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ACCRETION BY ROTATING MAGNETIC NEUTRON STARS. III. ACCRETION TORQUES AND PERIOD CHANGES IN PULSATING X-RAY SOURCES[1]

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ABSTRACT

We use the solutions of the two-dimensional hydromagnetic equations obtained previously to calculate the torque on a magnetic neutron star accreting from a Keplerian disk. We find that the magnetic coupling between the star and the plasma outside the inner edge of the disk is appreciable. As a result of this coupling the spin-up torque on fast rotators is substantially less than that on slow rotators; for sufficiently high stellar angular velocities or sufficiently low accretion rates this coupling dominates that due to the plasma and the magnetic field at the inner edge of the disk, braking the star's rotation even while accretion, and hence X-ray emission, continues.

We apply these results to pulsating X-ray sources, and show that the observed secular spin-up rates of all the sources in which this rate has been measured can be accounted for quantitatively if one assumes that these sources are accreting from Keplerian disks and have magnetic moments $\sim 10^{29}$–10^{32} gauss cm^3. The reduction of the torque on fast rotators provides a natural explanation of the spin-up rate of Her X-1, which is much below that expected for slow rotators. We show further that a simple relation between the secular spin-up rate $-\dot{P}$ and the quantity $PL^{3/7}$ adequately represents almost all the observational data, P and L being the pulse period and the luminosity of the source, respectively. This "universal" relation enables one to estimate any one of the parameters P, $\dot{P}$, and L for a given source if the other two are known. We show that the short-term period fluctuations observed in Her X-1, Cen X-3, Vela X-1, and X Per can be accounted for quite naturally as consequences of torque variations caused by fluctuations in the mass transfer rate. We also indicate how the spin-down torque at low luminosities found here may account for the paradoxical existence of a large number of long-period sources with short spin-up time scales. Finally, we stress the need for a sequence of simultaneous period and luminosity measurements of each source. Such measurements would provide a direct check on our theory, as well as valuable information about both the spin evolution of pulsating sources and temporal variations in the mass transfer process in accreting X-ray binaries.

Subject headings: hydromagnetics — stars: accretion — stars: magnetic — stars: neutron — X-rays: binaries

I. INTRODUCTION

The interpretation of most pulsating X-ray sources as accreting neutron stars, based on the qualitative features of their spectra and their secular spin-up rates, is now relatively secure. The period changes observed in these sources are of some importance because they offer the possibility of a direct, quantitative comparison of theoretical predictions with accurate observations. Furthermore, an understanding of this phenomenon would provide an important tool for exploring other outstanding problems, such as the characteristics of mass transfer in binaries and the properties of neutron stars. Thus, for example, observations of secular period changes probe the average circulation of the accreting plasma at the magnetospheric boundary, the strength of the star's dipole field, and the size of its moment of inertia, while measurements of short-term period fluctuations probe the stability of the accretion flow, the relative inertial moments of the crust and superfluid neutron core, and the frequencies of internal collective modes (Lamb 1977; Lamb, Pines, and Shaham 1978*a*, *b*).

This is the third in a series of papers in which we are developing a quantitative theory of accretion flows and period changes in pulsating X-ray sources. In the first paper (Ghosh, Lamb, and Pethick 1977, hereafter Paper I) we investigated the flow of accreting plasma and the configuration of the magnetic field inside the

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magnetosphere. There we showed that the early dimensional estimate (Pringle and Rees 1972; Lamb, Pethick, and Pines 1973) of the sign and magnitude of the torque on slowly rotating neutron stars is correct if the transition zone at the magnetospheric boundary is narrow and that in this case this same estimate is also approximately correct even for fast rotators. This result argued strongly that the transition zone is in fact broad, since otherwise the behavior of Her X-1, which almost certainly is accreting from a disk but has a secular spin-up rate ~40 times smaller than that predicted by the slow rotator estimate (Elsner and Lamb 1976), would be difficult to understand. Thus the results of Paper I underlined the importance of calculating the size and structure of the transition zone.

In a second paper (Ghosh and Lamb 1979, hereafter Paper II) we investigated the interaction between the stellar magnetic field and the disk plasma at the magnetospheric boundary. We found that, due to growth of the Kelvin-Helmholtz instability, turbulent diffusion, and magnetic field reconnection, the magnetospheric field readily invades the disk over a broad region near its inner edge. Assuming a stationary axisymmetric flow and treating the slippage of the stellar field lines through the disk plasma by an effective conductivity, we obtained solutions to the two-dimensional hydromagnetic equations which describe the radial and vertical structure of the transition zone. These solutions show that the inner radius of the transition zone is located where the integrated stress of the stellar magnetic field becomes comparable to the integrated material stress of the disk plasma, while the outer radius is located where the electrical currents flowing in the transition zone screen the stellar magnetic field to zero. The transition zone itself is composed of two qualitatively different parts, a broad outer part where the angular velocity is Keplerian, and a narrow inner part where it departs significantly from the Keplerian value.

In the present paper we use the accretion flow solutions obtained in Paper II to calculate the torque on the neutron star and discuss the implications for pulsating X-ray sources. In § II we describe the method we use to determine the accretion torque, discuss the results of this calculation, and compare our results with those of other workers, including Scharlemann (1978) and Ichimaru (1978). We discuss the general problem of interpreting period changes in pulsating X-ray sources in § III. In § IV we consider the limited evidence regarding accretion flow patterns which is provided by the relatively sparse period and X-ray flux measurements that have been made thus far, while in § V we estimate the dipole magnetic moments of nine pulsating X-ray sources by fitting the theoretical spin-up equation to estimates of the average luminosity and spin-up rate of each source. Accretion theory predicts that fluctuations in the mass accretion rate will cause fluctuations in both the accretion luminosity and the accretion torque. In § VI we show that torque variations that arise in this way can easily be large enough to account for the period wandering observed in the well-studied pulsating sources, if these sources are fed by disks. In § VII we consider the numerous long-period sources with relatively short spin-up times and describe how the existence of many such sources may be understood, given the braking torque found in the present calculations. Finally, in § VIII we summarize our results and point to a number of critical observations. A brief account of our results has been given previously in Ghosh and Lamb (1978*a*).

II. THE ACCRETION TORQUE

In this section, we use the two-dimensional flow solutions found in Paper II to calculate the accretion torque N acting on a magnetic neutron star accreting matter from a disk. First, we outline the method of calculation. Next, we explain the results in physical terms, describe the behavior of the torque as a function of the rotation period P, and the mass accretion rate $\dot{M}$, and discuss the generality of this behavior. Finally we compare our results with other work.

a) *Method of Calculation*

As in Paper II we assume that the stellar magnetic field is dipolar with moment μ, and that the flow is steady and has axial symmetry everywhere. We generally use cylindrical coordinates (ϖ, ϕ, z) centered on the neutron star and aligned with the stellar rotation axis, but sometimes also refer to the distance $r = (\varpi^2 + z^2)^{1/2}$ from the center of the star (note that in the disk plane, $r = \varpi$).

Figure 1 shows schematically the character of the flow solutions obtained in Paper II. Between the unperturbed accretion disk and the magnetosphere there is a broad transition zone where the stellar magnetic field threads the disk. This zone divides into two parts, a broad outer part stretching from the screening radius r_s inward to r_0, and a narrow inner part or boundary layer between r_0 and the corotation radius r_{co}, inside which plasma is forced to corotate with the star. The angular velocity is Keplerian outside r_0 but then falls sharply to the stellar angular velocity at r_{co}. Matter flows from the disk plane toward the star along the bundle of magnetic field lines that thread the boundary layer.

For steady accretion, the torque on the star is given by the integral of the angular momentum flux across any surface, S, enclosing the star. If the flow is also axisymmetric, this integral may be written in the form (Lamb 1977)

$$N = \int_S \left(-\rho \boldsymbol{v}_p \varpi^2 \Omega + \varpi \frac{\boldsymbol{B}_p B_\phi}{4\pi} + \eta \varpi^2 \nabla \Omega \right) \cdot \hat{\boldsymbol{n}} dS\,, \quad (1)$$

which displays explicitly the various stresses that contribute to the total torque. Here ρ is the mass density, $\boldsymbol{v}_p$ the poloidal velocity, and Ω the angular velocity of the plasma, $\boldsymbol{B}_p$ and B_ϕ are the poloidal and azimuthal components of the magnetic field, η is the effective dynamic viscosity, and $\boldsymbol{n}$ is a unit outward normal. The three terms on the right-hand side of equation (1) represent, in turn, the contributions of

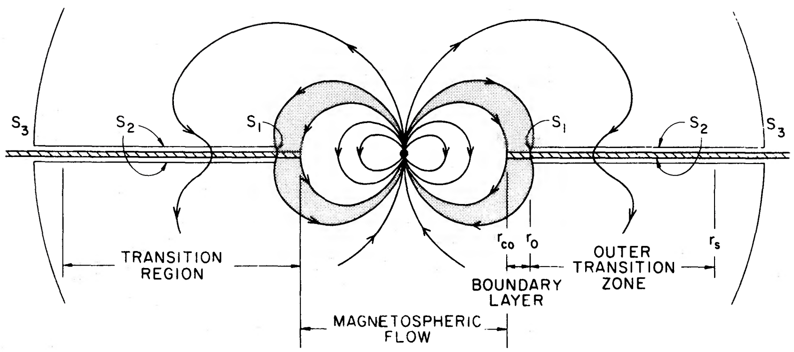

FIG. 1.—Side view of the accretion flow and the surfaces used to evaluate eq. (1), showing the transition region, composed of a broad outer zone where the angular velocity is Keplerian and a narrow boundary layer where it departs significantly from the Keplerian value, and the region of magnetospheric flow. The width $\delta = r_0 - r_{co}$ of the boundary layer is typically $\sim 0.04 r_0$ (see text). S_1 is a cylindrical surface of radius r_0 and height $2h$ while S_2 is composed of two plane surfaces just above and below the disk and S_3 comprises two hemispherical surfaces located at infinity.

the material, magnetic, and viscous stresses to the accretion torque. We note that the relative sizes of these three contributions depend on the surface used to evaluate the integral. Ultimately, however, the angular momentum flux is carried entirely by the magnetic stress in the sense that the other two contributions to the torque are completely negligible compared to the magnetic torque if the integral is evaluated on a surface that lies close to the surface of the neutron star.

In evaluating equation (1), it is convenient to choose the surface shown in Figure 1, which is composed of three parts: (1) a cylindrical surface S_1 of height $2h$ located at the radius r_0 that separates the boundary layer from the outer transition zone, (2) a surface S_1 consisting of two sheets running just above and below the disk from r_0 to infinity, and (3) two hemispherical surfaces at infinity. Here h is the semithickness of the disk. The integral over S_1 gives the torque $N_{\rm in}$ that is eventually communicated to the star by the magnetic field lines that thread the inner transition zone, while the integral over S_2 gives the torque $N_{\rm out}$ communicated by the magnetic field lines that thread the outer transition zone. The integral over S_3 vanishes.

To an excellent approximation the torque $N_{\rm in}$ is given by the material stress on S_1, since the viscous stress on S_1 is negligible by comparison (see § V of Paper II) while the magnetic stress has no component perpendicular to S_1. Now the angular velocity of the plasma at r_0 is closely Keplerian, by definition, so that

$$N_{\rm in} \approx -\rho v_r r_0^2 \Omega_{\rm K}(r_0) \cdot 2\pi r_0 \cdot 2h$$
$$= \dot{M}(GMr_0)^{1/2} \equiv N_0 \,, \tag{2}$$

where $\Omega_{\rm K}(r) = (GM/r^3)^{1/2}$ is the Keplerian angular velocity at r in terms of the mass M of the neutron star. The torque $N_{\rm out}$, on the other hand, is given by the magnetic stress on S_2, since the material stress on S_2 is negligible (no matter crosses it) while the viscous stress has no component perpendicular to S_2. Thus

$$N_{\rm out} = \int_{S_2} (rB_z B_\phi/4\pi) dS \,. \tag{3}$$

On combining contributions (2) and (3), one finds for the total torque on the star the result

$$N = N_0 + \int_{r_0}^{r_s} \gamma_\phi(r) B_z^2(r) r^2 dr \,, \tag{4}$$

where

$$\gamma_\phi \equiv -(B_\phi/B_z)_{z=h} = (B_\phi/B_z)_{z=-h} \tag{5}$$

is the average azimuthal pitch of the stellar magnetic field at the upper and lower surfaces of the disk and r_s is the outer radius of the transition zone, beyond which the stellar field is screened to zero. Equation (4) can be evaluated by using the poloidal magnetic field $B_z(r)$ given by equations (39)–(41) of Paper II and the azimuthal pitch, $\gamma_\phi(r)$, given by equation (37) of that paper. The result is

$$N = n(\omega_s) N_0 \,, \tag{6}$$

where the dimensionless accretion torque,

$$n(\omega_s) = 1 + \tfrac{1}{2}(1 - \omega_s)^{-1} \times \int_1^{y_s} b_{\rm out}(y)(y^{-3/2} - \omega_s) y^{-31/40} dy \,, \tag{7}$$

depends only on the fastness parameter (Elsner and Lamb 1977),

$$\omega_s \equiv \Omega_s / \Omega_{\rm K}(r_0) \,, \tag{8}$$

and the dimensionless outer radius of the transition zone, $y_s = r_s/r_0$. The function $b_{\rm out}(y)$ is the dimensionless poloidal magnetic field in the outer transition zone given by equation (40) of Paper II.

b) *Results*

The dimensionless accretion torque n is primarily a function of the fastness parameter ω_s, a fact we have emphasized by explicitly displaying this dependence in equation (7). For slow rotators ($\omega_s \ll 1$), one has $n(\omega_s) \approx 1.4$. For faster rotators spinning in the same direction as the disk flow, n decreases with increasing ω_s and vanishes at a certain critical fastness ω_c; for $\omega_s > \omega_c$, n is negative and becomes increasingly so with increasing ω_s. Finally, for ω_s greater than a certain maximum fastness $\omega_{\max}$ (typically ≈ 0.95) there are no stationary solutions to the two-dimensional flow equations of Paper II, and the torque on the star cannot be calculated in the manner described here.

The behavior of n as a function of ω_s can be understood as follows. The total accretion torque is the sum of the torque N_{in} eventually communicated to the star by the field lines that thread the inner transition zone and the torque N_{out} due to the twisted field lines threading the outer transition zone. N_{in} always acts to spin up a star rotating in the same sense as the disk flow, whereas N_{out} can have either sign. This is because the azimuthal pitch of the field lines threading the outer transition zone changes sign at the radius

$$r_c = (GM/\Omega_s^2)^{1/3}, \quad (9)$$

where the angular velocity Ω_K of the disk plasma is the same as that of the star, as shown in Figure 2*a*. The contribution to the torque from the field lines threading the disk between r_0 and r_c is positive whereas the contribution from the field lines threading the disk between r_c and r_s is negative. For slow rotators, $r_0 \ll r_c$, the positive part dominates the negative, and N_{out} adds a further spin-up torque $\approx 0.4 N_0$ to the torque N_{in} which is equal to N_0. In contrast, for fast rotators $r_0 \sim r_c$, the negative part dominates, and N_{out} contributes a spin-down torque which partly cancels N_{in}. For sufficiently fast rotators, the spin-down torque contributed by N_{out} dominates the spin-up torque contributed by N_{in} and there is a net spin-down torque on the star. The contribution to N_{out} made by the field lines threading the disk interior to a given radius is shown in Figure 2*b* for three values of ω_s.

For $\omega_s > \omega_{\max}$, the equations describing steady, axisymmetric inflow within the boundary layer have no solution (see § V of Paper II). The reason for this is that the centrifugal force and the force due to the magnetic pressure gradient, which is also outward, are so large that radial inflow is halted at r_0 in a distance small compared to the boundary layer width δ. We speculate that unsteady accretion may occur for values of ω_s larger than but comparable to $\omega_{\max}$, while for $\omega_s \gg \omega_{\max}$, disk accretion may cease altogether (compare Davidson and Ostriker 1973 and Lamb, Pethick, and Pines 1973).

In addition to its strong dependence on ω_s, the accretion torque also depends weakly on the four boundary layer constants C_b, C_ω, C_p, and γ_0 introduced in Paper II, through the weak dependence of y_s on these constants.[4] Although these constants are not determined by our model, appropriate values are expected to be of order unity. In order to give the reader a feeling for the uncertainty in the torque which stems from our lack of knowledge of the precise structure of the boundary layer, we show in Figures 3 and 4 the dependence of the torque curve $n(\omega_s)$ and the critical fastness ω_c on the value of γ_0, the constant to which they are most sensitive. Figure 3 shows that n is essentially independent of γ_0 for low angular velocities ($\omega_s \approx 0$) but becomes more sensitive to γ_0 as the angular velocity increases; however, even at the highest angular velocity for which there are stationary

[4] Actually, ω_s is also weakly dependent on the boundary layer constants, only three of which are linearly independent (see Paper II).

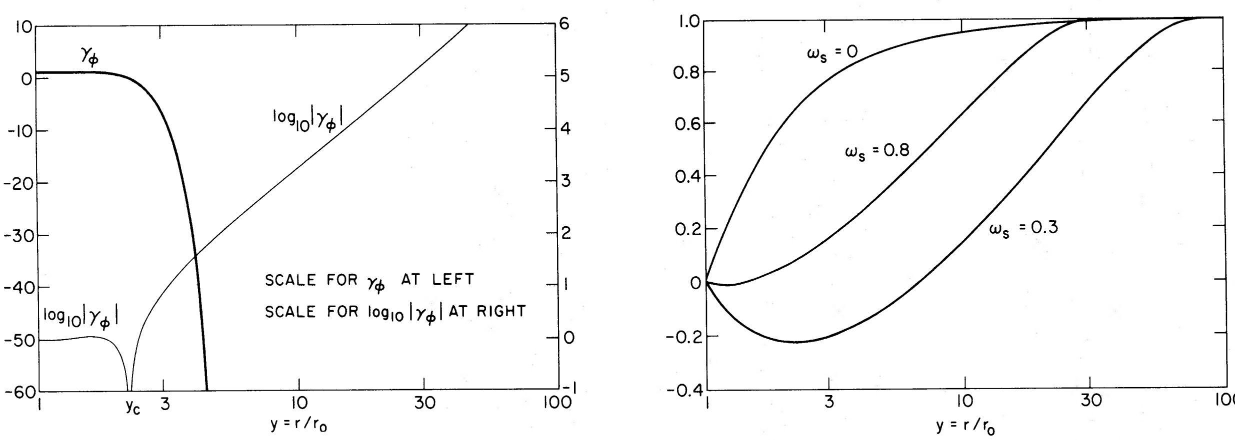

FIG. 2*a* FIG. 2*b*

FIG. 2.—Magnetic coupling between the outer transition zone and the star. (*a*) Azimuthal magnetic pitch in the outer transition zone as a function of the dimensionless radius $y = r/r_0$, for a star of fastness $\omega_s = 0.3$. The corotation point is at y_c. (*b*) Contribution to the torque N_{out} made by that part of the outer transition zone which is interior to radius y, in units of the total torque N_{out}, for three values of ω_s.

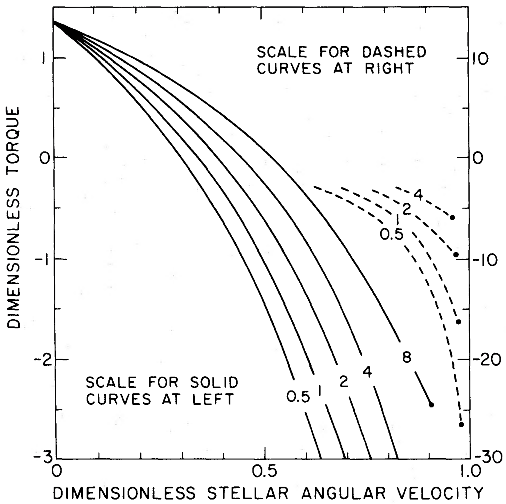

FIG. 3.—The dimensionless torque n as a function of the dimensionless stellar angular velocity or fastness ω_s, for five values of the magnetic pitch in the boundary layer, γ_0. Each curve is labeled with the corresponding value of the pitch. Those parts of the curves which continue off the bottom of the figure for large ω_s are shown as dashed curves on the reduced scale at right. The termination of each curve at the maximum fastness $\omega_{\max}$ for which a steady flow is possible, is indicated by a dot.

flow solutions ($\omega_s = \omega_{\max}$), n varies only by a factor ~ 2.7 for values of γ_0 in the range 0.5–2. The critical fastness ω_c is even less sensitive to γ_0, varying by only 20–30% for values of γ_0 in the range 0.5–2, as shown in Figure 4.[5] The boundary layer structure, and hence n, is significantly less sensitive to the other boundary

[5] A simple analytic approximation to ω_c, accurate to 1% for values of ω_c in the interval 0–0.8, is provided by the root of the equation

$$\alpha^{0.0986}(0.25 + 0.22\omega_c)\gamma_0^{0.123} = \omega_c(1 - \omega_c)^{0.173}\,,$$

where α is the viscosity parameter of the disk.

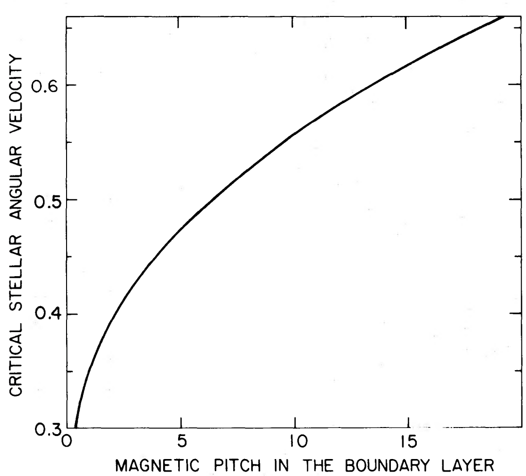

FIG. 4.—The critical value ω_c of the dimensionless stellar angular velocity or fastness parameter, as a function of the magnetic pitch in the boundary layer, γ_0.

layer constants (see Fig. 1 of Paper II). Thus the lack of precise knowledge of the boundary layer structure introduces a negligible uncertainty in the torque on slow rotators and only a moderate uncertainty in the torque on fast rotators. Throughout the remainder of this paper we shall assume $C_b = 2.5$, $C_p = 2$, and $\gamma_0 = 1$, the same values adopted for the bulk of the calculations presented in Paper II.

Once the boundary layer constants are fixed, y_s is a function only of ω_s (see Paper II) and hence the accretion torque N depends only on ω_s and N_0. A useful approximate expression for the dimensionless torque is

$$n(\omega_s) \approx 1.39\{1 - \omega_s[4.03(1 - \omega_s)^{0.173} - 0.878]\}(1 - \omega_s)^{-1}, \quad (10)$$

which is accurate to within 5% for $0 \leq \omega_s \leq 0.9$.

The fastness parameter ω_s can be expressed in terms of μ, M, Ω_s, and $\dot{M}$ by using expression (30) of Paper II for r_0, namely,

$$r_0 \approx 0.52 r_A^{(0)} = 0.52\mu^{4/7}(2GM)^{-1/7}\dot{M}^{-2/7}\,. \quad (11)$$

Here $r_A^{(0)}$ is the characteristic Alfvén radius for spherical accretion (Elsner and Lamb 1977). On substituting this expression into equation (8) one obtains

$$\omega_s = 1.19 P^{-1}\dot{M}_{17}^{-3/7}\mu_{30}^{6/7}(M/M_\odot)^{-5/7}\,, \quad (12)$$

where P is the spin period in seconds, $\dot{M}_{17}$ is $\dot{M}$ in units of 10^{17} g s^{-1}, and μ_{30} is μ in units of 10^{30} gauss cm^3. In equations (11) and (12) and those that follow, one should use a value of μ slightly larger than the unscreened dipole moment of the neutron star since the screening currents flowing in the boundary layer enhance the magnetic field within the magnetosphere. An accurate determination of the appropriate correction factor, which is expected to be of order unity (see Paper II), must await a detailed calculation of the accretion flow between r_0 and the flow-alignment radius r_f (again see Paper II).

c) *Discussion*

Equations (2) and (10)–(12) show that the accretion torque on a neutron star of given mass and magnetic moment depends on both the mass accretion rate, $\dot{M}$, and the star's spin period, P, through the dependence of N_0, r_0, and ω_s on these quantities. However, the behavior of the accretion torque can be simply described in the two following situations of some astrophysical interest.

Consider first a star rotating in the same direction as the disk and accreting at a constant rate. Such a star can experience either spin-up or spin-down, depending on its spin period. If P is sufficiently long, ω_s is small compared to unity, and the star experiences a strong spin-up torque $\sim 1.4N_0$. As P decreases, ω_s increases, and the spin-up torque falls, vanishing at the critical spin period P_c at which $\omega_s = \omega_c$. If, on the other hand,

P is less than P_c, the star experiences a spin-down torque. As P increases, ω_s decreases, and the spin-down torque diminishes, vanishing at P_c. Thus the spin period of such a star will approach the critical period P_c that corresponds to its accretion rate, and will then remain there. For P less than the period P_{min} at which $\omega_s = \omega_{max}$, accretion, if it occurs, is not steady.

In contrast, the accretion torque on a star of constant spin period varies with the accretion rate as shown in Figure 5. If $\dot{M}$ is sufficiently large, ω_s is small compared to unity, defining the star as a slow rotator, and the star experiences a strong spin-up torque $\sim 1.4N_0$. As $\dot{M}$ decreases, the fastness ω_s increases, and the spin-up torque falls, vanishing at the critical accretion rate $\dot{M}_c$ at which $\omega_s = \omega_c$. For accretion rates less than $\dot{M}_c$, the star experiences a spin-down torque, the magnitude of which increases steadily until $\dot{M}$ reaches the minimum accretion rate $\dot{M}_{min}$ consistent with steady accretion. At this accretion rate $\omega_s = \omega_{max}$. If accretion continues at mass flow rates less than $\dot{M}_{min}$, it is not steady.

To what extent does this behavior depend on the approximations inherent in the present model and to what extent is it likely to be a general feature of disk accretion? We consider the generality of our results first in the context of stationary axisymmetric flows and then in the wider context of more general flows.

As discussed in Paper II, the structure of the transition zone that forms the basis for our calculation of

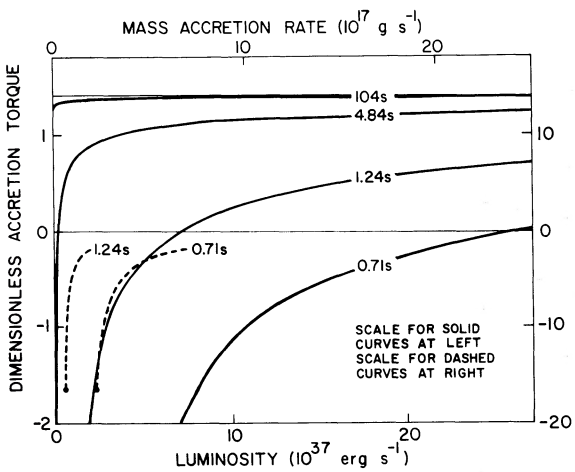

FIG. 5.—The dimensionless accretion torque n on model neutron stars of selected periods as a function of the mass accretion rate $\dot{M}$, in units of 10^{17} g s^{-1}, or the accretion luminosity L, in units of 10^{37} ergs s^{-1}. Each star has a magnetic moment $\mu = 10^{30}$ gauss cm^3 and a mass $M = 1.3\ M_\odot$, and has been constructed using the TI equation of state of Pandharipande, Pines, and Smith (1976). The period of each star is written on the corresponding curve. The periods are those of the sources SMC X-1, Her X-1, Cen X-3, and A0535+26. Those parts of the solid curves that go off scale for low accretion rates are shown as dashed curves on the reduced scale at right. The termination of each curve at the minimum accretion rate consistent with steady inflow is marked by a dot. The light solid line above the curve for 104 s is the asymptotic behavior obtained in the slow rotation limit, $P \rightarrow \infty$.

the torque, namely a narrow inner zone where most of the screening occurs together with a broad outer zone where the residual stellar flux threads the disk, appears to be a general feature of steady axisymmetric disk accretion. Moreover, the radius of the inner edge of the disk and the width of the inner transition zone do not depend on the details of the dissipative process in the boundary layer, but only on the approximate isotropy of the effective conductivity and the reasonable assumption that $\gamma_\phi \sim 1$ at the radius r_0 where the magnetic field begins to control the flow. Thus the contribution of N_{in} to the total torque is accurately given by equations (2) and (11). In contrast, N_{out} depends on the configuration of the magnetic field in the outer transition zone. Although the fraction (~ 0.2) of the total stellar flux that threads the outer transition zone appears to be relatively insensitive to the structure of the boundary layer, the azimuthal magnetic pitch in the outer zone is sensitive to the details of the magnetic field dissipation process there. Furthermore, according to the present model of the outer transition zone the azimuthal pitch increases steeply with radius beyond the disk corotation point. As a result, for some values of ω_s a substantial contribution to N_{out} comes from radii as large as 10–20r_0, where the pitch is very large. Therefore, should improvements in the model lead to a smaller outer transition zone (see Paper II), N_{out} would be somewhat reduced and ω_c somewhat increased.

These considerations suggest that for slow rotators, where N_{in} and N_{out} are additive and N_{out} is small, the torque given by the present model is likely to be fairly accurate. For fast rotators, on the other hand, N_{in} and N_{out} have opposite signs, N_{out} is large, and the present calculation is likely to be less accurate. Nevertheless, the qualitative behavior of the torque found here, including the braking torque on fast rotators, appears to be a general feature of steady axisymmetric disk accretion.

Finally, consider briefly the torque produced by more general accretion flows. In reality, disk accretion by aligned rotators is probably unsteady, at least on sufficiently small spatial and temporal scales. Even so, stationary flow models may provide an adequate description of the average accretion torque. In the case of oblique rotators, the flow is of necessity time-dependent and the coupling between the star and the disk altered from that of the aligned case (see Paper II). Nevertheless, we expect the qualitative structure of the flow to be similar to that of the present model, with a relatively narrow shear boundary layer and a more extended region where the disk and the star are magnetically coupled. If so, the accretion torque will be qualitatively similar to that found here.

d) *Comparison with Other Work*

The accretion torque on magnetic neutron stars was considered in the very first work on such stars. Lamb, Pethick, and Pines (1973; see also Pringle and Rees 1972) gave arguments which showed that the torque on slow rotators accreting from a disk is $\sim N_0$. In comparing observed changes in the rotation rates of accreting neutron stars with this theoretical estimate, Elsner and Lamb (1976) drew attention to the fact that Her X-1, which almost certainly is accreting from a disk, has a spin-up rate ~ 40 times smaller than that which would correspond to the torque N_0. As a possible explanation of this discrepancy they noted that if Her X-1 were a fast rotator ($\omega_s \sim 1$), the magnetic and viscous stresses at the inner edge of the disk would tend to cancel the material stress there even if matter were not ejected, so that the total torque would be less than the material torque N_0. They argued further that as the fast rotation limit is approached ($\omega_s \to 1$), the magnetic and viscous stresses might more than offset the material stress, causing a net braking torque on the star even while accretion continues.

Motivated in part by this discrepancy, in Paper I we sought to make the torque argument of Lamb, Pethick, and Pines more precise. There we showed that for a steady axisymmetric flow with a general dependence of Ω on r outside the Alfvén surface, the total torque is $\sim \dot{M} r_A{}^2 \Omega(r_A)$ if (1) the effective viscosity at r_A is less than a certain reference value or (2) the transition zone has a width less than or equal to its radius, magnetic stresses within it are negligible, and the stellar angular velocity is less than $\Omega(r_A)$. For stars accreting from a disk we showed further that if the transition zone were narrow compared to its radius, then the total accretion torque would be bounded above by the torque

$$N_{\max} = N_0 \tag{13}$$

and below by the torque

$$N_{\min} = \tfrac{1}{2}(1 + \omega_s) N_0 + O(h/r_0)^2 \,. \tag{14}$$

Thus, for disks rotating in the same sense as the star, a narrow transition zone would imply a spin-up torque $\sim N_0$ even on fast rotators. This result argued strongly that the transition zone in disk accretion must be broad, since otherwise the Her X-1 spin-up rate would be extremely difficult to understand.

As an alternative way to resolve the apparent discrepancy between the torque N_0 expected to act on slow rotators and the much smaller torque apparently experienced by Her X-1 and some other accreting neutron stars, Scharlemann (1978) suggested that the transition zone in disk accretion is narrow (width $\sim$ height) but that the torque is always comparable to

$$N_1 \equiv \dot{M} r_0{}^2 \Omega_s = \omega_s N_0 \,.$$

On this basis Scharlemann argued that all seven of the stars that he considered, including Her X-1, were slow rotators with weak dipole magnetic fields. Unfortunately, this suggestion cannot be correct, for the following reason. Scharlemann showed that if one makes the ad hoc assumption that the magnetic field lines threading the neutron star all have positive pitch ($\gamma_\phi \geq 0$), then the accretion torque cannot be *less* than N_1, but did not show that the torque N_1 could ever be achieved. In fact, for a narrow boundary layer and slow rotation the accretion torque can never be as small as N_1, since it is bounded below by the much larger torque $N_{\min}$, as shown in Paper I (a torque as small as N_1 would require a substantial violation of energy conservation).

In Ghosh and Lamb (1978*a*) and Paper II we showed that there need not be any discrepancy between the predicted and observed spin-up rates of sources like Her X-1, since the transition zone in disk accretion generally is *not* narrow, but is instead rather broad. Hence the bounds (13) and (14) do not apply. Here we have used the two-dimensional hydromagnetic flow solutions obtained in Paper II to calculate the accretion torque on the star. These calculations show that the magnetic coupling between the disk and the star due to the stellar field lines that thread the disk in the outer transition zone is appreciable and can be the dominant component of the accretion torque when the stellar angular velocity is high. The coupling via these field lines increases the spin-up torque on slow rotators, but reduces the torque on fast rotators and may even cause the total torque to become negative (braking the star's rotation) if the angular velocity is sufficiently high.

Recently Ichimaru (1978) attempted a calculation of the torque on a neutron star accreting from a disk. Unfortunately, Ichimaru's model is defective in important respects, some of which were noted in Paper II. Among those which are the most serious for his calculation of the accretion torque are the following. First, Ichimaru's equation (29) assumes that the flux of angular momentum toward the star is exactly zero. This assumption is far too restrictive; it implies that there is never any change in the angular momentum of the star and hence that the neutron star generally spins down as it accretes, since the added matter increases the star's moment of inertia. Second, Ichimaru assumes that the transition zone is narrow but that the torque on the star can be much less than the lower bound $N_{\min}$ given above. As noted above, this would require a substantial violation of energy conservation.

In closing this comparison with previous work, we note that the spin-down torque on fast rotators found here is quite distinct from possible spin-down torques associated with ejection of matter by very fast rotators which have been discussed previously by several authors (Davidson and Ostriker 1973; Illarionov and Sunyaev 1975; Fabian 1975; Shakura 1975; Kundt 1976; Savonije and van den Heuvel 1977). The spin-down torque found here operates on stars with angular velocities which lie between the critical angular velocity and the maximum allowable angular velocity consistent with steady accretion, does not involve any mass ejection from the vicinity of the neutron star, and acts even while steady accretion continues. On the other hand, both the magnetic spin-down torque suggested by Davidson and Ostriker (1973) and the "propeller" spin-down torques suggested by Illarionov and Sunyaev (1975) and Shakura (1975) were assumed to operate only on very fast rotators ($\omega_s > 1$), and both involve mass ejection from the vicinity of the neutron star in an essential way. Nevertheless, at the maximum allowable fastness, ω_{max}, the torque calculated here roughly agrees with the spin-down torques conjectured by Davidson and Ostriker, Illarionov and Sunyaev, and Shakura, evaluated at $\omega_s = 1$ (at this fastness all the latter torques are equal). Thus it may be possible to develop a consistent description of the torque on rotating stars by using the present model for $\omega_s < 1$ and models involving some mass ejection for $\omega_s > 1$.

III. INTERPRETING PERIOD CHANGES IN PULSATING X-RAY SOURCES

Most, and perhaps all, pulsating X-ray sources other than pulsars are accreting neutron stars in which the pulsation period is the rotation period of the neutron star crust (see Lamb 1977). In all sources that have been studied carefully, the pulsation period has been found to change with time (see Schreier 1977). These changes are thought to be due to changes in the angular momentum, and hence the rotation period, of the crust. According to current ideas, the change in period over a sufficiently long time is due to the action of the external accretion torque (Pringle and Rees 1972; Lamb, Pethick, and Pines 1973), while period fluctuations on shorter time scales are caused either by fluctuations in the accretion torque (Elsner and Lamb 1976) or by variations in the torque exerted on the crust by the liquid interior (Lamb, Pines, and Shaham 1978*a*).

The external torque depends on the inflow rate and flow pattern of the accreting plasma, while the internal torque depends on the strength and nature of the coupling between the liquid interior and the crust. The response of the neutron star to a torque acting on the crust, whether external or internal, is expected to depend on the dynamical properties of the star: the "applied signal" represented by the torque is, in effect, "filtered" by the coupled crust-core system to produce an "output" represented by changes in the pulsation period. Thus, a detailed study of period changes in a given X-ray source can provide valuable information both about the properties of the accretion flow onto the star and about the properties of the star itself.

In the following subsections we first describe the nature of the interpretational problem in more detail, outline a possible solution, and discuss the comparison of theory with observation. We then consider the pulsation period changes predicted by current models of Keplerian and non-Keplerian accretion flows. Finally, we summarize how the goals listed above can be accomplished.

a) *The Nature of the Problem and a Possible Solution*

Period measurements made at intervals ranging from days to years have been reported for nine of the 17 presently known pulsating X-ray sources (see Giacconi 1974; Fabbiano and Schreier 1977; Mason 1977; Ögelman *et al.* 1977; Rappaport and Joss 1977; Schreier 1977; White, Mason, and Sanford 1977; Becker *et al.* 1978; Charles *et al.* 1978; Rappaport *et al.* 1978; Boynton and Deeter 1979; Jernigan and Nugent 1979). The sources whose period derivatives have been measured in this way have so far been found to be spinning up, over the long term. In addition, the four sources that have been studied most carefully (Her X-1, Cen X-3, Vela X-1, and X Per) show substantial short-term period fluctuations with occasional episodes of spin down. Figure 6 shows the behavior of Cen X-3, which is typical. Figure 7 shows recent data on Vela X-1 which suggest that its period behavior is similar to that of Her X-1, Cen X-3, and X Per.

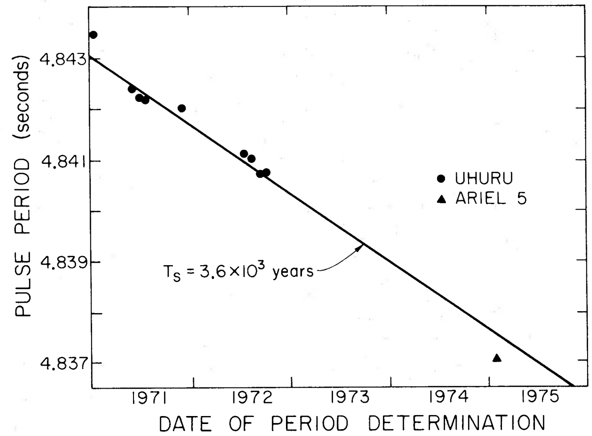

FIG. 6.—The behavior of the pulsation period of Cen X-3, showing the short-term period fluctuations and secular period decrease. Note the period increase of 1972 September-October. After Fabbiano and Schreier (1977).

The interpretation of these period changes is complicated by the fact that the response of the neutron star interior is not known *a priori*. Hence, such changes might be due either to the internal and external torques acting at the time, or to the response of the star to previous values of these torques. If the change in the rotation rate were smooth and the total observing interval were long, the likelihood that the star was

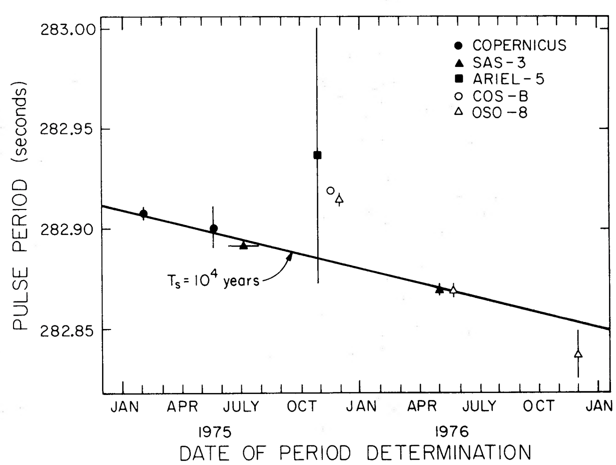

FIG. 7.—The behavior of the pulsation period of Vela X-1, showing the short-term period fluctuations and secular period decrease. Note the period increase of 1975 November. After Becker *et al.* (1978).

still responding to earlier values of these torques would be less, although the interpretation would still be ambiguous. In fact, the period changes in those sources that have been studied carefully are observed to be highly irregular.

Recently, Lamb, Pines, and Shaham (1978*a*) pointed out that if the torque fluctuations that cause the short-term period variations can be described by a simple noise process, then these interpretational difficulties can be overcome by using the period variations themselves to determine the dynamical response of the neutron star. These authors further suggested that either white or red torque fluctuations are physically plausible models and showed that the available data on Her X-1 and Cen X-3 are consistent with either type of torque noise. Motivated by these studies, Boynton and Deeter (1979) used the *Uhuru* data on Her X-1 to compute the power density spectrum of the pulse phase fluctuations in this source. They find that the torque fluctuations in Her X-1 are describable as white noise over a wide range of frequencies and derive significant constraints on the dynamical response of the neutron star from their analysis. Thus, this method of determining the neutron star response appears quite promising. Use of this method requires a dense, regular sequence of pulse phase measurements.

Measurements of the short-term behavior of the pulse phase and X-ray flux also provide the best type of data for probing the accretion flow pattern and the large-scale structure of the magnetosphere. First, accretion theory predicts that the change in spin period between two observations depends sensitively on the behavior of the accretion luminosity L during the interval. If L is highly variable, as it often is, spot checks at widely spaced times may lead to very large errors in the inferred luminosity history $L(t)$. Such errors can be minimized by measuring the period and flux at frequent intervals. Second, theoretical models necessarily include a number of poorly known parameters, such as the mass, radius, effective moment of inertia, and magnetic dipole moment of the star. Although the uncertainty in these parameters does not permit one to fit every possible value of P, $\dot{P}$, and L, in practice the possibility of adjusting some of these parameters introduces a substantial degree of freedom. If only the average values of $\dot{P}$ and L are known, there is often not enough information to test the theory. However, once these parameters are determined, there is no more freedom, so that additional measurements of P, $\dot{P}$, and L provide a quantitative test of the theory. Finally, since the effective inertial moment of the star generally is not constant, one must combine the theoretical relation for the accretion torque as a function of X-ray luminosity with the observed luminosity behavior $L(t)$ and a dynamical model of the neutron star crust and core in order to solve for the predicted period behavior $P(t)$ of the neutron star crust. Only then can a comparison be made with the observed period behavior. This procedure can be carried out only if a dense, regular sequence of period and flux measurements is available.

In summary, only studies of the short-term behavior of the pulse phase and flux of neutron star X-ray sources seem likely to provide the type of data required to test current theoretical models of their magnetospheres and interiors.

b) *Comparing Theory and Observation*

Although measurement of pulse period fluctuations appears to be the most promising method for obtaining information about the magnetospheric structure and accretion flow pattern of a given source, the extraction of this information from such measurements clearly requires some care. First, the period fluctuations must be shown to be due to fluctuations in the accretion torque rather than internal torques. Second, the response of the neutron star must be considered in interpreting the data, since the response time of the liquid interior may be comparable to the time scale for changes in the accretion torque.

Accretion torque fluctuations can potentially be identified by searching for correlated changes in the pulse period P and the X-ray flux F at Earth, since accretion theory predicts that fluctuations in the mass accretion rate will cause fluctuations in both the accretion luminosity and the accretion torque; correlated changes in P and F are much less likely if the period changes are caused by internal torque fluctuations.[6] Fluctuations in the accretion rate appear quite natural and, as we show in § VI, can produce torque variations large enough to account for for the period wandering observed in the well-studied pulsating sources. We shall therefore focus on torque variations that arise in this way.

To the extent that the dominant torque variations can be described by a simple noise process, one can disentangle the variations in the torque from the time-dependent response of the star in the manner described by Lamb, Pines, and Shaham (1978*a*). Once this is accomplished, a sequence of pulse period and X-ray flux measurements can potentially be used to (1) confirm that the X-ray source is indeed a neutron star, (2) determine whether the source is fed by a Keplerian accretion disk or by some other accretion flow pattern, (3) test quantitatively the theory of disk accretion, (4) determine accurately the dipole moment of the X-ray star, and (5) establish the nature of the accretion torque fluctuations.

The first step in achieving these goals is to construct a theoretical relation between the X-ray flux $F(t)$ at Earth and the pulse period $P(t)$. Such a relation can be constructed if one has available (1) a relation between F and the accretion luminosity L, (2) a relation between L and the mass accretion rate $\dot{M}$, (3) a relation between $\dot{M}$ and the torque N, and (4) a model for the change in the rotation of the neutron star crust caused by N. Assuming that the X-ray flux at Earth accurately reflects the X-ray luminosity and that the latter is

[6] We emphasize that the *absence* of correlated changes would not necessarily imply that accretion torque fluctuations are absent, since accretion flow variations different from those considered here could conceivably produce a change in the torque with little or no change in the mass accretion rate.

essentially the accretion luminosity L, then $F = L/4\pi D^2$, where D is the distance to the source, while $L = \dot{M}(GM/R)$. One can then turn to accretion theory for a relation between $\dot{M}$ and N, such as equation (6). Finally, the stellar properties required to determine the change in the rotation period P of the neutron star crust produced by the torque N are fixed by the power density spectrum of pulse phase fluctuations.

Once a theoretical relation between $F(t)$ and $P(t)$ has been constructed, it can be tested by comparison with a sequence of X-ray flux and pulse period measurements. To illustrate how this approach can be applied, we shall discuss briefly the spin-up equations given by accretion theory (*a*) for disk-fed sources and (*b*) for sources that accrete matter which has insufficient angular momentum to form a disk, which we refer to as "wind-fed" sources.[7] For simplicity we shall assume that the neutron star responds like a rigid body with an effective moment of inertia I_{eff}, but we note that such a simple model can only be justified by a measurement of the dynamical response of the star over the time scales of interest.

c) *Accretion from a Disk*

To the extent that the effective inertial moment of the neutron star is constant, the model of disk accretion described in Paper II predicts a simple relation between the spin-up rate, $-\dot{P}$, and the quantity $PL^{3/7}$, for a given source. This relation follows from equation (2) and the equation for the change in the stellar angular velocity Ω_s produced by the torque N. If we neglect the typically small effect of the change in the effective moment of inertia due to accretion (see Paper I), the latter becomes $\dot{\Omega}_s = N/I_{\text{eff}}$. Combining these two equations yields

$$-\dot{P} = 5.0 \times 10^{-5}\mu_{30}{}^{2/7}n(\omega_s)S_1(M)(PL_{37}{}^{3/7})^2 \text{ s yr}^{-1}\,, \tag{15}$$

where the function $n(\omega_s)$ is given by equation (7) or, approximately, by equation (10), and

$$\omega_s \approx 1.35\mu_{30}{}^{6/7}S_2(M)(PL_{37}{}^{3/7})^{-1}\,. \tag{16}$$

Here

$$S_1(M) = R_6{}^{6/7}(M/M_\odot)^{-3/7}I_{45}{}^{-1} \tag{17}$$

and

$$S_2(M) = R_6{}^{-3/7}(M/M_\odot)^{-2/7} \tag{18}$$

are structure functions that depend on the mass, equation of state, and dynamical response of the neutron star. L_{37} is the accretion luminosity in units of 10^{37} ergs s^{-1}, R_6 is the stellar radius R in units of 10^6 cm, and I_{45} is I_{eff} in units of 10^{45} g cm^2. Since n depends only on ω_s (see § II) and ω_s scales as $(PL^{3/7})^{-1}$, as shown by equation (16), *the value of* $\dot{P}$ *for a star of given mass and magnetic moment is a function only of* $PL^{3/7}$.

[7] Note that a neutron star whose binary companion is losing matter via a wind need not be wind-fed, since the matter that is accreted may still have sufficient angular momentum to form a disk (see, for example, Petterson 1978; Savonije 1978).

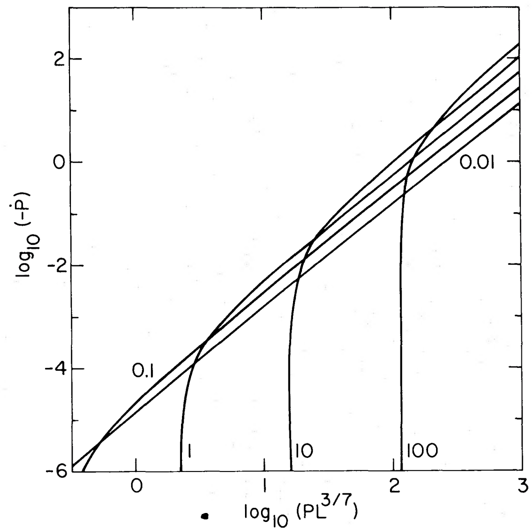

FIG. 8.—The theoretical relation between the spin-up rate $-\dot{P}$ of a 1.3 $M_\odot$ PPS neutron star and the quantity $PL^{3/7}$, for five values of the stellar dipole magnetic moment, μ. Each curve is labeled with the corresponding value of the magnetic moment in units of 10^{30} gauss cm^3. The units of $-\dot{P}$, P, and L are s yr^{-1}, s, and 10^{37} ergs s^{-1}, respectively.

The character of the relation between $-\dot{P}$ and $PL^{3/7}$ is shown in Figure 8. For large values of $PL^{3/7}$, the star is a slow rotator ($\omega_s \ll 1$), $n(\omega_s)$ is approximately constant (see eq. [10]), and $-\dot{P}$ scales as $(PL^{3/7})^2$. Thus, in a plot of log $(-\dot{P})$ versus log $(PL^{3/7})$, equation (15) predicts a straight line of slope 2 in the region of slow rotation. As $PL^{3/7}$ decreases, the fastness parameter ω_s becomes larger, $n(\omega_s)$ decreases, and log $(-\dot{P})$ falls below the extrapolation of this line. Finally, at the value of $PL^{3/7}$ for which $\omega_s = \omega_c$, $\dot{P}$ vanishes and log $(-\dot{P})$ diverges. The value of $PL^{3/7}$ at which the spin-up curve begins to fall below the extrapolated straight line depends on the magnetic moment of the star, and scales as $\mu^{6/7}$.

d) *Accretion from a Wind*

When an accretion disk does not form, as may happen if the neutron star accretes from a wind, the radial velocity of the accreting plasma near the accretion capture radius r_a is expected to be large, its shear is expected to be small, and the stellar magnetic field is expected to be confined to the interior of a magnetospheric cavity whose radius is much smaller than r_a (Lamb, Pethick, and Pines 1973; Arons and Lea 1976; Elsner and Lamb 1977). As a result, if the surface in equation (1) is placed at r_a, the material stress completely dominates and the torque is just

$$N = \dot{M}l_a\,, \tag{19}$$

where $\dot{M}$ is the mass capture rate and l_a is the specific angular momentum of the accreting plasma at r_a. The capture rate is given by

$$\dot{M} = \pi \rho_W v_0 r_a^2 , \qquad (20)$$

where ρ_W is the wind density and $v_0^2 \approx v_{\rm orb}^2 + v_W^2 + c_s^2$ is the square of the capture velocity. Here $v_{\rm orb}$ is the orbital velocity of the neutron star and v_W and c_s are the wind velocity with respect to the companion star and the local sound speed, evaluated at r_a. The specific angular momentum l_a can be expressed as (Shapiro and Lightman 1976)

$$l_a \approx \tfrac{1}{2} a v_{\rm orb} (r_a/a)^2 , \qquad (21)$$

where a is the binary separation. Finally, the capture radius is given by

$$r_a = \xi(2GM/v_0^2) , \qquad (22)$$

where ξ is a parameter of order unity. Thus equation (19) can be rewritten as

$$N = \pi^2 \xi^2 (2GM)^4 \rho_W v_0^{-7} P_{\rm orb}^{-1} , \qquad (23)$$

where $P_{\rm orb}$ is the orbital period of the binary system. An equation for wind accretion analogous to equation (15) for disk accretion is

$$-\dot{P} = 3.8 \times 10^{-5} R_6 (M/M_\odot)^{-1} I_{45}^{-1} \times (l_a/10^{17}\ {\rm cm^2\ s^{-1}}) P^2 L_{37}\ {\rm s\ yr^{-1}} . \qquad (24)$$

$\dot{P}$ is independent of the stellar magnetic moment because the magnetosphere plays no role in determining either $\dot{M}$ or l_a.

As the physical properties of the wind change in time, both $\dot{P}$ and L will vary. The behavior of $\dot{P}$ as a function of L depends on how ρ_W and v_0 vary with changes in the wind:

1. *The wind density ρ_W varies but $v_0 = const$.* This will be the case if $v_{\rm orb}^2 \gg v_W^2 + c_s^2$. In this case $r_a =$ const., $l_a =$ const., $\dot{M} \propto \rho_W$, and $-\dot{P} \propto L$.

2. *The velocity v_0 varies but $\rho_W = const$.* In this case $r_a \propto v_0^{-2}$, $l_a \propto v_0^{-4}$, $\dot{M} \propto v_0^{-3}$, and $-\dot{P} \propto L^{7/3}$.

3. *Both ρ_W and v_0 vary, but $\rho_W v_0 = const$.* In this case $r_a \propto \rho_W^2$, $l_a \propto \rho_W^4$, $\dot{M} \propto \rho_W^4$, and $-\dot{P} \propto L^2$.

The theory of winds in X-ray binaries is not sufficiently well developed to determine which of these (or other) possibilities for the variation of ρ_W and v_0 is correct (Conti 1978; Vitello 1979). Thus, at present wind theory cannot predict the behavior of $\dot{P}$ as a function of L for a given source.

e) Summary

These results point to the importance of obtaining a frequent, regular sequence of period and flux measurements in developing an understanding of a given source. Such a sequence can provide information about the accretion flow pattern and the X-ray star that is impossible to obtain from average values or from any single measurement.

If the effective moment of inertia of the neutron star is essentially constant over the time scales of interest, it is appropriate to plot the observed values of P, $\dot{P}$, and L on a graph of $\log(-\dot{P})$ vs. $\log(PL^{3/7})$. If the result can be described qualitatively by equation (15), this would constitute strong evidence that the X-ray source is a neutron star which is disk-fed and that the period variations are due largely to variations in the mass accretion rate. The shape of the curve would then give the size of the dipole magnetic moment and the stellar structure functions $S_1(M)$ and $S_2(M)$, while a detailed comparison of the data with the theoretical curve would provide a quantitative test of the theory.

If the inertial moment is *not* constant over the time scales of interest, one must instead combine the more fundamental equation for the accretion torque as a function of the mass accretion rate, equation (6), with the relation between $\dot{M}$ and L, the observed luminosity behavior $L(t)$ of the star, and the equations of motion of the neutron star crust and core, in order to solve for the rotation rate of the crust. The solution can then be compared with the observed period behavior $P(t)$ of the crust. Once again, if there is qualitative agreement between the predicted and observed period behavior, this would be strong evidence that the source is a disk-fed neutron star and that the period variations are due largely to variations in the mass accretion rate, and would allow a determination of the magnetic moment and structure of the star, as well as a quantitative test of the theory.

Finally, a search of pulse arrival times for relatively rare, very large pulse phase changes might allow one to resolve the torque noise process. If the noise process can be at least partially resolved, a detailed comparison of the time history of the pulse period and X-ray flux with the theoretical model can confirm that the largest torque fluctuations are caused by fluctuations in the mass accretion rate and determine the sign distribution of these torque excursions, their characteristic rise and fall times, and their mean rate of occurrence as a function of size. This information can then be compared with the expected properties of accretion torque fluctuations.

In the following sections we examine in turn the evidence furnished by the currently available data regarding accretion flow patterns, neutron star magnetic fields, and the origin of pulse period fluctuations. Where the value of $I_{\rm eff}$ is needed, we take it to be the inertial moment of the whole star.

IV. ACCRETION FROM DISKS OR WINDS?

In the previous section we have seen that a sequence of accurate period and luminosity measurements can establish unambiguously whether any given source is accreting from a disk. Unfortunately, such sequences are not yet available. Nevertheless, some evidence on accretion flow patterns can be obtained by comparing the currently available *time-average* values of P, $\dot{P}$, and L with theoretical predictions for a collection of sources. In the present section we consider this evidence.

a) The Disk Hypothesis

If we consider a *collection* of pulsating X-ray sources, equation (15) predicts that they would all lie

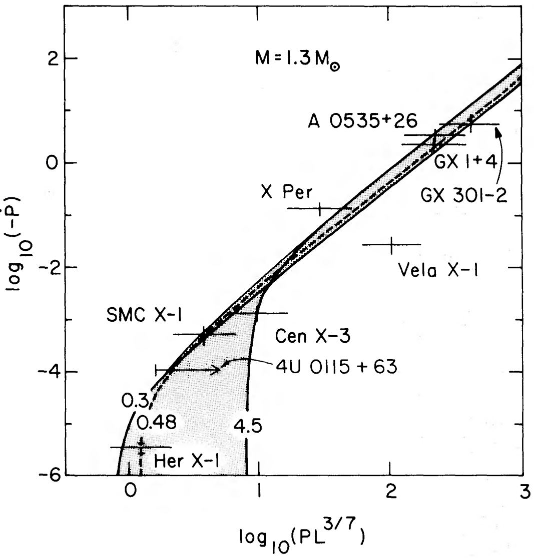

FIG. 9.—The theoretical relation between the spin-up rate, $-\dot{P}$, and the quantity $PL^{3/7}$, superposed on the data for nine pulsating X-ray sources. The units of $-\dot{P}$, P, and L are s yr^{-1}, s, and 10^{37} ergs s^{-1}, respectively. Shown are the theoretical curves for three values of the stellar magnetic moment, assuming a 1.3 $M_\odot$ PPS model for all neutron stars. Each curve is labeled with the corresponding value of the magnetic moment, in units of 10^{30} gauss cm^3. *Shaded area*, region spanned by the theoretical curves for $0.3 \leq \mu_{30} \leq 4.5$. *Dashed line*, theoretical curve for $\mu_{30} = 0.48$, which gives a rough best fit to the data. Because the curves corresponding to different magnetic moments cross (see text), the upper boundary of the shaded region is defined by the envelope of the curves (*light solid line*).

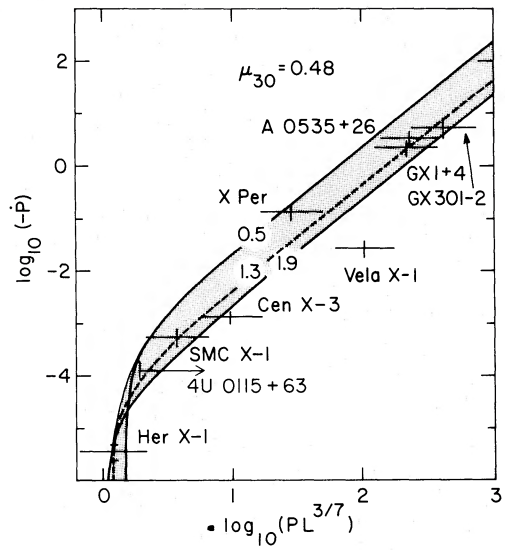

FIG. 10.—Same as Fig. 9, but showing the effect of varying the neutron star mass, assuming a stellar magnetic moment $\mu_{30} = 0.48$ and the PPS equation of state for all stars. Each curve is labeled with the corresponding value of $M/M_\odot$. *Shaded area*, region spanned by the theoretical curves for $0.5 \leq M/M_\odot \leq 1.9$. *Dashed line*, theoretical curve for $M/M_\odot = 1.3$, which gives a rough best fit to the data. Because the curves corresponding to different stellar masses cross, the upper boundary of the shaded region is defined by the envelope of the curves (*light solid line*).

on the same curve $-\dot{P} = f(PL^{3/7})$ if they all had (1) the same mass and (2) the same magnetic moment. Although all pulsating X-ray sources are not expected to have identical masses and magnetic moments, there should still be a correlation between $\dot{P}$ and $PL^{3/7}$ if the spread in masses and magnetic moments is not too large. It is therefore interesting to plot the logarithms of the observed values of $-\dot{P}$ against the logarithms of the observed values of $PL^{3/7}$, as in Figures 9 and 10. Such a plot tends to order the sources according to their fastness, since for fixed M and μ the fastness parameter ω_s is a function only of $PL^{3/7}$. The data used in constructing these figures are given in Table 1.

The effect of varying the stellar magnetic moment is shown in Figure 9, which displays the function $f(PL^{3/7})$ for $M = 1.3\ M_\odot$, assuming the tensor interaction (TI) equation of state of Pandharipande, Pines, and Smith (1976, hereafter PPS), and various values of the stellar magnetic moment. Also shown are the observed data on nine pulsating X-ray sources. The curve for $\mu_{30} = 0.48$ is a rough best fit to the data. Except for Vela X-1 = 4U 0900−40, all the sources lie in the region spanned by the curves corresponding to values of μ_{30} in the range 0.3–4.5. Note that the curves for different values of μ cross one another, so that the upper edge of this region is given by the envelope of the various curves, which is indicated by the light solid line in the figure. The crossing of the curves for different values of μ occurs because the higher moment gives the larger torque and hence the larger spin-up rate in the slow rotator region (large values of $PL^{3/7}$), but also gives the larger critical value of $PL^{3/7}$ (see eq. [16]), so that the spin-up rate in the fast rotator region falls more rapidly for the higher moment than for the lower one.

The effect of varying the stellar mass is illustrated in Figure 10, which shows the function $f(PL^{3/7})$ for $\mu_{30} = 0.48$, the best-fit value of Figure 9, and various values of the stellar mass, again assuming the TI equation of state. Once again the curves for different masses cross one another. All sources except Vela X-1 lie in the region spanned by stellar masses in the range 0.5–1.9 $M_\odot$ (the latter is the maximum stable mass for stars obeying the TI equation of state).

b) The Wind Hypothesis

Even though wind theory is not sufficiently advanced to predict the behavior of $\dot{P}$ as a function of the luminosity of any given source (see § III), nevertheless the wind hypothesis does, under some conditions, predict a correlation between $\dot{P}$ and other observable quantities when one compares a collection of sources, and it is therefore interesting to see if there is any evidence for such a correlation in the currently available data. In particular, the terminal velocities of winds from early-type stars like the massive companions of possibly wind-fed neutron stars are usually comparable to the escape velocities from these stars (Castor, Abbott, and Klein 1975; Abbott 1978). Moreover, the companion star must be inside its

Roche lobe, and the wind velocity near its terminal value at the accretion capture radius, if the flow is not to form an accretion disk around the neutron star (see Petterson 1978). Hence one expects $v_W \sim v_{esc}$, where $v_{esc} = (2GM_c/R_c)^{1/2}$ in terms of the mass M_c and radius R_c of the companion star. In addition, in most of these systems the binary separation a is comparable to R_c, so that the neutron star orbital velocity v_{orb} is also comparable to v_{esc}. Thus, if the local sound speed $c_s \lesssim (v_W{}^2 + v_{orb}{}^2)^{1/2}$, then the characteristic capture velocity v_0 (see § III) will scale from system to system as v_{esc} (Shapiro and Lightman 1976). And equation (24) can be rewritten in terms of the spin-up time scale $T_s \equiv -P/\dot{P}$ as

$$PLT_s = fP_{orb} \,, \quad (25)$$

where the function f is given by

$$f = (2GMI_{eff}/R)q^2R_c{}^{-2}\gamma^{-4} \,, \quad (26)$$

with $q = M_c/M$ and

$$\gamma \approx \xi^{1/2}[(v_W/v_{esc})^2 + (v_{orb}/v_{esc})^2]^{-1/2} \,. \quad (27)$$

Since the above scaling yields $\gamma \approx$ const., equation (25) predicts a correlation between the X-ray source properties P, L, T_s, and M, and the binary system properties P_{orb}, M_c, R_c, and γ. Given the current observational uncertainties in the quantities that enter f, one cannot hope to evaluate it for each individual source. However, to the extent that it also is roughly constant, there should be a correlation between PLT_s and P_{orb}.

Figure 11 shows the currently available data for seven well-studied X-ray sources on a plot of $\log(PLT_s)$ against $\log(P_{orb})$. The data used in constructing this figure are given in Table 1. Also shown in the figure is the straight line

$$\log[P(s)L_{37}T_s(y)] = \log[P_{orb}(d)] + 3.82 \,, \quad (28)$$

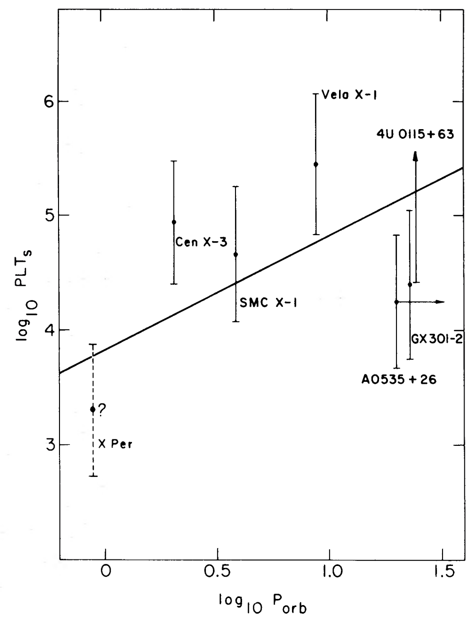

FIG. 11.—Observed values of the quantity PLT_s plotted against observed values of the binary orbital period P_{orb}, for seven pulsating X-ray sources. The units of P, L, T_s, and P_{orb} are seconds, 10^{37} ergs s^{-1}, years, and days, respectively. The straight line is the theoretical relation expected for wind-fed sources if $q = 15$, $R_c = 25\ R_\odot$, and $\gamma = 1$ (see text). The 22^h X-ray period is shown for X Per, although the evidence suggests that it is not the binary period (see Table 1).

which corresponds to $q = 15$ (see Cowley 1977), $R_c = 25\ R_\odot$ (see Ziółkowski 1977), and $\gamma = 1$. Although the observed values of PLT_s lie within an order of magnitude of the values expected theoretically, there is little evidence for any correlation with

TABLE 1

OBSERVED PROPERTIES OF NINE PULSATING X-RAY SOURCES[a]

Source	P (s)	Mean L (10^{37} ergs s^{-1})	$\bar{T}_s$ (years)	Binary Period (days)
4U 0115−73 = SMC X-1	0.71	50	$(1.3 \pm 0.4) \times 10^3$	3.89
4U 1653+35 = Her X-1	1.24	1	$(3.3 \pm 0.6) \times 10^5$	1.70
4U 0115+63	3.6[b]	≥0.9[c,d]	$\sim 3.1 \times 10^4$[e]	24.3[d]
4U 1118−60 = Cen X-3	4.84	5	$(3.6 \pm 0.6) \times 10^3$	2.09
A0535+26	104	6	29 ± 8	≥20[e]
4U 1728−24 = GX 1+4	121	4	50 ± 13	...
4U 0900−40 = Vela X-1	283	0.1	$(1.0 \pm 0.4) \times 10^4$[f]	8.97
4U 1223−62 = GX 301-2	700	0.3	120 ± 60	23[e]
4U 0352+30 = X Per	836	4×10^{-4}	$(5.9 \pm 1.5) \times 10^3$[g]	0.9[h]

[a] From Rappaport and Joss 1977, unless otherwise noted.

[b] Cominsky *et al.* 1978.

[c] Johnston *et al.* 1978.

[d] Rappaport *et al.* 1978.

[e] Bradt, Doxsey, and Jernigan 1979.

[f] Becker *et al.* 1978.

[g] White, Mason, and Sanford 1977; Jernigan and Nugent 1979.

[h] This period corresponds to a variation sometimes seen in the X-ray flux (see White *et al.* 1975 and Culhane *et al.* 1976), although current evidence suggests that it is not the binary period.

P_{orb} (the 22^h X-ray variation sometimes observed in X Per is shown in the figure, although present evidence suggests that it is not the binary period of the source; if this point is given a low weight, there is no evidence for any increase of PLT_s with P_{orb}).

c) *Discussion*

Figures 9 and 10 show that the present theory of disk accretion taken together with the available observational data is consistent with a relatively narrow range (~ 1 decade) of stellar magnetic moments for a given stellar mass or, alternatively, a narrow range ($\sim$ a factor of 4) of stellar masses for a given magnetic moment. The only source for which this is not the case is Vela X-1 which either has a much larger magnetic moment than the other sources or is not accreting from a disk.

Observational support for the existence of a significant region where the disk is magnetically coupled to the star and, ipso facto, support for the disk hypothesis comes from the fact that the full torque given by the present model (eq. [6]), which predicts a steep fall in $-\dot{P}$ for fast rotators, fits the data significantly better than the torque N_0 (eq. [2]).[8] If we exclude 4U 0900−40, then for a fixed mass and neutron star equation of state the present torque model can achieve agreement with the remaining sources with a factor of 15 variation in μ, while for fixed μ the model can achieve agreement with a factor of 3.8 variation in M. For comparison, the torque N_0, which neglects the disk-star magnetic coupling, would require, for a fixed mass and equation of state, a factor of 200 (if Her X-1 is excluded) or of 10^5 (if Her X-1 is included) variation in μ in order to achieve a similar agreement, while for fixed μ a factor of 20 (Her X-1 excluded) or of 10^3 (Her X-1 included) variation in M would be required, were this possible. The fall in $-\dot{P}$ at small values of $PL^{3/7}$, which is responsible for the better agreement of the present torque model with observation, is not expected in wind-fed sources. Conversely, the current observational data do not show the correlation with P_{orb} that might be expected for wind-fed sources, although the observational uncertainties are large and may mask such a correlation.

Considered as a group, the currently measured pulsating X-ray sources are remarkably well described by a single "universal" relation between P, $\dot{P}$, and L, obtained from equation (15) by assuming that all are 1.3 $M_\odot$ neutron stars with 5×10^{29} gauss cm^3 dipole moments accreting from disks. Although all pulsating sources are not actually expected to have identical masses and dipole moments, the good agreement of this relation with the data collected so far suggests that it holds approximately for almost all sources. If so, then given the values of two of the three quantities, P, $\dot{P}$, and L for a particular source, one can use this relation to estimate the value of the third parameter. The good agreement of this relation with the data also argues strongly that the nine sources considered here are indeed neutron stars, since the corresponding relation for magnetic white dwarfs predicts values of $\dot{P}$ which are several orders of magnitude smaller than the values observed in these sources (see Lamb 1977).

In summary, the striking agreement between the present torque model and the average spin-up rates of eight of the nine currently measured sources that is indicated in Figure 10 and the apparent absence of any similar correlation in Figure 11 suggests that these eight sources are disk-fed. In the remaining sections of the present paper we shall therefore adopt the disk hypothesis. However, given our limited knowledge of the magnetic moments, wind conditions, and binary system parameters of these sources, this evidence is far from decisive, a situation which underscores the importance of carrying out the potentially conclusive studies outlined in § III.

[8] For this reason, plotting log $(-\dot{P}/P)$ against log $(PL^{6/7})$, which tends to intermix fast rotators with slow ones, produces a markedly larger scatter in the data (see, for example, Rappaport and Joss 1977) than does a plot of log $(-\dot{P})$ against log $(PL^{3/7})$, which orders sources according to fastness.

V. NEUTRON STAR MAGNETIC FIELDS

In § III we showed that a sequence of accurate period and luminosity measurements can establish unambiguously that a given X-ray star is accreting from a disk and that such a sequence can then be used to determine accurately the magnetic moment of the star. Even without data of this quality, a first estimate of the magnetic moment can be obtained by fitting the theoretical spin-up equation to the *average* luminosity and spin-up rate of the source derived from the relatively sparse period determinations and X-ray flux measurements currently available (note, however, the cautionary remarks in § III*d*). In the present section we assume that all the observed sources are disk-fed and use their average spin-up time scales to infer their dipole moments.

a) *Spin-up Time Scale*

According to equation (15), the spin-up time scale $T_s \equiv -P/\dot{P}$ predicted by the present disk accretion model is

$$T_s = 2.0 \times 10^4 \mu_{30}{}^{-2/7} n^{-1} S_1{}^{-1} L_{37}{}^{-6/7} \text{ yr} , \quad (29)$$

where n is given by equation (7), and S_1 by equation (17). These equations show that for a given source (specified by a spin period P and luminosity L) and a given neutron star model (specified by a mass M, radius R, and effective inertial moment I_{eff}), the predicted value of T_s depends only on the magnetic moment μ of the star.

The behavior of T_s as a function of μ for a given source and neutron star model is illustrated in Figures 12 and 13, which show the functions $T_s(\mu)$ that correspond to the observed parameters of the sources Her X-1, GX 1+4, and X Per, for the TI neutron star models of PPS. For small values of μ, one has $T_s \propto \mu^{-2/7}$ and hence T_s decreases with increasing μ; this is the region of slow rotation. As μ is further

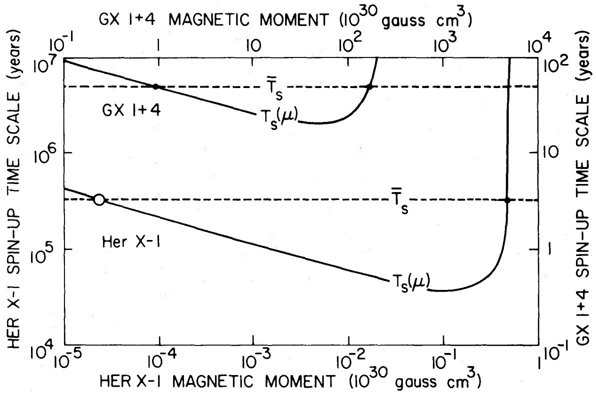

FIG. 12.—Spin-up time scale for GX 1+4 = 4U 1728−24 (*top curves*) and Her X-1 (*bottom curves*). *Solid curves*, theoretical spin-up time scales $T_s(\mu)$ for a 1.3 $M_\odot$ PPS neutron star. *Dashed lines*, observed average spin-up time scales $\overline{T}_s$. Theoretical and observed spin-up time scales agree at the points marked by circles. A filled circle indicates the physically acceptable solution for Her X-1. For GX 1+4, both solutions are physically acceptable.

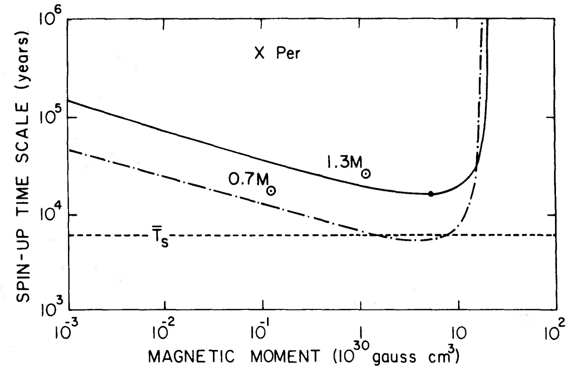

FIG. 13.—Same as Fig. 12, but for 4U 0352+30 = X Per. *Solid curve*, theoretical spin-up time scale $T_s(\mu)$ for a 1.3 $M_\odot$ PPS neutron star. *Dot-dashed curve*, theoretical spin-up time scale $T_s(\mu)$ for a 0.7 $M_\odot$ PPS neutron star. *Dashed line*, observed average spin-up time scale $\overline{T}_s$. A filled circle indicates the spin-up time scale and magnetic moment given in Table 2.

increased, T_s passes through a minimum and then begins to rise; the region of rising T_s is that of fast rotation. Finally, as μ approaches the critical value μ_c at which $\omega_s = \omega_c$, the accretion torque vanishes and hence T_s diverges. Thus, given a particular X-ray source with an observed average spin-up time scale $\overline{T}_s$ and a particular neutron star model, either there are two values of μ for which the theory gives a spin-up time scale in agreement with the observed value, if $\overline{T}_s$ lies above the minimum of the theoretical curve $T_s(\mu)$, or there are none, if $\overline{T}_s$ lies below the minimum.[9] When there are two solutions for μ, one is a "slow rotator" solution whereas the other corresponds to a "fast rotator." In practice, one of these two solutions can often be ruled out on other grounds; for example, a value of μ which corresponds to an inner disk radius r_0 less than or equal to the stellar radius R can be

[9] A single value of μ can also give agreement in the special case where $\overline{T}_s$ exactly equals the minimum value of $T_s(\mu)$. This is the case for SMC X-1.

TABLE 2

DERIVED PARAMETERS OF NINE PULSATING X-RAY SOURCES[a]

SOURCE	SLOW ROTATOR SOLUTION			FAST ROTATOR SOLUTION		
	μ_{30} [b]	r_0(10^8 cm)	ω_s	μ_{30} [b]	r_0 (10^8 cm)	ω_s
SMC X-1	0.50	0.36	0.11	0.50	0.36	0.11
Her X-1	(2.4×10^{-5})	(3.8×10^{-3})	(6.9×10^{-5})	0.47	1.1	0.35
4U 0115+63	2.9×10^{-3} (?)	6.0×10^{-2} (?)	1.5×10^{-3} (?)	1.4	2.0	0.30
Cen X-3	1.0×10^{-2} (?)	7.4×10^{-2} (?)	1.6×10^{-3} (?)	4.5	2.4	0.29
A0535+26	3.3	1.9	9.7×10^{-3}	148	17	0.25
GX 1+4	0.93	1.1	3.4×10^{-3}	170	21	0.29
Vela X-1	(2.7×10^{-5})	(7.8×10^{-3})	(9.0×10^{-7})	86	40	0.35
GX 301-2	0.3	1.2	6.7×10^{-4}	394	70	0.31
X Per	4.8[c]	38	0.10	4.8[c]	38	0.10

[a] Observed properties are taken from Table 1.

[b] For each source, the stellar magnetic moment was adjusted to obtain the best possible agreement between the observed value of T_s and the theoretical value of T_s for a 1.3 $M_\odot$ PPS neutron star. In general, two values of the moment give agreement for those sources for which an exact agreement is possible. For SMC X-1 these two values of μ coincide, as the observed value of T_s exactly equals the minimum theoretical value. The slow rotator solutions for Her X-1 and Vela X-1 (enclosed in parentheses) are ruled out because the corresponding inner disk radii would lie inside the neutron star. The slow rotator solutions for Cen X-3 and 4U 0115+63 are suspect because the corresponding inner disk radii would lie close to the stellar surface. If one chooses the underlined solutions for A0535+26, GX 1+4, and GX 301−2, then the magnetic moments of all sources except Vela X-1 would lie within an order of magnitude of one another. The inferred magnetic moments listed here differ slightly from the preliminary values reported in Ghosh and Lamb (1978*a*) because our preliminary calculations did not incorporate the self-consistency condition (eq. [34] of Paper II) between the four constants C_b, C_ω, C_p, and γ_0.

[c] For X Per, exact agreement between the observed and calculated values of T_s was not possible for a 1.3 $M_\odot$ PPS neutron star, since the minimum theoretical value, 1.5×10^4 years, was somewhat greater than the observed value, 5.9×10^3 years. The agreement is, however, well within the observational uncertainties (see text). The value of μ given for this source corresponds to the minimum theoretical value of T_s.

rejected (in Fig. 12 the "slow rotator" solution for Her X-1 is such a case).

b) *Inferred Dipole Moments*

As noted in § III, period measurements over relatively long time scales (~ months-years) have been made for nine of the 16 currently known pulsating X-ray sources. For each of these nine sources we have attempted to match the theoretical spin-up time scale given by equations (7), (17), and (29) to the observed average spin-up time scale by assuming that the neutron star has a mass of 1.3 $M_{\odot}$ and obeys the TI equation of state of PPS, and then adjusting μ. The results are summarized in Table 2. Two solutions for μ, corresponding to slow and fast rotation, are possible for all sources except SMC X-1, for which only a single value of μ fits the observations [$\bar{T}_s$ happens to equal the minimum value of $T_s(\mu)$], and X Per, for which no solution is possible given the current estimate of the mean source luminosity and the neutron star model chosen here [$\bar{T}_s$ lies below the minimum value of the curve $T_s(\mu)$]. The case of X Per is discussed further below.

Among those sources for which two solutions for μ are possible, the slow rotator solution is definitely ruled out for the two sources Her X-1 and Vela X-1 since it would imply that the inner radius of the disk lies inside the stellar surface. The rejected solutions are enclosed in parentheses in Table 2. In addition, the slow rotator solutions for Cen X-3 and 4U 0115+63 are very suspect, though not completely ruled out, since they correspond to inner disk radii only a few times larger than the stellar radius, and channelling of matter by the stellar magnetic field may be marginal in such a situation. For the other sources that admit of two solutions, there is as yet no compelling reason for rejecting either of them.

c) *Discussion*

First, consider the sources Vela X-1 and X Per, which are somewhat exceptional and deserve further comment.

In order to fit the relatively long spin-up time scale of Vela X-1, a magnetic moment ~10–10^2 times larger than that inferred for the other sources is required. While such a large magnetic moment is certainly allowed by our current understanding of neutron star magnetic fields, the fact that Vela X-1 alone requires such a large value of μ suggests the alternative possibility that Vela X-1 is wind-fed rather than disk-fed (Lamb 1977), in which case it could have a relatively long spin-up time scale even if its magnetic moment were similar to those of the other sources.

For X Per, on the other hand, the observed value of $\bar{T}_s$, 5.9×10^3 yr, lies somewhat below the minimum of the theoretical curve, which corresponds to 1.5×10^4 yr, and hence no solution is possible for current estimates of the mean source luminosity and the neutron star model chosen here (see Fig. 13; the value of μ given in Table 2 corresponds to the minimum of the theoretical curve). However, the present lack of intersection is not significant, since there are major uncertainties in several of the input parameters upon which the theoretical curve depends. First, the mean luminosity of the source over several years, which is required in the calculation, is very poorly known at present. The uncertainty in the current estimate (~4×10^{33} ergs s^{-1}) of the mean luminosity of X Per is not known. For the sake of definiteness, Rappaport and Joss (1977) *assumed* a plausible but arbitrary uncertainty of a factor of 3 in the mean luminosities of all sources, including X Per. If this uncertainty in luminosity is accepted, then solutions for X Per exist for values of the mean luminosity within this range but a factor of ~2.5 less than the estimate adopted by Rappaport and Joss. Second, for the sake of definiteness we *assumed* that *all* sources, including X Per, have a mass of 1.3 $M_{\odot}$. This is the approximate value inferred for Her X-1, the only source for which a fairly precise determination of this quantity is at present available (Middleditch and Nelson 1976; Bahcall and Chester 1977). However, the present observational bounds on the neutron star masses in other X-ray sources are very wide (Joss and Rappaport 1976; Avni 1978; Bahcall 1978). For a large range of values of the stellar mass within these bounds, the minimum theoretical value of T_s for X Per is below the observed value, giving two solutions. An example is presented in Figure 13, which shows the theoretical curves $T_s(\mu)$ corresponding to both 0.7 $M_{\odot}$ and 1.3 $M_{\odot}$ neutron stars. The former gives two solutions. Third, we *assumed* that neutron star matter follows the TI equation of state of PPS. However, the range of equations of state given by current conventional theories of many-body nuclear physics (see Baym and Pethick 1975, 1979) is wide enough to bring the minimum theoretical value of T_s well below the observed value for somewhat softer, but quite acceptable, equations of state, even for the values of luminosity and mass adopted here.

These remarks show that an acceptable fit is possible for all nine measured sources for magnetic moments in the range 3×10^{29}–4×10^{32} gauss cm^3. Such magnetic moments are consistent with our present meager knowledge of neutron star formation and evolution, and we therefore conclude that the observational data on these sources are *consistent* with their being disk-fed, although the secular change in period and the average luminosity are not by themselves sufficient to demonstrate that they are accreting from disks. If they *are* disk-fed, the possible value or values of μ for these sources are those given in Table 2. We note, however, that the inferred value of μ depends sensitively on the *mean* X-ray luminosity, averaged over the months or years involved in the secular spin-up time scale determination and that for many sources this important observational quantity is poorly known.

Furthermore, Table 2 shows that by adopting the acceptable solution for those sources in which only one solution is acceptable and then choosing one of the solutions for those sources in which two solutions are acceptable, one can find a set of solutions with magnetic moments all lying within the relatively narrow range $\sim 5 \times 10^{29}$–5×10^{30} gauss cm^3, that is consistent with the data on all the measured sources except Vela X-1. Where two are acceptable, the solution that corresponds to this set is underlined in Table 2. The fact that such a set of stellar magnetic moments can be found explains why the "universal" relation between the parameters $\dot{P}$ and $PL^{3/7}$ given in § IV, which assumes a single value of μ for all sources, describes the observational data so well.

Finally, we remark that the comparatively large dipole magnetic moments inferred for these accreting neutron stars (including Her X-1, which may be as old as $\sim 10^8$ years) argues against the universality of rapid magnetic field decay, a hypothesis that has been widely discussed as a possible explanation for some observed properties of pulsars (Gunn and Ostriker 1970; Lyne, Ritchings, and Smith 1975; Flowers and Ruderman 1977; Fujimura and Kennel 1979). The fact that the dipole moment inferred for Her X-1 is, nevertheless, substantially weaker than would be deduced by assuming that the stellar surface field inferred from apparent cyclotron features in the X-ray spectrum (Trümper *et al.* 1978) is dipolar in character, suggests that the surface field strength is largely in higher multipole moments (Lamb 1978). Elsner and Lamb (1976) have argued on different grounds that the surface magnetic fields of some X-ray stars are complex. The presence of higher multipole moments, which tend to decay faster than the dipole moment, would further constrain the magnetic field decay time scales in these sources.

These preliminary conclusions can be tested by a detailed comparison of the theoretical spin-up equation with sequences of accurate period and X-ray flux measurements, as discussed in § III.

VI. THE ORIGIN OF PULSE PERIOD FLUCTUATIONS

The well-studied pulsating X-ray sources have all been shown to exhibit short-term fluctuations or irregularities in their pulsation periods in addition to a secular spin-up trend. One possible explanation of this phenomenon is that it is caused by fluctuations in the accretion torque (Lamb, Pines, and Shaham 1976, 1978*a*, *b*). Here we show that if the present model applies, then torque fluctuations of the required size can be caused by fluctuations in the mass accretion rate. Since fluctuations in the accretion rate cause similar fluctuations in the luminosity of the source, this possibility can be checked directly by observation.

In order to relate the fluctuations in $\dot{M}$ to period and luminosity fluctuations, one needs a relation between $\dot{M}$ and L, a relation between $\dot{M}$ and N, and a model for the response of the neutron star to N. If we assume disk accretion and restrict ourselves to fluctuations on time scales longer than the inflow time through the transition zone ($\lesssim 3$ s) and the free-fall time from the inner radius of the disk ($\lesssim 0.1$ s), the stationary flow solutions presented in Paper II apply and the torque N as a function of $\dot{M}$ is given by equations (6) and (7). Thus, for a source of given P, L, M, and μ, these two equations, plus the accretion luminosity equation $L = (GM/R)\dot{M}$ and the neutron star equation of state, relate the relative fluctuation in the torque, $\delta N/N$, to the relative fluctuation in the accretion luminosity, $\delta L/L$. At the same time, the stellar model relates the relative fluctuation in the spin change rate, $\delta\dot{P}/\dot{P}$, to the relative fluctuation in the torque, $\delta N/N$. Hence one has a relation between $\delta\dot{P}/\dot{P}$ and $\delta L/L$. In Table 3 we give the ratio of $\delta\dot{P}/\dot{P}$ to $\delta L/L$ for the nine pulsating X-ray sources listed in Table 2, based on the choice of parameters given there and the assumption that the star responds as a rigid body. The behavior of fast rotators differs greatly from that of slow rotators. For slow rotators, one has $\delta\dot{P}/\dot{P} \sim \delta L/L$, whereas for fast rotators $\delta\dot{P}/\dot{P}$ may be orders of magnitude larger than $\delta L/L$. As examples, the chosen parameters give $\delta\dot{P}/\dot{P} = 2.8\delta L/L$ for Cen X-3, whereas for Her X-1, whose angular velocity is very close to the critical value, fluctuations in the accretion luminosity correspond to fluctuations in $\dot{P}$ which are 70 times larger.

Adopting the statistical description of torque fluctuations developed by Lamb, Pines, and Shaham (1978*a*) and the two-component model of the stellar response (Baym *et al.* 1969), and assuming a crustal moment of inertia comparable to that of the core, the condition for spin-down episodes to occur from time to time is that the root-mean-square relative torque fluctuation $\langle(\delta N/N)^2\rangle^{1/2}$ should exceed $(T/T_1)(RT)^{-1/2}$, for type 1 torque fluctuations (which correspond to white noise in N), or $(RT)^{-1/2}$, for type 2 fluctuations (which correspond to white noise in the time derivative of the torque, $\dot{N}$). Here R is the average rate of torque fluctuation events, T_1 is the duration of a type 1 event, and T (typically $\sim 3 \times 10^7$ to 10^8 s) is the length of the observing period.

In Her X-1, for example, the observational data show two spin-down episodes over a period ~ 14 months (Giacconi 1974). According to the above condition and the results in Table 3, type 1 fluctuations in $\dot{M}$ of relative size $\delta\dot{M}/\dot{M} \sim 1$ occurring about once every 10^3 s and lasting $\sim 2 \times 10^3$ s would account for these episodes. Alternatively, type 2 fluctuations at the same rate but of relative size $\sim 10^{-4}$ would suffice. In Cen X-3, on the other hand, Table 3 indicates that substantially larger fluctuations in $\dot{M}$ are needed to

TABLE 3

RATIOS OF FLUCTUATIONS IN $\dot{P}$ TO FLUCTUATIONS IN LUMINOSITY PREDICTED BY THE PRESENT MODEL

Source	$(\delta\dot{P}/\dot{P})/(\delta L/L)$
SMC X-1	1.0
Her X-1	70
4U 0115+63	3.2
Cen X-3	2.8
A0535+26	0.86
4U 1728−24	0.86
4U 0900−40	60
4U 1223−62	0.86
4U 0352+30	1.0

account for the spin-down episode observed in 1972 September-October (Fabbiano and Schreier 1977). There the observations lasted for a period ~ 21 months. Thus a type 1 fluctuation of relative size $\delta \dot{M}/\dot{M} \sim 1$ occurring once every 10^5 s and lasting $\sim 8 \times 10^5$ s would suffice, as would type 2 fluctuations of relative size $\sim 10^{-2}$ occurring at the same rate. Of course, fluctuations on shorter and longer time scales are also possible. If the observed period variations *are* caused by fluctuations in $\dot{M}$, the accretion luminosity should display similar fluctuations, with periods of reduced luminosity associated with periods of slower spin-up or, if the fluctuation is large enough, spin-down. We note that Fabbiano and Schreier have reported a possible decrease of relative size $\delta L/L \sim 0.5$ in the X-ray luminosity of Cen X-3 during the spin-down episode of 1972 September-October.

VII. THE NATURE OF THE LONG-PERIOD SOURCES

A large fraction of the known pulsating X-ray sources have long periods ($\geq 10^2$ s) but are observed to be spinning up on time scales ($\sim$ 50–100 years) which are extremely short compared to their expected lifetimes as bright X-ray sources ($\sim 10^3$–10^6 years; compare van den Heuvel 1977; Amnuel and Guseinov 1976; Ziółkowski 1977; and Savonije 1978). Examples include A0535+26, GX 1+4, and GX 301-2. This apparent paradox has been noted and discussed previously by numerous authors (see Illarionov and Sunyaev 1975; Lamb, Lamb, and Arnett 1975; Fabian 1975; Wickramasinghe and Whelan 1975; Kundt 1976; Lea 1976; Lipunov and Shakura 1976; Savonije and van den Heuvel 1977; Paper I; Holloway, Kundt and Wang 1978; Davies, Fabian, and Pringle 1979; Wang 1979). Almost all of these discussions have focused on mechanisms for producing a long period at the onset of accretion, when X-ray emission first begins. However, given the very short observed time scales noted above and the significant fraction of potential X-ray binaries which are actually emitting X-rays (see van den Heuvel 1977; Ziółkowski 1977), it appears that the slow rotation of many pulsating X-ray sources must somehow be maintained over evolutionary time scales, even though they are acted upon by strong spin-up torques. If so, the principal issue is how the sources *maintain* their long periods, rather than how these are produced initially; indeed, the solution of the former problem may partially solve the latter problem as well. As discussed in § IV, the existence of a universal relation between $\dot{P}$ and $PL^{3/7}$ which adequately reproduces almost all the available observational data suggests that most of the observed sources are accreting from disks. Thus, whatever the mechanism by which slow rotation is maintained, it should probably function for accreting sources with orbital rather than radial inflow.

The braking torque on fast rotators found in the present calculation suggests one way that slow spin rates can be maintained (Ghosh and Lamb 1978*a*). Thus, if a pulsating X-ray source has recurrent "low" states during which the accretion rate is much reduced, it would be subjected to strong, recurrent spin-down torques. Although the published data on the luminosity behavior of the long-period sources is extremely scant at present, there is some evidence for variations by factors ~ 30 or more (S. Holt, private communication). Depending on the luminosities in alternating high and low states and the relative durations of such states, the spin-down that occurs during the low states can, on average, equal or even exceed the spin-up that occurs during the high states. If the spin-down on average *equals* the spin-up, a neutron star born with a relatively long spin period or spun down to that period just after birth by electromagnetic and particle emission prior to the start of accretion will be maintained with a long spin period during much of its life as an accreting X-ray source. If the spin-down actually *exceeds* the spin-up, on average, even a neutron star born with a relatively short rotation period will, during the course of its life as an X-ray source, be spun down to a long period. In either case, the source would be much more readily observed during high states, when it is spinning up, than during low states, when it is spinning down. This would account for the fact that all nine sources with measured $\dot{P}$'s are usually observed to be spinning up.

The mechanism for maintaining slow spin rates suggested here would produce cyclic variations in the spin period of a given source together with a slow drift in the mean period on a time scale related to the time scale of the binary evolution. The lengths of the cycles in $P(t)$ would be determined by the durations of successive high and low states. Although in a certain sense this proposed mechanism simply transforms the problem of understanding the persistence of long spin periods into a problem of understanding the pattern in time of mass transfer in X-ray binaries, it can be directly tested independently of the development of the latter understanding. Thus, on a plot of observed luminosity versus spin period, such a source should trace out a nearly closed curve whose center drifts on an evolutionary time scale (Ghosh and Lamb 1978*b*). Theoretical evolutionary paths of this type which are suitable for comparison with the behavior of observed sources have been computed and will be published elsewhere (Elsner, Ghosh, and Lamb 1979). Some evidence for this picture is provided by Vela X-1, which shows variations by a factor $\gtrsim 30$ in luminosity together with alternating episodes of strong spin-up and spin-down (see Fig. 7), although as noted in § IV, this particular source may not be disk-fed.

In closing this discussion, we remark that the spin-down torque during low states may involve both the braking torque on fast rotators found here and the braking torque on very fast rotators due to mass ejection which has been suggested by other authors (Davidson and Ostriker 1973; Illarionov and Sunyaev 1975; Shakura 1975), if the mass accretion rate becomes sufficiently low. Our braking torque, which operates for values of ω_s between the critical fastness, ω_c, and the maximum allowable fastness for steady accretion, $\omega_{\max} \approx 1$, and involves no mass ejection from the vicinity of the magnetosphere, would act at luminosities between those corresponding to these two values of the fastness, whereas the braking torque suggested by the above authors, which is assumed to operate for $\omega_s > \omega_{max}$ and to involve mass ejection from the vicinity of the magnetosphere, would act at luminosities below that corresponding to $\omega_s = \omega_{max}$.

VIII. CONCLUDING REMARKS

In Paper I we showed that no model of axisymmetric stationary disk accretion by magnetic stars which involves a thin transition zone between the disk and the magnetosphere can produce an accretion torque significantly less than $\dot{M}(GMr_0)^{1/2}$ and hence that such models cannot account for the much smaller accretion torque acting on Her X-1. In Paper II we showed that in fact the transition zone in disk accretion is broad, owing to invasion of the disk plasma by the stellar magnetic field via the Kelvin-Helmholtz instability, turbulent diffusion, and magnetic flux reconnection. In the present paper we have used the solutions to the two-dimensional hydromagnetic equations obtained in Paper II to calculate the torque on the star. We find (1) that the magnetic coupling between the star and the plasma in the outer transition zone is appreciable; (2) that as a result of this coupling the spin-up torque on fast rotators is substantially less than that on slow rotators; and (3) that for sufficiently high stellar angular velocities or sufficiently low mass accretion rates the rotation of the star can be braked while accretion continues.

Applying these results to pulsating X-ray sources, we have shown that a star of given spin period rotating in the same direction as the disk can experience either spin-up or spin-down, depending on its luminosity. At high luminosities the star is a "slow rotator" and experiences a strong spin-up torque whereas at low luminosities it becomes a "fast rotator" and can experience a strong spin-down torque. If the accretion luminosity remains approximately constant, the spin period will eventually approach and then maintain the critical spin period at which the accretion torque vanishes.

We have discussed the general problem of interpreting period changes in pulsating X-ray sources and have described how one can disentangle the effect of the accretion torque from the effects of possible internal torques and the dynamical response of the neutron star utilizing a dense sequence of pulse period and X-ray flux measurements of each source. We have shown that the model of disk accretion developed in Paper II gives an equation for the accretion torque that can potentially be used to (1) establish the accretion flow pattern, (2) determine the properties of the X-ray star, and (3) establish the cause of pulse period fluctuations.

We have also considered the limited evidence regarding accretion flow patterns that is provided by the period and flux measurements that are currently available. Assuming that the effective inertial moments of the X-ray sources are roughly constant on the time scales of interest, disk-fed sources should show a particular correlation between $-\dot{P}$ and the quantity $PL^{3/7}$, and such a correlation is in fact indicated by the data. On the other hand wind-fed sources should, given certain assumptions, show a linear correlation between PLT_s and P_{orb}, the orbital period of the binary system. The current observational data do not show such a correlation, although the observational uncertainties are large and may mask it at present. The striking agreement of the observational data with the theoretical spin-up equation for disk accretion and the apparent absence of any correlation of PLT_s with P_{orb} suggests that most of the currently measured sources are disk-fed. The current evidence is, however, far from conclusive, a situation which underlines the importance of obtaining a regular sequence of accurate period and flux measurements for each source.

Assuming that a given source is disk-fed, its average luminosity and spin-up rate can be used to estimate its dipole magnetic moment. In most cases a single value of the magnetic moment is indicated, but in a few cases two solutions are possible. Acceptable solutions are possible for all the sources whose spin-up rates have so far been measured, including Her X-1, and yield magnetic moments in the range 3×10^{29}–4×10^{32} gauss cm^3. Solutions that correspond to magnetic moments in the much narrower range 5×10^{29}–5×10^{30} gauss cm^3 can be found for all sources except Vela X-1. The much larger magnetic moment required to fit Vela X-1 may indicate that this source is not disk-fed. The substantial values of the inferred magnetic moments argue against rapid, universal magnetic field decay in neutron stars. These preliminary conclusions can be tested by comparing the theoretical spin-up equation with regular sequences of accurate period and X-ray flux measurements.

The short-term period fluctuations and spin-down episodes observed in Her X-1, Cen X-3, Vela X-1, and X Per follow naturally from the present model of disk accretion as consequences of fluctuations in the mass accretion rate. In particular, the model predicts a braking torque sufficient to account for the observed spin-down episodes without ejection of mass from the vicinity of the neutron star, if there is a reduction in the accretion rate. Fast rotators differ dramatically from slow rotators in the size of the relative $\dot{P}$ fluctuation, $\delta\dot{P}/\dot{P}$, that accompanies a given relative fluctuation $\delta L/L$ in the accretion luminosity. For slow rotators one has $\delta\dot{P}/\dot{P} \sim \delta L/L$ whereas for fast rotators $\delta\dot{P}/\dot{P}$ may be many orders of magnitude larger than $\delta L/L$. Again, these predictions can be checked by comparison with a dense sequence of period and X-ray flux measurements.

Finally, we have pointed out that an understanding of the statistics of the long period pulsating sources requires not only mechanisms leading to slow rotation at the onset of accretion, when X-ray emission first begins, but also a means for maintaining long periods in the face of strong spin-up torques. We have noted that the braking torque at low accretion rates found in the present calculations could provide an explanation, if these sources have recurrent periods of low luminosity. The evolutionary tracks in the period-luminosity plane that are predicted by this hypothesis can be tested directly by comparison with a sequence of period and luminosity measurements.

The results presented here point clearly to the critical need for a regular sequence of accurate period and X-ray flux measurements of a number of sources. At a minimum, such sequences would make possible a better estimate of the mean luminosity of these sources and hence allow a more accurate comparison with the secular period changes predicted by theory. Far more interestingly, a regular sequence of measurements of each source would allow a determination of the dynamical response of the neutron star and a direct comparison with the quantitative theory of the accretion torque presented here. Such a comparison would provide a check on the theory while at the same time establishing which of these sources are accreting from disks and which are not. Assuming that the theory is confirmed, the observations could be used to accurately determine the dipole magnetic field of the neutron star and to probe the nature of short-term fluctuations in the accretion flow.

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