THE ASTROPHYSICAL JOURNAL, **240**:865–870, 1980 September 15

PERIOD STABILITY OF THE PULSATING WHITE DWARF R548 (=ZZ CETI)

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Received 1980 March 6; accepted 1980 March 20

ABSTRACT

R548 is a pulsating DA white dwarf and the prototype of the ZZ Ceti class of variable stars. R548 is pulsating in four pulsation modes simultaneously, the pulsations having periods of 212.77, 213.13, 274.25, and 274.77 s. We have combined all of the high-speed photometry of R548 acquired at McDonald and Cerro Tololo Inter-American Observatories. The data, which now cover the years from 1970 to 1978, demonstrate that the properties of the pulsations have remained remarkably constant during this time. There have been no detectable changes in either the pulsation periods or the amplitudes. The upper limit on the rate of change of the period of the 213.13 s pulsation is 2×10^{-13} s s^{-1}, which gives this pulsation the most stable period ever measured for a variable star at visual wavelengths.
Subject headings: stars: individual — stars: pulsation — stars: white dwarfs

I. INTRODUCTION

The ZZ Ceti stars are the pulsating white dwarfs lying within a narrow instability strip on the white dwarf cooling sequence (Robinson 1978; Van Horn 1978). This instability strip extends in temperature from about 10,500 to 13,500 K and in $B - V$ color from about +0.15 to +0.30. The luminosity variations of the ZZ Ceti stars can be exceedingly complex. Their light curves are always periodic or quasi-periodic, but dozens of periods can be present simultaneously. The amplitudes and exact periods of the variations can change on time scales as short as a few hours. The observed periods lie in the range 10^2 to 10^3 s. These periods are two orders of magnitude greater than the expected periods of radial pulsations in white dwarfs (Ostriker 1971), demonstrating that the observed pulsations must be nonradial. The obvious candidates for the pulsation modes are the g-mode pulsations since they can have very long periods, but it is not clear that even the g-mode pulsations can have isolated periods as long as the 1186 s period observed in the ZZ Ceti star GD 154 (Robinson *et al.* 1978).

The most thoroughly observed ZZ Ceti star is ZZ Ceti itself (=R548). R548, a DA white dwarf, was discovered to be a variable white dwarf by Lasker and Hesser (1970). Their analysis (Lasker and Hesser 1971, hereafter Paper I) demonstrated that the light curve of R548 has a relatively simple power spectrum consisting of just two low-amplitude pulsations with periods near 213 s and 274 s. The power spectrum varied on a time scale of about one day, but the data were insufficient to define the nature of the variations. Using a larger set of data, Robinson, Nather, and McGraw (1976, hereafter Paper II) showed that each pulsation found by Lasker and Hesser was actually an unresolved pair of pulsations with components separated by about 0.5 s in period. All four pulsations were constant in amplitude and period. The apparent variation of the power spectrum is caused solely by the slow beating between the unresolved components of the pairs. Using additional observations, Stover, Robinson, and Nather (1977, hereafter Paper III) improved the numerical accuracy of the model and gave an upper limit to the rate of change of the period of $|\dot{P}| < 10^{-11}$ s s^{-1}.

R548 is uniquely important among the ZZ Ceti stars because it is the only one whose light curve has been deciphered so completely, and it is the only one whose periods can be found reliably and accurately. These data are critically necessary for any theoretical interpretation of the ZZ Ceti stars. In this paper, the culmination of 9 years of work on R548, we combine the data in Papers I and III for the first time, and we add to these data many previously unpublished observations.

II. THE OBSERVATIONS

Table 1, with 58 entries, lists all of the observations included in the period stability analysis. Though obtained with a variety of photometers and telescopes, all of these observations were made without the use of color filters and will be referred to as the "white light" observations. Two of us (Hesser and Lasker) measured the light curve of R548 during the years 1970 through 1973 with the 0.9 m telescope of the Cerro Tololo

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TABLE 1

NIGHTLY OBSERVATIONS OF R548

	213 s PULSATION			274 s PULSATION		
RUN	Phase	Amplitude (mag)	T_{max} (EJD 2,440,000+)	Phase	Amplitude (mag)	T_{max} (EJD 2,440,000+)
HL 2109.....	0.00	0.0120	891.72958	0.80	0.0054	891.72983
HL 2111.....	0.66	0.0053	892.68475	0.37	0.0052	892.68596
HL 2112.....	0.94	0.0091	904.62390	0.54	<0.002	904.62682[a]
HL 2113.....	0.63	0.0069	905.61314	0.14	0.0086	905.61438
HL 2114.....	0.18	0.0112	913.61758	0.94	0.0075	913.61958
HL 2115.....	0.91	0.0124	914.66859	0.57	0.0013	914.66974
HL 2117.....	0.13	0.0087	936.59606	0.74	0.0043	936.59743
HL 2118.....	0.64	0.0073	954.62615	0.57	0.0037	954.62664
HL 2119.....	0.34	0.0059	955.64437	0.18	0.0072	955.64607
HL 2122.....	0.41	0.0040	958.62677	0.97	0.0095	958.62644
HL 2127.....	0.07	0.0119	1185.83779	0.41	0.0047	1185.83927
HL 2128.....	0.77	0.0092	1186.84714	0.02	0.0083	1186.84845
HL 2129.....	0.47	0.0056	1187.86070	0.62	0.0033	1187.86032
HL 2131.....	0.26	0.0081	1625.68777	0.54	0.0038	1625.68754
HL 2132A ...	0.90	0.0128	1626.60805	0.09	0.0081	1626.61165
HL 2132B ...	0.03	0.0172	1626.79294	0.20	0.0061	1626.79603
HL 2133.....	0.64	0.0051	1627.67396	0.73	0.0050	1627.67435
HL 2241.....	0.26	0.0107	1651.62864	0.12	0.0082	1651.63039
HL 2261.....	0.95	0.0127	1652.62051	0.71	0.0074	1652.62381
HL 2279.....	0.37	0.0077	1919.86005	0.19	0.0063	1919.86331
HL 2281.....	0.08	0.0102	1920.87392	0.80	0.0071	1920.87512
1432	0.39	0.0068	2337.84170	0.18	0.0059	2337.84161
1660	0.57	0.0061	2689.74986	0.51	<0.003	2689.75223[a]
1661A.......	0.28	0.0092	2690.78077	0.12	0.0090	2690.78075
1661B.......	0.38	0.0054	2690.92606	0.21	0.0068	2690.92683
1663A.......	0.98	0.0120	2691.78512	0.73	0.0072	2691.78350
1663B.......	0.08	0.0137	2691.93306	0.82	0.0091	2691.93603
1665	0.75	0.0093	2692.89034	0.39	0.0024	2692.89185
1667	0.45	0.0027	2693.89878	0.00	0.0095	2693.90095
S2163A......	0.81	0.0097	2694.42233	0.31	0.0052	2694.42162
S2163B......	0.90	0.0119	2694.55789	0.39	0.0037	2694.55808
1669	0.17	0.0107	2694.94007	0.62	0.0045	2694.93873
S2164A......	0.50	0.0048	2695.41610	0.91	0.0094	2695.41825
S2164B......	0.59	0.0055	2695.54952	0.99	0.0077	2695.54849
1671	0.79	0.0112	2695.83074	0.16	0.0066	2695.83128
1675	0.82	0.0116	2721.82102	0.76	0.0074	2721.82098
1684	0.87	0.0118	2724.77131	0.54	0.0032	2724.77301
1687	0.60	0.0037	2725.82987	0.17	0.0079	2725.83069
1690	0.26	0.0101	2726.78403	0.74	0.0068	2726.78210
1696	0.86	0.0116	2776.64345	0.69	0.0054	2776.64543
1701	0.55	<0.002	2777.63270[a]	0.28	0.0055	2777.63333
1706	0.93	0.0123	2779.63070	0.48	<0.003	2779.63304[a]
1709	0.71	0.0061	2783.63219	0.88	0.0083	2783.63225
1750	0.37	0.0065	2984.91601	0.75	0.0059	2984.91769
1751	0.76	0.0100	2986.91959	0.96	0.0078	2986.92066
1778	0.12	0.0119	3076.80007	0.93	0.0072	3076.80123
1781	0.78	0.0092	3077.74278	0.50	0.0030	3077.74436
1799	0.95	0.0116	3135.64607	0.28	0.0067	3135.64836
1802	0.63	0.0068	3136.62047	0.85	0.0069	3136.61885
1935	0.54	0.0051	3395.90455	0.56	<0.003	3395.90341[a]
1949	0.63	0.0069	3401.80041	0.09	0.0088	3401.80158
1951	0.43	0.0039	3405.83557	0.51	<0.004	3405.83566[a]
1954	0.57	0.0017	3434.86102	0.94	0.0075	3434.86059
1957	0.24	0.0109	3435.82725	0.49	<0.002	3435.82860[a]
1984	0.72	0.0098	3456.69680	0.06	0.0109	3456.69903
1999	0.19	0.0103	3458.81065	0.33	0.0071	3458.81057
2341	0.28	<0.002	3843.75419[a]	0.48	<0.002	3843.75475[a]
2373	0.71	0.0087	3874.63163	0.02	0.0104	3874.63011

[a] Amplitude given is an estimated upper limit, and T_{max} is the time predicted by our model.

Inter-American Observatory. Our observing technique has been described in Paper I and in Lasker (1971). The data consist of high-speed time-series observations taken with a conventional offset-guiding photometer and refrigerated blue-sensitive photomultiplier. The dates of the observations can be found in the first 21 entries of Table 1. The first six entries correspond to the six runs presented in Paper I. The remaining data in Table 1 include the observations presented in Paper III along with additional observations obtained with the 0.76 m and 2.1 m Struve telescopes at McDonald Observatory. The McDonald observations also consist of high-speed photometry made in unfiltered light with a blue-sensitive photomultiplier (Nather 1973). In total, the white light observations represent nearly 170 hours of observing on 51 nights. Additionally, color information for R548 was obtained on the nights of 1971 September 21–23, when it was observed simultaneously at two Cerro Tololo telescopes. At the 1.5 m telescope two channels of a three-channel photometer were used to monitor it through *B* and *V* filters, while white light observations were carried out with a single channel photometer on the 0.9 m telescope. These color data have been analyzed separately from the white light observations.

The time of arrival of the pulsations of R548 can be measured to about ± 10 s. In order to preserve this measurement accuracy over the 8 year time span covered by our observations, we have converted all of our timings from the UTC time scale to the more uniform Ephemeris Julian Date (EJD). Beginning with UTC, we compute the usual Heliocentric Julian date to which a correction is applied to convert the Julian Date to ephemeris time. The correction takes account of the fixed offset between UTC and ET as well as the change in length of the UT second prior to 1972 and the leap-seconds added to UTC after 1972. The resulting time scale should be uniform to better than 1 s.

III. ANALYSIS OF THE LIGHT CURVE

a) *Preparation of the Data*

Since our analysis techniques for the white light observations have been described in detail in Paper III, we will give only a brief review of the techniques here. First, we prepare each night's observations for analysis by removing contributions from sky background and dark noise and by making an approximate correction for atmospheric extinction. Positive magnitudes ($+2.5 \log N$) are computed for the observations so that the magnitude maximum corresponds to the intensity maximum. Then we apply a fast Fourier transform (FFT) filter designed to remove the mean and any remaining long-term trends from the data.

b) *Analysis of Individual Nights*

Our model for the light curve of R548 consists of two pairs of pulsations, one pair with periods near 213 s and one pair with periods near 274 s. Since the components of each pair are separated by only about 0.5 s, data obtained on a single night cannot resolve a pair into its two components. Instead, each pair appears as a single sinusoidal pulsation that is the sum of its two components. The two components beat together so that the sum pulsation varies in both amplitude and phase with a beat period of $1^{d}.44122$ for the 213 s pair and $1^{d}.66528$ for the 274 s pair. Because the beat periods are so long, a single night's observations can be correctly represented by a mean amplitude and a typical time of a maximum for each of the two summed pulsations. The only exceptions are nights in which the observations spanned 7 hours or more, allowing the night to be divided meaningfully into a first and a second half. The mean amplitudes and arrival times have been determined by fitting each night's FFT-filtered observations, $\Delta m(t)$, with two sine curves:

$$\Delta m(t) = \sum_{i=1}^{n} A_i \sin [\omega_i(t - t_{0_i})] , \qquad n = 2 . \quad (1)$$

The mean amplitudes are given by the A_i's, and the mean arrival times are given by $t_{\max} = t_{0_i} + \pi/(2\omega_i)$.

The results of fitting equation (1) to each night's data are recorded in Table 1 along with the phase of the beat cycle (calculated ex post facto). The estimated errors of the times of maximum depend on the amplitude of the observed pulsations, and range from about 10 s when the amplitudes are high to about 20 s when the amplitudes are low. For those runs marked with a superscript "a" in Table 1, the observed amplitudes were so low that a reliable time of arrival could not be determined. Instead, we have given an upper limit to the amplitude and the time of arrival predicted from the model of Table 2.

c) *Analysis of Pulsation Parameters*

To resolve fully the close pulsation pairs and to determine accurately all of the pulsation parameters, many nights' observations must be combined. The data given in Table 1 could be used to determine the amplitudes, periods, and phases of the four pulsations. However, we have employed a more direct method by collectively using all of the original data in a single least-squares analysis of the 12 model parameters. Specifically, we have directly fitted the 51,000 individual FFT-filtered white-light measurements with a model given by equation (1) with $n = 4$. As described in detail in Paper III, we have taken great care to ensure that no error in the pulsation cycle counts occurs between observing runs or between observing seasons. The parameters of the best fit are listed in Table 2.

d) *Analysis of $\dot{P}$*

We have attempted to measure the rate of change of the pulsation periods, $\dot{P}$, by fitting the data with a

TABLE 2

PARAMETERS OF THE LIGHT VARIATIONS OF R548

PARAMETER	PULSATION			
	1a	1b	2a	2b
Amplitude, A (mag)	0.0077 ±2	0.0044 ±2	0.0049 ±2	0.0034 ±2
Frequency, ω (s^{-1})	0.0294801695 ±2	0.0295306282 ±4	0.0229103616 ±4	0.0228666921 ±5
Period, P (s)	213.132605 ±2	212.768427 ±3	274.250814 ±4	274.774562 ±6
Epoch, t_0 (EJD 2,442,688+)	0.945106 ±8	0.945086 ±14	0.859541 ±16	0.859449 ±24
Period change, $\lvert\dot{P}\rvert$ (unitless)	$<2 \times 10^{-13}$	$<7 \times 10^{-13}$	$<3 \times 10^{-13}$	$<9 \times 10^{-13}$

model of the form

$$\Delta m(t) = \sum_{i=1}^{4} A_i \sin \{[\omega_i + \dot{\omega}_i(t - t_\omega)][t - t_{0_i}]\} . \quad (2)$$

The amplitudes A_i are fixed at the values determined by the pulsation analysis, and the single epoch t_ω is set equal to EJD 2,442,688.86. Therefore, this model also has 12 independent parameters. The value of $\dot{\omega}_i$ obtained from the fit of equation (2) were not significantly different from their estimated errors. Therefore, we give in the last row of Table 2 upper limits on $\dot{P}$ for each of the four pulsations.

e) *Analysis of Color Data*

The data included in the derivation of the model parameters listed in Table 2 were obtained with a variety of telescopes, photometers, and phototubes, each one with its own wavelength passband. This inhomogeneity in the data could lead to systematic errors in the derived model parameters if the phases of the pulsations observed in R548 are strongly wavelength dependent. Therefore, we have analyzed the B and V color observations obtained in 1971 September for wavelength-dependent phase differences by fitting to each color a model of the form

$$\Delta m(t) = A_{213} \sin [\omega_{213} t + \phi_{213}] + A_{274} \sin [\omega_{274} t + \phi_{274}] . \quad (3)$$

By fixing the two frequencies to be the average frequency of the respective pulsation pair the fitting procedure becomes a simple linear least-squares problem. To within the measurement errors, about ±15 s, we have not detected any systematic differences between the pulsation phases of either the B, V, or white-light measurements. Therefore, the inhomogeneity of the 9 years of data does not significantly influence the derived parameters of our model.

IV. DISCUSSION

The parameters given in Table 2 for our model of the light variations of R548 agree with the results of Paper III. However, the new results on the periods and period stabilities are much more accurate because the time

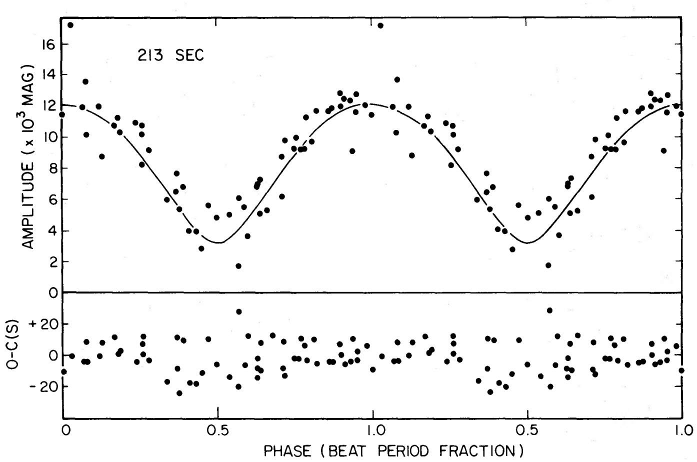

FIG. 1.—The upper half of the figure displays the measured amplitude of the 213 s sum pulsation folded on its 1ᵈ44122 beat period. The solid curve is the amplitude predicted by our model. The lower half of the figure displays the difference between the observed and predicted pulse arrival times ($O - C$) of the 213 s sum pulsation. The differences have been folded on the beat period.

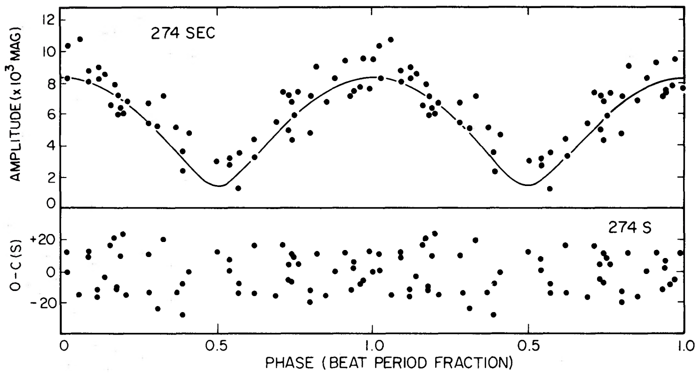

FIG. 2.—The upper half of the figure displays the measured amplitude of the 274 s sum pulsation folded on its 1ᵈ66528 beat period. The solid curve is the amplitude predicted by our model. The lower half of the figure displays the difference between the observed and predicted pulse arrival times ($O - C$) of the 274 s sum pulsation. The differences have been folded on the beat period.

interval covered in Paper III has been increased by about a factor of 7 in the present work. This large increase in the interval also requires us to retest the validity of the 12-parameter model we have adopted. There are several ways to test the model, but we believe that the most meaningful test is to examine the direct fit of the model to the data in Table 1. The upper halves of Figures 1 and 2 compare the amplitudes given in Table 1 with the amplitudes predicted by the model parameters given in Table 2. The amplitudes have been folded at the beat period. The solid curve is the amplitude predicted by the model. The model correctly reproduces both the shape and the range of the observed amplitude variations, with an rms deviation of the individual points from the curve of 0.0014 mag in both figures. There are no detectable systematic variations in the rms deviation over the 9 years of data, demonstrating that the model is valid for the entire interval.

The lower halves of Figures 1 and 2 display the difference ($O - C$) between the observed and predicted times of arrival of the maxima of the sum pulsations. The differences have been folded at the beat period. The residual differences have a zero mean in both figures and rms deviations of 11 s for the 213 s pair and 13 s for the 274 s pair. The rms residuals increase somewhat at beat phases when the amplitude is low, but there is no systematic variation in the mean of the residuals. Figures 3 and 4 plot the same ($O - C$) residuals against date of observation instead of phase of the beat cycle. No systematic trends are present in any of the figures, demonstrating again that the model is valid for the entire nine years of observations.

Figures 3 and 4 also show that the periods of the pulsations of R548 are constant to within the limits of measurement. From the direct fit of equation (2), we found the limits to be as small as $|\dot{P}| < 2 \times 10^{-13}$ s s^{-1} for the large-amplitude 213.13 s pulsation. This is the most stringent upper limit on the rate of change of a period in a periodic variable star ever to have been measured at visual wavelengths. For comparison, the 71 s oscillation in the old nova DQ Her has $\dot{P} = -5 \times 10^{-13}$ (Patterson, Robinson, and Nather 1978); the Crab pulsar has $\dot{P} = 4 \times 10^{-13}$ (Gullahorn *et al.* 1977); and the dwarf Cepheid EH Lib has $|\dot{P}| \lesssim 5 \times 10^{-13}$ (Percy, Matthews, and Wade 1979). It is instructive to note that the periodicities of DQ Her and the Crab pulsar are thought to be caused by rotation, whereas the periodicities of EH Lib and R548 are caused by pulsations. Obviously $\dot{P}$ cannot be used to discriminate rotation from pulsation in these four stars.

If R548 is evolving on a cooling time scale, the high

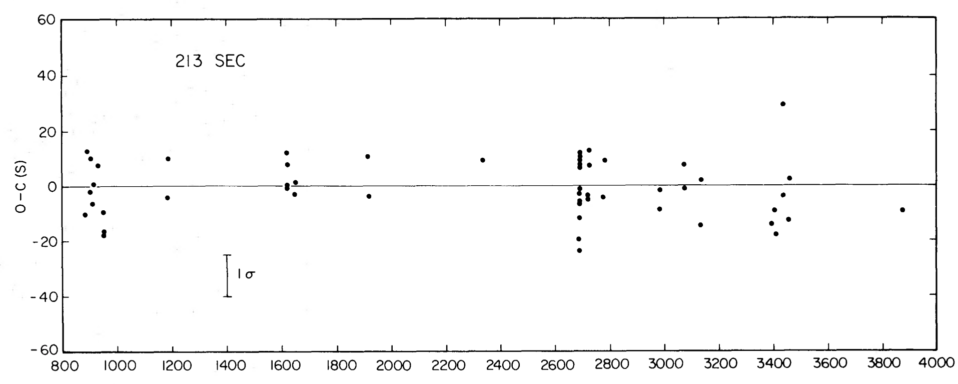

FIG. 3.—The dots are the difference between the observed and predicted pulse arrival times ($O - C$) of the 213 s sum pulsation. The differences are plotted against the date of observation.

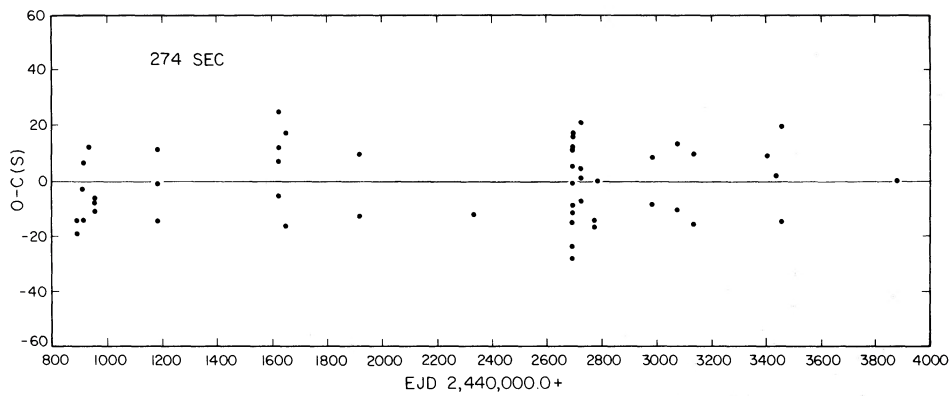

FIG. 4.—The dots are the difference between the observed and predicted pulse arrival times ($O - C$) of the 274 s sum pulsation. The differences are plotted against the date of observation.

stability of its pulsation periods is not surprising. The periods of *p*-mode pulsations are essentially independent of temperature in white dwarfs, and the periods of *g*-mode pulsations increase only very slowly with decreasing temperature. According to Van Horn (private communication), $\dot{P}$ should be of order 10^{-15} s s^{-1} for the *g*-modes of white dwarfs. Our measured upper limit on $\dot{P}$ is consistent with this cooling time scale. An actual measurement of $\dot{P}$ would be of interest, but patience and longevity will be required in order to obtain it. Barring a radical improvement in measurement techniques, the accuracy with which $\dot{P}$ can be measured improves as the square of the time interval over which the measurement is made. Thus, in order to measure $\dot{P}$ in R548 to an accuracy of 10^{-14} s s^{-1}, a time interval of over 100 years will be necessary.

In summary, then, our results are (*a*) the properties of the pulsations of R548 remained constant from 1970 through 1978; (*b*) the model presented in Papers II and III still provides a complete and accurate description of the light curve; and (*c*) the upper limit on the rate of change of the pulsation period has been reduced to as low as $|\dot{P}| = 2 \times 10^{-13}$ s s^{-1} for the 213.13 s pulsation.

This research was supported in part by NSF grants AST 76-23882 and AST-7906340.

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