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OUTER ATMOSPHERES OF COOL STARS. IV. A DISCUSSION OF COOL STELLAR WIND MODELS

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ABSTRACT

We investigate possible wind models for late-type stars which appear not to have hot coronae and transition regions. Taking Arcturus as our prototypical star, we consider wind models with $T \lesssim 20{,}000$ K and search for solutions with mass loss rates $\sim 10^{-9}\ M_\odot\ \mathrm{yr}^{-1}$. Thermally driven models which are spherically symmetric or include widely diverging geometries predict mass loss rates orders of magnitude less than $10^{-9}\ M_\odot\ \mathrm{yr}^{-1}$. We find that the radiation pressure of Lα resonance scattering can exceed the force of gravity in the chromosphere and initiate a net outflow, but it is insufficient to sustain a wind. If an additional momentum input term, such as Alfvén wave pressure, is also present, then Lα radiation pressure may play a crucial role in turning on strong winds in stars like Arcturus by producing a critical point in the chromosphere and a locally supersonic flow which the additional mechanism can further accelerate. We conclude that Lα radiation-pressure-initiated winds can occur in stars to the right of the Linsky-Haisch dividing line in the H-R diagram between stars with and without transition regions and presumably hot coronae, and that the existence of these winds may explain energetically the absence of hot coronae in these stars.

Subject headings: stars: chromospheres — stars: coronae — stars: late-type — stars: mass loss — stars: winds

I. INTRODUCTION

An important result of the first survey of late type stars by the *International Ultraviolet Explorer* (*IUE*) is the discovery by Linsky and Haisch (1979; hereafter called Paper I) that there exists a sharp division in the H-R diagram between stars which have outer atmospheres similar to the Sun and those which do not. The short-wavelength (1175–2000 Å) spectra of the transition region star group (called solar type in Paper I) contain chromospheric emission lines (Lα, C I, O I, Si II) indicative of 5000–10,000 K plasma as well as emission lines of He II, C II–IV, Si III–IV, N V indicative of 20,000–250,000 K plasma analogous to the solar transition region. The non-transition region star group (called nonsolar type in Paper I) has spectra containing the chromospheric lines, but none of the hotter transition region lines.[3] This discovery is especially interesting in light of the observational evidence in the visible spectrum for extensive mass loss from G and K giants and supergiants presented by Reimers (1977*a*) and by Stencel (1978). Their results also indicate a division into two fairly distinct groups in about the same place in the H-R diagram, a division characterized by the onset of large stellar winds.

The observations of Reimers (1977*a*) and Stencel (1978) were used by Mullan (1978) to derive a "supersonic transition locus" (STL) in the H-R diagram. In Mullan's picture, stellar wind flows are expected to become supersonic at the base of the corona along a locus of points in the H-R diagram corresponding to the observed mass-loss division. In the Sun, the "critical point," R_c, where the mass flow passes from subsonic to supersonic velocities is well above the solar surface at $R_c \approx 4\ R_\odot$. The location of this point depends on the temperature, density, and velocity structure of the outermost regions of a stellar envelope. Mullan combines coronal base pressures derived from the chromospheric models of Kelch *et al.* (1978) with the minimum flux corona theory of Hearn (1975) to derive R_c/R_* for G, K, and M stars. He then argues qualitatively that when this critical point approaches the stellar surface at some point in a star's evolution, stellar spicules, prominences, convective cells, flare material, and the like will well up into the region of supersonic wind flow and thus bring about large and discontinuous mass loss.

Mullan's STL is also in rough agreement with the dividing line between transition region star spectra and non-transition region star spectra in the H-R diagram proposed in Paper I. A plausible physical connection between these two ideas is that when a strong wind develops, it dominates the energy balance such that the major

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[3] We are adopting this change of terminology to avoid ambiguity and to explicitly state the qualitative difference in the *IUE* spectra of these stars.

portion of the available nonradiative heating above the chromosphere goes into driving the mass flow rather than into heating a transition region and corona. Thus late type stars either have a chromosphere, transition region, and hot corona, or they have a chromosphere and a large wind. Although this simple picture is appealing, there are potential problems. The minimum flux corona theory that forms the basis of Mullan's arguments incorporates various simplifications and approximations which may not be valid, especially in the limiting case used by Mullan (see Antiokos and Underwood 1978; Vaiana and Rosner 1978, and Mullan 1978 for discussion of the pros and cons of the minimum flux corona theory debate). Another potential difficulty is that one cannot simply push material through a critical point. A proper matching of densities, temperatures, and velocities is required to get through the "throat of the nozzle" (see Brandt 1970, pp. 72–75), and the addition of new material into the existing steady-state flow would change the temperature and velocity structure and upset the balance that defines the critical point in the first place. If Cannon and Thomas's (1977) argument that the nozzle is imperfect is correct, then this difficulty may not be real.

We have investigated in detail various possible stellar wind models for one of the *IUE* non–transition region stars, Arcturus (α Boo, K2 IIIp), which has been observed to have a variable chromospheric outflow (Chiu *et al.* 1977; van der Hucht *et al.* 1979) and which has a chromospheric model (Ayres and Linsky 1975; Haisch *et al.* 1977) that can be used as a starting point for stellar wind calculations. Our first models attempted to take into account the energy balance of mass flux, conductive flux, radiative cooling, and various forms of mechanical energy dissipation, but these models were found to have high-temperature coronae ($T > 3 \times 10^6$ K). Other models having various prespecified temperature distributions with $T_{\max} \leq 20{,}000$ K yielded analytical solutions for the mass flow based on the mass and momentum conservation equations alone which resulted in very low wind velocities near the stellar surface and very low mass loss rates. Models including divergence of the wind flow also failed by several orders of magnitude to account for the outflow rates sometimes seen in Arcturus by Chiu *et al.* (1977) of near sonic velocities in the upper chromosphere. These cool thermally driven models are discussed in §§ II and III. We conclude, therefore, that cool stellar winds which become supersonic in or near the chromosphere require a source of momentum input to the mass flow.

Radiation pressure on dust grains has been suggested as a possible source of momentum input in M supergiants (Jennings 1973; Kwok 1975; Hagan 1978), but K giants and supergiants do not usually exhibit infrared excesses or polarization indicative of dusty circumstellar envelopes. We decided to look elsewhere for an adequate source of radiation pressure on the gas, and were surprised to discover that the scattering of Lα photons in an optically thin envelope could provide a large force locally comparable to gravity.[4] A closer examination of a detailed chromospheric model for the Lα flux indicates that the radiation pressure gradient exceeds gravity by as much as a factor of 4 in the upper chromosphere, and provides a significant "push" at line-center optical depths greater than $\tau = 200$. Although locally large, the force due to the Lα flux is only significant over a limited region in the upper chromosphere, and thus by itself could not provide sufficient momentum to drive a wind. The nature of any stellar wind, however, must be strongly affected by the presence of the steep radiative pressure gradient, and the interaction of this effect with other sources of momentum input, such as Alfvén waves or acoustic waves, may bring about a multiple critical point topology possibly involving a stationary shock front. Thus the presence of a strong Lα force, although insufficient by itself to bring about a large mass loss, could be viewed as a sufficient condition for a stellar wind driven primarily by some other mechanism or mechanisms, since even a small outflow at some point in the atmosphere by continuity requires an outflow everywhere. This in turn may be possible only in conjunction with a much larger stellar wind. In the following sections we examine in detail the Lα flux radiation pressure, the Alfvén wave pressure, and the formation of a stationary shock in a possible cool wind model for Arcturus. Our results should be viewed as an exploratory attempt to delimit some of the constraints on cool wind flows and to present some schematic models for cool winds. Finally, we extrapolate conditions in Arcturus to other G, K, and M stars to delineate the region of the H-R diagram in which cool winds may be important.

II. RADIAL EVAPORATIVE WIND MODELS

We chose Arcturus as our prototype model for cool stellar winds in late type stars for several reasons: the star lies just to the right of the Linsky-Haisch division in the H-R diagram, it has been observed to have a variable chromospheric outflow on the order of 10–20 km s^{-1}, and it has a well modeled chromosphere. It is much easier to evaluate the importance of various terms in the flow equations when the stellar parameters are well specified, and only a detailed model atmosphere can provide the Lα fluxes needed for an examination of the radiative force in the upper chromosphere.

Throughout this paper we assume a maximum temperature $T_{\max} \approx 20{,}000$ K in the chromosphere or extended wind region, and we take as fundamental parameters $R_* = 1.9 \times 10^{12}$ cm (27.3 $R_\odot$), and $g = 50$ cm s^{-2} (Ayres and Linsky 1975). We extend the Ayres-Linsky model chromosphere to include a 20,000 K plateau. In this model $R(T = 20{,}000 \text{ K}) = 2.017 \times 10^{12}$ cm (1.06 R_*), $n(T = 20{,}000 \text{ K}) = 2 \times 10^8$ cm^{-3}. More than 98% of the

[4] We subsequently found that Lα radiation pressure had been suggested by Wilson (1959) as a possible mechanism for mass ejection in M stars.

hydrogen is ionized at this point, and thus $n_p \approx n_e \approx n$. We assume a mean particle weight $\mu = 0.7$, and for simplicity of notation we let $2\mu m_H = m = 2.33 \times 10^{-24}$ g, i.e., $\rho = nm$.[5]

We now ask the question, "How much radial wind flow can be generated by the thermal energy of the gas alone, with and without possible temperature gradients?" The equations of mass and momentum conservation are

$$nvr^2 = n_0 v_0 r_0^2 \,, \tag{1}$$

$$v\frac{dv}{dr} = \left[\frac{4kT(r)}{r} - \frac{GMm}{r^2} - 2k\frac{dT}{dr}\right] \Big/ \left[m - \frac{2kT(r)}{v^2}\right] . \tag{2}$$

We have used the perfect gas law, $P = 2nkT$, to replace the pressure gradient, and for simplicity bypass energy balance considerations by specifying *a priori* a temperature structure.

a) $T = 20{,}000$ K *Isothermal Model*

When we assume an isothermal 20,000 K outflow as the simplest of all possible models, (2) can readily be integrated to yield

$$\tfrac{1}{2}m(v_2^2 - v_1^2) - 2kT(\ln v_2 - \ln v_1) = 4kT(\ln r_2 - \ln r_1) + GMm\left(\frac{1}{r_2} - \frac{1}{r_1}\right) . \tag{3}$$

The critical point for a transonic flow occurs where the numerator and denominator of (2) simultaneously equal zero, and thus

$$R_c = \frac{GMm}{4kT} \,, \tag{4}$$

$$v_c = v(R_c) = \left(\frac{2kT}{m}\right)^{1/2} . \tag{5}$$

We find $R_c = 3.8 \times 10^{13}$ cm ($20\,R_*$) and $v_c = 15.4$ km s^{-1}. A thermally driven wind solution is possible at this temperature. However, letting $r_2 = R_c$, $v_2 = v_c$, we find from (3) that at the base of the plateau $v_1 = 9 \times 10^{-8}$ cm s^{-1} for a mass loss of $3 \times 10^{-23}\,M_\odot$ yr^{-1}. This is clearly not the sort of wind we have in mind.

b) $r_c \approx R_*$, $T_c = 20{,}000$ K, $dT/dr < 0$

The extreme assumption that the critical point is at the top of the chromosphere is consistent with the observations of Chiu *et al.* (1977) that large outflow velocities occur in the upper chromosphere. The critical point can be made to lie at $R_c = 2.017 \times 10^{12}$ cm by simply choosing $dT/dr|_{R_c} = -3.6 \times 10^{-7}$ K cm^{-1}. Above the critical point the denominator of (2) is positive for a transonic solution. Requiring the numerator to also be positive imposes the condition

$$\frac{4kT(r)}{r} - \frac{GMm}{r^2} - 2k\frac{dT}{dr} \geq 0 \,, \tag{6}$$

from which we find for $r \geq R_c$

$$T(r) \leq \frac{GMm}{6k}\left(\frac{1}{r} - \frac{r^2}{R_c^3}\right) + \frac{r^2}{R_c^2}T_c \,. \tag{7}$$

We find that $T(r) \to 0$ at a radius no larger than 2.073×10^{13} cm ($1.09\,R_*$). Furthermore, for v = constant as implied by (6), the temperature gradient at R_c for adiabatic expansion is only -1.4×10^{-8} K cm^{-1}, compared to $dT/dr = -3.6 \times 10^{-7}$ which was chosen to define the critical point; thus the requirement on dT/dr necessitates some cooling mechanism such as line emission.

c) 20,000 K *Plateau with* $T(r) \to 0$ *as* $r \to \infty$

If we set $T(\infty) = 0$, and require the critical point to be at some R_c to be determined by (7), we may then let $T = 20{,}000$ K for $r \leq R_c$. Rewriting (7), we find

$$T(r) = r^2\left(\frac{T_c}{R_c^2} - \frac{GMm}{6kR_c^3}\right) + \frac{GMm}{6kr} . \tag{8}$$

[5] The following discussion of various thermally driven models in this section, and of thermally driven plus geometrically divergent models in § III, will demonstrate that no acceptable temperature structure or nonradial flow can bring about the sort of stellar wind flow that we hope to model, i.e., $\sim 10^{-9}\,M_\odot$. A reader familiar with solar wind theory could proceed immediately to § IV at this point. The models in this section and the following one provide a step-by-step clarification of the various solutions of the stellar wind equation of motion and serve as a basis for the introduction of radiation pressure and Alfvén wave pressure in the subsequent discussion.

Thus $T(r) \to 0$ as $r \to \infty$ when

$$R_c = \frac{GMm}{6kT_c}; \tag{9}$$

and for $T_c = 20{,}000$ K, $R_c = 2.55 \times 10^{13}$ cm (13.4 R_*). An isothermal plateau from r_1 to R_c yields $v_1 = 1.0 \times 10^{-7}$ cm s^{-1} from (3) for a mass loss rate of 4×10^{-23} $M_\odot$ yr^{-1}.

d) 20,000 K *Plateau with* $T(R) \to 0$, R *Determined by Interstellar Pressure Requirements*

The pressure in a low-temperature supersonic wind at large distances from the star is determined by the balance with interstellar pressure through the impact pressure of matter flow. We thus have the condition

$$mn(R)v^2(R) \approx p_i\,, \tag{10}$$

where $p_i \leq 10^{-11}$ dyn cm^{-2} (Mihalas 1978, p. 533). We assume $T(r)$ from (7) and we look for solutions having an impact pressure $\sim p_i$ as $T \to 0$. Condition (7) implies $dv/dr = 0$ for $r > R_c$ and thus $v(R) = v_c$. Using the mass conservation requirement (1) to replace $n(R)$, we may rewrite (10) as

$$mn_0 v_0 r_0^2 v_c / R^2 \approx p_i\,. \tag{11}$$

Equation (7) yields the condition at R

$$R_c R^2 T_c - R^2 \frac{GMm}{6k} + R_c^3 \frac{GMm}{6kR} = 0\,. \tag{12}$$

Now for a given R and $T_c = 20{,}000$ K, we find R_c from (12). Assuming a 20,000 K plateau below R_c, we use the isothermal solution for v and (3) to find v_0. We then check condition (11) to determine that the impact pressure $\sim p_i$ at R to match the transition to the interstellar medium as $T \to 0$.

For R in the range $3.68 \times 10^{12} \leq R \leq 4.24 \times 10^{12}$ cm, R_c is found by (12) to be $3.5 \times 10^{12} \leq R_c \leq 4.0 \times 10^{12}$ cm, and v_0 is thus $0.30 \geq v_0 \geq 0.025$ cm s^{-1}. The impact pressure, P, at R is in the range $6.5 \times 10^{-11} \geq P \geq 4.0 \times 10^{-12}$ dyn cm^{-2}. We still derive very small flow velocities at the base in the upper chromosphere and mass loss rates less than 1.1×10^{-16} $M_\odot$ yr^{-1}.

III. WIND FLOWS WITH DIVERGING GEOMETRIES FOR $T_{\rm MAX} = 20{,}000$ K

It is probable that Arcturus, like the Sun, has an inhomogeneous chromosphere such that mass loss involves inhomogeneous flows. The possible existence of giant convective cells has been discussed by Schwarzschild (1975) and by Chiu *et al.* (1977) in connection with their observations that the Ca II K line profiles are variable. An excellent discussion of the effects of diverging geometries on solar and stellar winds is given by Holzer (1977). Assuming, as before, various given temperature structures with the constraint that $T_{\rm max} \leq 20{,}000$ K, we investigate the effect of divergence in the flow by assuming that the mass and momentum conservation equations may be written as

$$nvA = n_0 v_0 A_0\,, \tag{13}$$

$$v\frac{dv}{dr} = \left[\frac{2kT(r)}{A(r)}\frac{dA}{dr} - \frac{GMm}{r^2} - 2k\frac{dT}{dr}\right] \Big/ \left(m - \frac{2kT}{v^2}\right)\,. \tag{14}$$

The flow takes the form of diverging cones in this first-order approximation. We parametrize the cross section $A(r)$ of the flow as

$$A(r) = r^2 w(r)\,, \tag{15}$$

where the expansion factor $w(r)$ is allowed to have various assumed functional forms, and $w(r_0) = 1$, thus normalizing the flow to a rate per steradian.

a) $T = 20{,}000$ K *for* $r < R_c$

For any isothermal region (14) may be integrated to yield an expression analogous to (3),

$$\tfrac{1}{2}m(v_2^2 - v_1^2) - 2kT(\ln v_2 - \ln v_1) = 2kT[\ln A(r_2) - \ln A(r_1)] + GMm\left(\frac{1}{r_2} - \frac{1}{r_1}\right)\,. \tag{16}$$

When we assume a linear expansion factor

$$w(r) = 1 + (r - r_0)a\,, \tag{17}$$

the location of the critical point may be found, as before, by letting the numerator and denominator of (14) equal zero. The expansion velocities at the base, r_0, may then be found from (16). The critical points, R_c, for various values of a are given by

$$(6kTa)R_c^2 + [4kT(1 - ar_0) - GMma]R_c - GMm(1 - ar_0) = 0\,. \tag{18}$$

For a in the range $0 \leq a \leq 1.0 \times 10^{-12}$, the corresponding values of the expansion factor are $1 \leq w(R_c) \leq 24.1$. The case $w(R_c) = 1$ corresponds exactly to purely radial flow (see § II*a*), and the corresponding critical points and velocities are $R_c = 3.8 \times 10^{13}$ cm and $v_0 = 9.0 \times 10^{-8}$ cm s^{-1}. For diverging flows with $a > 0$ in the range specified above, the numerator of (14) is negative as is the denominator, and the flow velocities increase monotonically to $v_c = 15.4$ km s^{-1}. Some representative values of R_c and v_0 for various expansion factors are: $w(R_c) = 2.11$, $R_c = 2.98 \times 10^{13}$ cm, $v_0 = 2 \times 10^{-7}$ cm s^{-1}; $w(R_c) = 8.62$, $R_c = 2.584 \times 10^{13}$ cm, $v_0 = 9 \times 10^{-7}$ cm s^{-1}; $w(R_c) = 24.1$, $R_c = 2.153 \times 10^{13}$ cm, $v_0 = 3 \times 10^{-6}$ cm s^{-1}. Thus by allowing the flow to diverge in a constant $T = 20{,}000$ K plateau, the initial velocities can be increased over that in a radial flow, but only to values much less than 1 cm s^{-1}. Corresponding mass loss rates now, of course, depend on the actual area at the stellar surface in which the flow occurs, but obviously $w(r_0) < 4\pi/w(R_c)$, so that any highly divergent flow corresponds to a small area at the stellar surface.

b) $R_c \approx R_*$, $T = 20{,}000$ K

It is now possible to have a constant temperature flow with the critical point at the plateau (see § II*b*) due to the additional freedom allowed by varying the cross section of the flow. For a supersonic flow, the numerator of (14) must be greater than or equal to 0. The limiting case,

$$\frac{4kT}{r} + \frac{2kT}{w(r)}\frac{dw}{dr} - \frac{GMm}{r^2} \geq 0\,, \tag{19}$$

integrates to

$$\ln w(r) - \ln w(r_0) \geq -2(\ln r - \ln r_0) - \frac{GMm}{2kT}\left(\frac{1}{r} - \frac{1}{r_0}\right). \tag{20}$$

The solution of this equation for $r \geq R_c$ shows a very greatly diverging flow, $w(2R_*) \geq 5.4 \times 10^6$, and is thus unrealistic.

c) $R_c \approx R_*$, $dT/dr < 0$, $w(r) > 1$

A negative temperature gradient makes a supersonic flow possible with a less steeply diverging geometry. If we again impose the requirement that the numerator of (14) be greater than or equal to 0, and have a linearly increasing $w(r)$,

$$\frac{4kT}{r} + \frac{2kTa}{b + ar} - \frac{GMm}{r^2} - 2k\frac{dT}{dr} \geq 0\,, \tag{21}$$

where $b = 1 - ar_0$. A change of variable to $T' = T/r^2(b + ar)$ allows (21) to be rewritten as

$$\frac{dT'}{dr} \leq -\frac{GMm}{2k}\frac{1}{r^4(b + ar)}\,, \tag{22}$$

which may be integrated to yield, for $r \geq R_c$,

$$T(r) \leq \frac{r^2(b + ar)}{r_0^2(b + ar_0)}T_0 - r^2(b + ar)\frac{GMm}{2k}\left\{-\frac{1}{b^4}\left[\frac{(b + ar)^3}{3r^3} - \frac{3a(b + ar)^2}{2r^2} + \frac{3a^2(b + ar)}{r} - a^3\ln\left|\frac{b + ar}{r}\right|\right]\right\}_{r_0}^{r}. \tag{23}$$

All reasonable values of a show $T \rightarrow 0$ for $r < 1.1\,R_*$.

d) $R_c \approx R_*$, $T(r)$ *Specified, Variable* $w(r)$

If we allow the temperature to have a prespecified r dependence such that $T(r) \rightarrow 0$ as $r \rightarrow \infty$, (21) may be solved for $w(r)$. Integrating (21), we find

$$\ln w(r) - \ln w(r_0) \geq -2(\ln r - \ln r_0) + \ln(T - T_0) + \frac{GMm}{2k}\int_{r_0}^{r}\frac{dr}{r^2T(r)}\,. \tag{24}$$

For $T(r) = (r_0/r)T$ the last term in (24) is $(GMm/2k)[(\ln r - \ln r_0)/T_0 r_0]$, and we find that $w(r) \geq 3.5$ at $r = 1.1\,R_*$ and that it increases rapidly to $w(r = 1.5\,R_*) \geq 1.7 \times 10^5$.

Allowing $T(r)$ to decline more rapidly increases the expansion factor still more. For $T(r) = T_0(r_0/r)^2$, the last term in (24) is $(GMm/2k)[(r - r_0)/T_0 r_0^2]$, which yields $w(r = 1.5\,R_*) = 1.6 \times 10^6$. $T(r) = T_0(r_0/r)^3$ yields $(GMm/2k)[(r^2 - r_0^2)/2T_0 r_0^3]$, and $w(r = 1.5\,R_*) = 2.8 \times 10^7$.

IV. LYMAN-ALPHA RADIATION PRESSURE

It is clear from the results of the preceding two sections that evaporative models, even when combined with divergent geometries, cannot produce a stellar wind with large mass loss given the maximum temperature constraint. Some other momentum transfer mechanism is required to drive the mass flow, and a radiation pressure gradient is a possible source of momentum addition. For a spherically symmetric outflow the equation of momentum conservation, including the effect of radiation pressure, is

$$v\frac{dv}{dr} = \left[\frac{4kT(r)}{r} - \frac{GMm}{r^2} - 2k\frac{dT}{dr} - \frac{1}{n(r)}\frac{dP_R}{dr}\right] \Big/ \left[m - \frac{2kT(r)}{v^2}\right]. \tag{25}$$

As a first approximation we consider the effect of Lα resonance scattering in an optically thin envelope. Assuming that the Lα emission profile is sufficiently broad and flat relative to the possible Doppler shift of the outflowing atoms, we may approximate $\int F_\nu \sigma_\nu d\nu$ by $\bar{F}_\nu \sigma$, where F_ν is the Lα surface flux and σ is the total absorption cross section per neutral hydrogen atom. We then have

$$-\frac{1}{n(r)}\frac{dP_R}{dr} = x(r)\frac{\pi}{c}(0.011)\bar{F}_\nu(r) \equiv F_R\,, \tag{26}$$

where $x(r)$ is the fraction of neutral hydrogen and $x(r) \ll 1$. For Arcturus the observed Lα surface flux is on the order of $\pi F \approx 5 \times 10^4$ ergs cm^{-2} s^{-1} (McClintock *et al.* 1975; Haisch *et al.* 1977). Assuming a line width of about 3 Å, we find $\pi\bar{F}_\nu = 3 \times 10^{-9}$ ergs cm^{-2} s^{-1} Hz^{-1}. The dominant term in the numerator of (25) is the gravitational force, $-GMm/r^2$, assuming $T(r = 2.017 \times 10^{12}) = 20{,}000$ K and $|dT/dr| < 10^{-8}$. However, if $x \geq 0.01$, the radiation pressure gradient term begins to compete with the gravitational force. The numerator goes to 0 as $x \rightarrow 0.03$, i.e., a 3% fractional abundance of neutral hydrogen in an optically thin envelope.

Although a 3% fraction of neutral hydrogen is quite reasonable, the problem comes in keeping the term $x(\pi/c)(0.011)\bar{F}_\nu$ comparable with gravity over a large distance, since this results in large optical depth, and thus $r^2\bar{F}_\nu(r) \ll R_*^2\bar{F}_\nu(R_*)$, contradicting our original assumptions. Another way of looking at this is to examine the amount of momentum in the Lα radiation field. A comparison of the maximum amount of momentum available per second per cm^2 in the whole Lα flux, $\pi F/c = 1.7 \times 10^{-6}$ ergs cm^{-3}, to the rate of material momentum flux at the top of the chromosphere $n_0 m v_0^2(R_*^2/r_0^2)$ shows the lack of sufficient momentum to bring about large flow velocities, even when we ignore the force of gravity. For v_0 in the range 10–100 km s^{-1}, we find $10^{-13} < dM/dt < 10^{-12}\,M_\odot$ yr^{-1} when all the available radiative momentum is converted to mass outflow. Even though the momentum addition of Lα scattering is inadequate to drive a wind, the radiation force may be *locally* comparable to gravity. Thus there may be a critical point near the stellar surface, and this point merits further attention.

To examine this problem properly requires the specification of $\bar{F}_\nu$ and the ionization fraction in a model chromosphere. We therefore ran a five-level ionization equilibrium non–LTE radiative transfer program for hydrogen in two model chromospheres, one with a 20,000 K plateau and one with a 10,000 K plateau. The models were chosen to match the calculated emergent fluxes with the observed Lα surface fluxes. These extensions of the Ayres-Linsky model also successfully match observed fluxes in the Ca II H and K lines, the Ca II infrared triplet, the Mg II *h* and *k* lines, and O I. Figure 1 shows the temperature-density structure of the 20,000 K plateau model. The density in the 20,000 K plateau is $n = 2 \times 10^8$ cm^{-3}. We find an emergent Lα flux at line center, $\bar{F}_\nu = 1.9 \times 10^{-8}$ ergs cm^{-2} s^{-1} Hz^{-1}, which increases to a maximum of $F_\nu = 6.4 \times 10^{-8}$ at $\Delta\lambda = 0.15$ Å. As τ increases, the flux profile becomes flatter around line center, and we take as our mean flux $\bar{F}_\nu$, the flux at line center. Listed in Table 1 are $\bar{F}_\nu[\tau(r)]$ and $x[\tau(r)]$, where the optical depth τ is measured at line center. A comparison of $x(r)\pi c^{-1}(0.011)\bar{F}_\nu$ to gravity, GMm/r^2, shows that in the plateau region the radiative force exceeds gravity by as much as a factor of 4! Similar values are found in the cooler model.

The adopted model chromosphere, extending from the temperature minimum to the isothermal plateau, assumes hydrostatic equilibrium, and the large radiative force exceeding gravity will bring about some mass outflow. The extent to which a region of negative effective gravity will cause some readjustment of the upper chromosphere model and result in a much more extended chromosphere and perhaps less radiation force will be the subject of further study. At present we feel that since the observed Lα surface flux is well matched by the model and the observed flux requires a neutral hydrogen fraction of only $x = 0.005$ for the radiation force to cancel gravity, changing the atmospheric extension will not significantly alter the $\bar{F}_\nu(\tau)$ relationship, and thus a locally large net outward force will remain. It should be kept in mind, moreover, that it is the ratio of gravitational to radiation force *per particle* that matters, and thus the only possible effect of density changes might be on the functional dependence of $\bar{F}_\nu(r)$ and $x(r)$ on r.

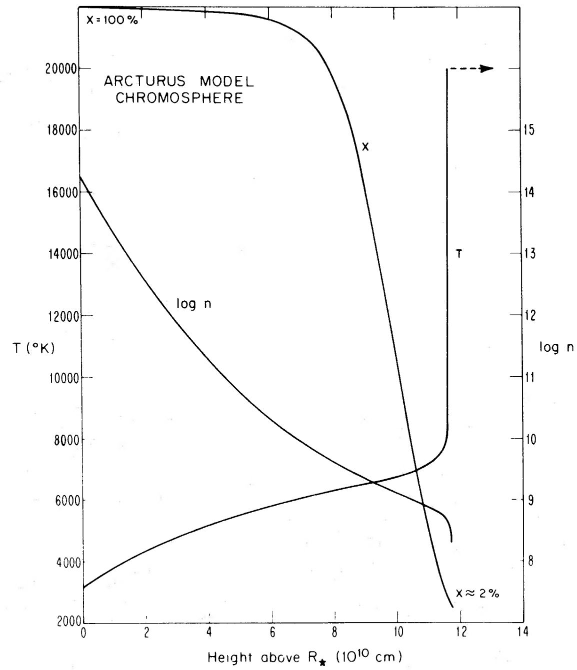

FIG. 1.—Distributions of total density, temperature, and fractional neutral hydrogen (×) in the 20,000 K plateau model chromosphere for Arcturus. The 10,000 K plateau model is almost identical throughout most of the chromosphere. The scale for × is linear, with 100% at the top and 0% at the bottom.

If we confine ourselves to spherically symmetric outflows, it is clear that the Lα force will initiate a net outflow, but this force by itself is incapable of sustaining the wind. In § VI we investigate the possible interaction of the locally large Lα force with some other possible driving mechanism, specifically the pressure gradient of radially propagating Alfvén waves. However, first we investigate the compatibility of an accelerating net outflow with the hydrostatic equilibrium density structure of the model chromosphere. If reasonable flow velocities $v(r)$ can be found from the equation of motion that are compatible with the hydrostatic equilibrium densities $n(r)$ throughout the model, and that result in a critical point in the isothermal plateau, then we have to first order a physically plausible wind model within the chromosphere, using the thermal energy of the gas alone to drive the wind up to the region where the Lα flux becomes dominant.

From the model chromosphere we have $T(r)$, in this case extending from the temperature minimum to the plateau. The sudden onset of a strong Lα force occurs at about $T = 10{,}000$ K in both models, at which point the denominator of (25) goes through zero due to the radiation pressure term. The critical point therefore occurs at $R_c = 2.0169 \times 10^{12}$ cm, where $v_c = (2kT_c/m)^{1/2} = 10.9$ km s^{-1}. Throughout the chromosphere, $r < R_c$, the Lα force is negligible, and we use $T(r)$ to numerically evaluate the equation of motion (2) to find $v(r)$. The results are shown in Table 2. As can be seen, the *prespecified* $n(r)$, which ranges over six orders of magnitude, and $v(r)$ calculated from the equation of motion combine to give the quantity nvr^2 which is fairly constant as required by conservation of mass flux. This indicates that the situation we have postulated is physically realistic.

In the 20,000 K plateau model shown in Figure 1 there is a steep temperature gradient between 10,000 K and 20,000 K. The 10,000 K plateau model is not pictured, but it has a nearly identical temperature-density structure.

TABLE 1

THE 20,000 K PLATEAU MODEL CHROMOSPHERE

r (cm)	τ	T (K)	$\bar{F}_\nu$ (ergs cm^{-2} s^{-1} Hz^{-1})	x	F_R (dyn cm^{-3})	$F_R/(GMm/r^2)$
2.01696 (12).......	0.02	20000	1.9 (−8)	0.020	4.3 (−22)	4.18
2.01695...........	0.86	20000	1.5 (−8)	0.016	2.7 (−22)	2.64
2.01694...........	1.44	20000	1.3 (−8)	0.014	2.2 (−22)	2.08
2.01693...........	3.07	16000	9.4 (−9)	0.019	2.0 (−22)	1.97
2.01692...........	5.60	10000	6.4 (−9)	0.031	2.3 (−22)	2.18
2.01685...........	42.5	9200	5.5 (−9)	0.025	1.6 (−22)	1.51
2.01675...........	96.3	8800	2.2 (−9)	0.025	6.2 (−23)	0.60
2.01656...........	217.	8400	3.7 (−10)	0.026	1.1 (−23)	0.11

TABLE 2

MASS FLOW IN THE MODEL CHROMOSPHERE

r (cm)	T (K)	v (cm s^{-1})	n (cm^{-3})	nvr^2 (s^{-1})
2.0169 (12).........	10000	10.9 (5)	3.8 (8)	1.7 (39)
2.0162.............	8000	5.6 (5)	5.3 (8)	1.2 (39)
2.0127.............	7380	4.0 (5)	6.7 (8)	1.1 (39)
2.0040.............	6916	2.3 (5)	1.1 (9)	1.0 (39)
1.9866.............	6452	7.6 (4)	2.7 (9)	0.8 (39)
1.9657.............	5987	1.9 (4)	1.2 (10)	0.9 (39)
1.9493.............	5523	5.8 (3)	6.7 (10)	1.5 (39)
1.9251.............	4593	7.1 (2)	1.5 (12)	3.9 (39)
1.9125.............	3974	1.8 (2)	1.2 (13)	7.9 (39)
1.9000.............	3200	3.5 (1)	1.7 (14)	21.5 (39)

The results in Table 2 are equally applicable to the chromospheres of both models. The temperature plateaus and accompanying steep temperature gradients should be viewed as schematic representations of the uppermost chromospheric regions. Both models correctly predict the observed Lα flux. We could just as well have modeled the chromospheric wind for the 20,000 K model by setting $R_c = r(20{,}000\ \mathrm{K})$ and $v_c = 15.4\ \mathrm{km\ s^{-1}}$, for example. The steep temperature gradient would then have resulted in a Δv of roughly a few km s^{-1} between $R(10{,}000\ \mathrm{K})$ and $R(20{,}000\ \mathrm{K})$, leaving the $v(r)$, $n(r)$ relationship in the rest of the chromosphere nearly unchanged. The important result is that mass outflow is compatible with the temperature-density structure of our model chromosphere, with the critical point occurring near the top of our model as required by the Lα force.

What happens above the critical point? The radiative force will provide some acceleration, but only over a very limited region. Model b in § II shows that a transonic wind cannot be sustained by a negative temperature gradient above R_c. Geometrical divergence, as discussed in § III, is also inadequate. We conclude that a wind accelerating smoothly through our model chromosphere and through a critical point is feasible, but that it will thereafter find itself in a transonic regime without the necessary "push" to remain supersonic. To resolve this dilemma we turn to the solution topologies of Holzer (1977). We consider as one possibility a shock transition, e.g., Figures 3c, 3e, and 3j in Holzer.

V. SHOCK TRANSITION

A stationary shock front may occur when the Lα force gives out and the wind, in effect, encounters an obstacle. Since the wind in our model is only barely supersonic due to the Lα force, we ask whether any further acceleration might occur to make the wind somewhat more supersonic before it shocks. Otherwise the shock transition will change the conditions very little, and the post-shock flow will still be quite close to the singular condition of the momentum equation. If we allow for adiabatic expansion of the gas after the Lα force ceases, we may write

$$P = P_0(\rho/\rho_0)^{5/3} . \tag{27}$$

The momentum equation for adiabatic expansion of the gas,

$$m\frac{dv}{dr} = -\frac{GMm}{vr^2} - \frac{1}{nv}\frac{dP}{dr}, \tag{28}$$

may be rewritten using (27) so that

$$-\frac{1}{nv}\frac{dP}{dr} = \frac{5}{3}\frac{p_0 v_0^{2/3} r_0^{4/3}}{n_0}\left(\frac{1}{v^{8/3}r^{4/3}}\frac{dv}{dr} + \frac{2}{v^{5/3}r^{7/3}}\right), \tag{29}$$

and thus

$$\left(m - \frac{5}{3}\frac{p_0 v_0^{2/3} r_0^{4/3}}{n_0}\frac{1}{v^{8/3}r^{4/3}}\right)v\frac{dv}{dr} = \frac{5}{3}\frac{p_0 v_0^{2/3} r_0^{4/3}}{n_0}\frac{2}{v^{2/3}r^{7/3}} - \frac{GMm}{r^2}, \tag{30}$$

where the subscript zero simply refers to some reference level. This may be integrated to yield

$$\tfrac{1}{2}m(v_2^2 - v_1^2) + 5kT_0 v_0^{2/3} r_0^{4/3}\left(\frac{1}{v_2^{2/3}r_2^{4/3}} - \frac{1}{v_1^{2/3}r_1^{4/3}}\right) = GMm\left(\frac{1}{r_2} - \frac{1}{r_1}\right). \tag{31}$$

Using as initial conditions $v_1 = 12\ \mathrm{km\ s^{-1}}$, $T_1 = 10{,}000\ \mathrm{K}$, $r_1 = 2.017 \times 10^{12}$ cm, and $n_1 = 3.8 \times 10^8\ \mathrm{cm^{-3}}$ as a continuation of the wind solution after the termination of the Lα force, we find a maximum $v_2 \approx 13\ \mathrm{km\ s^{-1}}$ using (31) at $r_2 = 2.018 \times 10^{12}$. If we choose $v_1 = 16\ \mathrm{km\ s^{-1}}$, $T_1 = 20{,}000\ \mathrm{K}$, $r_1 = 2.017 \times 10^{12}$ cm, $n_1 = 3 \times 10^8\ \mathrm{cm^{-3}}$, we again find a maximum v_2 which is not very much greater than v_1. Thus we have Mach numbers for the preshock flow, M, not much greater than unity. These specific values should be regarded as only schematic, but they do show the possibility of having the wind accelerated somewhat by expansion. Radiative cooling may accelerate the flow more.

An approximation for the density changes across the shock front is found by applying the Rankine-Hugoniot relations (see Mihalas 1978, pp. 519–520)

$$\frac{\rho_2}{\rho_1} = \frac{v_1}{v_2} = \frac{8}{3} M^2 \Big/ \left(\frac{2}{3} M^2 + 2\right), \tag{32}$$

$$\frac{T_2}{T_1} = \left(\frac{10}{3} M^2 - \frac{2}{3}\right) \Big/ \left(\frac{8}{3} \frac{\rho_2}{\rho_1}\right). \tag{33}$$

These show that the conditions do not change much across such a weak shock ($M \approx 1$). Neither the short-lived Lα acceleration nor a possible adiabatic expansion will accelerate the wind very much above the critical point. However, any type of shock, strong or weak, can change the topology of the wind model. The multiple critical point topologies investigated by Holzer (1977) show three possible solutions involving a shock discontinuity (see Fig. 3 in Holzer). We therefore ask whether it is possible for a wind solution such as ours to follow one of the Holzer tracks "around" a second critical point and through an outer critical point.

The temperature in the postshock region is not very different from the preshock temperature as the Rankine-Hugoniot relation shows. In the numerator of the equation of motion (2), the dominant term will therefore be the gravitational force, $-GMm/r^2$, out to a distance of many stellar radii. The numerator is thus fixed at a negative value. Passage through another critical point cannot occur until $4kT/r \approx GMm/r^2$, assuming the temperature gradient remains small as it must (cf. § II). Since $v \approx (2kT/m)^{1/2}$, the denominator is small relative to the numerator. Thus we cannot have the necessary condition, $v(dv/dr) \approx 0$, which would enable the wind solution to stay on a long, flat trajectory out to an outermost critical point at several stellar radii. Additional momentum input will be required to enable a multiple critical point type of wind solution to exist. There is insufficient thermal energy in the gas to sustain our wind flow, a conclusion we previously found in §§ II and III for single critical point wind flows.

VI. AN ALFVÉN WAVE DRIVEN WIND

Although other potential sources of momentum addition are possible, we investigate as one possibility the pressure due to outwardly propagating Alfvén waves. Alfvén waves have been identified in the solar wind (Unti and Neugebauer 1968; Coleman 1967; Belcher and Davis 1971), and are observed to be associated with, and perhaps may heat, active regions. Belcher and Olbert (1975) have studied the properties of stellar winds driven by Alfvén waves in a hot corona.

We are interested only in possible momentum addition by Alfvén waves, not heating. We thus wish to incorporate an Alfvén wave pressure gradient term into the stellar wind equation of motion, under conditions such that energy dissipation is negligible. We found in § IV that the thermal energy content of the gas is sufficient to bring about chromospheric mass flow consistent with a model chromosphere temperature-density structure, and with the location of a critical point at the top of the chromosphere as required by the Lα flux. We now propose using Alfvén wave pressure to keep the wind going where the evaporative model fails, but we need to examine this mechanism in the outer regions of the wind and to check the consistency of a new wind solution including Alfvén waves in the region where the evaporative model succeeds. We must therefore start over again, checking the consistency of an Alfvén wave model in the chromosphere, at a chromospheric critical point, at a possible shock front, and then in the region beyond the first critical point in a multiple critical point topology.

An excellent derivation of the basic equations governing Alfvén waves and their incorporation into a wind model is given by Hollweg (1973). We incorporate a wave pressure term into our momentum equation and find

$$\left(m - \frac{2kT}{v^2}\right)\frac{dv}{dr} = \frac{1}{v}\left[\frac{4kT(r)}{r} - \frac{GMm}{r^2} - 2k\frac{dT}{dr} - \frac{1}{n(r)}\frac{d}{dr}\frac{\langle \delta B^2(r)\rangle}{8\pi}\right], \tag{34}$$

where $\delta B(r)$ is the fluctuating part of the magnetic field. We further assume that the magnetic field $B(r)$ falls off as r^{-2},

$$B(r) = B(r_0)(r_0^2/r^2), \tag{35}$$

resulting in a radial Alfvén speed,

$$v_A(r) = \frac{B(r)}{[4\pi n(r)m]^{1/2}} = v_{A,0}\left(\frac{r_0}{r}\right)\left(\frac{v}{v_0}\right)^{1/2}, \tag{36}$$

where v and v_0 refer to the flow velocity of the matter. The amplitude of the magnetic field fluctuations varies as

$$\frac{\langle \delta B^2(r)\rangle}{\langle \delta B_0{}^2\rangle} = \left(\frac{n(r)}{n_0}\right)^{3/2}\left\{\frac{1 + v_{A,0}/v_0}{1 + v_A(r)/v(r)}\right\}^2, \tag{37}$$

where n_0, δB_0, $v_{A,0}$, v_0 refer to some arbitrary reference level. If we make the further assumption that $v_A(r) \gg v(r)$ everywhere, (37) simplifies to

$$\langle \delta B^2(r)\rangle = \langle \delta B_0{}^2\rangle(r_0/r)(v_0{}^{1/2}/v^{1/2}) \tag{38}$$

and permits us to rewrite (34) in the usual form from which the critical point, R_c, and the critical velocity, v_c, may later be determined:

$$v\frac{dv}{dr} = \left[\frac{4kT(r)}{r} - \frac{GMm}{r^2} - 2k\frac{dT}{dr} + \frac{\langle \delta B_0{}^2\rangle}{8\pi}\frac{1}{n_0 v_0{}^{1/2} r_0}v^{1/2}\right]\bigg/\left[m - \frac{2kT(r)}{v^2} - \frac{\langle \delta B_0{}^2\rangle}{8\pi}\frac{1}{n_0 v_0{}^{1/2} r_0}\frac{1}{2}\frac{r}{v^{3/2}}\right]. \tag{39}$$

Now B_0 and δB_0 are additional free parameters, with only δB_0 explicitly entering into the wind solution. Clearly we wish to have $\delta B_0 < B_0$; and since we are interested at this point only in the effects of wave pressure, we avoid solutions in which wave damping (heating) may occur. Thus we wish to have

$$\frac{\langle \delta B^2(r)\rangle}{B^2(r)} \ll 1\,. \tag{40}$$

The momentum equation (39) may then be integrated to yield

$$\tfrac{1}{2}m(v_2{}^2 - v_1{}^2) - 2k\int\frac{T}{v}dv - \frac{\langle \delta B_0{}^2\rangle}{8\pi}\frac{1}{n_0 v_0{}^{1/2} r_0}(r_2 v_2{}^{1/2} - r_1 v_1{}^{1/2}) = -2k(T_2 - T_1) + 4k\int\frac{T}{r}dr + GMm\left(\frac{1}{r_2} - \frac{1}{r_1}\right). \tag{41}$$

For mass flow in our model of the stellar chromosphere, we want the numerator and denominator of (39) to be negative below some R_c and go to zero simultaneously at R_c. As in § IV, near the 10,000 K or 20,000 K plateaus of our models the radiation force term $x(\pi/c)(0.011)\bar{F}_\nu$ should be added to the numerator of (39), forcing the numerator to go through a zero. As before, we therefore set $R_c = 2.017 \times 10^{12}$ cm, and choose δB_c and v_c such that the denominator also goes to 0 at R_c, that $n_0 v_0 R_*{}^2$ = fairly large mass loss rates, and that $v(r)$ is consistent with $n(r)$ in the prespecified chromosphere model. Given the available free parameters, this is clearly an overspecification of the problem. Thus any reasonable matching of all these conditions will be a good indication that the Alfvén wave driven winds are also capable of modeling the chromosphere in a physically realistic way.

Representative possible models for a chromospheric wind are shown in Table 3. If the sudden onset of a strong Lα force occurs at about T = 10,000 K, such that the numerator of (39) goes to zero at about this point, the choices $\langle \delta B^2(R_c)\rangle = 0.01$ and $v_c = 11.8$ km s^{-1}, $\langle \delta B^2(R_c)\rangle = 0.20$ and $v_c = 25.0$ km s^{-1}, and $\langle \delta B^2(R_c)\rangle = 1.0$ and $v_c = 46$ km s^{-1} result in the denominator also going to zero at this point. For $r < R_c$, the solution of (41) results in the $v(r)$ values presented in Table 3. The particle density, $n(r)$, taken from the model chromosphere is then combined with $v(r)$ to yield nvr^2 throughout the models. The reasonable constancy of nvr^2 through most of the chromosphere for the $\langle \delta B^2\rangle = 0.01$ model indicates that the flow velocities are fairly consistent with the prespecified density distribution (compare to Table 2). Only deep in the model, where n has changed by more than five orders of magnitude, does the mass flux constancy fail. The values of $\langle \delta B^2(r)\rangle$ may be found from (38).

TABLE 3

MASS FLOW IN THE MODEL CHROMOSPHERE WITH ALFVÉN WAVES

			$\langle \delta B^2(R_c)\rangle = 0.01$		$\langle \delta B^2(R_c)\rangle \approx 0.2$		$\langle \delta B^2(R_c)\rangle \approx 1.0$	
r (cm)	T (K)	n (cm^{-3})	v (cm s^{-1})	nvr^2 (s^{-1})	v (cm s^{-1})	nvr^2 (s^{-1})	v (cm s^{-1})	nvr^2 (s^{-1})
2.0169 (12)	10000	3.8 (8)	11.8 (5)	1.8 (39)	25.0 (5)	3.8 (39)	46 (5)	7.0 (39)
2.0162	8000	5.3 (8)	6.5 (5)	1.4	19.4 (5)	4.1	43 (5)	9.1
2.0127	7380	6.7 (8)	4.8 (5)	1.3	17.4 (5)	4.7	41 (5)	11.0
2.0040	6916	1.1 (9)	2.8 (5)	1.2	14.8 (5)	6.5	39 (5)	17.
1.9866	6452	2.7 (9)	1.1 (5)	1.2	11.3 (5)	12.0	37 (5)	40.
1.9657	5987	1.2 (10)	2.9 (4)	1.3	8.2 (5)	38	34 (5)	158.
1.9493	5523	6.7 (10)	9.2 (3)	2.3	6.3 (5)	160	32 (5)	815.
1.9251	4593	1.5 (12)	1.2 (3)	6.7	3.9 (5)	2170	29 (5)	16000
1.9125	3974	1.2 (13)	3.0 (2)	13.2	2.9 (5)	12700	28 (5)	121000
1.9000	3200	1.7 (14)	5.8 (1)	35.6	2.0 (5)	123000	26 (5)	1600000

For $\langle \delta B^2(R_c) \rangle = 0.01$ and $\langle \delta B^2(R_c) \rangle = 0.2$, they are physically plausible throughout the chromosphere; and if we assume that $B \gtrsim 10$ gauss, $v_A(r)$ is everywhere much greater than $v(r)$ and $\langle \delta B^2 \rangle / B^2 \ll 1$. The $\langle \delta B^2(R_c) \rangle = 0.01$ model presented here has not been rigorously selected to be an optimum representation for chromospheric mass flow, but it does show the feasibility of having an Alfvén wave driven wind consistent with a chromospheric model that successfully matches numerous chromospheric observed emission features (Lα, Ca II, Mg II, O I). The mass loss corresponding to $nvr^2 = 1.8 \times 10^{39}\ \mathrm{g\ s^{-1}}$ is $dM/dt = 10^{-9}\ M_\odot\ \mathrm{yr^{-1}}$. The important point is that a transonic solution can exist with a critical point at the top of the chromosphere as required by the sudden onset of the Lα force. A wind flow of this type can smoothly pass through the critical point and continue to be accelerated, albeit briefly, by the Lα force as well as by the Alfvén wave pressure gradient. The other two models clearly fail to satisfy mass flux conservation in the model chromosphere.

As discussed in § IV, the overall momentum input due to the Lα force is fairly small. Having been accelerated to a critical velocity v_c, the wind is not significantly accelerated further by the Lα flux. The wind soon finds itself as before with a supersonic velocity and no further outward force. We consider again a shock transition. The momentum equation for adiabatic expansion of the gas plus Alfvén wave pressure,

$$m \frac{dv}{dr} = -\frac{GMm}{vr^2} - \frac{1}{nv}\frac{dP}{dr} - \frac{1}{nv}\frac{d}{dr}\frac{\langle \delta B^2 \rangle}{8\pi}, \tag{42}$$

may be rewritten using (27) so that

$$-\frac{1}{nv}\frac{dP}{dr} = \frac{5}{3}\frac{p_0 v_0^{2/3} r_0^{4/3}}{n_0}\left[\frac{1}{v^{8/3} r^{4/3}}\frac{dv}{dr} + \frac{2}{v^{5/3} r^{7/3}}\right], \tag{43}$$

and thus

$$\left(m - \frac{5}{3}\frac{p_0 v_0^{2/3} r_0^{4/3}}{n_0}\frac{1}{v^{8/3} r^{4/3}} - \frac{\langle \delta B_0^2 \rangle}{8\pi}\frac{1}{n_0 v_0^{1/2} r_0}\frac{r}{2v^{3/2}}\right) v \frac{dv}{dr} = \frac{5}{3}\frac{p_0 v_0^{2/3} r_0^{4/3}}{n_0}\frac{2}{v^{2/3} r^{7/3}} - \frac{GMm}{r^2} + \frac{\langle \delta B_0^2 \rangle}{8\pi}\frac{v^{1/2}}{n_0 v_0^{1/2} r_0}, \tag{44}$$

where the subscript zero simply refers to some reference level. This may be integrated to yield

$$\tfrac{1}{2} m (v_2^2 - v_1^2) + 5kT_0 v_0^{2/3} r_0^{4/3}\left(\frac{1}{v_2^{2/3} r_2^{4/3}} - \frac{1}{v_1^{2/3} r_1^{4/3}}\right) - \frac{\langle \delta B_0^2 \rangle}{8\pi}\frac{1}{n_0 v_0^{1/2} r_0}(r_2 v_2^{1/2} - r_1 v_1^{1/2}) GMm\left(\frac{1}{r_2} - \frac{1}{r_1}\right) \cdot \tag{45}$$

Using as initial conditions for the $\langle \delta B_0^2 \rangle = 0.01$ model $v_1 = 12\ \mathrm{km\ s^{-1}}$, $T_1 = 20{,}000$ K, $r_1 = 2.017 \times 10^{12}$ cm, and $n_1 = 2 \times 10^8\ \mathrm{cm^{-3}}$, we find a maximum $v_2 = 18.1\ \mathrm{km\ s^{-1}}$ at $r_2 = 2.034 \times 10^{12}$ cm with $T_2 = 16{,}150$ K. Thus we have a Mach number for the preshock flow, $\mathscr{M} = 1.4$, a slightly stronger shock than the comparable case without Alfvén wave pressure.

The Rankine-Hugoniot relations (32) and (33) are not strictly valid in this approximation as they do not include the Alfvén wave pressure and energy terms. To first order, however, they result in a postshock temperature $T = 23{,}000$ K. We show in Figure 2 the stellar wind as it approaches the critical point, experiences a strong but short-lived acceleration due to the Lα force, expands adiabatically to about $\mathscr{M} \approx 1.4$, and then shocks. The insert, taken from Holzer (1977), suggests how the wind flow might continue, bypassing a second critical point, and finally becoming supersonic again after passing through a third critical point.

The $\langle \delta B_0^2 \rangle = 0.2$ and 1.0 models may have a shock transition as well, but rather than looking at the details of the preshock and postshock conditions, we concentrate on the more important question of whether or not any of these models might satisfy conditions appropriate for the Holzer topologies beyond the first critical point.

We want to have a very extended low temperature region out to several stellar radii. If we set $T \approx 20{,}000$ K and evaluate the integrated equation of motion (41) for the three Alfvén wave models, we obtain the $v(r)$ values shown in Table 4. For each of the models we have chosen a range of starting velocities $v_1 = v(r = 2.034 \times 10^{12}\ \mathrm{cm}) = v_{\mathrm{postshock}}$. Clearly the postshock velocity must be fixed by application of the appropriate shock conditions, but by examining the solution topology for a range of initial velocities we may determine whether or not any solutions exist at all. As shown in Table 4, only the $\langle \delta B_0^2 \rangle = 1.0$ model is able to maintain a wind flow out to $r > 5\ R_*$. It is beyond the scope of this analysis to evaluate all the conditions necessary for a Holzer multiple critical point topology and to find a solution that actually follows one of those trajectories, but clearly a strong Alfvén wave pressure gradient is required to maintain a wind out to a significant distance from the star. Unfortunately the conditions that enable a wind to exist at large distances from the star are inconsistent with the chromospheric requirements (see Table 3).

Depending on the choice of $\langle \delta B_0^2 \rangle$ and B, the energy flux associated with the Alfvén waves may substantially exceed the kinetic energy flux of the wind. This could lead to heating of the gas, a situation we wish to avoid. On the other hand, Hollweg (1974) has estimated that even in the solar wind at least 15% of the solar wind energy flux is associated with Alfvén wave pressures. We have taken as our criterion for no heating (40), and this may require $B(R_*) > 10$–100 gauss under some conditions. According to Belcher and Olbert (1975), "Alfvén waves

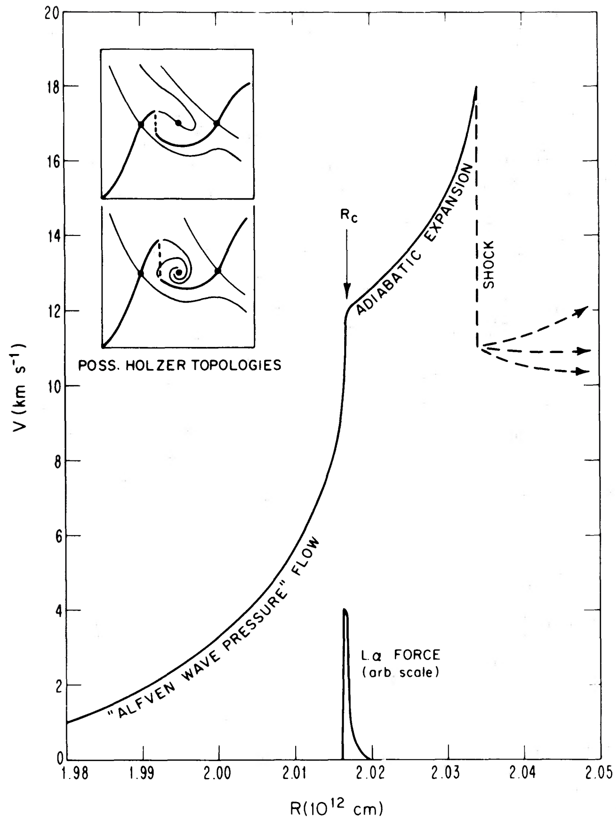

FIG. 2.—A schematic representation of a wind model for $\langle \delta B_0^2 \rangle = 0.01$ near the critical point showing the acceleration to the critical point, adiabatic expansion and a shock transition. R_c is the critical point, and possible Holzer topologies are indicated.

TABLE 4

POSTSHOCK WIND SOLUTIONS WITH ALFVÉN WAVES

r (cm)	v (km s^{-1})							
			$\langle \delta B_0^2 \rangle = 0.01$					
2.034 (12)............	1.0	5.0	7.5	10.0	15.0	20.0	30.0	40.0
2.040 (12)............	1.1	5.6	8.5	12.0	...	...	28.8	39.2
2.050 (12)............	1.3	6.7	11.0	...	...	...	26.5	37.9
2.100 (12)............	3.0	...	...	...	...	...	...	30.5
2.250 (12)............	...	...	...	...	...	...	...	...
			$\langle \delta B_0^2 \rangle = 0.2$					
2.034 (12)............	1.0	5.0	10.0	15.0	20.0	30.0	40.0	50.0
2.040 (12)............	1.1	5.3	10.5	16.0	22.2	...	38.9	49.3
2.050 (12)............	1.2	5.8	11.6	17.9	...	...	36.7	48.2
2.100 (12)............	2.1	8.9	18.8	...	...	...	...	41.5
2.250 (12)............	7.2	...	...	...	...	...	...	...
2.500 (12)............	...	...	...	...	...	...	...	...
			$\langle \delta B_0^2 \rangle = 1.0$					
2.034 (12)............	15.0	25.0	35.0	45.0	60.0	75.0	100.0	
2.040................	15.2	25.3	35.5	46.3	59.2	74.8	99.9	
2.050................	15.5	25.8	36.3	49.8	57.7	74.4	99.8	
2.100................	17.1	28.2	40.6	...	...	72.5	99.2	
2.250................	21.5	35.0	...	...	...	67.0	98.4	
2.500................	27.0	43.0	...	...	...	...	99.4	
3.000................	30.8	44.5	...	...	...	...	109.	
4.000................	26.4	33.2	...	...	...	...	140.	
5.000................	20.9	24.9	...	...	...	...	170.	
10.000..............	7.5	8.6	...	...	...	...	318.	

in astrophysical plasmas are notoriously difficult to damp." Since we have only delineated somewhat schematically the properties of a cool Alfvén wave driven wind, we defer further consideration regarding wave damping and consequent heating of the wind. Obviously, this question must be considered in some detail for a realistic wind model. Indeed, a full fledged consideration of radiative cooling, conductive heating, enthalpy of the gas, and wave damping will be called for in a complete model. We again strongly emphasize the exploratory nature of the present study.

VII. IMPLICATIONS FOR OTHER LATE-TYPE STARS

The role of the Lα flux in this paper is to define a condition which in turn can only be satisfied by the existence of a certain stellar wind. If we accept that the Lα force exceeds gravity somewhere in the stellar chromosphere, then mass loss and a transition through a singular point in the momentum equation both must occur. If we also require that the maximum temperature not exceed a certain value determined observationally, the constraints on possible wind models fulfilling these conditions are quite severe. We have here presented only schematic wind models—ones that rely on a combination of several different regimes in which one or the other type of mechanism dominates the solution. Other models must certainly be possible, for example with acoustic wave pressure instead of Alfvén wave pressure (see Ulmschneider 1979).

Reimers (1978*a*) finds evidence for chromospheric expansion velocities in late type stars as high as half the terminal speeds, that is, 5–50 km s^{-1}. Reimers has generally assumed a mass loss relation of the form

$$\frac{dM}{dt} = \frac{cL}{gR}, \tag{46}$$

where L, g, and R are stellar luminosities, gravities, and radii in solar units and c is an empirically derived constant. The value of c is uncertain, with values from 1×10^{-13} to 5.6×10^{-13} proposed at different times (Reimers 1975, 1977*b*, 1978*a*, *b*). This relation predicts for Arcturus $dM/dt = 0.3\text{–}2 \times 10^{-9}\, M_{\odot}\,\mathrm{yr}^{-1}$ (Reimer's most recent estimate of c results in the largest value of dM/dt), in agreement with the model presented here. Mullan (1978) derives the mass loss relation

$$\frac{dM}{dt} = 1.6 \times 10^{-9}\, MR^{1/2}, \tag{47}$$

where M and R are stellar masses and radii in solar units, for stars to the right of the "supersonic transition locus." This expression predicts $dM/dt = 9 \times 10^{-9}\, M_{\odot}\,\mathrm{yr}^{-1}$, assuming $M = 1.1$ (Ayres and Johnson 1977), which is an order of magnitude larger than our estimate for dM/dt. Cassinelli (1979) has reviewed our present understanding of winds in both early and late type stars.

We extrapolate our results to other late type stars by assuming similar ionization and temperature conditions, and derive a Lα "supersonic transition locus" by comparing Lα fluxes and stellar gravities. Linsky and Ayres (1978) and Basri and Linsky (1979) have compiled chromospheric Mg II emission fluxes observed by *IUE*, *OAO 2*, *Copernicus*, and BUSS for 40 stars including the Sun. Linsky and Ayres also give ratios of Lα to Mg II fluxes for seven stars. In Table 5 we list their tabulated values of $\pi F_{\nu}(\mathrm{Mg\ II})/\sigma T_{\mathrm{eff}}^4$ for those stars of spectral type G–M for which gravities are available. The observed Lα to Mg II flux ratios of six stars in their table have been used to extrapolate this ratio to the remaining stars. Apart from ϵ Eri, which has a ratio of 0.55, all the remaining ratios are in the range 0.20–0.33, and thus we expect this ratio to be fairly insensitive to changes in the chromospheric conditions appropriate to different late-type stars. Combining these two quantities enables us to estimate the Lα fluxes relative to Arcturus, $F(\mathrm{L}\alpha)/F(\mathrm{L}\alpha)_{\mathrm{ARC}}$.

The multicolor photometry of Johnson *et al.* (1966) has been used to provide V magnitudes and $(V - R)$ and $(V - I)$ colors. The latter was used to derive effective temperatures using the transformation of Johnson (1966). Parallaxes are taken from Hoffleit (1964), bolometric corrections are from Johnson (1966) yielding M_{bol}, and stellar gravities are taken from the references cited in Table 5.

We predict that those stars with $F_*(\mathrm{L}\alpha)/F(\mathrm{L}\alpha)_{\mathrm{ARC}} > g_*/g_{\mathrm{ARC}}$ will have cool winds in which Lα radiation pressure plays an important role. Column (11) of Table 5 summarizes our prediction as to whether or not a star should possess a cool wind. Two stars, α Aur and α Tau, are marginal cases. In Figure 3 we plot these stars in an H-R diagram with different symbols representing whether a star should or should not possess a cool wind. Also plotted is the dividing line between stars with transition regions and presumably hot coronae and those without transition regions and presumably with cool winds proposed in Paper I. Given the small data sample and approximations inherent in our predictions, we conclude that the Linsky and Haisch empirical dividing line is roughly consistent with our estimated dividing line between stars with and without Lα radiation pressure initiated cool winds.

We now ask whether Lα radiation pressure could be responsible for the absence of hot coronae in M supergiants like α Ori. Jennings and Dyke (1972) have shown that Ca II K line emission is relatively weak in those luminous M stars which show evidence of circumstellar envelopes with high grain densities. Since grains are effective cooling agents, their presence can reduce temperatures and densities in chromospheres (Jennings 1973) and thereby greatly reduce the Lα radiation pressure. This argument and Sanner's (1976) result that mass loss and

TABLE 5

LYMAN-ALPHA FLUXES AND IMPLICATIONS FOR WINDS IN OTHER STARS

Name	Sp. Type	B.S. #	T_{eff} (K)	$\frac{\pi F(\text{Mg II})}{\sigma T_{eff}^4}$	$\frac{\pi F(L\alpha)}{\pi F \text{ Mg II}}$	$\frac{F(L\alpha)}{F(L\alpha)_{ARC}}$*	g_*/g_{ARC}	V-R	M_{bol}	WIND	Ref.
SUN	G2 V		5770	2.0(-5)	0.22	4.9	550.	0.53	4.72	No	
ε Eri	K2 V	1084	4950	6.7(-5)	0.55	22.2	630.	0.72	5.91	No	(1)
α Aur	G5 III	1708	5280	8.2(-5)	0.20	12.7	8.3	0.60	-0.80	poss.	(3)
η Dra	G8 III	6132	5220	2.4(-5)	(0.20)	3.6	12.6	0.61	0.70	No	(5)
δ Dra	G9 III	7310	4910	2.7(-5)	(0.20)	3.2	10.0	0.70	0.10	No	(5)
β Gem	K0 III	2990	4830	1.1(-5)	0.20	1.2	15.9	0.75	0.68	No	(3)
α Boo	K2 III	5340	4240	1.1(-5)	0.28	--	--	0.97	-0.87	Yes	(2)
α Tau	K5 III	1457	3690	6.7(-6)	0.33	0.4	0.5	1.23	-1.81	poss.	(3)
β Dra	G2 II	6536	5010	7.0(-5)	(0.20)	8.9	0.45	0.68	-2.55	Yes	(1)
ε Gem	G8 Ib	2473	4300	3.0(-5)	(0.20)	2.1	0.14	0.96	-2.65	Yes	(1)
ζ Cep	K1 Ib	8465	3980	>2.7(-5)	(0.20)	<1.4	0.20	1.08	-0.86	Yes	(1)
ε Peg	K2 Ib	8308	4040	2.3(-5)	(0.20)	1.2	0.20	1.05	--	Yes	(1)
α Ori	M2 Iab	2061	3240	4.7(-6)	(0.30)	0.1	0.02	1.64	-7.99	Yes	(4)

(1) Luck, R. E. 1978, Ap. J., 219, 148.

(2) Ayres, T. R. and Linsky, J. L. 1975, Ap. J., 200, 660.

(3) Kelch, W. L. et al. 1978, Ap. J., 220, 962.

(4) Fay, T. D. and Johnson, H. R. 1973, Ap. J., 181, 851.

(5) Gustafsson, B., Kjaergaard, P. and Andersen, S. 1974, Astr. Ap., 34, 99.

REFERENCES TO TABLE 5.—(1) Luck 1978. (2) Ayres and Linsky 1975. (3) Kelch *et al.* 1978. (4) Faÿ and Johnson 1973. (5) Gustafsson *et al.* 1974.

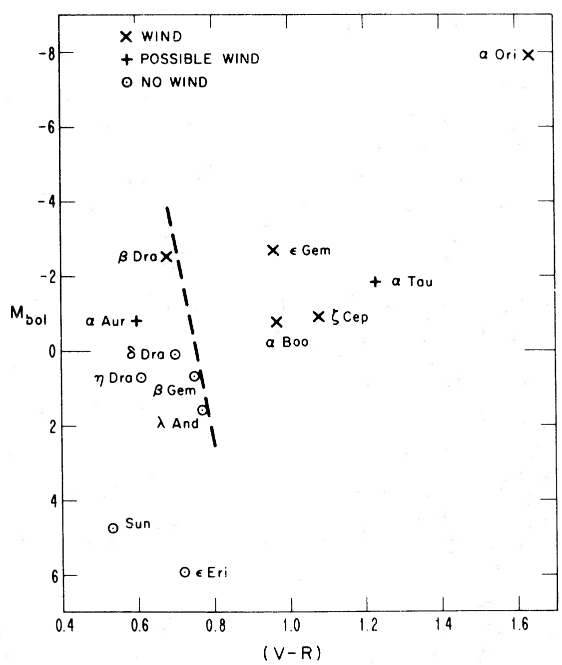

FIG. 3.—An H-R diagram for the stars in Table 5. The dashed line indicates the Linsky-Haisch empirical division between stars with hot coronae (to the left) and stars with no evidence of material hotter than 20,000 K (to the right).

Ca II K line emission are anticorrelated for M supergiants suggest that the Lα radiation pressure mechanism may be unimportant for M supergiants.

VIII. CONCLUSIONS

In this paper we have investigated possible wind models for late-type stars which appear not to have hot coronae and transition regions. Our conclusions are as follows:

1. Evaporative models of cool winds which are radially symmetric predict mass loss rates less than 10^{-16} $M_\odot$ yr^{-1}, far less than the rates $\sim 10^{-9}$ $M_\odot$ yr^{-1} seen in stars like Arcturus.
2. The incorporation of diverging geometries does not appreciably increase the computed mass loss rates.
3. An additional momentum transfer mechanism is needed to produce mass loss $\sim 10^{-9}$ $M_\odot$ yr^{-1} if we adopt the empirical constraint of $T \lesssim 20{,}000$ K in the outer atmospheres of stars to the right of the Linsky-Haisch dividing line in the H-R diagram.
4. The radiation pressure of Lα photons in the chromosphere of Arcturus is sufficient to produce negative effective gravities and a critical point in the chromosphere. Lα radiation pressure can thus initiate a net outflow, but it is insufficient in stars like Arcturus to sustain the wind.
5. An additional momentum source is necessary to maintain the wind. We find that Alfvén waves can provide sufficient momentum to produce a mass loss of 10^{-9} $M_\odot$ yr^{-1} for small values of the magnetic field, but that other waves may also be adequate for this purpose.
6. We consider 12 stars for which estimates of the gravity and Lα flux are available. We find that those stars to the right of the Linsky-Haisch dividing line can have Lα radiation pressure initiated cool winds, and with one exception those stars to the left of the Linsky-Haisch dividing line should not have cool winds. Thus the existence of cool winds may explain energetically the absence of transition regions and presumably hot coronae in stars to the right of the Linsky-Haisch dividing line.

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