THE ASTROPHYSICAL JOURNAL, 236:648–657, 1980 March 1

BARIUM STARS AND THE *s*-PROCESS

CHARLES R. COWLEY
Dominion Astrophysical Observatory; and University of Michigan
AND
PETER L. DOWNS
University of Michigan
Received 1979 June 11; accepted 1979 September 10

ABSTRACT

Theoretical *s*-process calculations are presented. Isotopic abundances are summed, so that atomic number Z becomes the independent variable. This form of representation of the calculations has distinct advantages in the interpretation of stellar data, since one generally does not have isotopic information. Predictions based on single neutron exposures as well as exponential exposure distributions are compared with the high-quality observations of Tech for ζ Cap and a few other stars. Tech's data provide a remarkably good fit of theory and observations. For the exposures of relevance to the barium stars, dysprosium and, indeed, heavier lanthanides are expected to be comparable in abundance to samarium and gadolinium. A very low Ce/Ba ratio ($\lesssim 10^{-1}$) is not expected if Ba and Sr have comparable abundances. It is concluded that the observations are not yet of high enough quality to allow one to discriminate among several possible exposure models, but that the prospect for doing so with improved observations is good.

Subject headings: nucleosynthesis — stars: Ba II

I. INTRODUCTION

According to standard ideas of nucleosynthesis, elements heavier than iron are synthesized in three distinct processes: the *r*-, the *s*-, and the *p*-process. Of these, the *s*-process is perhaps the best understood, from the point of view of both the nuclear physics and stellar evolution. The best observational data concerning the *s*-process are surely the solar system or standard abundances, for which information about isotopes is available. For the remainder of the universe, it is assumed, for lack of evidence to the contrary, that the solar system abundances may be considered representative (or cosmic).

Unlike many of the processes of nucleosynthesis, the *s*-process is presumed to take place in stable, or at least nonexploding, stars whose atmospheres would show the results of *s*-processing. Burbidge *et al.* (1957) summarized the observational basis for the conclusion that the S and Ba II stars exhibit *s*-processed material in their atmospheres. More recently, considerable attention has been given to FG Sge, whose time-variable spectrum has been interpreted in terms of *s*-processing followed by mixing to the surface.

Burbidge and Burbidge (1957), Warner (1964, 1965), and Danziger (1965) made detailed comparisons of their abundance distributions with those predicted by the theory of the *s*-process. Since that early work, stellar abundance workers have generally eschewed such detailed comparisons. An exception is the work of Butcher (1976), which we discuss briefly below. A number of factors discourage one from making a direct comparison of the theory of the *s*-process with stellar abundances. Of these, we mention only the fact that the theoretical predictions give $N\sigma$ as a function of the mass number A, while the stellar abundances are in the form of N as a function of atomic number Z.

In the *s*-process calculations of the present work, we sum the abundances of the individual isotopes and tabulate elemental abundances directly as a function of Z. This is done for the benefit of stellar abundance workers. Certainly, the loss of information on isotopes means that observational tests of the theory will be less rigorous than in the case of the solar system abundances. But stellar abundances may compensate for this loss by showing a *variety* of patterns.

Perhaps the best observational data for a Ba II star are those discussed by Tech (1971) for ζ Cap, the brightest of these objects. We shall compare Tech's results, as well as those for a few other Ba II stars, with theoretical predictions of *s*-process calculations. First, we give a brief discussion of the theory, and of the analysis of the stellar data, in order to clarify the procedure.

II. THEORETICAL PREDICTIONS

In principle, the *s*-process calculations are straightforward. Clayton's (1968) textbook covers all but the most recent developments. In practice, nontrivial problems arise because of uncertainties in the neutron capture cross sections. It is beyond the scope of the present paper to discuss these matters in detail. We have simply employed Newman's (1978) "adopted" cross sections, thereby making a standard assumption

that the canonical, 30 keV, thermal averaged cross sections which are apposite to the solar system abundances apply also to the Ba II stars. We must refer the reader to Newman's paper for a more detailed discussion. We mention here that the primary sources of experimental and theoretical cross sections are respectively Allen, Gibbons, and Macklin (1971), and Holms, Woosley, Fowler, and Zimmerman (1976).

Another matter that is of special relevance in the application of *s*-process theory to *stellar* data concerns the model assumed for the neutron exposures. Most theoretical calculations done to date have been undertaken in an attempt to fit the solar system *s*-process abundances, for which it has long been recognized that a *distribution* of exposures to neutrons is required. Recent work (e.g., Ulrich 1973; Truran and Iben 1977) has shown how a distribution of exposures can arise naturally as a result of helium shell flashes in massive, asymptotic-branch stars, but the relevance of these calculations to the Ba II stars is unclear (see Iben and Truran 1978).

Thus it cannot be ruled out that a single neutron exposure is relevant to the *s*-process abundances of the Ba II stars. Indeed, this assumption was made in the classical analyses of the Ba II stars by Warner (1965; see § III below) and is the basis for a rather detailed mixing model by Tomkin and Lambert (1979).

The variety of neutron exposure models of conceivable relevance to stellar abundances is infinite. However, the observations show a limited range of peculiarities, and for this reason we restrict ourselves to two models: First, we use exponential exposure distribution models, for which Clayton and Ward (1974) give an exact solution. Second, we use a single exposure model with the approximate solution introduced by Clayton *et al.* (1961). This so-called CFHZ approximation has been thoroughly investigated by Newman (1978), who finds errors of the order of 10% or less—negligible, for our purposes.

Butcher (1976) has considered "flat" exposure distributions, ranging from infinite widths to delta functions. Such distributions are, to some extent, bracketed by the kinds of solutions considered here.

Computer codes exist which treat the differential equations of the *s*-process in terms of a reaction network which is solved numerically (see Truran and Iben 1977). The major advantage of such codes is that they allow naturally for branching of the *s*-process path. It is also somewhat more straightforward to obtain solutions when a nonzero initial abundance distribution of heavy nuclides is assumed, although this problem may also be treated by an extension of the analytical procedures of Clayton (1968) and Clayton and Ward (1974).

The "exposure" to neutrons is measured by the quantity

$$\tau = \int_0^t nv dt ,$$

where n is the neutron number density, v an appropriately averaged relative velocity, and t is an interval of time. The parameter τ is dimensionally equal to a reciprocal area, and it is convenient to use the unit millibarn^{-1} (mb^{-1}).

For relatively high neutron exposures, τ of the order of a few tenths millibarns^{-1} or more, the results of the network calculations do not appear to differ significantly from those that may be obtained by analytic means. Since relatively high exposures are relevant for the Ba II stars, the most important distinction among the theoretical models is that of a single exposure versus an exposure distribution. A (decreasing) exponential exposure distribution will never make more cerium than strontium, for example, while this is entirely possible for a single exposure.

Several unstable isotopes that are involved in the *s*-process can decay to two different nuclides, giving rise to branching of the path. Only one such case deserves to be mentioned here, that of ^{152}Eu, which can decay to ^{152}Sm or ^{152}Gd. The difference in assuming one path or the other makes only a small change in the gadolinium abundance. The results reported here are for the "primary" path through ^{152}Sm, which is taken 73% of the time.

The (present) calculations assume an ^{56}Fe seed nucleus, with all other initial abundances set equal to zero, so it is relevant to consider the importance of heavy nuclei which may in reality be present prior to the *s*-process neutron exposure under consideration. For a single exposure τ, original nuclides decline in abundance by a factor $\exp(-\sigma\tau)$. If there is a distribution of exposures, $\rho(\tau) \sim \exp(-\tau/\tau_0)$, the original nuclides decline by a factor $1/(1 + \sigma\tau_0)$.

Perhaps the most relevant nuclides are ^{151}Eu and ^{153}Eu which, in the solar system abundances, are predominantly due to the *r*-process. It is well known that europium shows little or no enhancement in the Ba II stars, and it is usually assumed that the amounts of this element that are present are due to an admixture of material with "solar" composition. However, if all of the material in the stellar envelope had been subjected to a distribution of neutron exposures, the "solar" fraction of europium could have been substantially reduced because of the large neutron capture cross sections of both stable isotopes. If one then tried to subtract out the initial solar abundances from the present stellar abundances (see § III below), one could be left with a negative europium abundance. In fact, this may have happened in HD 46407 and HD 92626 (see Warner 1965, Table 9).

The weakness of Eu II in the barium stars, with the simultaneous strengthening of lines of Sr, Ba, La, Pr, Ce, etc., which all have small neutron capture cross sections, is a primitive but powerful argument that the peculiar abundances in these stars are due to the *s*-process. Warner (1965) and Danziger (1965) sought a more refined confirmation of this theory when they examined the *distribution* of abundances of elements, for indications of the influence of the $N = 50$ or $N = 82$ neutron shell closings. We shall see that those shell closings show a somewhat different aspect when Z rather than A is the independent variable.

TABLE 1

s-PROCESS SOLUTIONS

	(a) Clayton - Ward						(b) CFHZ					
Element	τ_0 = 0.08	0.10	0.20	0.30	0.40	0.60	τ = 10.80	0.90	1.00	1.10	1.20	1.40
FE	23.34	22.79	21.57	21.12	20.89	20.64	18.11	17.71	17.38	17.09	16.85	16.46
CO	22.00	21.53	20.48	20.10	19.89	19.68	17.60	17.19	16.84	16.54	16.28	15.84
NI	22.31	21.90	20.99	20.66	20.49	20.30	18.82	18.44	18.10	17.79	17.52	17.05
CU	21.53	21.17	20.37	20.08	19.92	19.76	18.53	18.16	17.82	17.52	17.24	16.75
ZN	21.43	21.14	20.52	20.29	20.17	20.04	19.27	18.94	18.65	18.38	18.13	17.67
GA	20.39	20.15	19.63	19.43	19.32	19.21	18.63	18.34	18.06	17.80	17.56	17.11
GE	20.87	20.67	20.22	20.05	19.96	19.87	19.45	19.18	18.93	18.70	18.47	18.04
AS	19.13	18.96	18.58	18.43	18.35	18.26	17.94	17.69	17.46	17.23	17.01	16.59
SE	20.45	20.32	20.01	19.89	19.83	19.76	19.55	19.34	19.14	18.94	18.74	18.34
BR	18.82	18.71	18.46	18.36	18.30	18.24	18.10	17.91	17.72	17.53	17.34	16.96
KR	19.99	19.92	19.74	19.66	19.62	19.57	19.51	19.37	19.21	19.05	18.89	18.55
RB	18.81	18.75	18.60	18.53	18.49	18.45	18.41	18.28	18.13	17.98	17.82	17.49
SR	20.00	20.00	20.00	20.00	20.00	20.00	20.00	20.00	20.00	20.00	20.00	20.00
Y	19.14	19.20	19.31	19.36	19.38	19.40	19.40	19.46	19.51	19.55	19.59	19.64
ZR	19.28	19.39	19.66	19.76	19.82	19.87	19.78	19.93	20.05	20.17	20.27	20.44
MO	18.03	18.22	18.64	18.80	18.88	18.97	18.76	18.98	19.17	19.34	19.48	19.72
TC	16.71	16.91	17.35	17.51	17.60	17.69	17.46	17.69	17.89	18.06	18.21	18.47
RU	17.68	17.90	18.36	18.53	18.62	18.71	18.46	18.70	18.91	19.09	19.25	19.51
RH	16.66	16.88	17.35	17.52	17.61	17.71	17.45	17.69	17.91	18.09	18.25	18.52
PD	17.75	17.98	18.48	18.66	18.75	18.85	18.56	18.82	19.05	19.24	19.41	19.69
AG	16.63	16.87	17.39	17.57	17.67	17.77	17.46	17.73	17.96	18.17	18.34	18.63
CD	17.63	17.87	18.41	18.61	18.71	18.81	18.48	18.76	19.00	19.21	19.40	19.70
IN	16.45	16.70	17.26	17.46	17.57	17.67	17.31	17.61	17.86	18.07	18.26	18.57
SN	17.88	18.17	18.79	19.01	19.12	19.24	18.79	19.12	19.40	19.65	19.86	20.21
SB	16.13	16.44	17.10	17.34	17.46	17.59	17.07	17.42	17.73	17.99	18.21	18.59
TE	17.41	17.74	18.44	18.69	18.82	18.95	18.57	18.74	19.07	19.34	19.58	19.98
I	16.02	16.36	17.08	17.34	17.47	17.61	16.99	17.38	17.71	18.00	18.25	18.66
XE	17.23	17.58	18.34	18.61	18.74	18.89	18.21	18.62	18.97	19.27	19.53	19.96
CS	15.92	16.28	17.06	17.34	17.48	17.63	16.91	17.33	17.69	18.00	18.27	18.72
BA	17.55	17.98	18.91	19.26	19.44	19.63	18.56	19.05	19.48	19.86	20.21	20.81
LA	16.39	16.87	17.91	18.29	18.49	18.69	17.41	17.95	18.43	18.85	19.23	19.89
CE	16.68	17.21	18.37	18.81	19.03	19.27	17.66	18.26	18.79	19.27	19.70	20.46
PR	15.60	16.13	17.31	17.75	17.98	18.22	16.57	17.18	17.72	18.20	18.64	19.41
ND	16.13	16.69	17.93	18.39	18.63	18.87	17.08	17.72	18.28	18.79	19.25	20.06
SM	15.40	15.98	17.27	17.74	17.99	18.24	16.32	16.99	17.58	18.10	18.58	19.42
EU	14.20	14.78	16.07	16.55	16.80	17.05	15.11	15.78	16.38	16.91	17.39	18.23
GD	15.12	15.72	17.02	17.49	17.74	18.00	16.03	16.71	17.31	17.84	18.33	19.17
TB	14.02	14.62	15.92	16.40	16.65	16.91	14.93	15.61	16.21	16.74	17.23	18.08
DY	15.28	15.88	17.20	17.68	17.94	18.20	16.17	16.86	17.47	18.01	18.50	19.36
HO	14.17	14.77	16.10	16.59	16.84	17.11	15.06	15.75	16.36	16.91	17.41	18.27
ER	15.07	15.68	17.02	17.51	17.77	18.04	15.96	16.66	17.27	17.83	18.32	19.20
TM	14.06	14.67	16.01	16.51	16.76	17.03	14.94	15.64	16.26	16.81	17.31	18.19
YB	15.23	15.86	17.21	17.71	17.97	18.24	16.10	16.81	17.44	18.00	18.51	19.39
LU	14.03	14.66	16.03	16.53	16.79	17.06	14.90	15.61	16.25	16.81	17.32	18.21
HF	15.07	15.70	17.08	17.59	17.85	18.12	15.92	16.64	17.28	17.85	18.37	19.27
TA	14.13	14.76	16.15	16.66	16.93	17.20	14.97	15.70	16.34	16.92	17.44	18.34
W	15.13	15.77	17.18	17.69	17.96	18.24	15.96	16.70	17.35	17.93	18.46	19.37
RE	13.87	14.52	15.93	16.45	16.72	17.00	14.69	15.44	16.09	16.68	17.21	18.13
OS	15.02	15.67	17.10	17.62	17.89	18.17	15.83	16.58	17.24	17.84	18.37	19.30
IR	13.72	14.38	15.82	16.34	16.61	16.90	14.52	15.28	15.95	16.55	17.08	18.02
PT	15.10	15.76	17.22	17.75	18.03	18.31	15.88	16.65	17.33	17.93	18.48	19.43
AU	14.15	14.83	16.30	16.83	17.11	17.40	14.92	15.70	16.39	17.00	17.55	18.51
HG	15.26	15.95	17.46	18.01	18.29	18.59	15.99	16.79	17.49	18.12	18.69	19.67

Table 1 gives the results of the theoretical calculations for several neutron exposure parameters τ (for CFHZ solutions) and τ_0 (for Clayton and Ward solutions). All solutions are normalized to log N(Sr) = 20.00. We point out the following salient features of these calculations:

1. For the exposure parameters most relevant to Ba II stars, Ce has an abundance only slightly less than Ba. Thus the "$N = 82$ shell-closing break" occurs *after* Ce.

2. For most exposures of interest, the elements beyond the $N = 82$ shell closing have comparable abundances (note especially Sm, Gd, Dy, and Er). The reasons for these features are readily understood in terms of the relevant neutron capture cross sections.

We list no entry for niobium. Our assumed path passes through ^{93}Zr, which has a half-life of 9.5×10^5 years, and bypasses ^{93}Nb (the only stable isotope) completely. Ultimately, the abundance of this element will increase when ^{93}Zr decays—it is a question of

how old the *s*-processed material is. In stars where Tc is present, one might expect a low abundance of Nb, but the effect would not be simple to establish observationally.

Butcher (1976) has given a detailed discussion of branching of the *s*-process path at ^{93}Zr and its observational consequences.

III. ANALYSIS OF THE STELLAR DATA

In the early work of the Burbidges (1957) it was assumed that the Ba II star abundances are the sum of (1) matter with solar abundances plus (2) matter that had been exposed to slow neutrons. The former material contained *r*-, *s*-, and *p*-nuclides; the latter, only *s*-nuclides.

Let S, R, and P stand for the ratios of the number densities of all solar system isotopes of a given element *relative to iron*, due to the *s*-, *r*-, and *p*-processes. Let S' be a similar ratio for the same element due to the "extra" *s*-processing in the star. Then the observed abundance excess y, relative to analogous quantities in the Sun, is

$$y = \frac{S + S' + R + P}{(S_\odot + R_\odot + P_\odot)} . \quad (1)$$

We note that $\log y = [\mathrm{M/Fe}]_\odot$, a notation commonly in use by abundance workers. The Burbidges also defined

$$y' = \frac{S + S'}{S_\odot} . \quad (2)$$

If we assume that the quantities S, R, and P are the *same* as those in the Sun, then it is easily seen that

$$y' = \frac{R_\odot + P_\odot}{S_\odot} (y - 1) + y . \quad (3)$$

It is then reasonable to consider the quantity

$$(y' - 1) = S'/S_\odot . \quad (4)$$

We have used the r/s ratios given by Seeger, Fowler, and Clayton (1965). All solar system abundances are from Cameron (1973).

Danziger (1965) used y' directly, while Warner (1965) studied the distribution of $(y' - 1)$. In most instances, the difference in the two procedures is masked by observational uncertainties ($y' \gtrsim 2$).

Butcher (1976) also considered only the newly synthesized *s*-process elements (indicated by a prime in our notation). However, his comparison of theory with observation involved ratios of averaged abundances of adjacent elements, e.g., $\langle \mathrm{SrYZr} \rangle / \langle \mathrm{BaLaCe} \rangle$. His methodology is therefore somewhat different from ours, although our conclusions are essentially the same in the domain where our techniques overlap.

One must at least consider more general assumptions that those which led to equation (1). For example, the *entire* material of the stellar envelope may have been subject to a distribution of neutron fluxes. In this case, abundances of iron group elements would be modified. The calculations of Lamb *et al.* (1977) are relevant. Their results show that the iron peak and scandium "low" can both be displaced toward larger Z.

It is important to note the depth of the minimum in solar abundances which occurs at scandium. Scandium has essentially the same "cosmic" abundances as the "*s*-process elements" strontium and zirconium, and neutron addition processes which affect the latter can surely change the Sc abundance. But there is exiguous evidence in the Ba II stars of departures of the iron group elements, including Sc, from a solar abundance pattern. We therefore assume that insofar as a distribution of exposures is relevant to heavy elements, it does not apply to the observed iron group elements. This is an important constraint on mixing models, as we shall see below.

IV. RESULTS FOR SELECTED STARS

a) ζ Capricorni

For eight elements, abundance ratios relative to iron were calculated using the method introduced by Cowley and Hartoog (1972). Equivalent widths were taken from Tech. Absolute *gf*-values for Fe I were taken from Corliss and Tech (1968, 1972) after application of the recommended correction. For the lanthanides, the corrections to the Corliss-Bozman (1962) tabulation were applied based on recent lifetime measurements by Arnesen *et al.* (1977), Andersen *et al.* (1975), and Lage and Whaling (1976).

Table 2 gives the results of these calculations along with Tech's results (last column) relative to the Sun (col. [2] of his Table 7.5). Cameron's (1973) "solar" abundances were assumed. It is easily seen that the second decimal is unjustified, and we drop it in subsequent tabulations.

We now compare the ζ Cap abundances with theoretical calculations. Table 3 gives the relevant data. The r to s ratio is based on Table 5 of Seeger, Fowler, and Clayton (1965), and we assume that $N(r + p) \approx N(r)$. Log y is obtained directly from Tech's Table 7.5, and from it y' is formed using equation (3). In order to remove the solar (s/Fe), we multiply y' and $y' - 1$ by Seeger's $N(s)$ and divide by 8.3×10^5,

TABLE 2

LOGARITHMIC RELATIVE ABUNDANCES FOR ζ CAPRICORNI

Element	No. Line Pairs	[E1/Fe] = log y Line Pairs	Tech
Y.....	2	1.09	1.06
Zr.....	1	0.54	0.86
Ba.....	1	1.87	1.38
La.....	14	1.21	0.93
Ce.....	17	1.34	0.98
Pr.....	2	0.88	0.88
Nd.....	12	1.47	0.97
Sm.....	1	0.49	0.82

TABLE 3

s-PROCESS DATA FOR ζ CAPRICORNI

Element	y	y'	log $[y'(s/\mathrm{Fe}_\odot)]$	log $[(y'-1)(s/\mathrm{Fe}_\odot)]$	log (N/Fe)
Sr	10.2	10.6	−3.5	−3.6	−3.5
Y	11.5	13.6	−4.2	−4.2	−4.2
Zr	7.2	8.2	−3.9	−3.9	−3.6
Nb	1.7	3.3	−5.9	−6.1	−5.5
Mo	4.9	17.2	−5.0	−5.0	−4.3
Ru	2.7	4.0	−5.4	−5.6	−5.2
Ba	24.0	28.2	−3.9	−3.9	−3.9
La	8.5	14.3	−5.4	−5.4	−5.3
Ce	9.6	11.4	−4.9	−5.0	−4.9
Pr	7.6	12.5	−5.9	−6.0	−5.9
Nd	9.3	13.5	−5.2	−5.2	−5.1
Sm	6.6	19.8	−5.8	−5.8	−5.7
Eu	1.3	4.0	−7.5	−7.6	−6.9
Gd	2.1	10.3	−6.3	−6.4	−6.1
Dy	6.2	43.6	−5.7	−5.7	−5.6

Cameron's (1973) iron abundance with Si = 10^6. Finally, we give log (N/Fe).

With the exception of the entries for Eu, the last three columns of Table 3 are essentially the same, and this is characteristic of most of the Ba II stars. In Figure 1, we compare the observations with Clayton-Ward predictions for $\tau_0 = 0.3$. The crosses and circles represent the data from the penultimate and ultimate columns, respectively, of Table 3.

In order to have some measure of the goodness of fits in these figures, we have computed the quantity

$$D = \sum_{i=1}^{N} (O_i + \delta - T_i)^2 . \qquad (5)$$

The quantities O_i and T_i represent observational results and theoretical predictions, while δ is chosen in such a way as to minimize D. Clearly

$$\delta = \frac{1}{N} \sum_{i=1}^{N} (T_i - O_i) . \qquad (6)$$

Table 4 gives the quantity D for the observations listed in columns (5) and (6) of Table 3, viz., log $(y'-1)(s/\mathrm{Fe})_\odot]$ and log (N/Fe). These data were compared with predictions using the Clayton-Ward and CFHZ solutions (see § II). All columns show distinct minima; the best fit criterion does not lead to multivalued solutions for an optimum τ or τ_0.

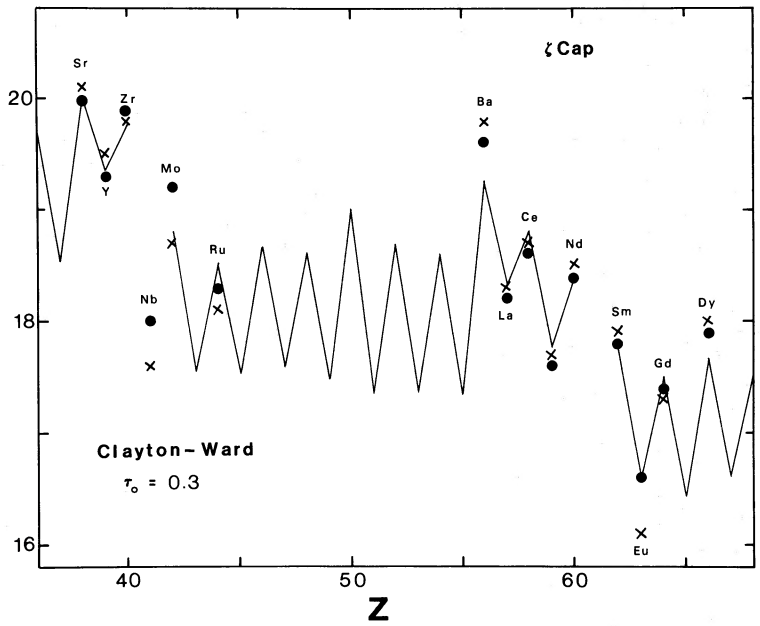

FIG. 1.—Tech's abundances for ζ Cap compared with the Clayton-Ward solution for an exponential distribution of neutron exposures with $\tau_0 = 0.3$ mb^{-1}. The ordinate for the filled circles is log (N/Fe) + 23.5, and for the crosses is log $(y'-1)(s/\mathrm{Fe})_\odot$ + 23.7. The solid line represents the theoretical calculations which have been normalized to 20.0 for strontium. See text for further explanation.

TABLE 4

GOODNESS OF s-PROCESS SOLUTION FITS

CLAYTON-WARD			CFHZ		
τ_0	log (N/Fe)	log $(y' - 1)(s/Fe)_\odot$	τ	log (N/Fe)	log $(y' - 1)(s/Fe)_\odot$
0.08	10.8	11.2	0.8	5.76	6.31
0.10	6.24	6.69	0.9	2.46	3.06
0.20	0.77	1.31	1.0	0.91	1.56
0.30	0.34	0.91	1.1	0.64	1.34
0.40	0.45	1.04	1.2	1.34	2.06
0.60	0.82	1.42	1.4	4.72	5.49

NOTE.—Entries are the parameter D of equation (5).

Let us for the moment, take the entries in Table 4 at their face values. The following conclusions would then be drawn:

1) The best fits are those with the Clayton-Ward solutions.

2) The best fits are the ones where the logarithm of the element-to-iron ratio is used; the standard algorithm (§ III) to correct for some material with cosmic abundances does not improve the fit.

These conclusions would follow naturally if the entire material in the envelope of ζ Cap had been subject to an exponential distribution of neutron exposures with parameter $\tau_0 \approx 0.3$. However, the abundances of iron group elements preclude this interpretation; the expected value of log (Sr/Fe) for $\tau_0 = 0.3$ is -1.2, for example, which is much larger than the observed value. Moreover, as we discussed earlier, we would expect the Sc low to be filled in, and there is no indication of this.

The simplest model compatible with conclusions (1) and (2) is that we see a mixture of (*a*) material with solar iron group elements but little or no heavy elements, and (*b*) a smaller amount of material which originally consisted of ^{56}Fe, or more realistically, iron peak elements, which experienced the *distribution* of exposures with parameter $\tau_0 \approx 0.3$. This model, however, is difficult to integrate with common notions of stellar structure and evolution. Where would matter with these ad hoc compositions come from, and how would the mixture get into the envelope of ζ Cap?

Let us then ask ourselves how *significant* the entries in Table 4 are. If we make certain assumptions about the uncertainties in the quantities $(O_i + \delta - T_i)$, a straightforward mathematical analysis is possible. The results of this analysis are detailed in the Appendix, where it is shown that the accuracy of the observations is not yet good enough to allow one to make an incisive choice among the theoretical calculations with the several methods outlined for treating the observations. However, this accuracy, which is illustrated—perhaps conservatively—in Table 2, is only slightly less than that which would support more interesting conclusions. The incentive for continued efforts on this problem is therefore high.

For the present, we draw the following conclusions from Table 4:

1) Satisfactory fits for the elements from Sr through Dy may be obtained using either the Clayton-Ward or the CFHZ solutions.

2) Allowance for an admixture of "solar" material does not improve the fits.

It seems wisest to conclude simply that the present analysis strongly supports the hypothesis that the peculiar heavy-element abundances in ζ Cap are due to *s*-processing. It is not yet possible to be more specific about the scenario under which the neutron addition took place.

Figure 1 shows that the dysprosium abundance found by Tech is in good accord with the theory of the *s*-process. Contrary to a commonly held notion, this element is expected to be present in the Ba II stars and with an abundance comparable to those of Sm or Gd. Figure 1 shows that a search for still heavier lanthanides would not be unrealistic. Indeed, Warner (1964) attributed a single line to Yb I in several of his stars.

b) HD 46407

This star was analyzed by the Burbidges (1957), Danziger (1965), and Warner (1965). We have adopted Warner's abundances, applying the correction for Pr estimated by Allen and Cowley (1974). We assumed that his standard star α Ind has solar abundances. For HD 46407 and the following stars, simple "by eye" fits were made since we did not judge the data to warrant optomized fits such as were used for ζ Cap. Figure 2 shows the fit of the stellar data to the CFHZ solution for $\tau = 1.1$. Values of log (E1/Fe) are shown as filled circles; crosses are for log $(y' - 1)(s/Fe)_\odot$. The results with a Clayton-Ward solution are of similar quality and will not be shown separately.

c) HR 774

This bright Ba II star was studied by Cowley (1968), Nishimura (1966), Pilachowski (1977), and most recently by Tomkin and Lambert (1979). According to the latter authors, the earlier analyses contain serious errors, possibly attributable to the modest spectral dispersion employed. We compare Cowley's results with Clayton-Ward prediction for $\tau_0 = 0.3$ in Figure 3. The points for Pr are from Nishimura, who used weaker Pr II lines in the green that are less sensitive to hyperfine structure. The Ba result is also from

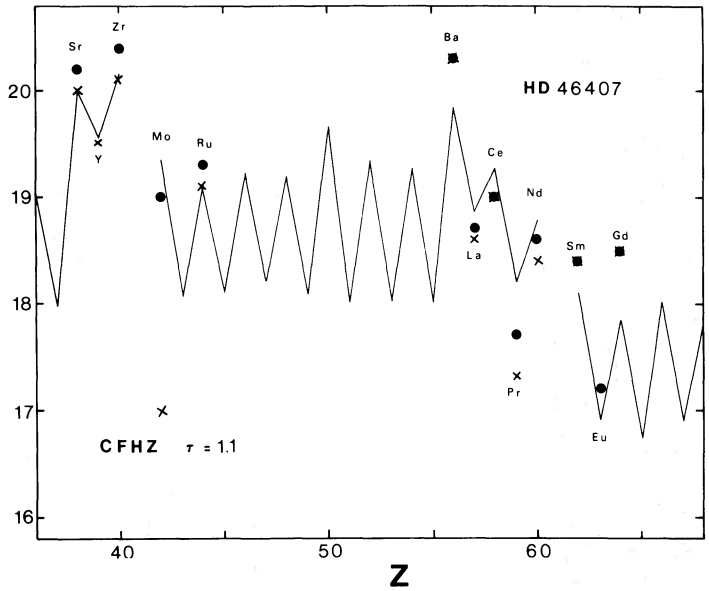

FIG. 2.—Warner's abundances for HD 46407 compared with the CFHZ solution for $\tau = 1.1$ mb^{-1}. The point for praseodymium was adjusted downward by 0.85 dex as recommended by Allen and Cowley (1974); a smaller correction is indicated. Ordinates for the filled circles are log $(N/\mathrm{Fe}) + 24.3$; for the crosses, log $(y' - 1)(s/\mathrm{Fe})_\odot + 24.4$.

Nishimura, since Cowley did not make a quantitative determination for this element.

These data appear to be in satisfactory agreement with the theoretical predictions. Tomkin and Lambert's abundances average a factor of 6 larger than those of Cowley. If a more or less uniform correction were to be applied to the abundances adopted here, the fit to the theoretical calculations would not be badly degraded. Quite a nice fit of Tomkin and Lambert's five points is also obtained with the CFHZ solution for $\tau = 1.2$ (see Fig. 4). It is clear that more detailed high-resolution work needs to be done on HR 774 and, indeed, on all Ba II stars, since virtually all previous (low resolution) work except that of Tech has been called into question by the results of Tomkin and Lambert.

d) γ Pavonis: A Poor Fit

It might be thought, certainly in view of the uncertainties surrounding HR 774, that any stellar abun-

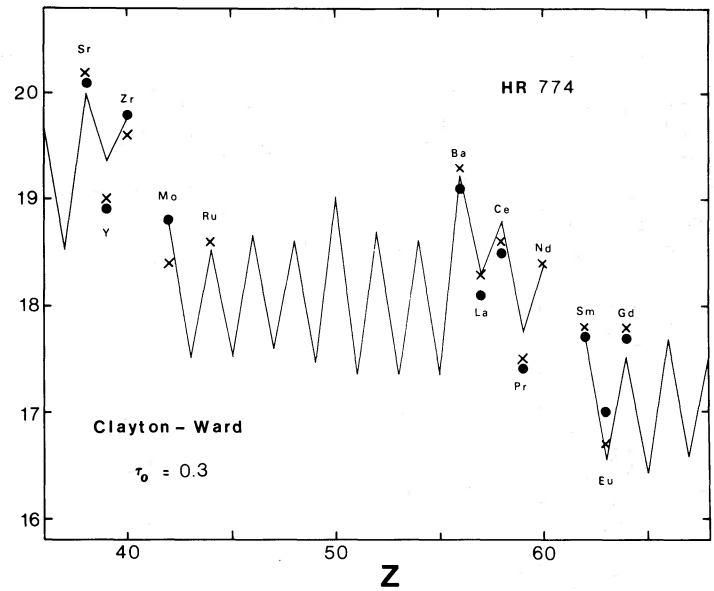

FIG. 3.—Cowley and Nishimura's abundances for HR 774 compared with the Clayton-Ward solution for $\tau_0 = 0.3$ mb^{-1}. Ordinates for the filled circles are log $(N/\mathrm{Fe}) + 23.8$; for the crosses, log $(y' - 1)(s/\mathrm{Fe})_\odot + 24.1$.

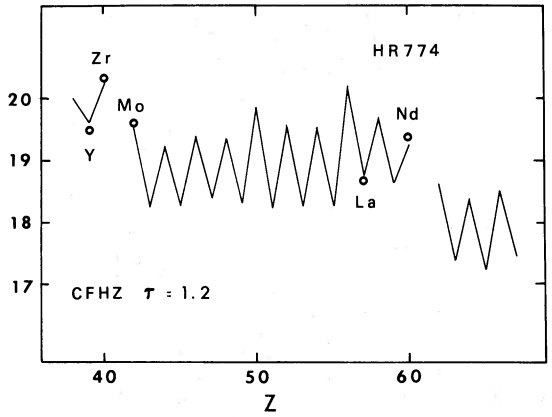

FIG. 4.—Tomkin and Lambert's recent abundances for HR 774 fit well with the CFHZ solution for $\tau = 1.2$. The abundances, however, are about a factor of 5 larger than those of Cowley and Nishimura. See discussion in text. Ordinates for the open circles are log (N/Fe) + 23.9.

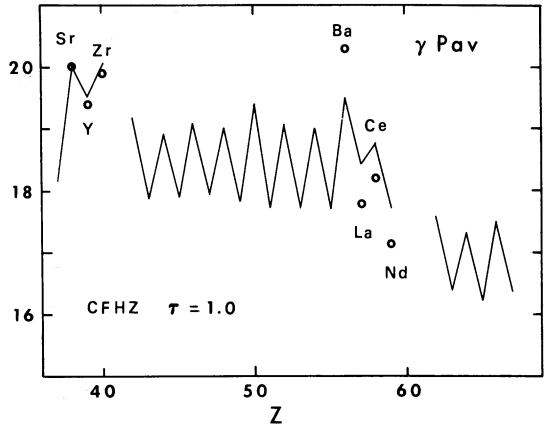

FIG. 5.—Danziger's abundances for γ Pav do not provide a close fit with any of the s-process solutions of this study. The abundance of Ba is too large with respect to those of the other elements plotted. The CFHZ, $\tau = 1.0$, solution is shown. Ordinates for the open circles are log (N/Fe) + 24.4.

dance might show a good fit with some theoretical s-process prediction. This is not the case, as may be demonstrated with Danziger's (1966) data for the subdwarf γ Pav in Figure 5. These particular results have been frequently cited as demonstrating that s-processing has influenced stellar abundances. While an interpretation is still possible in terms of the s-process with low neutron exposures, it is necessary to assume that the Ba to (Ce + La + Pr) ratio has been overestimated by roughly an order of magnitude.

We see here the advantage of employing the theory in the form that fits the observational data directly. According to the s-process theory, a discontinuity is expected near A = 138–140 (Ba, La, Ce), and it was inferred in early work that this discontinuity had been observed in γ Pav. However, the predictions are somewhat different when Z is taken as the independent variable. Because ^{140}Ce has a neutron capture cross section even smaller than that of ^{148}Ba, a large Ba/Ce ratio is *not* expected when the neutron exposures are such that Ba and Sr are comparable in abundance.

V. CONCLUSIONS

The overall conclusions to be drawn from the present work are that the theory of the s-process and its relationship to Ba II stars, as expounded by the Burbidges (1957), Warner (1965), Danziger (1965), and others, is well founded. Tech's (1971) high-resolution study of the brightest Ba II star ζ Cap leads to a very close fit with s-process theory. This fit includes the element dysprosium.

The accuracy of the observations is not yet good enough for this study to discriminate among exposure models. A single exposure or an exponential distribution of exposures provides comparable fits in the domain between strontium and the lanthanides. Exposure parameters τ of the order of 1.0 ± 0.1 mb^{-1} are appropriate for fits with CFHZ solutions. If we use the CFHZ (1961) calibration of the average number of neutrons captured n_c as a function of τ, we find values in the range 50–100, somewhat larger than previous estimates.

Abundance studies of the Ba II stars provide a close and fruitful link between observational astronomy, stellar evolution, and nuclear astrophysics. There is a great need for more high-quality observations which could enable us to fix parameters of a detailed model such as that proposed by Tomkin and Lambert (1979). The possibility of discovering details of internal stellar structure from observations of surficial abundances is an exciting prospect.

It is a pleasure to thank J. B. Blake and J. L. Tech for reading an early version of the present paper and offering a number of valuable comments. We thank K. E. Guire of the University of Michigan Statistical Research Laboratory for his advice with the data analysis. This research was supported by the National Science Foundation.

APPENDIX

GOODNESS OF FIT OF s-PROCESS SOLUTIONS

A straightforward criterion for the "goodness" of fits such as those illustrated in Figure 1 of the text may be derived, provided we are willing to make certain assumptions about the accuracy of the individual determinations.

The fits in Figure 1 were obtained by finding a number δ such that the sum of the squares of the deviations of the observed points from the theoretical predictions was minimized. Let O_i and T_i be observed and theoretical values. Define

$$x_i \equiv (O_i + \delta - T_i), \tag{A1}$$

and assume that the (probability) density functions for x_i are normal with standard deviation σ; the deviations of the logarithms are assumed to be Gaussian distributed. If, further, we assume the x_i's are independent of one another and (see below) that the σ's are all the same, then it is straightforward to show that the density function of

$$D \equiv \sum_{i=1}^{N} (O_i + \delta - T_i)^2 \tag{A2}$$

is given by

$$f(D) = \frac{1}{2^{N/2}\sigma^N \Gamma(N/2)} D^{(N-2)/2} \exp(-D/2\sigma^2), \quad D \geq 0. \tag{A3}$$

We omit details of this and other derivations which may be found in standard works on probability and statistics (see, e.g., Papoulis 1965, § 8-4). The probability that D will exceed some value D_1, is then

$$P(D \geq D_1) = \int_{D_1}^{\infty} f(y)dy. \tag{A4}$$

Using (A3) and changing the variable of integration to $w = D/\sigma^2$, it follows that $P(D > D_1)$ is the same integral that appears in χ^2 tests, with the lower limit of the integral, usually called χ^2_0, equal to D_1/σ^2.

Only one further point must be noted before we make use of standard tables. Because δ was chosen in such a way as to minimize D, the x_i's are not independent. Indeed,

$$(O_1 + \delta - T_1) = -\sum_{i=2}^{N} (O_i + \delta - T_i). \tag{A5}$$

One so-called "degree of freedom" (df) has been lost, and the appropriate tabular entries for us are at df = $N - 1$.

The expected value of D is

$$\langle D \rangle = \int_0^{\infty} yf(y)dy = (N - 1)\sigma^2 \tag{A6}$$

for $(N - 1)$ degrees of freedom.

We must now choose σ. Realistically, σ differs from element to element. It is straightforward to incorporate this into the analysis by redefining the x_i's as x_i/σ_i; we continue here with a single "representative" σ. For the ζ Cap abundances, Tech (1971) gives "estimated errors" for the elements ftom Sr to Dy of 0.2–0.5 dex. By equation (A6), the expected values of D would be (for 13 df) 0.52 and 3.25. These figures should be compared with the best-fit entries of Table 4, which range from 0.34 to 1.34.

The standard procedure in statistics is to adopt some "null hypothesis," and test the deviations from the predictions of that hypothesis to see if there is a basis for rejecting it. By analogy, our null hypotheses are that the best-fit theories correctly predict the abundances, apart from observational error. Certainly if we take a conservative position about the observational errors, $\sigma \sim 0.5$, there is no basis for rejecting any of the four combinations of two theories with two treatments of the observations, since $D = 13\sigma^2$ is greater than any of the computed best-fit entries.

TABLE 5

ACCURACY NECESSARY FOR REJECTION OF NULL HYPOTHESIS AT 99% CONFIDENCE

Null Hypothesis	Min y	τ or τ_0	$\sigma = (\min D/27.69)^{1/2}$
Clayton-Ward (N/Fe)	0.34	0.3	0.11
Clayton-Ward ($y' - 1$) $(s/\mathrm{Fe})_\odot$	0.91	0.3	0.18
CFHZ (N/Fe)	0.64	1.1	0.15
CFHZ ($y' - 1$) $(s/\mathrm{Fe})_\odot$	1.34	1.1	0.22

NOTE.—Table 5 shows that we have good prospects for improving our techniques to the point where conclusions of considerable interest might be drawn. Even with the present observations one might argue that the fourth null hypothesis is excluded, since the errors of Tech's careful *differential* analysis may already be closer to 0.2 dex than the 0.3 dex indicated by Table 2 of our text.

It is important to know *how* good the observations must be in order for us to draw some incisive conclusions. We ask: What must σ be in order for us to reject one of the hypotheses at some "confidence level"? This question may be readily answered with a standard χ^2 table. We list results in Table 5 for the four "null hypotheses" leading to minimum values of y in Table 4. Entries in the final column of Table 5 are the rejection σ's at 99% confidence.

REFERENCES

Allen, M. S., and Cowley, C. R. 1974, *Ap. J.*, **190**, 601.
Allen, B. J., Gibbons, J. H., and Macklin, R. L. 1971, *Advances in Nuclear Physics*, ed. M. Baranger and E. Vogt (New York: Plenum), Vol. **4**, p. 205.
Andersen, T., Poulsen, O., Ramanujam, P. S., and Petkov, A. P. 1975, *Solar Phys.*, **44**, 257.
Arnesen, A., Bengtsson, A., Hallen, R., Lindskog, J., Nordling, C., and Noreland, T. 1977, *Phys. Scripta*, **16**, 31.
Burbidge, E. M., and Burbidge, G. R. 1957, *Ap. J.*, **126**, 357.
Burbidge, E. M., Burbidge, G. R., Fowler, W. A., and Hoyle, F. 1957, *Rev. Mod. Phys.*, **29**, 547.
Butcher, H. R. 1976, *Ap. J.*, **210**, 489.
Cameron, A. G. W. 1973, *Space Sci. Rev.*, **15**, 121.
Clayton, D. D. 1968, *Principles of Stellar Evolution and Nucleosynthesis* (New York: McGraw-Hill).
Clayton, D. D., Fowler, W. A., Hull, T. E., and Zimmerman, B. A. 1961, *Ann. Phys.*, **12**, 331 (CFHZ).
Clayton, D. D., and Ward, R. A. 1974, *Ap. J.*, **193**, 397.
Corliss, C. H., and Bozman, W. R. 1962, *NBS Monog.*, No. 53.
Corliss, C. H., and Tech, J. L. 1968, *NBS Monog.*, No. 108.
———. 1976, *J. Res. NBS*, **80A**, 787.
Cowley, C. R. 1968, *Ap. J.*, **153**, 169.
Cowley, C. R., and Hartoog, M. R. 1972, *Ap. J.* (*Letters*), **178**, L9.
Danziger, I. J. 1965, *M.N.R.A.S.*, **131**, 51.
Danzinger, I. J. 1966, *Ap. J.*, **143**, 527.
Holmes, J. A., Woosley, S. E., Fowler, W. A., and Zimmerman, B. A. 1976, *At. Data Nucl. Data Tables*, **18**, 305.
Iben, I., and Truran, J. W. 1978, *Ap. J.*, **220**, 980.
Lamb, S. A., Howard, W. M., Truran, J. W., and Iben, I. 1977, *Ap. J.*, **217**, 213.
Lage, C. S., and Whaling, W. 1976, *J. Quant. Spectrosc. Rad. Transf.*, **16**, 537.
Newman, M. J. 1978, *Ap. J.*, **219**, 676.
Nishimura, S. 1966, in *Colloquium on Late-Type Stars*, ed. Margherita Hack (Trieste: Osservatorio Astronomico di Trieste), p. 125.
Papoulis, A. 1965, *Probability, Random Variables, and Stochastic Processes* (New York: McGraw-Hill).
Pilachowski, C. A. 1977, *Astr. Ap.*, **54**, 465.
Seeger, P. A., Fowler, W. A., and Clayton, D. D. 1965, *Ap. J. Suppl.*, **11**, 121.
Tech, J. L. 1971, *NBS Monog.*, No. 119.
Tomkin, J., and Lambert, D. 1979, *Ap. J.*, **227**, 209.
Truran, J. W., and Iben, I. 1977, *Ap. J.*, **216**, 797.
Ulrich, R. K. 1973, in *Explosive Nucleosynthesis*, ed. D. N. Schramm and W. D. Arnett (Austin: University of Texas Press), p. 139.
Warner, B. 1964, *Comm. Univ. London Obs.*, No. 65.
———. 1965, *M.N.R.A.S.*, **129**, 263.

CHARLES L. COWLEY and PETER L. DOWNS: Department of Astronomy, University of Michigan, Ann Arbor, MI 48109