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CARBON IN WHITE DWARFS[1]

FRANCESCA D'ANTONA AND ITALO MAZZITELLI
Astronomy Department, University of Illinois

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ABSTRACT

The problem of obtaining a considerable carbon enhancement in nova envelopes to account for the occurrence of fast novae is examined in detail. No mechanism seems completely satisfactory from the points of view of stellar evolution and white dwarf structure, even considering evidence from single white dwarf observations.

A further mechanism for carbon enhancement is proposed and its limits discussed in detail. The problem of neon enhancement in some novae is also considered and a qualitative explanation is found within the proposed scheme for carbon enhancement.

It is suggested that sedimentation of carbon in the prenovae stage may influence the outburst modalities, thereby further enhancing the carbon abundance in the shell undergoing thermonuclear runaway.

Subject headings: stars: abundances — stars: interiors — stars: novae —
stars: white dwarfs

I. INTRODUCTION

The source of energy in nova outbursts is commonly identified as the thermonuclear energy released by the ignition of hydrogen at the surface of white dwarfs which accrete matter in a close binary system (Giannone and Weigert 1967; Nomoto, Nariai, and Sugimoto 1979; Starrfield, Sparks, and Truran 1978). The main problem yet unresolved in the study of nova outbursts is the following: It seems straightforward to reproduce the main characteristics (ejection velocities, light curve) of slow or moderately fast novae (Prialnik, Shara, and Shaviv 1978, 1979; Sparks, Starrfield, and Truran 1978; Nariai, Nomoto, and Sugimoto 1979), but the only successful models of "very fast" novae (Truran 1979) require considerable CNO enhancement in the hydrogen envelope in order to release the energy required to eject an optically thick envelope in a short time scale (Starrfield, Sparks, and Truran 1978).

The required CNO enhancement seems to be roughly in agreement with some observations (Ferland and Shields 1978), although the interpretation of the observational results must be regarded with caution because of the complex physics involved in the study of the recombination lines in nova ejecta. Furthermore, it is unclear whether a one-to-one correlation between the speed class of the nova and CNO abundance in the ejecta may exist: for instance, the very slow nova DQ Her shows very large carbon abundance in its ejecta (Williams *et al.* 1978). A tentative combination of the parameters influencing the speed class of the nova (mass of the white dwarf and of the envelope, CNO enhancement) has been given by Shara and Shaviv (1979).

Until a better understanding of both observations and theory is available, we thought it worthwhile to examine the possible mechanisms for CNO enhancement of the nova envelope, in light of the current theories both of stellar evolution and of white dwarf envelopes.

We emphasize that the current theory on the structure of white dwarf external layers is successful in explaining the observational features of white dwarfs, and is in good agreement with the classical theories of stellar evolution, so that it cannot be ignored, whenever it is relevant, in the study of white dwarfs in novae. In particular, all the results concerning interstellar accretion (D'Antona and Mazzitelli 1974, 1975; Koester 1976), helium envelope thickness (D'Antona and Mazzitelli 1979, hereafter DM), and deepening of convection in the envelope (Fountaine and Van Horn 1976; DM) cannot be easily dismissed.

We are aware that nova white dwarfs belong to short-period binaries, and their evolutionary history can be claimed to be different from single-star evolution. However, the most plausible theory for the formation of cataclysmic binaries (Ritter 1976; Webbink 1975) implies that they have been very long-period systems, in which the evolution of the present white dwarf is not much different from a single-star evolution.

In this framework, the implications of which are described in § II, we analyze the mechanisms proposed until now for CNO enrichment of novae (§ III) and report some problems regarding each of them. As a corollary of this discussion we briefly examine the problem, and its possible solution, of carbon in single, helium-atmosphere white dwarfs (§ IV). We then propose an alternative mechanism for CNO enhancement

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in novae, one consistent with the current theories of stellar evolution. We shall show that even under the most favorable conditions, there exists a discrepancy greater than a factor of 2 between the maximum CNO enhancement which this scheme can provide and the average enhancement required by the fast nova theory. In § V we examine the problem of the large neon abundance in Nova Cygni 1975 (Ferland and Shields 1978), which is shown to be a serious one for each theory concerning the composition of novae envelopes. We finally discuss the possible role of sedimentation of heavy elements (mainly of CNO) in nova envelopes.

II. FRAMEWORK OF PRENOVA EVOLUTION

It seems reasonably certain that white dwarfs belonging to nova systems are massive, in excess of 1 $M_\odot$ (Robinson 1976). Furthermore, theoretical arguments require that fast novae be massive (Prialnik, Shara, and Shaviv 1979; Truran 1979). This implies that they are carbon-oxygen core white dwarfs, remnants of the evolution of much more massive stars ($M_{\mathrm{initial}} \sim 3$–$5\ M_\odot$, if steady mass loss and planetary nebula ejection are taken into account; Iben and Truran 1978). According to several authors (e.g., Paczyński 1976; Ritter 1976; Webbink 1975) the evolution proceeded as follows: in a very long-period binary ($P \sim 10$–50 yr) the most massive component ($M > 3\ M_\odot$) evolves quite as a single star through the phases of double shell burning (steady hydrogen burning plus helium-shell pulses; Schwarzschild and Härm 1967; Iben 1975*a*, *b*; 1976, 1977). When the radius of the giant reaches the Roche-lobe radius of the system, mass transfer onto the companion and mass loss from the system occur on a very fast time scale (Webbink 1979), soon leading to the formation of a "common envelope" system. The degenerate core and the secondary interact with this envelope, and a rapid shrinking of the binary follows, leading to the cataclysmic binary period (few hours) in less than 10^3 yr (Meyer and Meyer-Hofmeister 1979).

What is of interest for our following discussion is that the structure of the red giant which will evolve into a white dwarf consists of a C-O core plus a helium envelope having already a small white dwarf radius ($\sim 10^{-2}\ R_\odot$) and of an extended hydrogen envelope. Therefore, the mass transferred or lost from the primary does not affect the helium layers (see also Lauterborn 1970). This implies that the final white dwarf still has a helium envelope of mass consistent with the helium remnant of a single star evolution. In DM we showed that the helium-mass surviving at the end of the pre–white dwarf evolution, including nuclear burning and mass loss, cannot be much smaller than the mass contained in the helium intershell during the double-shell burning phases. This latter mass is a function mainly of the core mass (Paczyński 1975). Therefore, each white dwarf, either single or belonging to a cataclysmic binary, has a helium envelope whose mass is a function of the white dwarf mass, i.e., the previous core of the red giant, and cannot be far different from the masses estimated by Paczyński (1975). Another important point to be kept in mind is the chemical composition of the helium remnant and of the possible hydrogen remnant. The helium intershell has been enriched in carbon during the thermal pulses, from 10 to 30% according to the white dwarf C-O core mass (Iben 1977), and the hydrogen layer has dredged up the products of nuclear burning (carbon, helium, *s*-process elements, neon) as shown in Iben and Truran (1978).

III. MECHANISMS FOR CNO ENHANCEMENT

Starrfield, Sparks, and Truran (1978) make a list of some possibilities for a CNO enhancement in the hydrogen layers of white dwarfs undergoing a nova outburst. These possibilities can be grouped into two main categories: (*a*) mechanisms involving mixing of the hydrogen layer with the C-O core of the white dwarf; and (*b*) mechanisms related to "dredging up" of carbon-rich matter following helium-shell pulses or flashes.

In category (*a*) carbon enhancement can be obtained if (i) accretion occurs on a pure C-O white dwarf (no helium layer at the surface) causing convective mixing (Colvin *et al.* 1977); (ii) there is convective overshooting between the hydrogen shell and the carbon core during the nova outbursts; or (iii) accretion from a disk causes the formation of an "accretion belt" around the white dwarf; the shear instability between the rapid rotating disk and the slowly rotating white dwarf can reach the depth of the helium-carbon interface (Kippenhahn and Thomas 1978) and cause mixing with the carbon-rich material.

Let us examine these mechanisms in detail. The occurrence of accretion on a pure-carbon-oxygen white dwarf implies, first of all, the existence of such stars. Actually, in § II, we recalled that according to the theory of stellar evolution, a helium layer is left on the surface of the white dwarf even in binary evolution. As an alternative possibility one might think that the surface chemical composition of the white dwarf could be turned into almost pure carbon if the helium remnant were very thin in mass. In this situation, helium convection, during the star cooling, could reach the bottom of the helium layer. In this case a deep carbon convection zone has been shown to exist under the helium and complete mixing is ensured (DM; Vauclair and Fontaine 1979). Because the actual occurrence of this possibility depends on the relative sizes of the helium convective zone and the helium remnant, a brief discussion of these parameters is required.

The maximum thickness of the convective helium layers ($M_{\mathrm{He,ce}}{}^{\mathrm{max}}$)—reached when the star has $T_{\mathrm{eff}} < 12{,}000$ K—is known with reasonable accuracy (DM; Fontaine and Van Horn 1976) and is substantially independent of the treatment of superadiabatic convection and of the metal content of the envelope. It is a function of the white dwarf mass

(M_{wd}), the convective mass being smaller for larger white dwarf masses. From DM we can derive an approximate relation valid between $M_{\mathrm{wd}} = 0.6$ and $1.2\, M_{\odot}$:

$$\log M_{\mathrm{He,ce}}{}^{\mathrm{max}} \approx -3.68 M_{\mathrm{wd}} - 3.25\, (M_{\odot})\,, \quad (1)$$

but, for larger white-dwarf masses, the values of $M_{\mathrm{He,ce}}{}^{\mathrm{max}}$ computed from (1) are much in excess (a factor of ~ 30 for $M_{\mathrm{wd}} = 1.35\, M_{\odot}$) of the values obtained from the computation of the models.

Comparing these results with Paczyński's (1975) results on the remnant mass of the helium envelope from the preceding evolution, as a function of the core mass (which is practically the mass of the white dwarf to be), one can easily see that the remnant helium mass is always at least three orders of magnitude larger than $M_{\mathrm{He,ce}}{}^{\mathrm{max}}$. Even if the remnant helium mass were greatly reduced (even by a factor of 10, which is not likely to happen; see DM), during the last phases of evolution to a white dwarf, it would remain much larger than the mass which ensures the appearance of C-O atmosphere white dwarfs.

The third possible explanation of the accretion of hydrogen onto a pure carbon white dwarf requires the star to have lost more mass than it has accreted during each of the preceding outbursts. This has been proposed by Ferland (1979) to explain the tentative inverse correlation which the data suggest between helium abundance in the ejecta and speed class of the nova. In the course of $\sim 10^3$ outbursts the nova could shed its helium envelope, thereby exposing its carbon core. Another requirement of this tempting suggestion is that there be substantial mixing of the accreting hydrogen with the helium matter before the outburst. This can be obtained only if accretion occurs from a Keplerian disk causing a shear instability in the external layers of the white dwarf (Durisen 1977). We shall later discuss the difficulties inherent in the "accretion belt" model (Kippenhahn and Thomas 1978). For the present we note just one obvious exception to Ferland's proposal: for the nova DQ Her, whose ejecta show the largest carbon abundance found among novae (Williams *et al.* 1978), the accretion near the white dwarf is surely dominated by the large magnetic field (10^7 gauss) of the star (Angel 1978), so that no shear instability can be invoked to obtain mixing through the helium zone and the ultimate expulsion of the whole helium envelope. Furthermore, it has not been demonstrated that slow novae lose more matter belonging to the helium envelope than the helium they produce during the outburst itself.

In § IV we shall discuss the consequence of the discussion on carbon-envelope white dwarfs for the interpretation of single white-dwarf spectra. For the present purpose, it is enough to know that no pure carbon atmosphere white dwarfs are observed, so that the possibility (*a*[i]) of carbon enhancement can be dismissed as unrealistic on the basis of both theory and observation. Furthermore, Colvin *et al.* (1977) have shown that, even in the theoretical case of hydrogen accretion onto a pure carbon white dwarf, a small hydrogen abundance is able to stop the carbon convection. As a result, further mixing is not allowed and carbon enhancement is achieved only in a small fraction of the hydrogen-rich envelope, very far from the requirements of Starrfield, Sparks, and Truran (1978).

In regard to point (*a*[ii]), we notice that model computations *do not* predict overshooting into the hydrogen region from the small helium convective region which develops in the inner layers during the flash. The physical reason for this impossibility is the very large inverse temperature gradient between the hydrogen shell and the matter underneath (Starrfield, Sparks, and Truran 1978). If overshooting were possible, it would occur between the hydrogen and helium layers, not with the carbon core. However, since the helium region is significantly enriched in carbon (§ II), considerable carbon enhancement could be achieved. This would happen at the expense of the hydrogen concentration: if carbon-enriched matter has to play a role in the thermonuclear runaway, this matter would consist of helium and carbon, plus a few percent of hydrogen in mass. Such models have never been computed, so that it is difficult to understand whether the outburst would resemble a fast nova event.

Mechanism (*a*[iii]) deals with a very different kind of picture of accretion on white dwarfs, and therefore with problems very different from the ones relevant to single white dwarf structures. Until now, the nova runaway has been computed either starting from a hydrogen envelope already in place on the star (Starrfield, Sparks, and Truran 1978, and references therein), or computing spherically symmetric accretion (Nariai *et al.* 1979). Only recently the effect of accretion from a rotating Keplerian disk on a slowly rotating white dwarf has been considered, following Durisen (1977) and the "accretion belt" model by Kippenhahn and Thomas (1978). The application of such models to nova envelopes is still in progress (Sparks and Kutter 1979).

A detailed discussion of the formation and stability of a rotating Keplerian envelope and of the mechanism of angular momentum transfer between the envelope and the inner white dwarf is beyond the scope of this paper. We refer the reader to Durisen (1977), who discusses all the physical uncertainties pertaining to these models. We notice here the following simple reason for which the application of these models to novae should be treated with extreme care. From Kippenhahn and Thomas (1978) it results that the dissipation time scale due to circulation is $\sim 2 \times 10^5$ yr. They further state that the dissipation time due to viscosity is $\sim 2.5 \times 10^7$ yr, when computing it at densities $\rho \sim 100\ \mathrm{g\ cm^{-3}}$. But the typical densities at the bottom of a critical layer for the ignition are $\rho \sim 2$–$3 \times 10^4\ \mathrm{g\ cm^{-3}}$. In this case, since the molecular viscosity depends on the 5/3 power of the density (Durisen 1973), the dissipation time scale is decreased to $\sim 10^6$ years. These two possibilities of dissipation give a time scale comparable to or shorter than the time of accretion of a critical envelope, when the accretion rate is $\sim 10^{-10}\ M_{\odot}\ \mathrm{yr}^{-1}$ (Taam and Faulkner 1975; Taam 1979). This rate of accretion is probably typical for nova binary systems having a low–main-sequence companion, and consequently short periods. In this case, the angular momentum losses by gravitational radiation are dominant (Paczyński 1967; Faulkner 1971) and mass transfer rates (0.2–$2 \times 10^{-10}\ M_{\odot}\ \mathrm{yr}^{-1}$) are inferred (Faulkner 1974). Therefore, the actual occurrence of considerable mixing by shear instability in nova envelopes is not definitely settled.

Let us examine in detail (*b*) type mechanisms. Starrfield, Sparks, and Truran (1978) quote two possibilities:

i) The hydrogen layer undergoing the nova event is a remnant of the preceding evolution during which carbon has been enhanced by dredging up following thermal pulses in the helium shell (Iben 1975*a*, *b*, 1976; Iben and Truran 1978).

ii) The hydrogen layers are enriched by dredging up following a helium-shell flash occurring during the nova lifetime.

These two mechanisms meet another kind of difficulty. Ford (1978) and Bath and Shaviv (1978) have shown that the statistics pertaining to nova events imply that they are recurrent 10^3–10^4 times for each cataclysmic system. In case (i) only the first nova outburst would occur in carbon-enriched material, in case (ii) only the outburst following a helium-shell flash. We cannot expect helium-shell flashes to be very frequent in novae. The study of helium-shell pulses in giants (Iben 1977) shows that a helium pulse occurs when the helium intershell mass has reached about 100 times the mass in the hydrogen shell. Even if all the hydrogen accreted on the white dwarf where converted into helium, this would imply one helium-shell flash for each 100 nova events. The actual frequency is probably lower since a fraction of the accreted hydrogen is lost following the outbursts. If only the nova outbursts following a helium-shell flash were fast novae, the frequency of fast novae would be insignificant, unless helium ignition were to occur at the top of the helium layer triggered by the nova explosion itself, as suggested by Webbink, Truran, and Gallagher (1978). Unfortunately, this possibility has not yet been confirmed by model computation.

IV. THE PROBLEM OF CARBON IN SINGLE WHITE DWARFS

It is well known that a group of helium-atmosphere white dwarfs (the λ4670 group) shows carbon lines or bands. The analysis of atmospheres gives mass abundance of carbon $Z_c \sim 10^{-3}$–10^{-4} in these stars (Bues 1973, 1979). These features have been commonly taken as proof that in these stars some carbon is picked up from the core by convective mixing, an interpretation that supported the theory of carbon enrichment in novae from the core. The models by DM and Vauclair and Fontaine (1979) have convincingly shown that this explanation for λ4670 has to be ruled out, as convective mixing would ensure the appearance of pure carbon spectra.

However, in the absence of a better explanation, Vauclair and Fontaine (1979) suggested that perhaps the λ4670 group was the class of white dwarfs in which the helium remnant was just equal to the maximum convective mass. Under these circumstances, only a small quantity of carbon would be convected to the surface. But, of course, if there are white dwarfs with envelopes just as massive as $M_{\mathrm{He,ce}}{}^{\max}$ one could argue that there must be others with smaller helium envelopes, which have been turned into pure carbon atmospheres: perhaps these white dwarfs exist but have not been identified.

Therefore, it is important to find and alternative satisfactory explanation for the λ4670 group, in order to remove any doubt that stellar evolution predicts the correct helium envelope remnants on carbon-core white dwarfs.

An alternative mechanism, proposed by Greenstein (1976) to account for the presence of metals in some WD atmospheres, and by DM to explain mainly the λ4670 group, is the competition between diffusion of metals in the envelope of the white dwarf and convective mixing. The scheme is the following: in the first phases of the white dwarf lifetimes, sedimentation in the atmosphere and subatmosphere causes the heavy elements to be hidden with respect to the main constituent. When convection appears and sinks into the star, it can reach a fraction of the sinking elements and bring them back to the surface. The main problem is that the study of sedimentation in white dwarfs is in good qualitative agreement with observations, as this process can well explain the observed monoelemental character of white dwarf spectra, but the computed time scales of diffusion are so short with respect to the evolutionary time scale that even the trace heavy elements present in white dwarf atmosphere are unexplainable (e.g., Fontaine and Michaud 1979*a*). However, the theory is still so uncertain (for a discussion, see Fontaine and Michaud 1979*b*) and the problem to so complex (e.g., Muchmore 1979) that it is still unclear whether one has to invoke accretion of heavy elements (coupled with no accretion of hydrogen!) in order to explain the presence of metals in WD atmospheres (Vauclair 1979). However, the feeling is that not even accretion can explain the presence of carbon in λ4670, as the ratio of carbon to oxygen in these atmospheres is very large (C/O > 10; Bues 1973) whereas the relative diffusion velocities of carbon and oxygen cannot be very different, whatever settling theory is used.

DM proposed the convection-diffusion competition mechanism for λ4670 stars, noticing that the discrepancy with the short sedimentation time scales can be resolved if one considers that carbon was not a trace element in the helium intershell of the double shell burning stars progenitors of these white dwarfs. Iben

(1977) estimates that carbon can be 10–30% in mass fraction during the thermal pulse phases. In these conditions, the gradient of the element concentration, which counteracts sedimentation, cannot be neglected as has been done until now in all computations of the diffusion time scales. Furthermore, it is required that the diffusion velocities be very large, otherwise excessive carbon abundances would be predicted by stellar evolution. The problem of the C/O ratio finds in this scheme an obvious explanation, as carbon but not oxygen is produced during helium-shell flashes.

We thought it worthwhile to insert this discussion in a paper mainly dedicated to the study of novae, as it shows that the knowledge both of stellar evolution and of single white dwarf structure are important for the understanding of what happens in novae as well.

V. A FURTHER MECHANISM FOR CARBON ENHANCEMENT IN PRENOVAE

In the preceding sections we have shown that, whatever the mechanism for CNO enrichment in nova white dwarfs may be, it must (i) involve no mixing with carbon from the core; and (ii) operate recurrently or continuously. The following possibility appears to us to satisfy the conditions stated: *the hydrogen-rich matter accreted on the white dwarf must be* CNO *enriched.* This situation cannot be achieved by means of the evolution of the secondary itself, which is always a small-mass dwarf or a subgiant, but it can probably be achieved if the now-white dwarf star has substantially polluted the external layers of the secondary component during the phases of mass exchange and mass accretion in a common envelope which occur in the formation of the cataclysmic system.

We already stated that the hydrogen envelope of the pre–white dwarf has been contaminated by carbon during the dredge-up phases following helium-shell thermal pulses. The envelope of the secondary star, as a result of the accretion of mass from the red giant, may assume the chemical composition of the latter. In this situation, at the beginning of the second phase of mass exchange, from the secondary to the now-white dwarf, the matter accreted is carbon enhanced. This situation resembles mechanism (*b*[i]). The difference is that now the carbon enhancement is a recurrent feature of the model.

To understand whether this mechanism can give CNO abundances in agreement with the theoretical requirements for very fast novae, we shall make the idealized assumption that the envelope of the secondary has obtained the maximum carbon enhancement achieved by the primary during the thermal pulses. The most drastic hypothesis is that the matter transferred to the secondary has not been diluted into the secondary. This case is probably not realistic since the secondary may have a deep convective envelope if its mass is low. Furthermore, Webbink (1977) has shown that the mass transfer itself can induce deep convection in the secondary. On the other hand, since it seems very likely that the last phases of mass loss during the formation of the cataclysmic system occur from the secondary itself (Meyer and Meyer-Hofmeister 1979) it is very important that the secondary become deeply convective, so that its deep layers, which will not be immediately lost, must be contaminated with the primary envelope composition. However, simply in order to compute the upper limit to the possible carbon enhancement, we will assume that the secondary will obtain the composition of the primary envelope.

Another important point to remember is that the process of formation of a cataclysmic binary is very fast. During the 10^3 years of formation, it may be that some last helium-shell pulses occur in the primary star. If the core is massive ($M \sim 1.25\ M_\odot$) the interpulse duration can be as short as 10^2 years (Iben 1977). Therefore, even if the sudden mass loss from the star does not influence its shell behavior, no more than ~ 10 pulses can be expected to occur during these last phases. Considering that about 2000 pulses have been suffered from the star in the course of formation of its massive core, it is clear that neither the chemical composition of the envelope nor the core mass can be further changed during the evolution to a cataclysmic binary. Consequently, the chemical composition of the envelope is fixed by its single-star evolution, and is a function only of the initial mass of the star and of the core mass at the moment in which mass exchange begins.

It is therefore simple to read the CNO enhancement expected on the basis of the Iben and Truran (1978) computation. The maximum enhancement will occur if the pre–white dwarf reaches the Roche-lobe radius just before a planetary nebula ejection would take place, when the initial mass is $\sim 5\ M_\odot$, leaving a white dwarf mass of 1.36 $M_\odot$. The carbon is in this case about 13 times the solar value. Nitrogen plus oxygen do not vary too much during the evolution, so that the total CNO enhancement is less than a factor 5, much to low with respect to the expectation of Starrfield, Sparks, and Truran (1978).

As this is the enhancement mechanism which we find most satisfactory from an evolutionary point of view, we decided it was worthwhile to examine the details of Iben and Truran's assumptions on the evolution which generated the quoted numbers, and to try to understand in what instances possibly more satisfactory CNO enhancement could be achieved.

It appears that the most uncertain parameter (and the most sensitive one) in the computation by Iben and Truran is the so-called "dredge-up law," the dependence on the core mass of the ratio of the dredged up mass ($\delta M_{\mathrm{dredge}}$) with respect to the mass processed by the hydrogen shell (ΔM_c) during the interpulse phase. This parameter $\lambda = \delta M_{\mathrm{dredge}}/\Delta M_c$ is 0.33 when $M_c = 0.96\ M_\odot$ (Iben 1976). According to Iben and Truran (1978), for larger M_c only lower limits are known, as the computations of pulsing phases costs dearly in computer time. Iben and Truran (1978) choose λ to increase with a sine law from 0.33 to 0.5 as M_c varies

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from 0.96 to 1.4 $M_\odot$. Since the theory is uncertain, one might consider another, more liberal law, allowing, for example, λ to approach 1 with a sine law when M_c approaches 1.4 $M_\odot$. The program and the program input of Iben and Truran (1978) have been used to check this possibility. Among the possible laws for λ, we chose a new sine law between $M_c = 0.96$ ($\lambda = 0.33$) and $M_c = 1.4$ ($\lambda = 1.0$). It is clear that the greater the value of λ, the greater the amount of matter which is dredged up and the larger the carbon abundance in the hydrogen envelope for each given M_c. This is not the only effect. Another consequence of changing the dredge-up law is the following: If λ increases, the fraction of the processed material which goes to increasing the carbon core decreases. If λ were 1, the core would not increase at all, as all the processed material would be dredged up. In other words, whatever the initial mass of the star may be, the carbon core would not grow to 1.4 $M_\odot$ and, as steady mass loss continues, a white dwarf remnant would be left. In order to obtain a more realistic situation, we have one possibility: choose a maximum initial mass for planetary nebulae ejection (and white dwarf formation) and adjust the λ-law so that the competition between steady mass loss and core growth may be in agreement with this choice. We decided to set the maximum initial mass for white dwarf formation equal to 6 $M_\odot$, and the resulting dredging-up law is a sine law between $M_c = 0.96$ and $M_c = 1.2$ $M_\odot$, whereas λ is fixed at 0.83 for $M_c > 1.2$ $M_\odot$.

With this choice, and leaving the other parameters as defined in Iben and Truran (1978), we get the maximum carbon enhancement before the ejection of a planetary nebula from the $M_{\rm initial} = 6$ $M_\odot$, and it is $\bar{Y}_{12} = C_{12}/C_{12\odot} \approx 33$ and $\bar{Y}_{\rm CNO} = {\rm CNO}/{\rm CNO}_\odot = 13$. The resulting white dwarf has a mass $M = 1.37$ $M_\odot$.

Needless to say, this value can be regarded as the maximum CNO enhancement obtainable with this evolutionary scheme. It is still unclear whether it is enough to reproduce the features of a very fast nova (e.g., Shara and Shaviv 1979). In Figure 1 we show, as a function of the core mass, the CNO enhancement predicted with our choice for the dredge-up law, starting from different initial masses. A wide spread in CNO abundances can be found for a given core mass, but only the largest cores ($M \gtrsim 1.35$) reach a factor of ~10 enhancement.

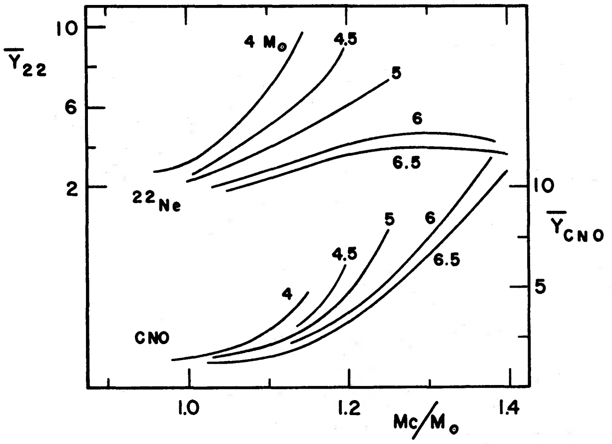

FIG. 1.—The CNO and ^{22}Ne enhancement in the envelope of pulsing stars is shown for several labeled initial masses. It is obtained following the prescriptions of Iben and Truran (1978), but adopting as dredge-up law, for core masses $M_c > 0.96$, $\lambda = 0.33 + 0.67 \sin[(\pi/0.88)(M_c - 0.96)]$ for $M_c < 1.2$ $M_\odot$; $\lambda = 0.83$ for $1.2 \le M_c < 1.4$ $M_\odot$.

VI. THE PROBLEM OF NEON

Ferland and Shields's (1978) analysis of the ejecta of the very fast Nova Cygni 1975 leads to a total CNO enhancement of a factor of ~25 with respect to the solar value. With this CNO enhancement it seems to be possible to model the Nova Cygni light curve (Kenyon and Truran 1980). But, if one wants to believe the analysis of observations, one has to face the problem that neon also is enhanced in the ejecta, namely $({\rm Ne/H})_{\rm Nova\,Cygni} = 10({\rm Ne/H})_\odot$. Neon seems to be enhanced also in some other novae, as in RR Pictoris 1925 (Williams and Gallagher 1979), in the Orion spectrum of Nova Aquilae 1945 (McLaughlin 1960), or in the nebular stage of the GK Persei nova (Payne-Gaposchkin 1957). On the basis of only the abundance determined in Nova Cygni, is there any possible explanation for this anomaly?

An overabundance of neon can result either from our proposed scheme of evolution or during a helium-shell flash in the nova stage. Following Iben (1975*a*, *b*) the source of neutrons for producing *s*-process elements in pulsing stars can be identified with the result of the reaction $^{22}{\rm Ne}(\alpha, n)^{25}{\rm Mg}$ (Cameron 1960). ^{22}Ne is produced by the burning of ^{14}N when the ^{14}N left from CNO cycling is convected into the helium shell during a thermal pulse. After each pulse, dredging up results in bringing to the surface the ^{22}Ne produced during this cycle. Therefore, the enhancement in neon in the envelope of the progenitor of the white dwarf depends on the relative rates of ^{22}Ne production and destruction during the thermal pulse. When the core mass is relatively small (up to ~1.1 $M_\odot$) the ^{22}Ne produced is not burned and comes to the surface. For $M_c \ge 1.2$ $M_\odot$ the ^{22}Ne is burned into ^{25}Mg. From Iben and Truran (1978) one can find that the maximum ^{22}Ne is 5 times solar, resulting from the evolution of the 4 $M_\odot$ model. We show our result in Figure 1. The ^{22}Ne can reach a factor of ~10 times $^{22}{\rm Ne}_\odot$, from the evolution of an initial 4 $M_\odot$. This enhancement can approximately double the total Ne abundance in the envelope. But from this kind of evolution CNO enhancement is very small (a factor of <5). Even if it were possible to adjust the parameters regulating ^{22}Ne burning in order to have a somewhat larger neon enhancement, it seems unavoidable that the core masses for which one can obtain the largest CNO enhancement are not the same masses for which the maximum neon enhancement can be obtained. On the one hand, this would agree with the moderate nitrogen enhancement of the slow nova RR Pic, but, on the other hand, a factor of 10 enhancement in neon as in Nova Cygni 1975 is to be completely ruled out from our scheme. In other words, a small neon enhancement (up to a factor of ~ 2) is possible in the matter accreting on the white dwarf, but the agreement with Nova Cygni 1975 is only qualitative. The other possibility of obtaining a considerable amount of neon is the ^{14}N burning to ^{22}Ne chain occurring during a helium flash in the nova stage itself. In this case the dilution into the hydrogen envelope would be small, and a large amount of neon could be present, together with a large carbon abundance. As we discussed, not all very fast novae can be related to the nova outburst following a helium-shell flash, as the frequency of fast novae would be too low, but we have no accurate statistics on the occurrence of neon enhancement, so that we cannot dismiss this possibility for some novae. As a word of caution, though, we must warn that in this case the helium abundance in the nova ejecta would be much larger than solar. This is in agreement with Nova RR Pic analysis (Williams and Gallagher 1979) but the analysis of Nova Cygni 1975 shows "normal" helium! In any case, observations would not agree with any theory! Therefore, it seems that at present a better understanding of the observations is needed before any further speculation or computation will be worthwhile.

VIII. ROLE OF SEDIMENTATION IN PRENOVA ENVELOPES

In § IV we mentioned that diffusion of heavy ions in the atmospheres and envelopes of single white dwarfs is surely an efficient mechanism, although at present many uncertainties bear on its quantitative understanding. In § III we mentioned that it has not been determined with certainty whether the shear instability due to accretion from a rotating disk occurs deep in the white dwarf envelope. If the angular momentum of the accreting matter is dissipated in the upper layers of the star, diffusion can be an important mechanism in the envelopes of prenovae, and it is worthwhile to try to understand its possible effect on the nova behavior. Unfortunately, at present it is hopeless to try to obtain reliable sedimentation time scales deep in the envelope of the white dwarf. The only possibility of getting an idea of the relevant time scales would be to make the usual approximation of a bi-component gas (hydrogen plus carbon), whereas the matter accreting on the white dwarf consists of hydrogen, large quantities of helium, and relevant fractions of carbon and other heavy elements (as an example, enhanced *s*-process elements). It would be important, for instance, to understand the diffusion of helium with respect to hydrogen, which could explain the "quite normal" helium abundances in some nova ejecta, compared to the enormous anomalies in CNO elements (Ferland 1979).

Furthermore, even for correct use of the bi-component gas approximation one has to know the correct density-temperature stratification into the envelope, which may be influenced by nonstatic heating effects. To make things simple, one must compare the diffusion time scale at a given mass point M_r in the white dwarf envelope with the time necessary to accrete M_r at the rate of mass exchange between the components of the binary system. Taking $M = 10^{-10}\ M_\odot\ \mathrm{yr}^{-1}$ it results that for massive ($M > 1.3\ M_\odot$) white dwarfs the time scale of diffusion is of the same order of magnitude as the accretion time along the whole envelope. At the base of a massive envelope ($M_e \sim 10^{-4}\ M_\odot$) the true time scale is probably much longer, as the pure Coulomb potential approximation is surely wrong (Fontaine and Michaud 1979*a*). But if the outburst occurs off the base, as suggested by the models by Nariai *et al.* (1979) or by the hydrostatic computations at Giannone and Weigert (1967), Taam and Faulkner (1975), and Taam (1979), the diffusion effect can be relevant.

Now, if the ignition occurs at half the critical accreted mass, is the CNO abundance reduced due to sedimentation? It is worth predicting here that this is not the case and, furthermore, that the abundances could be enhanced just in the shell undergoing the outburst, for the following reasons. The thermonuclear reactions begin outside the base of the envelope when the release of gravitational energy is important in modifying the thermal structure of the white dwarf in the envelope. In this case, the temperature in the envelope can be much larger than the core temperature of the white dwarf (e.g., Giannone and Weigert 1967) and a negative temperature gradient is formed between the envelope and the core. It is well known that gravitational settling and thermal diffusion are of the same order of magnitude in white dwarfs (Fontaine and Michaud 1979*a*). The negative temperature gradient can easily counteract the effect of gravitational settling. In order for the total velocity of diffusion to be zero it must be

$$\frac{\partial \ln P}{\partial r}\left[\frac{m_2}{m_1}(1 + Z_1) - Z_2 - 1\right] + \frac{\partial \ln T}{\partial r}\alpha_{12}' = 0\,,$$

where $m_1(Z_1)$ and $m_2(Z_2)$, respectively, refer to the hydrogen and carbon mass (charge) and α_{12}' is the thermal diffusion coefficient. Therefore, it must be

$$\frac{\partial \ln T}{\partial \ln P} = \frac{(m_2/m_1)(1 + Z_1) - Z_2 - 1}{\alpha_{12}'}\,.$$

Taking for α_{12}' the asymptotic expression

$$\alpha_{12}' = 3.15Z_2^{\,2} - 0.806Z_2$$

(Fontaine and Michaud 1970*a*), and assuming fully ionized carbon, it must be

$$\frac{\partial \ln T}{\partial \ln P} = -0.16\,.$$

This requirement is not very strong. The negative temperature gradient should have the effect of making the CNO elements concentrate in the shell which will undergo the outburst.

It is difficult to envisage the quantitative effectiveness of the enhancement, but it is probably lower than a factor of 2, taking into account that the shell will have about the same mass as the matter which lies on it. However, the effective CNO abundance could be doubled.

Does this enhancement have any effect on the outburst strength? According to Starrfield, Sparks, and Truran (1978), a deep external convection develops in the very first moment of the outburst, so that there would be no difference whether the carbon is all concentrated on the burning shell or spread out, but computations using time-dependent mixing are needed to show whether the spreading of the carbon-rich material could be delayed enough and allow a more energetic flash to occur.

In the most favorable case, therefore, if carbon-rich matter is accreted from the companion, its abundance can be doubled in the burning shell, possibly producing interesting consequences on the nova explosion.

VIII. SUMMARY

We have investigated the possibilities for CNO enrichment in the envelopes of prenovae, keeping in mind the prescriptions dictated by the current theories of stellar evolution and of white dwarf structure. We point out several difficulties concerning each of these. We propose another mechanism which can enhance CNO up to about 12 times the solar value, and suggest that sedimentation of carbon, together with an off-base ignition of hydrogen, could produce abundances twice this great during the outburst. As a corollary we stress that the explanation for carbon abundances in the $\lambda4670$ group of white dwarfs must be found in the diffusion-convection competition mechanism. Finally, we remark that the problem of the neon enhancement in Nova Cygni is qualitatively in agreement with the proposal that neon comes from the secondary together with carbon-enriched material, but it is not in quantitative agreement either with this or any other suggestion.

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REFERENCES

Angel, J. R. P. 1978, *Ann. Rev. Astr. Ap.*, **16**, 487.
Bath, G. T., and Shaviv, G. 1978, *M.N.R.A.S.*, **183**, 515.
Bues, I. 1973, *Astr. Ap.*, **28**, 181.
———. 1979, in *IAU Colloquium No. 53, White Dwarfs and Variable Degenerate Stars*, Rochester, New York.
Cameron, A. G. W. 1960, *A.J.*, **65**, 485.
Colvin, J., Van Horn, H M., Starrfield, S., and Truran, J. W. 1977, *Ap. J.*, **212**, 791.
D'Antona, F., and Mazzitelli, I. 1974, *Ap. Space Sci.*, **27**, 137.
———. 1975, *Astr. Ap.*, **44**, 253.
———. 1979, *Astr. Ap.*, **74**, 161 (DM).
Durisen, R. H. 1973, *Ap. J.*, **183**, 205.
———. 1977, *Ap. J.*, **213**, 145.
Faulkner, J. 1971, *Ap. J. (Letters)*, **170**, L99.
———. 1974, in *IAU Symposium No. 66, Late Stages of Stellar Evolution*, ed. R. J. Rayler and J. H. Hesser (Dordrecht: Reidel).
Ferland, G. J. 1979, *Ap. J.*, **231**, 781.
Ferland, G. J., and Shields, G. A. 1978, *Ap. J.*, **226**, 172.
Fontaine, G., and Michaud, G. 1979*a*, *Ap. J.*, **231**, 826.
———. 1979*b*, in *IAU Colloqium No. 53, White Dwarfs and Variable Degenerate Stars*, Rochester, New York.
Fontaine, G., and Van Horn, H. M. 1976, *Ap. J. Suppl.*, **31**, 467.
Ford, H. C. 1978, *Ap. J.*, **219**, 595.
Giannone, P., and Weigert, A. 1967, *Zs. Ap.*, **67**, 41.
Greenstein, J. L. 1976, *Ap. J.*, **210**, 524.
Iben, I., Jr. 1975*a*, *Ap. J.*, **196**, 525.
———. 1975*b*, *Ap. J.*, **196**, 549.
———. 1976, *Ap. J.*, **208**, 765.
———. 1977, *Ap. J.*, **217**, 788.
Iben, I., and Truran, J. W. 1978, *Ap. J.*, **220**, 980.
Kenyon, S., and Truran, J. W. 1980, in preparation.
Kippenhahn, R., and Thomas, H. C. 1978, *Astr. Ap.*, **63**, 265.
Koester, D. 1976, *Astr. Ap.*, **52**, 615.
Lauterborn, D. 1970, *Astr. Ap.*, **7**, 150.
McLaughlin, D. B. 1960, *Ap. J.*, **131**, 739.
Meyer, H., and Meyer-Hofmeister, E. 1979, *Astr. Ap.*, **78**, 176.
Muchmore, D. 1979, in *IAU Colloquium No. 53, White Dwarfs and Variable Degenerate Stars*, Rochester, New York.
Nariai, K., Nomoto, K., and Sugiomoto, D. 1979, *Pub. Astr. Soc. Japan*, in press.
Nomoto, K., Nariai, K., and Sugimoto, D. 1979, *Pub. Astr. Soc. Japan*, in press.
Paczyński, B. 1967, *Acta Astr.*, **17**, 287.
———. 1975, *Ap. J.*, **202**, 558.
———. 1976, in *IAU Symposium No. 78, Structure and Evolution of Close Binary Systems* (Dordrecht: Reidel), p. 75.
Payne-Gaposchkin, C. 1957, *The Galactic Novae* (Amsterdam: North Holland).
Prialnik, D., Shara, M. H., and Shaviv, G. 1978, *Astr. Ap.*, **62**, 339.
———. 1979, *Astr. Ap.*, **72**, 192.
Ritter, H. 1976, *M.N.R.A.S.*, **175**, 279.
Robinson, E. L. 1976, *Ann. Rev. Astr. Ap.*, **14**, 119.
Schwarzschild, M., and Härm, R. 1967, *Ap. J.*, **150**, 961.
Shara, M. H., and Shaviv, G. 1979, 4th Annual Workshop on Cataclysmic Binaries, Rochester, New York.
Sparks, W. M., and Kutter, G. S. 1979, in *IAU Colloquium No. 53, White Dwarfs and Variable Degenerate Stars*, Rochester, New York.
Sparks, W. M., Starrfield, S., and Truran, J. W. 1978, *Ap. J.*, **220**, 1063.
Starrfield, S., Sparks, W. M., and Truran, J. W. 1978, *Ap. J.*, **226**, 186.
Tamm, R. E. 1979, *Ap. Space Sci.*, **66**, 467.
Taam, R. E., and Faulkner, J. 1975, *Ap. J.*, **198**, 435.
Truran, J. W. 1979, 4th Annual Workshop on Cataclysmic Binaries, Rochester, New York.
Vauclair, G. 1979, in *IAU Colloquium No. 53, White Dwarfs and Variable Degenerate Stars*, Rochester, New York.
Vauclair, G., and Fontaine, G. 1979, *Ap. J.*, **230**, 563.
Webbink, R. 1975, Ph.D. thesis, Cambridge University.

Webbink, R. 1977, *Ap. J.*, **211**, 486.
———. 1979, in *IAU Colloquium No. 46, Changing Trends in Variable Star Research*, ed. F. M. Bateson, J. Smak, and I. H. Urch (Hamilton, New Zealand: University of Waikato).
Webbink, R., Truran, J. W., and Gallagher, J. 1978, *Bull. AAS*, **10**, 438.
Williams, R. E., and Gallagher, J. S. 1979, *Ap. J.*, **228**, 482.
Williams, R. E., Woolf, N. J., Hegel, E. K., Moore, R. L., and Kopriva, D. A. 1978, *Ap. J.*, **224**, 171.

Francesca D'Antona: Osservatorio Astronomico di Roma, I-00040 Monte Porzio, Rome, Italy

Italo Mazzitelli: Laboratorio di Astrofisica Spaziale del C.N.R., C.P. 67, I-00044 Frascati, Rome, Italy