1\text{–}2) \times 10^{11}$ Hz when high-altitute measurements are made to avoid the effects of absorption by oxygen and water vapor. Thus, using the suggested method can constitute a more convincing determination of $T$ than previously possible. Knowing $T$ to the above accuracy will then allow a determination of the value of $y$ to between 0.005 and 0.01. While in relative terms this might not be too impressive an accuracy, it is still much better than results obtained using absolute measurements at the higher frequencies ($\nu \gtrsim 3 \times 10^{11}$ Hz).

TABLE 1

THE (MAINLY) TEMPERATURE-SENSITIVE RATIO $\Delta I_1/\Delta I_2$

| | | $y = 0$ | | $y = 0.005$ | | $y = 0.01$ | | $y = 0.02$ | | $y = 0.03$ | | $y = 0.04$ | | $y = 0.05$ | | $y = 0.06$ | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $T$(K) | $x_1$ | $F$ | $\Delta I_1/\Delta I_2$ | $F$ | $\Delta I_1/\Delta I_2$ | $F$ | $\Delta I_1/\Delta I_2$ | $F$ | $\Delta I_1/\Delta I_2$ | $F$ | $\Delta I_1/\Delta I_2$ | $F$ | $\Delta I_1/\Delta I_2$ | $F$ | $\Delta I_1/\Delta I_2$ | $F$ | $\Delta I_1/\Delta I_2$ |
| 2.50....... | 1.537 | 3.16 | 1.11 | 3.10 | 1.10 | 3.04 | 1.10 | 2.92 | 1.08 | 2.81 | 1.06 | 2.70 | 1.04 | 2.60 | 1.02 | 2.50 | .99 |
| 2.55....... | 1.507 | 3.08 | 1.02 | 3.02 | 1.02 | 2.97 | 1.01 | 2.86[a] | 1.00[a] | 2.75 | 0.99 | 2.64 | 0.98 | 2.55 | 0.96 | 2.45 | 0.94 |
| 2.60....... | 1.478 | 3.01 | 0.95 | 2.95 | 0.95 | 2.90 | 0.95 | 2.79 | 0.94 | 2.69[a] | 0.93[a] | 2.59 | 0.92 | 2.49 | 0.91 | 2.40 | 0.89 |
| 2.65....... | 1.450 | 2.94 | 0.89 | 2.89 | 0.89 | 2.83 | 0.89 | 2.73 | 0.89 | 2.63 | 0.88 | 2.53[a] | 0.87[a] | 2.44 | 0.86 | 2.35 | 0.85 |
| 2.70....... | 1.424 | 2.87 | 0.83 | 2.82 | 0.84 | 2.77 | 0.84 | 2.67 | 0.84 | 2.57 | 0.83 | 2.48 | 0.83 | 2.39 | 0.82 | 2.30 | 0.81 |
| 2.75....... | 1.398 | 2.80 | 0.79 | 2.75 | 0.79 | 2.70 | 0.79 | 2.61 | 0.80 | 2.51 | 0.79 | 2.42 | 0.79 | 2.33[a] | 0.79[a] | 2.25 | 0.78 |
| 2.80....... | 1.373 | 2.74 | 0.75 | 2.69 | 0.75 | 2.64 | 0.76 | 2.55 | 0.76 | 2.45 | 0.76 | 2.37 | 0.76 | 2.28 | 0.76 | 2.20[a] | 0.75[a] |
| 2.85....... | 1.349 | 2.67 | 0.71 | 2.62 | 0.72 | 2.58 | 0.72 | 2.49 | 0.73 | 2.40 | 0.73 | 2.32 | 0.73 | 2.23 | 0.73 | 2.16 | 0.72 |
| 2.90....... | 1.325 | 2.61 | 0.68 | 2.56 | 0.69 | 2.52 | 0.69 | 2.43 | 0.70 | 2.35 | 0.70 | 2.26 | 0.70 | 2.18 | 0.70 | 2.11 | 0.70 |
| 2.95....... | 1.303 | 2.55 | 0.65 | 2.50 | 0.66 | 2.46 | 0.66 | 2.37 | 0.67 | 2.29 | 0.67 | 2.21 | 0.68 | 2.14 | 0.68 | 2.06 | 0.68 |
| 3.00....... | 1.281 | 2.48 | 0.63 | 2.44 | 0.64 | 2.40 | 0.64 | 2.32 | 0.65 | 2.24 | 0.65 | 2.16 | 0.65 | 2.09 | 0.66 | 2.02 | 0.66 |
| 3.05....... | 1.260 | 2.43 | 0.61 | 2.39 | 0.61 | 2.35 | 0.62 | 2.27 | 0.62 | 2.19 | 0.63 | 2.12 | 0.63 | 2.04 | 0.64 | 1.98 | 0.64 |
| 3.10....... | 1.240 | 2.37 | 0.59 | 2.33 | 0.59 | 2.29 | 0.60 | 2.22 | 0.61 | 2.14 | 0.61 | 2.07 | 0.62 | 2.01 | 0.62 | 1.93 | 0.62 |

NOTE.—$F \equiv -x_1{}^3\Delta\bar{n}(x, y)/y_c$.

[a] The minimum value of $T$ allowed for a given value of $y$, the criterion being that $T_{R-J} = Te^{-2y} > 2.45$ K.

TABLE 2

THE (MAINLY) $y$-SENSITIVE RATIO $\Delta I_3/\Delta I_1$

| $T$ (K) | $x_1$ | $-\Delta I_3/\Delta I_1$ | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | $y = 0$ | $y = 0.005$ | $y = 0.01$ | $y = 0.02$ | $y = 0.03$ | $y = 0.04$ | $y = 0.05$ | $y = 0.06$ |
| 2.50 | 1.537 | 1.99 | 1.89 | 1.80 | 1.63 | 1.48 | 1.34 | 1.21 | 1.09 |
| 2.55 | 1.507 | 1.98 | 1.88 | 1.78 | 1.61[a] | 1.45 | 1.30 | 1.17 | 1.04 |
| 2.60 | 1.478 | 1.96 | 1.86 | 1.76 | 1.58 | 1.41[a] | 1.26 | 1.12 | 0.99 |
| 2.65 | 1.450 | 1.94 | 1.83 | 1.73 | 1.54 | 1.37 | 1.22[a] | 1.08 | 0.94 |
| 2.70 | 1.424 | 1.91 | 1.79 | 1.69 | 1.50 | 1.32 | 1.17 | 1.02 | 0.89 |
| 2.75 | 1.398 | 1.87 | 1.75 | 1.65 | 1.45 | 1.27 | 1.11 | 0.97[a] | 0.83 |
| 2.80 | 1.373 | 1.82 | 1.71 | 1.60 | 1.40 | 1.22 | 1.06 | 0.91 | 0.77[a] |
| 2.85 | 1.349 | 1.77 | 1.65 | 1.54 | 1.34 | 1.16 | 1.00 | 0.85 | 0.71 |
| 2.90 | 1.325 | 1.71 | 1.59 | 1.48 | 1.28 | 1.10 | 0.94 | 0.79 | 0.65 |
| 2.95 | 1.303 | 1.65 | 1.53 | 1.42 | 1.21 | 1.03 | 0.87 | 0.72 | 0.58 |
| 3.00 | 1.281 | 1.58 | 1.46 | 1.35 | 1.15 | 0.96 | 0.80 | 0.65 | 0.51 |
| 3.05 | 1.260 | 1.50 | 1.38 | 1.27 | 1.07 | 0.89 | 0.73 | 0.58 | 0.44 |
| 3.10 | 1.240 | 1.42 | 1.30 | 1.20 | 1.00 | 0.82 | 0.65 | 0.50 | 0.37 |

[a] The minimum value of $T$ allowed for a given value of $y$, the criterion being that $T_{\text{R-J}} = Te^{-2y} > 2.45$ K.

We emphasize that the required accuracy of 5% in the measured value of $\Delta I$ is by no means easy to achieve. Also, we have not mentioned all the observational considerations which may affect the experimental error. For example, achieving the same cluster beam coverage at different frequencies may be difficult.

The dependence on the cluster gas properties is of practical importance because of the required accuracy in the measurement of $\Delta I$. This can be more easily achieved for larger values of $y_c$, i.e., for a cluster in which the gas pressure is high. This requirement constitutes the only restriction on the beam size. If the reference beam, which ideally should be centered on a line of sight away from the cluster, includes nonnegligible part of the gas-containing region in the cluster, then $\Delta I$ will be smaller than its possible maximum value. This can lead to an underestimate of the cluster integrated gas pressure (Gould and Rephaeli 1978). However, the ratio of intensity changes will, in principle, be independent of whether the reference beam is totally out of the effective radius of the gas distribution.

Note that since $x$ is independent of the redshift and since the determination of $T$ involves essentially a ratio of occupation numbers, the method is independent of the cluster's redshift. For (practical) beam separation of less than 15′, the likelihood that the reference beam will itself be spectrally distorted by gas in another cluster along its direction, can be easily seen to be remote.

Measurements of the expected diminution (in the R-J region) of the CBR in a direction to some clusters have already been reported (Parijskij 1973, 1974; Gull and Northover 1975; Lake and Partridge 1977, 1980). These observations were motivated mainly by a desire to obtain independent evidence for the existence of hot intracluster (IC) gas, and they attest to the feasibility of observing relative intensity changes of as low as $10^{-4}$ in the R-J region. In the millimetric region the uncertainty is expected to be lower (Houck 1979), perhaps as low as few percent for few hours of integration time, as claimed by Fabbri, Melchiorri, and Natale (1978).

Incidentally, when making cluster measurements for the above purpose of studying the IC gas properties, it should be remembered that $y$ is known only to be $\leq 0.05$. Assuming $y = 0$ may introduce an error which amounts to a factor of up to $e^{-0.1} = 0.90$ in the calculated value of $\Delta I$ in the R-J region alone. As can be seen from Figure 1, much larger error may result around the peak and at the Wien side of the spectrum.

## IV. DISCUSSION

Some of the general advantages of the method suggested here for the determination of the CBR temperature and degree of cosmological distortions are clear and have already been touched upon in the Introduction. It is also clear that this method does not remove the need for absolute measurements of the spectrum in order to determine its *actual* shape. Woody and Richard's (1979) measurements are consistent at the 80% confidence level with a distorted Planckian. However, the distortions cannot be readily fitted to the form expected from cosmologically Compton-scattered blackbody radiation, especially because of the curious decrease of the intensity below that of 2.96 K blackbody in the interval $2.7 \times 10^{11}$–$3.5 \times 10^{11}$ Hz. It should be remembered, however, that the night-sky emission dominates above $\sim 3 \times 10^{11}$ Hz, so that this feature is somewhat uncertain. The dominance of the night-sky emission at a region where the intensity is strongly dependent on $T$ and $y$ calls for a method, such as the one described here, involving differential, rather than absolute, flux measurements.

Fabbri, Melchiorri, and Natale (1978) have mentioned the possibility that Compton-produced distortions can be used to determine the temperature. They suggest (observationally) finding the frequency, $\nu_0$, at which the distortions vanish. Practically, such an observational procedure is cumbersome compared to the one suggested here, for one has to measure the intensity changes at various frequencies in order to determine $\nu_0$. Since long integration times (> 1 hr) may be required for these types of measurements, and in view of the fact that smaller intensity changes have to be measured with correspondingly larger uncertainties, we conclude that the method suggested here is more observationally desirable. In addition, Fabbri, Melchiorri, and Natale's (1978) assumption of negligible $y$ leads to an error of as much as 10% in the deduced value of $T$, even before accounting for experimental uncertainties. This stems from the fact that $x_0$ depends on $y$: $x_0 = 3.83$ for $y = 0$, while $x_0 = 4.23$ for $y = 0.05$.

In attributing all the cluster-produced distortions to Compton scattering with hot gas, we have ignored the possibility of radio emission (from nonthermal sources as well as thermal emission from a colder gas component) and millimetric dust emission. Since the suggested method involves measurements at $\nu \geq 8 \times 10^{10}$ Hz, nonnegligible contribution from radio sources is clearly unlikely. Aiello, Melchiorri, and Mencaraglia (1978) considered the possibility of IC dust emission at a sufficient level to affect the Compton interpretation of the distortions. They concluded that for this to occur, dust grains of conventional size, composed of graphite and silicate particles, must be present at densities comparable with those of the densest Galactic regions. In view of the theoretical uncertainty concerning the survival of grains immersed in a relatively dense hot gas (Silk and Burke 1974), dust emission was ignored altogether, even though small (radii $\lesssim 7 \times 10^{-7}$ cm), spinning charged grains could, in principle, emit in the relevant frequency interval of concern here.

The Kompaneets (1957) equation was derived in the limiting case $kT_e \ll m_c^2$ when the mean energy transfer from an electron to a photon is small compared to a typical photon energy. For small Compton optical depth the applicability of the equation is restricted to the nonrelativistic limit. Wright (1979) has calculated the Compton distortions for interaction with an electron gas at a mean temperature of $1.5 \times 10^9$ K using the exact photon frequency redistribution function. He finds that the degree of distortion is noticeably different from that calculated using the Kompaneets equation, a difference which generally amounts to less than 0.1 K in the corresponding Planckian temperature. The particular value of $1.5 \times 10^9$ K is the mean temperature of the intergalactic gas (IGG) if it were to account for *all* the X-ray background between 2 and 100 keV. The gas temperature is deduced from a fit to the X-ray measurements when assuming that the IGG was heated by galactic explosions back to a redshift $z = 3$. The reasons upon which we justify our using the nonrelativistic Kompaneets equation are the following. First, in our method only ratios of intensity changes are used; these are still less sensitive to the nonrelativistic assumption even if $\langle T_e(z) \rangle \sim 10^9$ K. Second, for the exact calculation of the Compton distored spectrum, $y$, $\tau$, $T$, and $T_e(z)$ have to be specified, in contrast to the nonrelativistic case for which only $y$ and $T$ are needed to determine the spectrum. Third, it is not at all clear that all the X-ray background originates in a hot IGG; a significant fraction may originate in already identified X-ray sources (Marshall *et al.* 1980; Turner and Geller 1980), though this does not necessarily mean lower temperature for the IGG. Fourth, it is quite uncertain whether it is energetically probable to have such a hot IGG at a density close to the closure value. In effect, what we are saying amounts to a de facto assumption, implicit also in the work of Zel'dovich and Sunyaev, that $T_e(z) < 10^9$ K (in all known X-ray clusters $T_{e,c} < 10^9$ K).

Finally, our suggested method for the determination of the CBR temperature can also be viewed as a test for the universality of the radiation. For if measurements of distortions of the spectrum in a direction to a distant cluster of galaxies are successfully used to determine the radiation temperature, one will have strong evidence for the universality of the CBR.

The author thanks Drs. J. Houck, E. Salpeter and I. Wasserman and the referee for their comments, and Dr. R. B. Partridge for bringing to his attention the work of Fabbri, Melchiorri, and Natale (1978). This research was supported by National Science Foundation grant AST 78-20708 at Cornell University.

REFERENCES

Aiello, A., Melchiorri, F., and Mencaraglia, F. 1978, *Ap. Space Sci.*, **53**, 403.
Chan, K. L., and Jones, B. J. T. 1975*a*, *Ap. J.*, **195**, 1.
———. 1975*b*, *Ap. J.*, **198**, 245.
Danese, L., and De Zotti, G. 1977, *Nuovo Cimento*, **7**, 277.
Fabbri, R., Melchiorri, F., and Natale, V. 1978, *Ap. Space Sci.*, **59**, 223.
Field, G. B., and Perrenod, S. C. 1977, *Ap. J.*, **215**, 717.
Gould, R. J., and Rephaeli, Y. 1978, *Ap. J.*, **219**, 12.
Gull, S. F., and Northover, K. J. E. 1975, *M.N.R.A.S.*, **173**, 585.
Gursky, H., and Schwartz, D. A. 1977, *Ann. Rev. Astr. Ap.*, **15**, 541.
Houck, J. R. 1979, private communication.
Kompaneets, A. S. 1957, Soviet Phys.—JETP, **4**, 730.
Lake, G., and Partridge, R. B. 1977, *Nature*, **270**, 502.
———. 1980, *Ap. J.*, **237**, 378.
Marshall, F. E., Boldt, E. A., Holt, S. S., Miller, R. B., Mushotzky, R. F., Rose, L. A., Rothchild, R. E., and Serleimitsos, P. J. 1980, *Ap. J.*, **235**, 4.
Parijskij, Y. N. 1973, *Ap. J.* (*Letters*), **180**, L47.
———. 1974, *Ap. J.* (*Letters*), **188**, L113.
Peebles, P. J. E. 1971, *Physical Cosmology* (Princeton: Princeton University Press).
Penzias, A. A., and Wilson, R. W. 1965, *Ap. J.*, **142**, 419.
Silk, J., and Burke, J. R. 1974, *Ap. J.*, **190**, 11.
Sunyaev, R. A., and Zel'dovich, Ya. B., 1972, *Comments Ap. Space Phys.*, **4**, 173.
Turner, E. L., and Geller, M. J. 1980, *Ap. J.*, **236**, 1.
Woody, D. P., Mather, J. C., Nishioka, N. S., and Richards, P. L. 1975, *Phys. Rev. Letters*, **34**, 1036.
Woody, D. P., and Richards, P. L. 1979, *Phys. Rev. Letters*, **42**, 925.
Wright, E. L. 1979, *Ap. J.*, **232**, 348.
Zel'dovich, Ya. B., and Sunyaev, R. A. 1969, *Ap. Space Sci.*, **4**, 301.

YOEL REPHAELI: Institute for Theoretical Physics, University of California, Santa Barbara, CA 93106