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ON THE WARPED OPTICAL PLANE OF M33

Allan Sandage

Hale Observatories,1 Carnegie Institution of Washington

AND

ROBERTA M. HUMPHREYS University of Minnesota, Minneapolis Received 1979 September 24; accepted 1979 November 9

ABSTRACT

The shapes of the spiral arms of M33 betray a severely warped plane over most of the optical image. Ten spiral arms are identified, five on each side of the center. The inclination of the arms and of the optical disk to the plane of the sky increases outward from $\omega = 40^{\circ}$ near the center to $\omega \approx 67^{\circ}$ at $r \approx 30'$, while the position angle of the major axis rotates from $\sim 50^{\circ}$ toward P.A. $\simeq 0^{\circ}$. The optical warp begins within 0.25 of the Holmberg radius.

The unusual optical features are consistent with the tilted-ring radio model of Rogstad, Wright, and Lockhart, except that the optical plane begins to warp closer to the center, and our central tilt of $\omega = 40^{\circ}$ differs from $\omega = 55^{\circ}$ adopted there. The consequences of the smaller ω value are (1) the M33 rotation curve is steeper for r < 10' than current models, and (2) the calibration of the Tully-Fisher 21 cm line width-absolute magnitude relation may be affected.

Subject headings: galaxies: individual — galaxies: structure — cosmology

I. INTRODUCTION

Warps occur in the outer H I disks of many nearby spiral galaxies. Burke (1957) and Kerr (1957) discovered that the outer disk of our Galaxy is bent and reaches heights of $z \ge 1$ kpc at radii of only twice the solar circle (cf. Gum, Kerr, and Westerhout 1960, Figs. 1 and 6; Kerr 1969, Fig. 6). Discovery of similar warps in other galaxies followed the observation by Lewis (1968) of distorted H I isovelocity contours for the southern galaxies NGC 300 and NGC 5236 (M83). The kinematic data were extended by Rogstad, Lockhart, and Wright (1974), who introduced a model of concentric rings that are progressively tilted from a flat central region, and where the line of nodes rotates outward.

Similarly, early H I radio maps of M33 showed a velocity field that differs from a flat disk in circular differential rotation. The pronounced change in P.A. of both the isovelocity and the H I contours led Gordon (1971) and de Jager and Davies (1971) to suggest a warped H I plane for $r \ge 30'$ (i.e., 6 kpc). More detailed maps by Wright, Warner, and Baldwin (1972), by Huchtmeier (1973, 1978), and by Reakes and Newton (1978), show the extended symmetrical NE and SW wings that start ~40' (8 kpc) from the center and reach 70' radius (14 kpc) at P.A. $\approx -20^{\circ}$ rather than $+50^{\circ}$ that applies to the inner optical contours. New data by Rogstad, Wright, and Lockhart (1976) give additional evidence for a severely warped H I plane.

Similar models have been proposed for other galaxies from equally persuasive kinematic data. The north and south ends of the M31 disk, starting at about half the Holmberg (1950) radius, are warped (Roberts and

¹Operated jointly by the Carnegie Institution of Washington and the California Institute of Technology. Whitehurst 1975; Newton and Emerson 1977). Optical evidence for a warped M31 plane using H II regions was discussed earlier by Arp (1964). The data and discussion by Bosma (1977) give evidence for distorted outer H I planes in NGC 2841, NGC 5033, NGC 5055, M83, and NGC 7331. And the most direct evidence, independent of any modeling, is that of Sancisi (1976, 1977) from the H I contours for the edge-on galaxies NGC 4244, 4565, 4631, and 5907.

II. THE OPTICAL ARM SYSTEM

We have been led to the warp of the optical disk in M33 from a study of its brightest blue and red stars. A catalog of ~ 1000 such stars has been prepared for another purpose, and parts of the spiral structure have been mapped using 143 stellar associations (Humphreys and Sandage 1978, Fig. 2).

A system of five spiral arms has been traced over the optical face to radii of $r \approx 30'$. The prominent inner arms are the easiest to trace. The other arms are not well outlined by the associations and the blue stars alone. The difficulty can be seen from Figure 1 of Madore (1979), where additional information such as the surface-brightness distribution from fainter stars is clearly needed.

Figure 1 (Plates L1 and L2) is deep blue exposure of M33 taken with the Palomar 1.2 m Schmidt. The obvious system of outer arms shown in the left panel is marked on the right. The major axis is drawn as a dotted line that passes through the extremities of the five-arm system.

The spiral structure of the two prominent innermost arms is evident on short-exposure plates (cf. Roberts 1899; Hubble 1926, Plate XIV; Walker 1964, Fig. 1;



FIG. 1*a*.—Reproduction of a deeply exposed blue plate (Eastman Kodak 103a-O + Wratten 2c) taken with the Palomar 1.2 m Schmidt SANDAGE AND HUMPHREYS (see page L1)



FIG. 1b.—Lighter print of the same plate with the system of five arms marked. The major axis, put approximately through the extremities of the arm system, is drawn as a dotted line.

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Sandage and Johnson 1974). They occur within $r \approx 11'$ of the center and are well shown in the Ritchey photograph given by Hubble (1926, Plate XIV).

In his study of the inclinations of nearby spirals, Danver (1942) measured the inner arms alone and obtained an inclination of $\omega = 40^{\circ}.4$, at P.A. = 48°9 for the line of nodes. However, the surface photometry by de Vaucouleurs (1959) extends to radii of ~40', and from the axial ratio and the orientation of the outer isophotes, he obtained the quite different values of $\omega = 55^{\circ} \pm 1^{\circ}$, P.A. = $23^{\circ} \pm 1^{\circ}$. But because the values of de Vaucouleurs were used in the warped-plane models of Rogstad, Wright, and Lockhart (1976) and Reakes and Newton (1978, Fig. 10) for the *entire flat disk* with r < 25', we also began with $\omega = 55^{\circ}$, P.A. = 23° in a study of the spiral pattern.

III. THE TILT OF THE OPTICAL PLANE

The disk that is inside arms IN and IS is more circular than the isophotes farther out. This is why shortexposure plates of M33 show a galaxy that is less inclined than the impression given by deeply exposed images. Even causal inspection of Figure 1 shows that the major axis of the inner *disk* isophotes is near Danver's value of P.A. $\sim 50^{\circ}$, rather than P.A. $\sim 20^{\circ}$ which applies to the much more elongated outer optical isophotes.

These features, now so evident in Figure 1, were not recognized at first, but were seen only after we initially failed to rectify properly the arm system to a face-on orientation. We began the rectification by determining the positions for arms IN and IS from the associations alone (Humphreys and Sandage 1980). The polar coordinates, ρ and φ in the plane of the sky, measured from the major axis of the projected galaxy image, were then put into the tilt equations of von der Pahlen (1911), using $\omega = 55^{\circ}$, P.A. = 23° to determine r and θ in the plane of the galaxy.

The resulting face-on picture was not correct; the arms had too wide an opening angle; they were unlike any arms in well-known nearby face-on galaxies. We then tried Danver's values of $\omega = 40^{\circ}$, P.A. = 49°, and these quite correctly rectified the inner arms, *but not* arms II-V, for which increasing values of ω and decreasing position angles are needed.

Only as the first inklings of this requirement were dawning did we recall the tilting-ring diagram of Rogstad, Wright, and Lockhart (1976, Fig. 14). This model demands (1) the increased ellipticity of the isophotes for increasing r, and (2) the drastic change in position angle of the major axis. These features of Figure 1 so strongly resemble those in Figure 14 of Rogstad, Wright, and Lockhart (1976) that we began the complete rectification calculations in earnest.

IV. PRELIMINARY PARAMETERS OF THE ARMS

Following the justifications given by von der Pahlen (1911), Groot (1925, 1926), Reynolds (1925), and Danver (1942), we assume that equiangular (logarithmic) spirals of the form $r = r_0 \exp b\theta$ represent real spiral arms seen face-on. An objection is, of course, that in

both the density wave theory and the stochastic star formation model of Gerola and Seiden (1978), the pitch angle u (given by $u = \tan^{-1} [b^{-1} (\pi/180) \log e]$ for logarithmic spirals) changes gradually outward—hence b is predicted to be a slow function of θ . However, the variation is weak enough so that the assumption of constant b suffices here.

The search for the values of ω_i and $(P.A.)_i$ for any arm *i* is made by constructing a family of face-on spirals using ρ and φ in the projected image, by letting ω and P.A. vary. The test is to find ω_i and $(P.A.)_i$ from the best linear regression of ln *r* on θ .

An example is shown in Figure 2. The top panel is the family of face-on patterns for $55^{\circ} \leq \omega \leq 75^{\circ}$ with P.A. fixed at 23°. The lower panel is the family when P.A. is changed between 10° and 42°, with ω fixed at 65°. An acceptable fit is $\omega \approx 65^{\circ}$, P.A. $\approx 15^{\circ}$, but a larger family of curves could possibly provide a somewhat finer determination. With this caveat, we give the parameters of the 10 optical arms in M33 in Table 1. The positions of the major-axis crossings of the arms are given in columns (2) and (3). The calculated P.A. and ω values are in columns (4) and (5). The scale factor b is in column (6). The pitch angle as defined by Danver is in column (7). The scale factor in $r = r_0 \exp b\theta$ is in column (8), and the angles (north through east) over which the particular arms exist are in columns (9) and (10).

The resulting face-on picture of the five-arm system, shown as if the warp were removed, is shown in Figure 3 as a continuous line. For comparison, the logarithmic spirals, using the b and r_0 values from Table 1, are shown for each arm by the open circles.

The accuracy of the listed P.A. and ω values in Table 1 is estimated to be $\sim \pm 5^{\circ}$. Hence there is no question that a change in both ω and P.A. outward is needed, but the change may be more gradual than indicated in Table 1. We have not yet made more refined calculations to optimize the model by constructing a wider family of curves for each arm similar to Figure 2.

V. SUMMARY AND DISCUSSION

1. The *optical* plane defined by the young spiral arms and the bright underlying disk in M33 begins to warp as close to the center as the first arm system at $r \approx 2 \text{ kpc}$ (10'), which is only 0.25 of the Holmberg radius. Warps in the H I disks of other galaxies begin farther out. Bosma (1977) finds the warp to begin at 0.5 $R_{\rm H}$ in NGC 7331, 0.7 $R_{\rm H}$ in NGC 5033, and 0.8 $R_{\rm H}$ in NGC 2841, NGC 5055, M83, and NGC 7731. The warp in our Galaxy begins at the solar circle (10 kpc). Since the galactic Holmberg radius is 17 kpc on this scale (Tammann 1975), the warp appears at ~0.5 $R_{\rm H}$. The warp found directly by Sancisi (1976) in NGC 5907 begins "at the edge of the visible optical disk," which must be well within the Holmberg radius, perhaps at about $r_w = 0.5 R_{\rm H}$. The warp in M31 (Roberts and Whitehurst 1975) also begins near 0.5 $R_{\rm H}$. Hence, M33 is a severely warped galaxy.

2. The inclination angle of the central optical plane



FIG. 2.—The family of curves in the (ln r, θ)-plane for arm IIIN obtained by varying ω and the position angle. An approximation of $\omega = 65^{\circ}$, P.A. = 15° gives a satisfactory linear regression.

Акм (1)	ARM CROSSINGS		P.A. - N→E	ω		uª	r ₀	$\Delta P.A.$	
								θ_1	θ_2
	(arcmin) (2)	(kpc) (3)	(deg) (4)	(deg) (5)	b ^a (6)	(deg) (7)	(arcsec) (8)	(deg) (9)	(deg) (10)
				N	orth				
I	11.0	2.2	49	40	0.00762	45	472	310	70
II	17.2	3.4	15	65	0.01117	34	672	340	60
III.	21.8	4.4	15	65	0.01389	29	724	310	70
IV	33.0	6.6	14	67	0.00783	44	1510	280	50
V	~ 38.0	~ 7.6	15	63	0.00642	50	2039	300	10
				Sc	outh				
I	11.0	2.2	49	40	0.00684	48	141	150	280
II	15.0	3.0	15	55	0.01055	36	88	150	250
III	19.0	3.8	23	60	0.00826	43	183	210	275
IV	27.0	5.4	18	67	0.00630	50	429	110	280
V	30.0	6.0	18	63	0.00693	48	451	130	210

TABLE 1

^a tan $u = b^{-1}(\pi/180) \log e$.

is Danver's value of $\omega = 40^{\circ}$. This increases to $\omega \approx 70^{\circ}$ at $\sim 30'$ (6 kpc) and from the radio data, even more at larger distances. Hence, parts of arms IV and V bend out of the plane by ~ 3 kpc and would appear at an M33 galactic latitude of $\sim 30^{\circ}$ as seen from the center.

3. The optical warp in M33 is not subtle but is a major feature of the optical image. Figure 1 (right panel) displays the diagnostic signal, which is the systematic change of P.A. with radius and the symmetrical behavior of this change on the north and south sides. Comparison of Figure 1 with Figure 14 of Rog-stad, Wright, and Lockhart (1976) shows a close correspondence.

4. Our value of $\omega = 40^{\circ}$ for the central disk differs from $\omega = 55^{\circ}$ adopted by Rogstad, Wright, and Lockhart (1976) and by Reakes and Newton (their Fig. 14) at the center. The practical effects of this less inclined central plane and steeper warp at intermediate distances are that (1) the derived rotation curves of Rogstad, Wright, and Lockhart (1976) and of Reakes and Newton (1978) must be steepened for $r \leq 10'$ and made flatter for $r \geq 25'$; and (2) the integrated 21 cm velocity profile must be modeled because 2 v_{max} is now to be found by an integral over the entire warped plane.

Point (1) has consequences for the comparison of Freeman's (1970) prediction of the velocity curve for a truly exponential mass distribution in a disk. Point (2) has consequences when M33 is used as a calibrator for the Tully-Fisher (1977) line width-absolute magnitude relation. Distances from that method may be systematically affected in studies where M33 has high weight in the calibration.

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FIG. 3.—Face-on appearance of the arm system brought into the same plane. The lines are the arms as drawn in Fig. 1, but projected to face-on by the ω and P.A. values listed in Table 1. The open circles are the logarithmic spirals defined by the r_0 and b values over the range of angles listed in the table.

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REFERENCES

Arp, H. C. 1964, Ap, J., 139, 1045. Bosma, A. 1977, Ph.D. thesis, Groningen. Burke, B. F. 1957, A.J., 62, 90. Danver, G. C. 1942, Ann. Obs. Lund, No. 10. de Jager, G., and Davies, R. D. 1971, M.N.R.A.S., 153, 9. de Vaucouleurs, G. 1959, Ap, J., 130, 728. Freeman, K. C. 1970, Ap, J., 160, 811. Gerola, H., and Seiden, P. E. 1978, Ap, J., 223, 129. Gordon, K. I. 1971, Ap, J., 169, 235. Gordon, K. J. 1971, Ap. J., 169, 235. Groot, H. 1925, M.N.R.A.S., 85, 535. Gum, C. S., Kerr, F. J., and Westerhout, G. 1960, *M.N.R.A.S.*, 121, 132. Holmberg, E. 1950, Medd. Lund Astr. Obs., Ser. 2, No. 128.
Hubble, E. 1926, Ap. J., 63, 236.
Huchtmeier, W. 1973, Astr. Ap., 22, 91.
——. 1978, in IAU Symposium No. 77, Structure and Properties of Nearby Galaxies, ed. E. M. Berkhuijsen and R. Wielebinski (Dortheld & Dirich) (Dordrecht: Reidel), p. 191. Humphreys, R. H., and Sandage, A. 1978, Ann. Rep. Carnegie Inst., 77, 180.

——. 1980, in preparation. Kerr, F. J. 1957, A.J., 62, 93.

- -. 1969, Ann. Rev. Astr. Ap., 7, 39.

Lewis, B. M. 1968, Proc. Astr. Soc. Australia, 1, 104. Madore, B. F. 1979, Observatory, 98, 169. Newton, K., and Emerson, D. T. 1977, M.N.R.A.S., 181, 573. Reakes, M. L., and Newton, K. 1978, M.N.R.A.S., 185, 277. Reynolds, J. H. 1925, M.N.R.A.S., 85, 1014. Roberts, I. 1899, Photographs of Stars, Star-Clusters, and Nebulae, Vol. 2 (London: Universal Press) Roberts, I. 1899, Photographs of Stars, Star-Clusters, and Nebulae, Vol. 2 (London: Universal Press).
Roberts, M. S., and Whitehurst, R. N. 1975, Ap. J., 201, 327.
Rogstad, D. H., Lockhart, I. A., and Wright, M. C. H. 1974, Ap. J., 193, 309.
Rogstad, D. H., Wright, M. C. H., and Lockhart, I. A. 1976, Ap. J., 204, 703.
Sancisi, R. 1976, Astr. Ap., 53, 159.
——. 1977, in Topics in Interstellar Matter, ed. H. van Woerden (Dordrecht: Reidel), p. 225.
Sandage, A., and Johnson, H. L. 1974, Ap. J., 191, 63.

- Sandage, A., and Johnson, H. L. 1974, Ap. J., 191, 63. Tammann, G. A. 1975, Conference on Optical Observing Programs on Galactic Structure and Dynamics, ed. Th. Schmidt-Kaler (Bochum).
- Tully, R. B., and Fisher, J. R. 1977, Astr. Ap., 54, 661.

- Walker, M. F. 1961, J. K. 1977, Nach., 188, 249.
 Walker, M. F. 1964, A.J., 69, 744.
 Wright, M. C. H., Warner, P. J., and Baldwin, J. E. 1972, M.N.R.A.S., 155, 337.

ROBERTA M. HUMPHREYS: Department of Astronomy, University of Minnesota, Minneapolis, MN 55455

ALLAN SANDAGE: Hale Observatories, 813 Santa Barbara Street, Pasadena, CA 91101