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GRAVITATIONAL RADIATION AND THE EVOLUTION OF CATACLYSMIC BINARIES

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ABSTRACT

Results are presented for several evolutionary sequences of close binary systems which resemble dwarf novae. The systems, containing white dwarfs and main-sequence companions, have masses less than 3.4 $M_\odot$ and periods less than 17 hours. The loss of angular momentum by gravitational radiation is included in the calculations. It is found that a system can be captured as a result of evolution dominated by gravitational radiation losses when the primary encounters its Roche lobe while near the main sequence. In such a case, the system evolves to even shorter periods, with the nuclear time scale of the primary increasing so that the latter is unable to exhaust hydrogen.

Other sequences do not result in capture (1) when the primary is sufficiently evolved that it can exhaust hydrogen during mass exchange and (2) when we consider a simple model for mass and angular momentum losses associated with nova outbursts from the accreting white dwarf.

Subject headings: gravitation — stars: binaries — stars: dwarf novae — stars: evolution

I. INTRODUCTION

In this paper we investigate the effects of gravitational radiation on the evolution of short-period, semidetached, cataclysmic variable stars. The possible importance of gravitational radiation for these short period systems was first pointed out by Kraft, Mathews, and Greenstein (1962). In general this study is an extension of an earlier paper by Faulkner (1971) in which it was shown that gravitational radiation, by removing orbital angular momentum, could stimulate mass transfer from the main-sequence red component at a rate which causes the binary to evolve more rapidly than nuclear evolution would dictate. Results from that study indicated mass transfer rates in the range 10^{-9} to 10^{-10} $M_\odot$ yr^{-1}, and predicted that the orbital periods would continuously decrease until the red star's structure switched to degenerate electron pressure support. This ultimate, doubly degenerate configuration was suggested as a model for HZ 29, a binary with a period of only 18 minutes and no detectable hydrogen (Faulkner, Flannery, and Warner 1972).

Faulkner's earlier study was based on the response to mass transfer of equilibrium models for the red star. In fact, the structure of the red star is altered from thermal equilibrium by the mass transfer. Since the mass transfer rate, the structure of the red star, and the binary orbital changes are totally interrelated, we have calculated the time-dependent evolution of the red component in order to assess the effects of nonequilibrium and nonzero age structure on the overall evolutionary picture. A brief summary of our work has already been given in an earlier communication (Faulkner 1976). The discussion of similar effects has also been presented in an independent study by Chau and Lauterborn (1977).

In the next section the method of calculation and the basic assumptions are stated; in § III the results are presented and described, and finally in § IV we discuss the implications of these results and make some concluding remarks.

II. METHOD AND ASSUMPTIONS

As is usual in double star calculations (see Paczyński 1971), we evolve the red star, also referred to as the primary, as though it were a single, spherically symmetric star. The influence of the binary companion, assumed to be a white dwarf, enters by constraining the red star's radius, R, to remain inside its volume equivalent Roche radius, R_L. If at any time step R would exceed R_L, we force the primary to lose mass at a rate such that $R = R_L$. Again as is customary, to evaluate the Roche radius throughout the evolution, it is assumed that the binary orbit remains circular, that the red star corotates, and that the (small) angular

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momentum associated with rotation of the components can be neglected. From the known mass ratio and the variation in orbital separation, described below, the Roche radius is calculated using standard approximations.

To determine the evolution, the variation in orbital parameters in response to mass loss from the primary must be prescribed. For circular orbits the period P, masses M_1 (primary, red star) and M_2 (secondary, white dwarf), and orbital angular momentum J are related by

$$J^3 = G^2(P/2\pi)\frac{(M_1 M_2)^3}{M_1 + M_2}\cdot \tag{1}$$

The separation follows from Kepler's third law. If one makes the standard assumption that both total mass and orbital angular momentum are conserved, then both the period and separation reach minima when the mass ratio is unity. However, the primary's Roche radius reaches a minimum when its mass fraction, $M_1/(M_1 + M_2) \approx 0.44$. In all our calculations we include the loss of angular momentum associated with the emission of gravitational radiation, given by

$$\frac{dJ}{dt} = -\frac{32}{5}\frac{(2\pi G)^{7/3}}{c^5}\frac{(M_1 M_2)^2}{(M_1 + M_2)^{2/3}}\frac{1}{P^{7/3}}\cdot \tag{2}$$

One can define a characteristic time scale for evolution in response to gravitational radiation $\tau_{\rm GR}$, by

$$\frac{d \ln J}{dt} \equiv -\frac{1}{\tau_{\rm GR}} = -1.27 \times 10^{-8} \times \frac{m_1 m_2}{(m_1 + m_2)^{1/3}}\frac{1}{P_{\rm hr}{}^{8/3}}\ {\rm yr}^{-1}\ . \tag{3}$$

Here m_1 and m_2 are the masses in units of a solar mass. Note that the strong dependence on period makes such losses most effective in short-period binary stars. To illustrate the relevant time scales, consider a binary containing two 1 $M_\odot$ components. Evolution by gravitational radiation will occur on a time scale shorter than the nuclear evolution of the main-sequence primary (10^{10} years) when the binary period is less than 6 hours, and losses by gravitational radiation dominate thermal evolution (10^8 years) when the period is less than 1 hour. The loss of angular momentum always forces the period and separation of the binary to reach smaller minima than would obtain with completely conservative assumptions.

Once the primary begins to lose mass, the orbit varies in response to the shifting mass ratio according to equation (1). One can define a time scale for mass loss from the primary by

$$\frac{d \ln M_1}{dt} = -\frac{1}{\tau_M}\cdot \tag{4}$$

We investigate two modes for the transfer of mass from the primary to the white dwarf. In the first case we assume that the system conserves mass and loses angular momentum only by gravitational radiation; then the period changes according to

Case 1:

$$\frac{d \ln P}{dt} = 3\left[-\frac{1}{\tau_{\rm GR}} + \frac{(M_2 - M_1)}{M_2}\frac{1}{\tau_M}\right]\cdot \tag{5}$$

Losses by gravitational radiation always tend to decrease the period; but when the mass ratio favors the white dwarf, mass loss from the red star acts to increase the period.

The fate of the matter accreted by the white dwarf from the red star remains uncertain. If the hydrogen-rich matter remains on the white dwarf, it must ignite in fusion reactions which may release sufficient energy to eject the mass from the system. Furthermore, during outbursts novae appear to expel some mass at velocities large compared to orbital speeds. Although the fraction of accreted matter which remains bound to the white dwarf is unknown, it is possible that the white dwarf can be driven over the critical Chandrasekhar mass, inducing a supernova.

We have also considered an ad hoc model of possible mass loss from the system. In this second mode of mass transfer, we assume that mass flowing from the primary is temporarily accreted by the white dwarf which then completely ejects the accreted mass from the system. It is also assumed that the escaping mass carries off the specific angular momentum appropriate to the white dwarf. This crudely mimics a picture of nova outbursts recurring frequently relative to the overall evolutionary time scale of the binary. Mass from the primary temporarily falls onto the white dwarf, with the binary orbit remaining circular; then in repeating episodes a nova event expels the material accumulated since the last outburst. In this case the period variation is given by

Case 2:

$$\frac{d \ln P}{dt} = 3\left\{-\frac{1}{\tau_{\rm GR}} + \frac{M_2}{M_1 + M_2} \times \left[-\left(\frac{M_1}{M_2}\right)^2 + \frac{2}{3}\left(\frac{M_1}{M_2}\right) + 1\right]\frac{1}{\tau_M}\right\}\cdot \tag{6}$$

From the quadratic factor in the second term one can easily show that mass loss from the primary already begins to favor an increasing period for $M_1/M_2 \lesssim 1.39$, compared with $M_1/M_2 < 1$ in case 1. Note that the ratio of the specific angular momentum of the white dwarf relative to the specific angular momentum of the binary is simply the mass ratio M_1/M_2 (where 2 denotes the white dwarf); thus, mass loss increases the specific angular momentum of the remaining matter when the white dwarf is the more massive component. It is entirely possible that nature utilizes specifications completely unlike either of our mass transfer modes, but the two cases, here, have the virtue of being plausible and well defined.

Finally, one must contrive to specify initial configurations from which to commence evolutionary sequences, even though the progenitors of the dwarf novae are not known. In normal classical calculations of the evolution of double systems with constant mass and angular momentum, and in which the system never evolves through a period of contact, it is impossible to produce such a short-period system, near unit mass ratio, containing a white dwarf. The incubation of a white dwarf implies a previous stage when the binary would require a period of at least decades to accommodate the giant progenitor. However, it is well known that many observed binary stars must have evolved into contact with probably severe losses of mass and angular momentum (Flannery and Ulrich 1977). Though no remotely relevant calculations exist, speculation and observation suggest that, for some classes of progenitors, dwarf novae emerge from the contact system (see, e.g., the discussions in Ostriker 1975; Paczyński 1976; Flannery and Ulrich 1977; Flannery 1978; Taam, Bodenheimer, and Ostriker 1978).

We chose to model cases in which the binary contains a white dwarf paired with a normal, unevolved, main-sequence companion. The initial system not only has the virtue of simplicity, but it also has observational justification in some of the best studied systems. In particular, in the dwarf novae Z Cam (Kraft, Krzeminski, and Mumford 1969) and EM Cyg (Robinson 1973) the mass, radius, and luminosity of the red component closely resemble those of a main-sequence object. Admittedly in some other cases, such as the longer period systems T CrB and GK Per, the red star shows giant characteristics. The entire question is discussed in Robinson (1976). While the mechanism which produces such an initial system remains unknown, our initial models do agree with at least some observed systems.

III. RESULTS

The evolution of the primary component was followed utilizing the stellar evolution program developed and described by Eggleton (1971, 1972). The code was modified to incorporate the constraint on the stellar radius as described in the previous section. A Population I composition ($X = 0.70$, $Y = 0.28$, $Z = 0.02$) was adopted for all calculations. Seven evolutionary sequences were calculated, each distinguished by the initial period, total mass, and the mode of mass transfer. A summary of the initial parameters of each sequence is given in Table 1. In order to reveal the main physical effects of gravitational radiation on the binary evolutions, we have divided the discussion of the results into separate cases where gravitational radiation leads to capture (evolution into a shorter period system) and to other cases where capture is avoided.

a) Case 1: Evolution Leading to Capture

Evolutionary sequences 1 and 5 with initial periods of 9.42 hours and 10.51 hours, respectively, resulted in gravitational radiation capture. The primary of mass 1.2 $M_\odot$ reached its Roche radius at a less evolved phase of evolution than would have been the case had gravitational radiation losses been omitted. Mass transfer was initiated onto the 0.8 $M_\odot$ white dwarf secondary as the Roche lobe (shrinking due to the loss of orbital angular momentum) met the expanding stellar radius. We denote this point of contact as point A on the H-R diagram (Fig. 1). As the mass transfer rate increased to a brief maximum rate of $3.5 \times 10^{-9}\ M_\odot\ \mathrm{yr}^{-1}$ ($4.0 \times 10^{-9}\ M_\odot\ \mathrm{yr}^{-1}$) for sequence 1 (5), the primary slid down roughly parallel to the zero-age main sequence. Thermal equilibrium of the primary was disturbed and was not restored until point B when the mass transfer rate had abated. During this thermal disequilibrium phase the central regions underwent a nonhomologous contraction. Because of the reduction in pressure associated with the cumulative effects of the rapid mass loss phase, the central temperatures in sequence 1 decreased quite strongly with increasing density (see Fig. 2). Consequently, the nuclear energy generated in the core was noticeably reduced, leading to a greater longevity for the binary system. In the surface layers, we note that the luminosity decreased much faster than in the central regions because of the luminosity deficit in the expanding envelope. In sequence 5, on the other hand, the variations in the central temperature were less marked, mainly due to the greater tendency for contraction to

TABLE I

Sequence	$M_{1i}\,(M_\odot)$	$M_{2i}\,(M_\odot)$	P_c (hours)	X_{ci}	τ_c (years)	β
1	1.20	0.80	9.42	0.56	8.90 (8)	0
2	1.20	0.80	12.30	0.25	2.75 (9)	0
3	1.20	0.80	15.93	0.00	3.70 (9)	0
4	1.20	0.40	14.96	0.00	3.70 (9)	0
5	1.20	0.80	10.51	0.42	1.81 (9)	0
6	2.00	1.33	11.15	0.66	8.35 (7)	1
7	1.20	0.80	9.42	0.56	8.90 (8)	1

NOTE.—Column headings have the following meanings: M_{1i}, initial primary mass; M_{2i}, initial secondary mass; P_c, period of the binary system at the onset of mass transfer; X_{ci}, central hydrogen content at the onset of mass transfer; τ_c, age of the system at the onset of mass transfer; and $\beta = 0$ (1) corresponds to conservative (nonconservative) assumptions.

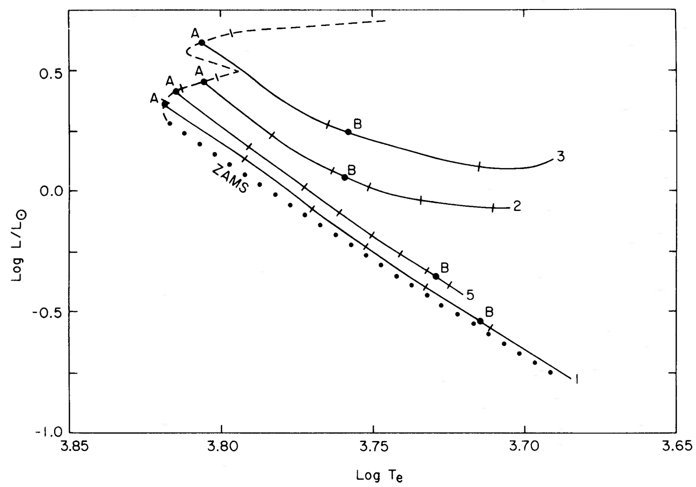

FIG. 1.—The evolutionary tracks in the H-R diagram for selected sequences with identical initial primary mass (1.2 $M_\odot$), labeled by model sequence numbers. Each tick mark corresponds to intervals of 10^9 years. Points marked A on the evolutionary track of a single star of constant mass (dashed curve) indicate where mass transfer is initiated in the binary sequences. Points B correspond to the restoration of thermal equilibrium.

occur in regions of varying molecular weight. From Fig. 3, it is found that the slope of the mass-radius relation for the primary in its nonequilibrium phase of evolution, especially for sequence 1, was nearly identical to that given for the zero-age main sequence. This, in effect, justifies the assumptions made in the work of Faulkner (1971). By the time thermal equilibrium was restored, the primary was the less massive component; however, subsequent evolution proceeded on a time scale dictated by the mass transfer stimulated by the gravitational radiation losses rather than by nuclear evolution.

The period and Roche radius continued to decrease throughout the evolution. In fact, even for a mass ratio (primary to secondary) less than unity, the loss of orbital angular momentum more than offset the compensating effect of mass transfer (see Figs. 4 and 5). For sequence 1, comparison between the mass transfer and gravitational radiation time scales (calculated for unit mass ratio) yields $\tau_M/\tau_{GR} \sim 0.35$. Although this is small, mass transfer does not dominate; the coefficient $1 - M_1/M_2$ in equation (5) remains smaller (i.e., < 0.35) for some time after mass ratio reversal. Therefore, the period decreases further. Consequently, τ_{GR} itself continued to decrease. Gravitational radiation, once dominant, becomes increasingly so, and the main-sequence component of the binary is unable to escape from its shrinking fate by nuclear means.

In Table 2, the parameters of the evolution of the primary as well as those of the binary system are summarized. We remark that the differences in the

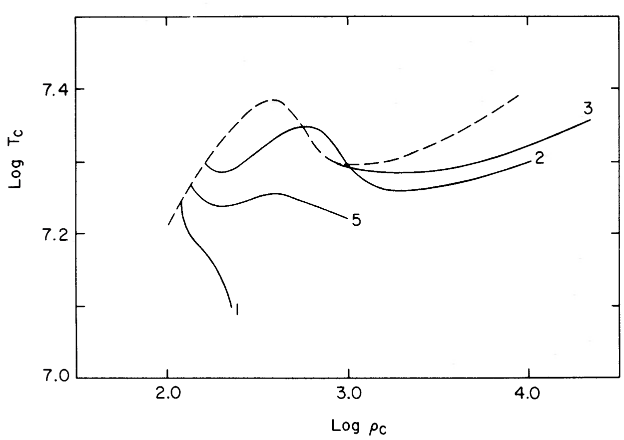

FIG. 2.—The evolution of the central regions for selected sequences are labeled by sequence number. The dashed-line track corresponds to the evolution with no mass loss.

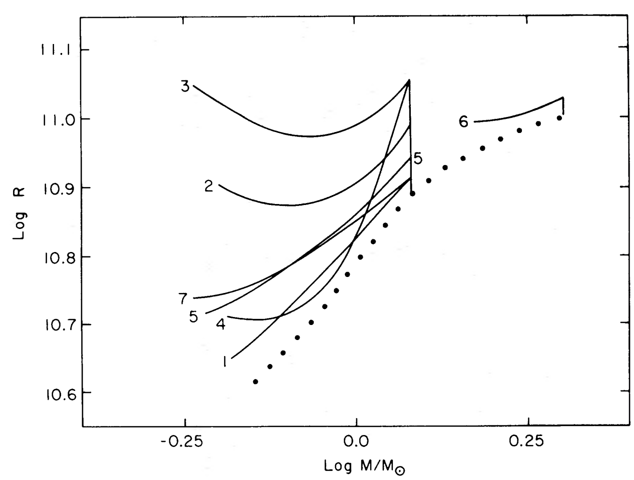

FIG. 3.—The mass-radius diagram for all sequences. The dotted line indicates the mass radius relation for the zero-age main sequence. Vertical sections correspond to evolution prior to mass transfer.

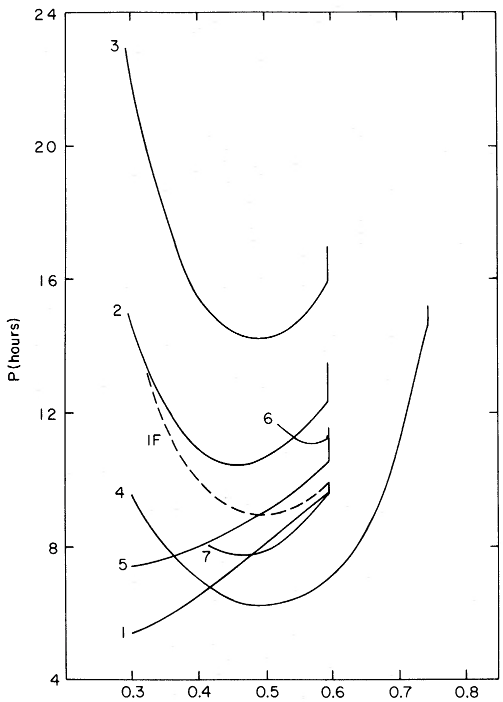

FIG. 4

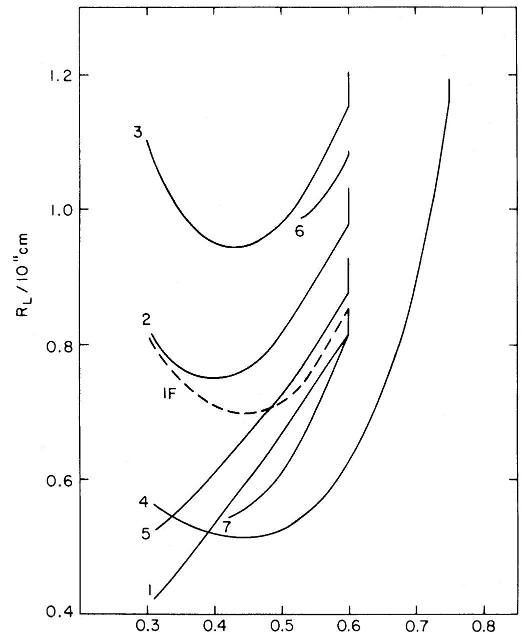

FIG. 5

FIG. 4.—Period versus mass fraction, $M_1/(M_1 + M_2)$ for all sequences (labeled by sequence number). The fiducial line labeled (1F) corresponds to the case where mass and angular momentum are conserved for sequence 1. Vertical portions correspond to evolution of the system period (due to gravitational radiation losses) prior to mass transfer.

FIG. 5.—Roche radius versus mass fraction, $M_1/(M_1 + M_2)$ for all sequences (labeled by sequence number). The fiducial curve (labeled 1F) corresponds to constant mass and angular momentum for sequence 1.

evolution between sequences 1 and 5, due solely to the more evolved state of sequence 5 at the initiation of mass loss and subsequently, is one of degree rather than of kind. In sequence 5, the primary was on the verge of hydrogen exhaustion ($X_c = 0.001$) when the evolution was terminated. However, for sequence 1 the hydrogen fraction in the core remains large, $X_c = 0.28$, so that nuclear evolution will never dominate the continuously decreasing time scale for evolution in response to gravitational radiation. Somewhat arbitrarily, we terminated the sequence at this point, in part because the assumption of conserved mass now results

TABLE 2

Parameter	Sequence 1	2	3	4	5	6	7
M_{1f} ($M_\odot$)	0.62	0.61	0.61	0.49	0.60	1.49	0.58
M_{Tf} ($M_\odot$)	2.00	2.00	2.00	1.60	2.00	2.82	1.38
P_f (hours)	5.42	14.85	22.33	9.41	7.50	11.62	7.99
X_{cf}	0.28	0.00	0.00	0.00	0.00	0.56	0.00
q_{He}	0.00	0.13	0.20	0.07	0.00	0.00	0.02
P_{min} (hours)	...	10.43	14.12	6.24	...	11.00	7.64
$\dot{M}_{max}$ ($M_\odot$ yr^{-1})	3.5 (−9)	4.0 (−9)	9.2 (−9)	3.1 (−7)	4.0 (−9)	1.4 (−8)	2.8 (−10)
τ_M (years)	8.7 (9)	6.9 (9)	2.2 (9)	1.0 (7)	8.5 (9)	5.9 (8)[a]	1.4 (10)
τ_{GR} (years)	2.5 (10)	5.5 (10)	1.2 (11)	1.4 (10)	3.5 (10)	2.9 (10)[a]	3.5 (10)
τ_f (years)	5.9 (9)	7.1 (9)	5.4 (9)	5.3 (9)	8.7 (9)	3.7 (8)	1.1 (10)

NOTE.—Row headings have the following meanings: M_{1f}, final mass of the primary; M_{Tf}, final total mass of the binary; P_f, period at the termination of the calculation; X_{cf}, final central hydrogen content; q_{He}, mass fraction in the helium core; P_{min}, the minimum binary period; $\dot{M}_{max}$, the maximum mass loss rate from the primary; $\tau_M(\tau_{GR})$, the mass transfer (gravitational radiation) time scale calculated at unit mass ratio; and τ_f, the final age of the binary sequence.

[a] Calculated at minimum period.

in the white dwarf exceeding its Chandrasekhar mass. This sequence, in which the primary encounters its Roche radius before appreciable nuclear evolution can occur, demonstrates that it is possible for the system to evolve to ever shorter periods without the primary leaving the main sequence. In close detail this sequence reproduces the earlier results obtained by Faulkner (1971) under the assumption that the primary always obeys a main-sequence mass-radius relation. Our results, and those in an analogous sequence discussed by Chau and Lauterborn (1977), show that Faulkner's approximation is self-consistent. Such a sequence can reproduce the known range of periods in dwarf novae as a straightforward evolutionary progression from long to short periods induced by gravitational radiation losses at a rate which can "freeze" the normal nuclear evolution of the primary. On the other hand, an extremely short period system such as HZ 29 could not result, because hydrogen will never be exhausted in the primary.

b) *Case 2: Evolution Avoiding Capture*

Evolutionary sequences 2, 3, 4, 6, and 7 do not lead to gravitational radiation capture. In the following we distinguish between the cases for which the total mass is conserved or not conserved in the evolution.

i) *Total Mass Conserved*

Sequences 2 and 3 illustrate the sensitivity of the results to the value of the binary period when the "initial" system of white dwarf and main-sequence star is first realized. The longer periods permit a greater time for the evolution of the primary as an essentially single star, and commensurately reduce the period-sensitive losses from gravitational radiation. Consequently these sequences, differing from sequence 1 in the substantially greater reduction in central hydrogen content at the onset of mass exchange, result in evolution little different from the conventional mass transfer case AB (Ziółkowski 1970) and case B (Kippenhahn, Kohl, and Weigert 1967; Refsdal and Weigert 1969; Giannone, Refsdal, and Weigert 1970) respectively. The onset of mass transfer for each sequence is, again, illustrated in Figure 1.

In sequence 2, it was found that the mass transfer time scale was significantly shorter than the gravitational radiation time scale at unit mass ratio (see Table 2). After mass ratio reversal τ_M decreased further, the enhanced mass transfer rates being a result of the exhaustion of hydrogen in the center with consequent formation of a hydrogen-burning shell. During this phase a secondary maximum mass transfer rate of $7.9 \times 10^{-10}\ M_\odot\ \mathrm{yr}^{-1}$ was achieved. Sequence 3 was qualitatively similar to sequence 2 with τ_M much shorter than τ_{GR} since most of the mass exchange occurred during the establishment of the hydrogen shell source burning phase. This corroborates the result that evolution in case B is faster than that in case AB for a system of fixed mass.

In sequence 3 mass transfer was initiated at 3.70×10^9 yr. We investigated the sensitivity of the results to initial mass ratio for mass transfer beginning at this age, by constructing sequence 4 with initial white dwarf mass equal to $0.4\ M_\odot$ rather than $0.8\ M_\odot$. Because of the change in companion mass, the corresponding binary period at the onset of mass exchange is slightly shorter than in sequence 3. Since the Roche radius is a strongly decreasing function of the mass ratio for large values of the mass ratio, the stellar radius, reduced substantially, approaching a value nearly appropriate to that for a main-sequence star of the same mass (see Fig. 3). Consequently, the mass transfer rates were about an order of magnitude larger than those attained in sequence 3, leading to $\tau_M/\tau_{\mathrm{GR}} \sim 0.001$. Although the minimum period reached 6.24 hours, the time the binary system spent at short periods was insufficient to permit the angular momentum losses due to gravitational radiation to be effective. In all of sequences 2, 3, and 4 the final slow mass transfer phase proceeded, in contrast to sequences 1 and 5, on a nuclear evolutionary time scale.

ii) *Evolution of the System with Mass Loss*

The prescription for the loss of mass and angular momentum from the binary system as described in § II was adopted for the last two evolutionary sequences. In evolutionary sequence 7 the maximum mass transfer rate was about an order of magnitude lower than that of sequence 1, although the initial stellar models are identical (see Table 1). This slow phase of mass transfer is attributable to the fact that the period and Roche radius decrease less rapidly with respect to the primary's mass when mass loss is included. Although $\tau_M/\tau_{\mathrm{GR}} \sim 0.40$ at unit mass ratio, it was found that gravitational radiation losses were unimportant. The mass that was lost after the mass ratio was reversed removed less than the average angular momentum per unit mass of the binary. Thus, the combined effects of the decrease in total mass with the tendency for the increase of the angular momentum per unit mass of the binary outweighed the effect of gravitational radiation losses, and the two components of the binary separated. To determine the effect of the total mass on the course of evolution, a sequence (numbered 6) was calculated in which the total mass of the system was increased to $3.33\ M_\odot$. The mass ratio was kept at 1.5 with the primary mass taken to be $2\ M_\odot$. The mass loss rate increased rapidly to about $10^{-8}\ M_\odot\ \mathrm{yr}^{-1}$ within 7×10^6 years after contact with the Roche surface. The accelerated rate of evolution is characteristic of a more massive primary since the thermal time scale, which determines the rapid mass loss rate, is shorter. Gravitational radiation losses were found to be unimportant as $\tau_M/\tau_{\mathrm{GR}}$ (calculated at the point of minimum period) was about 0.02. From equation (6) the minimum period occurs near $M_1 = 1.39\ M_2$ or where the mass fraction $\mu = M_1/(M_1 + M_2) = 0.58$ (see Fig. 4). Although the specific angular momentum removed by the mass in the rapid phase (from $\mu = 0.6$ to 0.57) was 1.5–1.3 times the average value for the binary, the mass loss, per se, dictated that the period increase for μ less than this value. Thus, gravitational radiation capture did not result for moderate mass systems because (*a*) the orbital period at the onset of mass transfer was too long and (*b*) the mass loss rates were too high at the phase of minimum period. In other words, for sequences 2, 3, 4, 6, and 7, either the radiation rate was too low or, if it was high, the time spent by the binary at this phase was so short that the net angular momentum radiated was small.

IV. CONCLUSION

We have shown that, for initial systems which agree with the observed structure of at least some dwarf novae, the loss of angular momentum by gravitational radiation can force the binary system to evolve with a continuously decreasing period. As a result of the loss of mass by the primary its nuclear evolution is arrested to the extent that the star will never exhaust hydrogen in its core. The results for this sequence substantiate the treatment of Faulkner (1971), who correctly assumed that the mass-radius relation of the primary would closely resemble that of a main-sequence star. They also agree with the analogous time-dependent calculations of Chau and Lauterborn (1977). Such a sequence provides a natural explanation for at least some of the dwarf nova systems as an evolutionary progression from long to short periods. However, this explanation cannot account for the ultrashort-period binary system HZ 29, in which both components are degenerate dwarfs lacking hydrogen, because the primary will never exhaust hydrogen.

On the other hand, several evolutionary sequences indicate that, if the primary can evolve appreciably enough that hydrogen can be exhausted, the resulting evolution will force the binary to separate sufficiently that gravitational radiation will be of little consequence. This occurs in part because the binary period must be longer to accommodate an evolved component, and losses by gravitational radiation decrease rapidly with increasing period. In addition, an evolved component tends to lose mass more rapidly when it encounters its Roche lobe since it has a shorter thermal time scale. This forces the binary to evolve more rapidly through its phase of minimum period without spending a long enough time at short period for gravitational losses to substantially reduce the angular momentum.

Finally, we have investigated sequences in which the binary loses both mass and angular momentum from the system. As a model of nova outbursts, all the mass lost from the primary was assumed to land on the white dwarf temporarily, and then to be expelled from the system, carrying off the specific angular momentum appropriate to the white dwarf. In this case two new effects operate: (1) mass loss from the system reduces gravitational binding and therefore favors an increasing separation, and (2) when the white dwarf is the more massive component, the lost mass carries off less than the average specific angular momentum, leaving the remaining mass with increased specific angular momentum. Both of these effects tend to decrease the influence of gravitational radiation losses.

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