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PULSAR TIMING. III. TIMING NOISE OF 50 PULSARS

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ABSTRACT

From an examination of the timing behavior of 50 pulsars, we have found that timing activity is $\gtrsim 50\%$ correlated with pulsar period derivative, weakly correlated with period, and uncorrelated with radio luminosity, galactic altitude, and other source parameters. We place an upper limit on the timing noise for the binary pulsar PSR 1913+16 that is consistent with its small period derivative. A detailed analysis of 11 pulsars indicates that random walks in the rotational phase (2 objects), frequency (4–7 objects), and frequency derivative (2 objects) are consistent with the data. Timing activity in general (apart from pulse shape changes and the glitches of the Crab and Vela pulsars and PSR 1641−45) appears to be consistent with a random walk origin. We explicitly show that the frequency second derivatives (apart from that of the Crab pulsar) and apparent steps in frequency that have been reported for some pulsars are probable manifestations of random walk processes. We constrain the rate of random walk steps between many steps per day and one every few years.

Subject heading: pulsars

I. INTRODUCTION

The precise rotation rates of pulsars, combined with their secular decrease, are the hallmark of the rotating searchlight interpretation of the observed pulse periodicities. The phase stability of the pulsation (defined as the rms phase divided by the total accumulated phase) is indeed at least as accurate as one part in 10^9 over a period of 5 years and, in some cases, is more stable by four orders of magnitude. At these levels of accuracy, however, some pulsars show deviations from a deterministic or predictable spin-down which appear to be random, either in the sense that discontinuous steps in the rotation frequency (glitches) occur at unpredictable times or that they represent the accrued phase change of an ongoing random process.

The present paper is the third of a series on pulsar timing. In Paper I (Helfand *et al.* 1980), data were presented which show that phase deviations are a common phenomenon. Paper II (Cordes 1980) presented a technique, similar to that used by Groth (1975*b*) in his study of the Crab pulsar, for comparing observed phase deviations with those of random walk processes. In the present paper, we describe the application of this technique to 11 pulsars and link this to a more general study of the rotational stability of 39 other pulsars. We also consider the possibility that systematic frequency second derivatives or steps in frequency account for the phase residuals of some pulsars (Gullahorn and Rankin 1978, 1979), but we find that both of these can be understood as fluctuations of random walks. A detailed discussion of physical models for timing noise will appear in the fourth paper of this series (Cordes and Greenstein 1980).

II. TIMING ACTIVITY OF 50 PULSARS

a) *Activity Parameter Definition*

Our aim here is to quantify the timing irregularities of a large sample of pulsars in a general fashion. We do so by defining an activity parameter

$$A = \log\,[\sigma_{\mathscr{R}}(m, T)/\sigma_{\mathscr{R}}(m, T)_{\rm Crab}]\,, \qquad (1)$$

where $\sigma_{\mathscr{R}}(m, T)$ is the rms residual phase from a least-squares polynomial fit of order m over an interval of length T. It is assumed that the contribution of measurement error to the rms residual has been subtracted quadratically from the observed rms residual to arrive at $\sigma_{\mathscr{R}}(m, T)$. Our treatment considers $\sigma_{\mathscr{R}}$ to be in units of time rather than in cycles because one aim of our analysis is to determine whether A is correlated with rotation period or period derivative; expressing $\sigma_{\mathscr{R}}$ in periods would automatically impose a period dependence on A. Use of the Crab pulsar for normalization is purely arbitrary, of course, but the Crab is well studied and therefore provides perspective on the activity parameters of other pulsars.

We apply the activity parameter for $m = 2$, and we restrict the range of T in order to have as uniform a measure of timing activity as possible. If different pulsars have different kinds of timing noise, then A will be a function of T and using different data span lengths will introduce systematic errors in the set of activity parameters. This can be illustrated as follows. If timing noise is due to a random walk process then, *on average*, $\langle\sigma_{\mathscr{R}}(m, T)\rangle = C_m T^k$, where k varies with type of random walk. On the basis of simulations of random walks discussed in Paper II, we know that the actual rms residual for a given realization of a random walk will vary from the average by a factor of ξ where, typically $\xi \lesssim 3$. Therefore, the total error in the activity parameter is

$$\delta A \leq (k - k_{\rm Crab})\delta \log T + \delta \log \xi\,, \tag{2}$$

where the first term is the systematic error associated with using different time spans T. Errors in A would be purely random if all pulsars were to have the same kind of random process. As we show in § IV, however, the type of random walk varies from source to source such that $|k - k_{\rm Crab}| \leq 1$ because we observe only random walks in the phase (PN, $k = \frac{1}{2}$), frequency (FN, $k = k_{\rm Crab} = \frac{3}{2}$), and frequency derivative (SN, $k = \frac{5}{2}$). Consequently, we can allow a variation in T as large as a factor of ξ in order to constrain the systematic error to be less than the random error. Most of the time spans are within a factor of 2 of the 1628 d span for the Crab pulsar. For the Crab pulsar we adopt a value of $\sigma_{\mathscr{R}}(2, T = 1628 \text{ days}) = 12$ ms. This value cannot be directly measured from a second order fit because the residuals to such a fit are dominated by terms proportional to $\ddot{\nu}$ and $\dddot{\nu}$ that describe the deterministic spin down. We have therefore estimated $\sigma_{\mathscr{R}}(2, T)$ for the Crab by scaling $\sigma_{\mathscr{R}}(4, T)$, the rms residual from a fourth order fit (which measures only the timing noise), on the basis of simulations.

b) Correlations with Other Pulsar Parameters

Table 1 lists parameters for 50 pulsars. Columns (1)–(10) respectively are the pulsar, time span, rms residual after a second-order fit, rms residual after a third-order fit, activity parameter (A), period (P), period derivative ($\dot{P}$), spin-down age ($\tau = P/2\dot{P}$), distance above the galactic plane (z), and the radio luminosity (after Taylor and Manchester 1975). Column (11) lists the source of the data, some of which came from the Arecibo timing program (Gullahorn and Rankin 1978), the JPL/Goldstone program (Downs 1979), and the Princeton optical timing data of the Crab pulsar (Groth 1975*a*) in addition to the FCRAO/NRAO data described in Paper I. We have included one pulsar (PSR 0943+10) for which the residuals may be due to pulse shape changes that are random from one timing data point to the next (Gullahorn 1979). Although such behavior is qualitatively different from the timing activity of other pulsars (Paper I), we have nonetheless included the source in our analysis here because the pulse shape change interpretation has not been proven.

Correlation coefficients were calculated between A and the logarithms of various quantities, including those in Table 1 and also the surface magnetic field, $B_S \propto (P\dot{P})^{1/2}$, derived by assuming that magnetic dipole radiation (Pacini 1968) or homopolar currents (Goldreich and Julian 1969) spin down the star. An additional correlation was performed between A and an inferred z velocity

$$V_z = z/t\,, \tag{3}$$

where t is the chronological age defined by

$$t = \tfrac{1}{2}\tau_D \ln(1 + 2\tau/\tau_D)\,, \tag{4}$$

with τ_D being the e-folding time for torque decay (Helfand and Tademaru 1977). A value of $\tau_D = 5 \times 10^6$ yr was adopted because it is evident that the spin-down age τ becomes unrelated to the chronological age for $\tau \gtrsim \tau_D$. Finally, A was correlated with 100 realizations of white noise to illustrate the fact that the correlation coefficient of A with some quantity X,

$$\rho_{AX} \equiv \frac{\sum_j (A_j - \bar{A})(X_j - \bar{X})}{[\sum_j (A_j - \bar{A})^2 \sum_k (X_k - \bar{X})^2]^{1/2}} \tag{5}$$

(where barred quantities are the sample means), has an estimation error $\sigma_\rho(N) \approx N^{-1/2}$. Even if there is no correlation between two quantities, estimated correlation coefficients from a finite sample of 48 pulsars will be as large as $\sigma_\rho(48) \approx 0.15$. We have excluded the Vela (PSR 0833−45) and binary (PSR 1913+16) pulsars from the correlation analysis (see discussion below on these two sources).

Two sets of correlation coefficients are given in Table 2. The first set was calculated from 48 pulsars (excluding Vela and the binary) even though 16 of them only have upper limits on the activity parameter. The second set of correlation coefficients in parentheses do not include these 16 pulsars. *A priori*, one might expect that inclusion or exclusion of those pulsars with upper limits will bias the correlation coefficients. However, the majority of pulsars appear to be active, and therefore the actual activity parameters may not be appreciably less than the upper limits.

The largest correlation coefficient in Table 2 is that for A and chronological age. Generally, quantities that are functions of $\dot{P}$ are those most correlated with A, whereas P appears to be at most weakly correlated with A. We correlated A with $\log(P^\alpha \dot{P}^\beta)$ while varying the exponents α and β in increments of 0.25. As a function of α and β, the correlation coefficient exhibits a broad maximum near $\alpha = -1.25 \pm 1.0$ and $\beta = -0.75 \pm 0.50$ with a peak value of 0.58 (for 48 pulsars). The correlation coefficient is more sensitive to β (for nonzero α) than to α (for nonzero β), suggesting that the period derivative $\dot{P}$, frequency derivative $\dot{\nu} = -P^2\dot{P}$, or age $\tau = P/2\dot{P}$ is the quantity that is physically related to timing activity. In Figure 1, A is plotted against $\dot{P}$ for the 50 pulsars in Table 1. The slope of the least-squares line fit to the points in Figure 1 is 0.41 if all sources are included, compared to a value of 0.57 if

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TABLE 1

PARAMETERS OF 50 PULSARS

Pulsar (1)	T (d) (2)	$\sigma_R(2,T)$ (ms) (3)	$\sigma_R(3,T)$ (ms) (4)	Activity Parameter (5)	P (s) (6)	$\dot{P}$ ($10^{-15}ss^{-1}$) (7)	log τ (yrs) (8)	|z| (pc) (9)	log L (erg sec^{-1}) (10)	Reference for Timing Data[a] (11)
0031-07	2488	<1.00	---	<-1.4	0.943	0.400	7.6	420	26.4	
0301+19	1566	0.22	---	-1.7	1.388	1.290	7.2	340	26.1	A
0329+54	2859	2.31	0.74	-1.1	0.715	2.050	6.7	28	28.6	
0355+54	1862	<0.77	---	<-1.3	0.156	4.390	5.8	21	27.5	
0450-18	1646	<1.88	---	<-0.8	0.549	5.780	6.2	1400	28.5	
0525+21	1162	0.80	---	-1.0	3.745	40.060	6.2	230	27.9	A
0531+21	1628	11.97	6.82	0.0	0.033	422.689	3.1	200	29.1	P
0540+23	1368	0.52	0.07	-1.3	0.246	15.430	5.4	160	28.2	A
0611+22	1586	107.00	62.20	1.0	0.335	59.730	5.0	150	28.2	
0809+74	2778	<1.58	---	<-1.2	1.292	0.160	8.1	110	26.2	
0818-13	2175	<0.20	---	<-2.0	1.238	2.110	6.0	350	27.4	
0823+26	1834	12.55	3.06	-0.1	0.531	1.680	6.7	420	26.5	
0833-45	~1000	4.2-32.0	---	-0.2→1.1	0.089	125.030	4.0	24	28.2	JPL
0834+06	1355	<0.03	---	<-2.5	1.274	6.800	6.5	210	26.5	A
0943+10	1399	3.63	---	-0.4	1.098	3.530	6.7	430	26.3	A
0950+08	1563	0.45	0.18	-1.4	0.253	0.230	7.2	71	26.1	A
1133+16	2946	0.34	0.24	-1.9	1.188	3.730	6.7	160	25.7	
1237+25	2447	0.07	---	-2.5	1.382	0.956	7.4	370	26.1	A
1508+55	2865	5.67	1.21	-0.7	0.740	5.040	6.4	730	26.9	
1541+09	1669	0.95	0.61	-1.1	0.748	0.410	7.5	1800	28.4	A
1604-00	2177	0.33	---	-1.8	0.422	0.310	7.3	230	25.9	
1642-03	2560	2.66	1.75	-1.0	0.388	1.780	6.5	72	26.1	
1706-16	2559	9.15	2.73	-0.4	0.653	6.370	6.2	38	25.5	
1818-04	2560	10.70	3.61	-0.3	0.598	6.320	6.2	260	28.3	
1859+03	1227	3.98	1.98	-0.3	0.655	7.500	6.1	160	30.3	A
1907+10	1107	1.40	0.24	-0.7	0.284	2.690	6.2	84	28.8	A
1910+20	1117	0.16	---	-1.6	2.233	9.500	6.6	280	26.8	A
1911-04	2558	0.97	0.67	-1.4	0.826	4.060	6.5	460	27.0	
1913+16	334	<0.02	---	<-1.7	0.059	0.009	8.0	230	28.7	TFM
1915+13	1603	1.30	0.44	-1.0	0.195	7.200	5.6	35	28.2	A
1916+14	366	6.30	---	0.7	1.180	211.000	5.0	16	25.9	A
1917+00	1319	<0.12	---	<-1.9	1.272	7.670	6.4	360	27.6	A
1918+19	923	<0.14	---	<-1.6	0.821	0.800	7.2	240	28.6	A
1919+21	2792	<0.13	---	<-2.3	1.337	1.350	7.2	26	26.7	
1929+10	1334	0.38	0.23	-1.4	0.227	1.160	6.5	7	25.8	A
1920+21	1007	0.23	---	-1.4	1.078	8.200	6.3	470	28.5	A
1933+16	1124	0.14	---	-1.7	0.359	6.000	6.0	220	28.9	A
1944+17	1639	<0.14	---	<-1.9	0.441	0.024	8.5	34	27.1	A
1946+35	984	<0.05	---	<-2.1	0.717	7.050	6.2	480	29.1	A
1952+29	1409	<0.14	---	<-1.8	0.427	0.002	9.5	9	26.5	A
2002+31	1607	1.39	0.16	-0.9	2.111	74.580	5.7	3	28.3	A
2016+28	2790	0.47	0.41	-1.8	0.558	0.150	7.8	33	27.1	
2020+28	2047	0.51	0.38	-1.5	0.343	1.900	6.5	160	28.7	
2021+51	2585	4.82	0.54	-0.7	0.529	3.040	6.4	120	27.3	
2045-16	2487	1.16	0.93	-1.3	1.962	10.970	6.5	230	26.5	
2111+46	2458	<0.77	---	<-1.5	1.015	0.720	7.4	88	28.4	
2154+40	1649	<0.69	---	<-1.3	1.525	3.420	6.9	630	28.2	
2217+47	2865	0.82	0.29	-1.5	0.538	2.760	6.5	210	27.5	
2303+30	1596	<0.18	---	<-1.8	1.576	2.910	7.0	1400	27.5	A
2319+60	1577	<1.50	---	<-0.9	2.256	7.040	6.7	25	27.6	

[a] Note: A denotes Arecibo (Gullahorn and Rankin 1978).

JPL denotes data from the Jet Propulsion Laboratory (Downs 1979).

P denotes Princeton (Groth 1975a).

TFM denotes Taylor, Fowler, and McCulloch (1979).

All other data are from the FCRAO/NRAO timing program described in Paper I.

TABLE 2

CORRELATIONS OF ACTIVITY PARAMETER WITH OTHER PULSAR PARAMETERS

Parameter	Correlation Coefficient
Period P	−0.27 (−0.23)
Period derivative $\dot{P}$	0.51 (0.60)
Spindown age $P/2\dot{P}$	−0.57 (−0.63)
Chronological age t	−0.61 (−0.66)
Magnetic field $B_S \propto (P\dot{P})^{1/2}$	0.37 (0.43)
Galactic height z	−0.08 (−0.14)
Z velocity V_z	0.33 (0.37)
Luminosity L	0.18 (0.20)
White noise	0 ± 0.15 (0 ± 0.18)

upper limits are excluded from the fit. This suggests that the strength of the random process [which is proportional to $\sigma_{\mathscr{R}}^2(m, T)$] is proportional to $\dot{P}$.

None of the other correlation coefficients appear to be significant. Timing activity is not significantly correlated with radio luminosity or with galactic height. The large correlation of A with $\log V_z$ is probably an artifact of the correlation of A with $\log \tau$. We illustrate this by predicting the correlation of A with $\log V_z$ under the assumption that neither A nor $\log \tau$ is correlated with $\log V_z$. It can be shown in this case that the predicted correlation, $\hat{\rho}_{AV_z}$, is

$$\hat{\rho}_{AV_z} = -\rho_{At}(1 + \sigma_z^2/\sigma_t^2)^{-1/2}\,, \qquad (6)$$

where σ_z and σ_t are the standard deviations of $\log z$ and $\log t$, respectively. Excluding the Crab pulsar because of its anomalously large z velocity, we obtain $\hat{\rho}_{AV_z} = 0.38$, which is slightly larger than the measured value of 0.33.

c) The Vela and Binary Pulsars

The Vela pulsar (PSR 0833−45) and the binary pulsar PSR 1913+16 have respectively the fourth largest and second smallest values of $\dot{P}$ (Manchester and Taylor 1977). Although data were not available on these pulsars as they were for the others, it is possible to make estimates of their activity parameters. The results conform to the aforementioned relationship of $\dot{P}$ to timing activity.

The Vela pulsar has experienced four large glitches ($\Delta\nu/\nu \approx 2 \times 10^{-6}$) in the last 10 years separated by intervals of 900 to 1500 days of less active behavior. By estimating the rms residual from fits (provided by G. Downs, private communication) made over subintervals of 90 to 145 days, we find the activity parameter to be between −0.15 and 1.1, thus making Vela at least the fifth most active pulsar of those we have studied.

The binary pulsar must be treated in a unique fashion because the arrival time measurements are fitted by an 18-parameter function that describes the orbital as well as the spin-down behavior (see, e.g., Taylor *et al.* 1976). However, the spin-down parameters ν and $\dot{\nu}$ are not very strongly coupled to the orbital terms, which are periodic in harmonics of the 8 hour orbital period, P_b. Therefore the orbital terms will be much less responsive to the random walk than the polynomial terms (described by ν and $\dot{\nu}$) which better approximate the low-order terms of a random walk.

The presence of a random walk in the timing of PSR 1913+16 could be detected in two quantities: the rms residual and the frequency. We examined the residual phase from a fit to a 334 day span of the most recently obtained timing data (J. H. Taylor, private communication) by calculating the autocorrelation function of the residuals at lags of integer multiples of 5 minutes (the typical separation of arrival time measurements). The autocorrelation function is consistent with there being only random residual phases and therefore an upper limit on the rms residual contributed by a random walk is

$$\sigma_{\mathscr{R}} \lesssim \sigma_M N^{-1/4} \approx 20\ \mu\text{s}\,, \qquad (7)$$

where $\sigma_M \approx 80\ \mu$s (Taylor, Fowler, and McCulloch 1979) is the rms measurement error and $N = 338$ is the number of lagged products. The upper limit on the activity parameter is $A \leq -1.7$ which (see Fig. 1) is in accord with the small value of $\dot{P}$ (8.8×10^{-18} s s^{-1}) for this pulsar.

The residuals discussed above were obtained from a 334 day fit for ν, $\dot{\nu}$, and P_b, while all other parameters were held fixed to values obtained from a fit to a nonoverlapping span of 1282 days of data. The data points in the first fit of 334 days are clumped into three observing sessions separated by ~160 days. Therefore it is possible that the νt and $\dot{\nu}t^2/2$ terms of the fitting function have "absorbed" any random walk that was occurring. This does not appear to be the case, however, since the derived values of $\dot{\nu}$ obtained from the 334 day and 1282 day spans are equal to within the measurement errors. The difference between the values is $\Delta\dot{\nu} \approx 3 \times 10^{-17}$ Hz s^{-1} whereas the expected

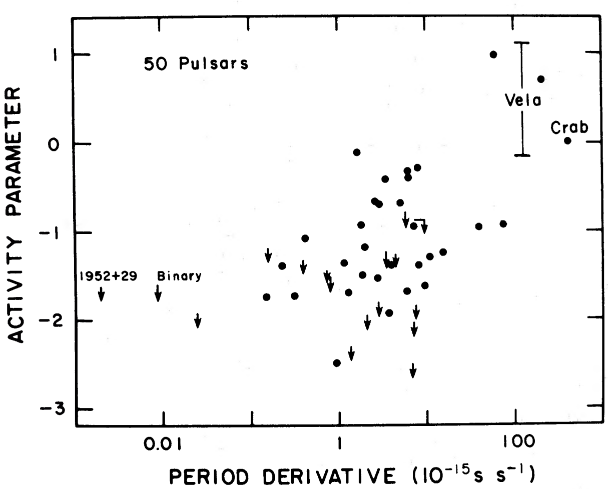

FIG. 1.—Plot of activity parameter (eq. [1]) versus period derivative. Arrows denote upper limits.

rms value of $\dot{\nu}$ due to a random walk in frequency (FN) is

$$\dot{\nu}_{\rm rms} \approx T^{-1/2} S_{\rm FN}^{1/2} . \quad (8)$$

Equating $\Delta\dot{\nu}$ and $\dot{\nu}_{\rm rms}$ and solving for the strength $S_{\rm FN}$, we find again that an upper limit on the activity parameter is between -1.4 and -1.7, consistent with the upper limit obtained from the residuals.

III. TYPES OF TIMING ACTIVITY

As we have seen, timing activity is easy to detect as departures of the rotational phase from a low-order polynomial. However, some care is needed to pin down the nature of the timing activity. We will consider here several possibilities. First is white noise in the phase (other than measurement errors) that may be due to random pulse shape changes. Such noise, which is stationary, does not account for most timing noise which appears to be nonstationary. Second, glitches—steps in the rotation frequency large enough to be individually measured—have been suggested. Third, it has been proposed (Gullahorn and Rankin 1979) that systematic frequency second derivatives account for the residuals for some pulsars, even though such second derivatives imply braking indexes that are sometimes negative with magnitudes in excess of 10^3. Finally, there are the three random walk processes—phase noise (PN), frequency noise (FN), and slowing-down noise (SN)—that are produced respectively by microscopic steps in the phase, the frequency, and the frequency derivative. In § IV we demonstrate in detail—for 11 pulsars with adequate data bases—that random walk processes are consistent with the data. Presently, we establish that the apparent frequency second derivatives and apparent isolated frequency steps that have been measured can be understood as manifestations of random walk processes.

a) Apparent Frequency Second Derivatives

For a sample of 22 pulsars, the residuals after a second order polynomial fit are significant in that the reduced χ^2 of the fit is large. Therefore, a third order fit can be performed, yielding a coefficient that can be inferred to be a frequency second derivative $\ddot{\nu}$. The question is whether the measured $\ddot{\nu}$ is to be equated with the systematic frequency second derivative of the torque process. If so, then the braking index

$$n = \nu\ddot{\nu}/\dot{\nu}^2 \quad (9)$$

can be calculated as has been done for the Crab pulsar. Such computations have been made on 19 pulsars by Gullahorn and Rankin (1979), the results yielding eight negative values for n and magnitudes between 4.2 and 10^5. While these measurements are generally inconsistent with the braking indices of simple torque processes (which range between 1 and 5), it is always possible that some previously unconsidered process can account for them. Our aim here, however, is to show that random walk processes cannot be rejected as the cause for the large residuals after a second order fit.

If third order fits are made to random walks over finite time spans, the derived second derivatives $\ddot{\nu}_R$ (R for random) can be either positive or negative; they will not necessarily be different for contiguous nonoverlapping time spans, and the residuals will yield a large reduced χ^2 if the fourth and higher order components of the random walk are larger than random measurement errors. For random walks the rms residual after a third-order fit $\sigma_{\mathscr{R}}(3, T)$ will be smaller than the rms residual after a second-order fit $\sigma_{\mathscr{R}}(2, T)$ by a predictable amount, on average. Therefore to test the apparent values of $\ddot{\nu}$ we will use the diagnostic quantity

$$r_{32} = \sigma_{\mathscr{R}}(3, T)/\sigma_{\mathscr{R}}(2, T) . \quad (10)$$

For the three kinds of random walks and for white noise we have

$$\begin{aligned} r_{32} &= [(N-4)/(N-3)]^{1/2} && \text{(white noise)} \\ &= 0.79 \pm 0.21 && \text{(PN)} \\ &= 0.47 \pm 0.30 && \text{(FN)} \\ &= 0.36 \pm 0.32 && \text{(SN)} , \quad (11) \end{aligned}$$

whereas if only a true frequency second derivative and measurement error cause the residuals after a second-order fit, then

$$0 \le r_{32} \lesssim 120 \times 7^{1/2} \sigma_M / \ddot{\nu} T^3 (N-4)^{1/2} . \quad (12)$$

Here N is the number of data points, assumed to be large ($N \gg 4$) for the random walks, and T is the time span of the fit. The mean and range ($\pm 1\ \sigma$) of r_{32} specified for the random walks were determined directly from simulations. The upper limit on r_{32} for the second frequency derivative case derives from the fact that the accuracy of r_{32} is limited by the estimation error of the estimate of σ_M.

In Figure 2 we have plotted r_{32} versus $\dot{P}$ for 23 pulsars. Apart from the Crab pulsar for which $r_{32} = 0.002$, all of the apparent second derivatives are consistent with a random walk origin because the values of r_{32} are well within the ranges of r_{32} for the random walks (also shown in Fig. 2). Excluding the Crab, the mean and rms of r_{32} are $\langle r_{32} \rangle = 0.45$ and $\sigma_{r_{32}} = 0.24$.

Unfortunately, the quantity r_{32} is of no use in distinguishing between the three random walk processes because of the overlap of the distributions of r_{32} for the different random walks. The large variation of r_{32} occurs because the power in a polynomial component can vary over a large range from one realization to the next. It can be shown in the continuous limit (i.e., an infinite number of data points) that the expansion coefficients C_k for a random walk in a Legendre polynomial series have moments that satisfy

$$\langle C_k^4 \rangle / \langle C_k^2 \rangle^2 = 3 + O[\langle a^4 \rangle / RT \langle a^2 \rangle^2] , \quad (13)$$

where R is the step rate of the random walk, T is the time interval, and a is the step amplitude in the

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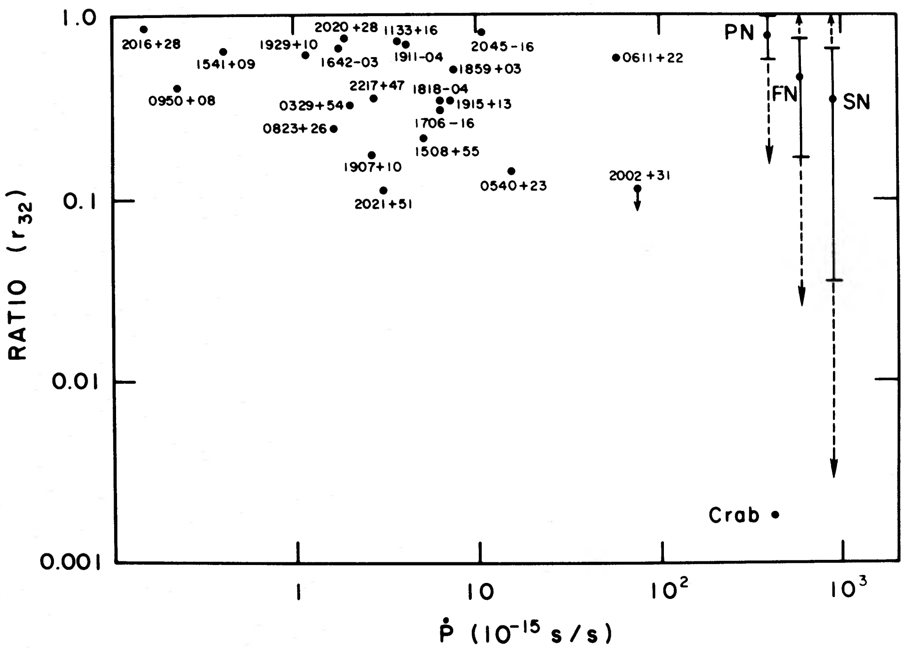

FIG. 2.—Plot of the ratio r_{32} (eq. [10]) versus $\dot{P}$. The solid bars represent the 1 σ range of values of r_{32} obtained from 100 realizations of phase noise (PN), frequency noise (FN), and slowing-down noise (SN). The dashed lines are the total ranges of r_{32} found from the 100 realizations.

appropriate units. For $RT \to \infty$, the expansion coefficients behave like Gaussian random variables. Hence the ratio r_{32} is expected to vary by an amount commensurate with the variation of the second and third order coefficients and according to the fraction of the "power" of the random walk these two components possess relative to higher order components.

In order to distinguish PN from FN or SN, etc., it is necessary to have many independent estimates of r_{32} or of some other quantity that parametrizes the random walk. Paper II outlined the procedure by which random walks can be distinguished, the application of which appears in the next section.

b) Apparent Frequency Steps ("Glitches") of 11 Pulsars

i) *Fast and Slow Glitches*

The notion of glitches arose from the observed discontinuities in rotation frequency from the Crab and Vela pulsars. Such discontinuities have very small rise times ($\lesssim$ few days) and they have all represented *increases* in rotation frequency with amplitudes $\Delta\nu/\nu = 4 \times 10^{-8}$–$10^{-9}$ for the Crab glitches of 1969 September and 1975 March and $\Delta\nu/\nu = 2 \times 10^{-6}$ for Vela. Greenstein (1979) has discussed how the rise time may depend strongly on the temperature of the neutron star, therefore allowing the possibility of *positive* frequency steps with very long (days to decades) rise times for low-temperature stars. Such frequency steps we will call slow glitches. Independent of Greenstein's work, Manchester and Taylor (1974) and Gullahorn and Rankin (1978, 1979) have isolated frequency steps for 11 pulsars with magnitudes in the range $2 \times 10^{-8} \leq |\Delta\nu/\nu| \leq 10^{-10}$. While such steps may correspond to slow glitches, it should be noted that six of the attributed steps are negative—a result not to be expected if glitches arise from a transfer of angular momentum from the superfluid interior of a neutron star to a more slowly rotating crust.

In this section we test whether or not the frequency steps for the above mentioned 11 pulsars are merely fluctuations of a random walk process. Given an infinite amount of data, it would be easy to distinguish a random walk from a single frequency step because the rms phase of a random walk diverges monotonically as $T \to \infty$ whereas the rms phase of a single frequency step would first increase and then asymptotically tend to zero. The problem at hand arises primarily because we have only a finite number of data and therefore the polynomial components of the rotational phase for data containing a frequency step may be similar to those of a random walk. In the following we develop a criterion for judging whether a given frequency step is merely a fluctuation of a random walk.

ii) *Random Walks*

First, it is clear that a random walk can produce changes in slope of the phase. In Figure 3 we show simulated random walks of frequency noise computed with $T = 1000$ d, a rate of one step per day, and steps with a uniform, zero-mean amplitude distribution. Several abrupt slope changes are evident, but what we need to know is the *distribution* of slope changes for a given random walk strength. For our purposes, it is

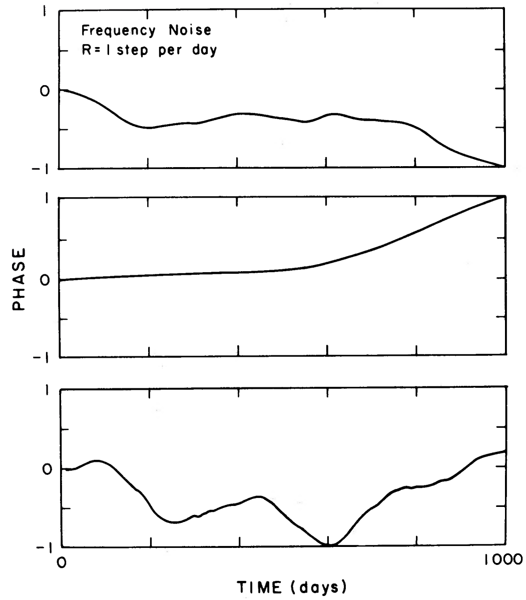

FIG. 3.—Three realizations of a random walk in the frequency (FN) computed with zero-mean steps, an average rate of 1 step per day, and a time span of 1000 days.

sufficient to calculate the moments of the quantity $\Delta\nu \equiv \nu(t + \Delta t) - \nu(t)$. For frequency noise we have

$$\nu(t) = \sum_j \delta\nu_j H(t - t_j) , \quad (14)$$

where $H(t)$ is the unit step function. It can be shown that $\Delta\nu$ has a standard deviation $\sigma_{\Delta\nu} = (S_{FN}\Delta t)^{1/2}$, where $S_{FN} = R\langle\delta\nu^2\rangle$ is the random walk strength and R is the rate. Also $\Delta\nu$ has a Gaussian probability density function as long as $R\Delta t \gg \langle\delta\nu^4\rangle/\langle\delta\nu^2\rangle^2$; this constraint on $R\Delta t$ is essentially the condition for the Central Limit Theorem to hold.

For the Crab pulsar, whose timing noise we know is consistent with frequency noise, we predict slope changes as large as $3\sigma_{\Delta\nu} \approx 6 \times 10^{-9}(\Delta t/1 \text{ day})^{1/2}$ Hz. Inspection of the phase residuals for Crab optical timing data (see Fig. 4 of Groth 1975*c*) shows positive and negative slope changes with magnitudes as large as 1.3×10^{-8} Hz over $\Delta t \approx 10$ days. These are within the range expected solely from the random walk.

iii) *Tests of Apparent Frequency Steps*

For an arbitrary pulsar we must test whether a particular slope change is larger than that expected for a given strength. To do so, we estimate a strength from the slope change,

$$\hat{S}_{FN,\Delta\nu} = (\Delta\nu/3)^2/\Delta t , \quad (15)$$

and compare it to a strength estimate determined from the residuals to an mth order fit,

$$\hat{S}_{FN,\mathscr{R}} = 24C_m{}^2\sigma_{\mathscr{R}}{}^2(T)/T^3P^2 , \quad (16)$$

where $\sigma_{\mathscr{R}}{}^2$ is the residual variance (in excess of measurement errors), C_m is a correction factor for the fit (see Paper II), and T is the length of the span of data *which does not include the slope change*. If $r = \hat{S}_{FN,\Delta\nu}/\hat{S}_{FN,\mathscr{R}} \gg 1$, then the slope change is too large to be considered a fluctuation of the random walk. By this criterion, the glitch of the Crab pulsar in 1975 March (Lohsen 1975; Helfand 1977) with $\Delta\nu = 1.2 \times 10^{-6}$ Hz and $\Delta t = 5$ days has $r = 7000$. The 1969 September Crab glitch has a similarly large value of r, but the alleged glitch of 1972 (Lohsen 1972) has $r \approx 1$ and therefore appears to be a manifestation of the random walk, as suggested by Groth (1975*c*).

In Table 3 we estimate r for the 10 frequency steps discussed by Gullahorn and Rankin (1978) and for PSR 1508+55 (Manchester and Taylor 1974). The step times for the frequency steps, Δt, were estimated from residual curves (Gullahorn 1979 and unpublished work) and are not well defined because of the paucity of data points. Generally we have attempted to be conservative by adopting the smallest possible estimates for Δt.

The resultant values of the ratio of strengths indicate to us that none of the steps can confidently be considered a glitch. Even though values of r as large as 9 are obtained, we feel that the uncertainty in r may be as large as an order of magnitude. The random error is expected to be large because r will vary according to the F distribution with 1 and 1 degrees of freedom if it

TABLE 3

TEST OF FREQUENCY STEPS FOR 11 PULSARS

Pulsar	$\Delta\nu$ (Hz)	Δt (d)	$\hat{S}_{FN,\Delta\nu}$ (Hz^2 s^{-1})	$\hat{S}_{FN,\mathscr{R}}$ (Hz^2 s^{-1})	$r = \hat{S}_{FN,\Delta\nu}/\hat{S}_{FN,\mathscr{R}}$
0525+21	2.6×10^{-10}	50	1.7×10^{-27}	2.6×10^{-28}	6.5
0611+21	1.0×10^{-7}	100	1.3×10^{-22}	$(0.25\text{–}2.3) \times 10^{-23}$	5.5–58
0823+26	-5.5×10^{-9}	200	2.0×10^{-25}	$(0.072\text{–}2.8) \times 10^{-25}$	0.7–27
1508+55	1.4×10^{-9}	115	2.2×10^{-26}	6.8×10^{-27}	3.3
1859+03	-2.0×10^{-9}	100	5.1×10^{-26}	1.8×10^{-25}	0.3
1900+01	-3.0×10^{-10}	100	1.2×10^{-27}	$\lesssim 1.3 \times 10^{-28}$	$\gtrsim 9.2$
1907+00	0.5×10^{-9}	100	3.2×10^{-27}	1.8×10^{-27}	1.8
1916+14	-1.7×10^{-8}	50	7.4×10^{-24}	5.6×10^{-24}	1.3
1929+20	-1.3×10^{-9}	60	2.8×10^{-24}	1×10^{-24}	2.8
1933+16	2.8×10^{-10}	100	1.0×10^{-27}	9.7×10^{-28}	1.0
1946+35	-2.0×10^{-10}	$\lesssim 200$	2.5×10^{-28}	4.6×10^{-29}	$\gtrsim 5.4$

is assumed that the strength estimates are independent. For such an F distribution, values as large as 9.5 can occur at the 20% level. Second, a systematic error, which will tend to bias r to larger values, derives from the very act of selecting slope changes out of the data. We have accounted for the fact that the largest slope changes are extraordinary by considering them to be 3 σ values (cf. eq. [15]), but we have not accounted for the fact that the data used to calculate $\hat{S}_{\mathrm{FN},\mathscr{R}}$ are likely to occur in an interval of less than average power in the random walk. Finally, not all of the pulsars are expected to behave according to the predictions based on frequency noise. Pulsars 0611+22 and 0823+26, for example, are apparently consistent with slowing-down noise (Table 4), and our implied estimates for predicted slope changes therefore may be only dimensionally correct.

We conclude that none of the frequency steps can be confidently interpreted as anything other than fluctuations of a random walk process. This conclusion includes PSR 1907+00 for which a frequency step was inferred by Gullahorn *et al.* (1976). In order for a frequency step to be considered a real event (as opposed to a superposition of many smaller events), it would have to be of much larger amplitude or have a much smaller rise time than any of those in Table 3. The discontinuity reported by Manchester *et al.* (1978) for PSR 1641−45 is large enough ($\Delta\nu/\nu \sim 2 \times 10^{-7}$) to imply a ratio $r \geq 600$ even though the rise time is known only to be less than 120 days.

IV. RANDOM WALK ANALYSIS

a) Technique

Techniques developed in Paper II enable us to test whether a random walk is consistent with the timing activity of a pulsar. To do so requires that the phase deviations be larger than measurement errors on a time span that is much shorter than the time span of the entire data set; then both short-time structure and long-time structure of the phase deviations can be compared with the predictions of a random walk, thus affording a strong test of the model. This criterion limits us to a study of 10 pulsars in addition to the Crab pulsar.

The method consists of finding the rms residual from an mth order polynomial fit, $\sigma_{\mathscr{R}}(m, T)$, calculating strength parameters for each of the three kinds of random walks, and determining whether the strength parameters are constant (within estimation errors) in T. As before, we obtain $\sigma_{\mathscr{R}}(m, T)$ by quadratically subtracting the measurement error σ_M from the actual rms residual. We estimate σ_M from the residuals by subtracting a linear trend and finding the variance from that trend, as discussed in Paper II. The data were divided into $N_{\min}$ equal size blocks of size $T_{\min}$. These blocks yield $N_{\min}$ strength parameters whose average is

$$\bar{S}(T_{\min}) = \frac{1}{N_{\min}} \sum_{j=1}^{N_{\min}} S_j(T_{\min}) . \qquad (17)$$

Larger blocks composed of 2, 3, . . . , $N_{\min}$ subblocks were also analyzed. The largest yields a strength estimate, $S(T_{\max})$. If a random walk is inconsistent with the data, then the derived strengths will vary strongly and (on average) monotonically with T. The strength estimates for a consistent random walk should be roughly constant in T. In Paper II we argued that a single strength estimate may be considered a χ^2 random variable with 1 degree of freedom. Therefore the ratio $S(T_{\max})/\bar{S}(T_{\min})$ should be of order unity for a consistent random walk and it will vary according to an F distribution with 1 and $N_{\min}$ degrees of freedom if $S(T_{\max})$ and $\bar{S}(T_{\min})$ are statistically independent. If $T_{\min} \ll T_{\max}$, then the strength estimate $\bar{S}(T_{\min})$ will measure structure in the random walk that varies much faster and has much less variance (power) than the larger scale structure that dominates $S(T_{\max})$. Consequently we expect $S(T_{\max})$ and $\bar{S}(T_{\min})$ to be essentially independent, and we therefore may apply the F test to the quantity $F = S(T_{\max})/\bar{S}(T_{\min})$.

b) Results

Results of the random walk analysis for 11 pulsars are given in Table 4. Columns (1)–(9) are, respectively, the pulsar; the time spans $T_{\max}$ and $T_{\min}$; the number of blocks of length $T_{\min}$, $N_{\min}$; the quantity $F = S(T_{\max})/\bar{S}(T_{\min})$ for the three random walks; the most probable random walk; and the adopted strength for the most probable random walk. Also in parentheses in columns (5)–(7) are the probabilities of obtaining F values less than those observed. If this probability is different from 0 or 1 by a small amount ϵ, then the random walk can be rejected at the 100ϵ percentage level. If the probability is of order 0.5, then the random walk is not inconsistent with the data.

It is clear from Table 4 that the strongest F tests obtain for the largest ratios of $T_{\max}/T_{\min}$. This is true because if one of the random walks is consistent with the data, then the F values for the other two random walks will vary at least as fast as $(T_{\max}/T_{\min})^2$. For pulsars 0329+54, 0531+21, 0611+22, 0823+26, 1133+16, 1508+55, 2016+28, and 2217+47, the results are unambiguous as to which random walk is consistent with the data. Four of these are consistent with frequency noise, two with slowing-down noise, and two with phase noise. In all eight cases, the strengths of the inconsistent random walks vary monotonically with T and over ranges consistent with the values of $T_{\max}/T_{\min}$. The remaining three pulsars, 1915+13, 2002+31, and 2020+28, have timing noise weak enough that $T_{\max}/T_{\min}$ is too small to afford a definitive F test. However, we have tentatively attributed frequency noise to the timing noise for these three pulsars.

c) Discussion

The basic conclusions to be made from Table 4 are that three kinds of random walks are required to account for the timing noise of the 11 pulsars and that

TABLE 4

RANDOM WALK ANALYSIS OF 11 PULSARS

PULSAR (1)	T_{max} (d) (2)	T_{min} (d) (3)	N_{min} (4)	RATIO OF STRENGTHS $S(T_{max})/\bar{S}(T_{min})$ PN (5)	FN (6)	SN (7)	MOST PROBABLE RANDOM WALK (8)	STRENGTH (9)
0329+54	2859	450	6	98	0.41	0.058	FN	$7.0 \pm 4.0 \times 10^{-27}$ Hz^2 s^{-1}
				($1–6 \times 10^{-5}$	0.45	0.18)		
0531+21	1628	100	9	267	1.96	0.024	FN	$6.6 \pm 3.0 \times 10^{-23}$ Hz^2 s^{-1}
				($1–10^{-6}$	0.80	0.12)		
0611+22	1586	275	3	550	19.3	0.62	SN	$1.3 \pm 0.9 \times 10^{-37}$ Hz^2 s^{-3}
				($1–2 \times 10^{-4}$	0.978	0.51)		
0823+26	1834	400	4	153	8.2	0.38	SN	$2.0 \pm 1.3 \times 10^{-40}$ Hz^2 s^{-3}
				($1–3 \times 10^{-4}$	0.95	0.43)		
	1834	900	3	27	7.2	1.84		
				(0.99	0.93	0.73)		
1133+16	2946	425	6	0.83	0.023	0.00059	PN	$1.5 \pm 0.9 \times 10^{-14}$ s^{-1}
				(0.60	0.12	0.02)		
1508+55	2865	425	6	40.0	1.18	0.031	FN	$1.0 \pm 0.6 \times 10^{-26}$ Hz^2 s^{-1}
				(0.999	0.68	0.13)		
1915+13	1602	500	3	3.9	0.58	0.059	(FN)	$1.1 \pm 0.7 \times 10^{-25}$ Hz^2 s^{-1}
				(0.86	0.50	0.18)		
2002+31	1606	500	3	$\gtrsim 1.41$	$\gtrsim 0.21$	$\gtrsim 0.030$	(FN)	$1.0 \pm 0.7 \times 10^{-27}$ Hz^2 s^{-1}
				($\gtrsim 0.68$	$\gtrsim 0.32$	$\gtrsim 0.13$)		
2016+28	2790	850	5	11.8	0.50	0.056	FN	$2.0 \pm 1.2 \times 10^{-28}$ Hz^2 s^{-1}
				(0.98	0.49	0.18)		
2020+28	2047	1000	2	2.2	0.60	0.13	(FN)	$2.0 \pm 1.6 \times 10^{-27}$ Hz^2 s^{-1}
				(0.72	0.48	0.25)		
2217+47	2865	450	6	1.82	0.036	0.00067	PN	$1.6 \pm 0.9 \times 10^{-13}$ s^{-1}
				(0.77	0.14	0.02)		

these encompass a range of strengths. The strengths are second-order quantities and as such give no information on the rate or mean amplitude of the steps of the random walks. Here we will discuss limits that can be placed on these quantities.

i) *Limits on the Rate*

First, if we find that the random walk strength is the same (within the expected errors) for blocks of length T_{min} as it is for a block of length T_{max}, then a limit on the rate is

$$R \gtrsim R_{min} = T_{min}^{-1} . \tag{18}$$

Also, if the step amplitudes have a nonzero mean (e.g., if they are all positive frequency steps such as are expected from starquakes which presumably always decrease the moment of inertia of the star), then the quantities

$$\begin{aligned} R\langle \delta\phi \rangle &= \langle \nu_R \rangle \quad \text{(PN)} \\ R\langle \delta\nu \rangle &= \langle \dot{\nu}_R \rangle \quad \text{(FN)} \\ R\langle \delta\dot{\nu} \rangle &= \langle \ddot{\nu}_R \rangle \quad \text{(SN)} \end{aligned} \tag{19}$$

represent average noise induced frequencies and frequency derivatives that will contribute to the net measured values of these quantities.

For SN, a systematic frequency second derivative is predicted that would increase the residuals from a second-order fit over and above those for a random walk with zero-mean steps ($\langle \delta\dot{\nu} \rangle = 0$). If $\langle \ddot{\nu}_R \rangle$ were significant for those pulsars that are consistent with SN (PSR 0611+22 and PSR 0823+26), then the values of r_{32} obtained for them (see Fig. 2 and discussion in § III*a*) would be smaller than expected for a zero-mean random walk. That r_{32} for these pulsars is consistent with a zero-mean random walk implies that the residuals to a second-order fit are dominated by the second-order terms of the random walk and therefore

$$R\langle \delta\dot{\nu} \rangle \lesssim \frac{120\sqrt{7}}{T^3 P} \sigma_{\mathscr{R}}(2, T) . \tag{20}$$

This inequality was obtained by calculating (in the continuous limit) the residual $\sigma_{\mathscr{R}}(2, T)$ expected for a systematic second derivative. For the above two pulsars this becomes (evaluating inequality [20] using values from Table 4)

$$\begin{aligned} R\langle \delta\dot{\nu} \rangle &\lesssim 3.9 \times 10^{-23}\ \text{Hz s}^{-2} \quad \text{(PSR 0611+22)} \\ &\lesssim 1.9 \times 10^{-24}\ \text{Hz s}^{-2} \quad \text{(PSR 0823+26)} . \end{aligned}$$

Now we can combine this constraint with the observed strength if we assume that all step amplitudes are approximately the same. That is, we write $\langle \delta\dot{\nu} \rangle = \alpha \langle \delta\dot{\nu}^2 \rangle^{1/2}$ with $\alpha \approx 1$. Then $R\langle \delta\dot{\nu} \rangle = \alpha R^{1/2} S_{SN}{}^{1/2}$, and inequality (20) becomes an upper limit on the rate

$$R \lesssim \left[\frac{120\sqrt{7}}{T^3 P} \sigma_{\mathscr{R}}(2, T) \alpha^{-1} \right]^2 S_{SN}{}^{-1} , \tag{21}$$

or $R \lesssim 10^{-8}\ s^{-1}$ to $10^{-7.7}\ s^{-1}$ for both pulsars; thus steps must occur no more often than once every 1–3 years. This upper limit on R is marginally consistent with the lower limit obtained from inequality (18) which implies that steps occur as often as once every 1–2 years. Therefore, if all steps are of the same sign, then they must occur on the order of once a year.

These constraints on R are interesting in that Downs (1979) has identified nine steps in 10 years of JPL timing data for PSR 0823+26; six of these events are steps in ν while three are steps in $\dot{\nu}$. The JPL timing data have measurement errors that are an order of magnitude smaller than the data analyzed here, thus allowing the identification of these events. If all nine events produce phase changes of the same sign, then the similarity of our constraints on the rate with the implied rate of the nine steps attests to the reality of those steps. However, the six steps in ν and three steps in $\dot{\nu}$ must have "conspired" to appear as slowing-down noise in our analysis. An alternative situation is possible. Steps in $\dot{\nu}$ may be occurring with both signs (such that $\alpha \ll 1$) and with a rate much larger than once a year. Then the events identified by Downs are spurious, as were the apparent frequency steps discussed in § III*b*. To determine the actual situation, the JPL timing data—especially the intervals *between* events examined by Downs—will have to be carefully analyzed.

Similarly, a limit on R can be made for those pulsars for which FN is consistent with their timing noise. Frequency noise generates $\langle \dot{\nu}_R \rangle = R\langle \delta\nu \rangle > 0$ (if all steps are spin-ups) whereas the measured values of the frequency first derivative $\dot{\nu}_m$ are always negative. Since we have found that the random walk strength is larger for larger values of $|\dot{\nu}_m|$, it appears that $\langle \dot{\nu}_R \rangle \ll |\dot{\nu}_m|$. Again assuming that step amplitudes are approximately equal (i.e., $\langle \delta\nu \rangle \equiv \alpha \langle \delta\nu^2 \rangle^{1/2}$), we have

$$R \lesssim \alpha^{-2} S_{\mathrm{FN}}^{-1} \dot{\nu}_m^{2}, \qquad (22)$$

which yields upper limits on R between 10^{-1} and $10^{-3}\ s^{-1}$ for the seven pulsars in Table 4 that are most probably FN. Therefore we find that the rate of steps is constrained between many per day and one every few hundred days.

ii) *Can Other Effects Mimic Random Walks?*

The detailed analysis of this section has shown the consistency of random walks with the timing noise of 11 pulsars. In a preceding section we demonstrated—via the ratio of rms residuals after third and second order fits—that a larger sample of sources is also consistent with random walks. Our analysis therefore *suggests* that only two phenomena are needed to account for the observed rotational irregularities: large-amplitude discontinuous increases of the rotation frequency (glitches) for the Crab and Vela pulsars and PSR 1641−45 and random walks. We note, however, that timing activity of some pulsars may not reflect rotational irregularities but may signify pulse shape changes, as for PSR 0943+10 (Gullahorn 1979), or orbital motion in a binary system. Orbital motion with an orbital period larger than the data span length T could mimic a random walk because the rms residual of a polynomial fit would probably increase with data span length. There also may be additional types of rotational irregularities that have not been theoretically or observationally recognized. One theoretical possibility (Greenstein 1979) is that of very slow glitches with rise times of years which, like long-period orbital motion, might mimic a random walk over finite spans of data. For well sampled data with timing noise larger than measurement errors, random walks can be distinguished from orbital motion or a very slow glitch. However, one must admit the possibility of effects other than random walks for those pulsars that are poorly sampled, even though their timing is consistent with a random walk interpretation. Gullahorn and Rankin (1979), for example, have argued that timing activity in general may result from torque variations induced by density irregularities in the interstellar medium. They have accepted the reality of frequency second derivatives and apparent frequency steps (similar to those shown in § III*b* to represent chance fluctuations produced by an ensemble of events) and attributed them respectively to either slow or fast variations of the interstellar density.

V. CONCLUSIONS

We have shown that timing noise is common among pulsars, that its strength is correlated with period derivative and weakly correlated with period, and that random walks are consistent with the timing noise of 11 pulsars that were studied in detail. Random walks are not inconsistent with the timing behavior of most of the other pulsars in our sample.

Although a detailed discussion of the physics of timing noise is deferred to Paper IV of this series, we will make some brief comments here. First, the correlation of timing activity with $\dot{P}$ suggests that timing activity is related to the rotational energy loss, $\dot{E} = I\Omega\dot{\Omega}$, or to the change in stellar oblateness. Whatever the case, it is important to note that there is a large scatter about any straight line fit to the points in Figure 1. At a constant $\dot{P}$, there appears to be a spread in activity parameter of 2, corresponding to a variation of two orders of magnitude in the rms residual. Although some of this spread is estimation error of the activity parameter, some of it probably reflects the distribution of masses, magnetic fields, etc., over the set of pulsars and therefore, given a physical model for timing activity, limits on the mass range, etc., may be possible.

In the starquake model (e.g., Baym and Pines 1971; Pines and Shaham 1972), stresses in the neutron star crust are induced by the change in equilibrium oblateness and are relieved by crustal cracking. The amplitudes of frequency steps are initially proportional to the changes in moment of inertia and then evolve according to the structurally dependent response of the star to such perturbations. According to the two component model developed by Baym *et al.* (1969), a starquake at $t = 0$ will produce a frequency perturbation of the form

$$\Delta\nu(t) = \Delta\nu(0)(Qe^{-t/\tau} + 1 - Q)H(t)\,, \qquad (23)$$

where $H(t)$ is the unit step function and τ is a decay time determined by the frictional coupling of the superfluid neutrons and the crust. The fraction of the frequency jump that is permanent is $1 - Q$, where Q is constrained between 0 and 1 and is large for the least massive stars and small for the most massive stars.

Groth has pointed out that an ensemble of frequency perturbations of the form of equation (23) can produce either phase noise (for light neutron stars with $Q = 1$) or frequency noise (for heavy stars with $Q = 0$). However, there is no way of producing slowing-down noise (SN) in this model because SN requires frequency perturbations which increase linearly with time. A frequency perturbation with a large rise time (compared to the time span T of the data) would mimic SN, but this is not possible with starquakes for which the crust rotational frequency changes almost instantaneously.

It therefore appears that starquakes in and of themselves cannot produce all the kinds of timing noise we have observed. The same conclusion holds for accretion because the response of the star to infalling matter with net angular momentum will also be of the form of equation (23).

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