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NUMERICAL TAXONOMY OF Ap AND Am STARS

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ABSTRACT

High-dispersion spectrograms of 40 Ap and Am stars have been classified on the basis of the second spectra of their lanthanide and iron-group elements with the help of cluster analysis. The atomic spectra have been described by the S parameters from wavelength-coincidence statistics. Useful classifications emerge from either the iron-group or lanthanide elements, but with the exception of the Am stars there is little coherence between these two groups of elements. The diversity among Ap stars is shown clearly.

Subject headings: stars: abundances — stars: metallic-line — stars: peculiar A — stars: spectral classification

I. INTRODUCTION

Ap and Am stars exhibit complex and variegated spectra. Descriptive methods, in particular classification, will remain an important tool in the study of such objects until a comprehensive theoretical understanding of them is at hand.

The need for new classification schemes for Ap and Am stars has come about as a result of the increased information content in high-dispersion studies of their spectra. The high-dispersion work may be grouped into two categories: abundance determinations (Adelman 1973*a*, *b*; Smith 1971, 1973; and many others), and the more descriptive work such as the Jascheks' (1960) on Am stars, or Babcock's (1958) and Bidelman's (1966, 1968) on Ap stars.

Bidelman demonstrated the power of very high dispersion (~ 2 Å mm^{-1}) in the descriptive study of stars with sharp-lined spectra, and in a series of papers (cf. Cowley 1978*a*), Cowley, Aikman, and their collaborators have attempted to systematize and automate Bidelman's procedures so that they could be applied to a large number of stars. It was already evident from Bidelman's work that the late-B through F stars were a far more heterogeneous mixtures than might have been thought from the classifications based on low-dispersion spectra, which group all of these objects into a half-dozen or so categories. Our own work made this abundantly clear, but the question remained of finding a fruitful way to systematize and pursue these results.

In a preliminary study (Cowley 1978*b*; henceforth Paper I) we introduced a classification scheme based entirely on the second spectra of the lanthanides. That work was qualitative in nature, using what might be called "classical" methods of classification. In the present paper, we shall make use of a relatively new technique of statistical multivariate analysis known as cluster analysis. Its usefulness in astronomical contexts is just beginning to be explored (cf. Heck 1978; Heck *et al.* 1977), but it is an established tool in the burgeoning field of numerical taxonomy (Sneath and Sokel 1973). Numerical methods of classification have the advantage that they allow one to systematically and simultaneously treat a large number of variables. This makes them especially valuable in the analysis of high-dispersion spectra of Ap and Am stars where 20 or more species of atomic spectra are frequently present.

Additional variables such as $v \sin i$ or surface magnetic field strength may also be introduced, although we have not done so in the present work. Eventually, the task of keeping track of all of the relevant data simply transcends the power of the human mind, and recourse to computerized techniques is a natural step.

II. CLUSTER ANALYSIS

Cluster analysis is a relatively recent addition to the techniques of multivariate analysis (see Anderberg 1973 or Everitt 1974). In the present application we have applied the technique to our measures of the significance parameters S which are determined in each of our stars for a variety of atomic spectra. The determination of this parameter is described in detail by Hartoog, Cowley, and Cowley (1973). It is related to the strength with which a particular atomic spectrum is represented in a stellar spectrum, and for the present it is useful to think of it simply as a "strength parameter."

The chief advantage of S is that it is available for immediate analysis. Our experience is that S is correlated with line strengths and therefore with abundances. However, we also know very well that S is related to a number of other stellar parameters which are *not* directly connected to abundances, such as effective temperatures and the general "richness" of the stellar spectra in lines. We can attempt to minimize the effects of those known variables other than abundances by the selection of data for analysis. For example, we can analyze only those stars whose spectra have similar line densities ("richness"), and we can choose ions which represent a major ionization stage throughout the range of surface temperatures spanned by the sample of stars. In this study, we have attempted to minimize the effects of variables other than abundances, but it has not been possible to do this in a precise way. Our work is therefore very much in the spirit of some photometric methods which are chosen to be most sensitive to certain physical variables such as T_e, $\log g$, or [Fe/H], but which require extensive calibrations to delineate the precise interrelationships among the variables.

The value of S for a given atomic spectrum, say Ti II, may be thought of as a coordinate in "Ti II-space." Each star may be represented by a point in N-dimensional space, N being the number of S values considered for each star. For convenience, we shall refer to this space as "variable space," or simply "v-space." The objectives of cluster analysis are to attempt to group the points in v-space in some meaningful way that will suggest associations among the different stars.

We used a program for cluster analysis developed by the Statistics Department of the University of Michigan. Our thanks are due to Dr. K. E. Guire for valuable discussions concerning this program.

In the present work, distances between points are calculated as Euclidian distances, which seemed to be the most straightforward of a variety of choices that may be used. Several options are available for forming the clusters. In this paper we will discuss only results based on "average" distances in which the criterion of membership of a given point with a cluster of points is based on the average distance of that point to all of the other points in the cluster.

The program locates the closest points and fuses them to form the first (binary) cluster. The position point for the first cluster is replaced by the average coordinate for the two points fused. The next cluster is then formed in the same way as the first. Two other "close" points may be fused or a third point fused to the original two. This procedure is repeated until all the points are joined in one final cluster. The program generates plots which illustrate formation of the clusters, and at each step, or fusing of points, the Euclidian distance is given. In this way it can be seen that the last points fused are much further apart in v-space than the first.

III. THE STELLAR DATA

The basic data upon which the cluster analysis has been performed are given in Table 1. The stars are listed in order of the classification categories discussed in Paper I and summarized in Table 2 below. Paper I and references cited therein give spectral type, $b - y$, $v \sin i$, and relevant plate epochs for all but three of the stars given in Table 1. We list supplemental data for these objects in Table 3.

The parameter S has been represented schematically for small and negative values. In these cases we use our parameter p, which gives a Monte Carlo estimate, based on 200 trials, of the probability that an observed number of wavelength coincidences, within ± 0.06 Å, is due to chance. All cases with $4.0 > S$ and $p \leq 0.005$ have been assigned the value $S = 2.5$. For $0.005 < p \leq 0.05$, S is set equal to 1.0; for $p > 0.05$, $S = 0$.

The present work deals entirely with second spectra of iron-group elements, yttrium, and the lanthanides, which have numerous strong lines. Eu II has been specifically excluded from analysis, since the parameter S is not a satisfactory measure of the strength of the Eu II lines (cf. Paper I).

IV. RESULTS

Three cluster analyses were performed. In the first, only the second spectra of yttrium and the lanthanides were used (viz., Y II, La II, Ce II, Pr II, Nd II, Sm II, Gd II, Tb II, Dy II, Ho II, Er II). The second analysis was based only on iron-group spectra (viz., Sc II, Ti II, V II, Cr II, Mn II, Fe II, Co II, Ni II). In a third analysis these spectra were considered simultaneously.

We examine first the cluster based on the lanthanides, which is shown graphically in Figure 1. The classifications of Paper I are given beside each star name. One would expect a good degree of coherence between the results of the cluster analysis and the subjective classifications of Paper I, since the same basic data were involved. Figure 1 shows that this is indeed the case, although some of the groups have been divided.

A standard method of using dendrograms or trees such as Figure 1 is to make a cut at some appropriate place the position of which will depend on the number of categories one wishes to generate. An example of this procedure is illustrated in Figure 1. The cut is made after 30 steps, or fusings, and 10 categories are formed from the 40 stars. The different categories are separated by a space in Figure 1. It is well to be aware that some arbitrary decisions are made in the construction of a diagram of this kind. From a given stem, the order of listing is simply a numerical one devoid of physical meaning; a rotation could be performed about any stem without changing the valid inferences that can be made. Thus HR 465(a) might have been listed after HD 101065; the fact that the latter is adjacent to HR 6958 does not mean that HD 101065 and HR 6958 are more closely related than, for example, HD 6958 and HR 465(a). On the other hand, the juxtaposition of the stars from HR 6958 to HR 465(b) is no accident. These objects are all from the same "stem" and are in some sense related. It is also valid to assume that the lanthanide spectra of HR 465(b) are more closely comparable to those of HD 200311 than the other stars in the subgroup. An extensive discussion of the interpretation of such trees may be found in the books by Sneath and Sokal (1973) or Anderberg (1973).

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TABLE 1

Basic Data for Cluster Analysis

STAR	N	Y II	LAII	CEII	PRII	NDII	SMII	GDII	TBII	DYII	HOII	ERII	SCII	TIII	V II	CRII	MNII	FEII	COII	NIII
HR 465(A)	4293	00.0	00.0	05.9	01.0	10.0	13.0	11.8	04.4	10.7	04.6	06.2	00.0	08.9	04.2	08.6	02.5	10.0	04.4	00.0
HR 6958	1437	00.0	00.0	12.8	01.0	06.5	02.5	02.5	01.0	05.6	00.0	00.0	01.0	18.9	01.0	12.2	01.0	12.6	00.0	00.0
HD 25354A	1396	00.0	00.0	02.5	01.0	09.7	04.6	01.0	00.0	02.5	00.0	00.0	00.0	14.5	00.0	09.6	06.4	11.0	02.5	00.0
HD 25354B	1353	00.0	00.0	06.0	04.3	06.7	05.2	02.5	00.0	02.5	00.0	00.0	00.0	11.2	00.0	09.2	05.4	09.3	01.0	00.0
HD 51418	1444	00.0	01.0	00.0	01.0	05.0	02.5	01.0	01.0	02.5	04.4	00.0	00.0	06.9	05.2	01.0	00.0	05.9	01.0	00.0
HD 101065	2821	04.6	08.0	10.9	05.9	09.9	11.8	06.7	01.0	05.9	05.2	06.8	00.0	02.5	00.0	02.5	00.0	00.0	00.0	00.0
HD 192913	1149	00.0	00.0	01.0	01.0	04.9	01.0	01.0	01.0	04.1	06.6	04.8	00.0	11.6	00.0	11.7	02.5	14.9	00.0	00.0
HD 200311	1119	00.0	01.0	13.5	02.5	00.0	00.0	01.0	00.0	00.0	00.0	00.0	01.0	13.6	00.0	17.4	02.5	14.3	06.3	00.0
HD 221568B	3332	01.0	00.0	07.7	00.0	07.1	08.0	07.0	00.0	03.9	01.0	02.5	01.0	07.6	02.5	06.2	04.8	06.8	00.0	00.0
HD 221568A	2792	03.6	00.0	10.8	00.0	05.6	01.0	06.9	00.0	01.0	00.0	00.0	01.0	11.1	02.5	08.6	04.9	08.3	00.0	00.0
10 AQL	2872	02.5	02.5	08.2	00.0	02.5	08.2	07.1	00.0	02.5	00.0	00.0	01.0	10.9	02.5	07.6	01.0	07.7	04.4	00.0
BETA CRB	2610	02.5	04.0	08.4	00.0	00.0	00.0	05.2	00.0	00.0	00.0	00.0	02.5	10.1	01.0	08.8	00.0	07.8	00.0	01.0
52 HER	1124	00.0	01.0	04.2	00.0	00.0	00.0	02.5	00.0	00.0	00.0	00.0	00.0	11.7	00.0	07.2	00.0	08.6	00.0	00.0
HR 7575	2485	00.0	05.4	07.5	00.0	00.0	01.0	09.1	01.0	00.0	00.0	00.0	02.5	10.8	00.0	06.4	04.3	08.2	01.0	00.0
HD 71866	1353	01.0	05.0	07.1	00.0	01.0	01.0	02.5	01.0	00.0	00.0	00.0	02.5	08.6	00.0	11.5	06.0	10.3	00.0	00.0
53 CAM	1099	00.0	08.4	06.1	00.0	00.0	00.0	00.0	00.0	00.0	01.0	01.0	00.0	11.0	00.0	07.2	00.0	08.4	00.0	00.0
49 CNC	0954	00.0	01.0	02.5	05.7	00.0	01.0	00.0	00.0	00.0	00.0	00.0	00.0	13.8	05.4	11.8	05.1	13.1	00.0	00.0
ALPHA2 CVN	1115	00.0	00.0	07.3	01.0	01.0	00.0	00.0	00.0	00.0	00.0	00.0	00.0	11.5	00.0	09.3	01.0	11.5	00.0	00.0
45 LEO	1041	00.0	01.0	07.6	02.5	00.0	00.0	00.0	00.0	00.0	00.0	00.0	00.0	15.6	00.0	15.5	01.0	15.9	00.0	00.0
41 TAU	0367	00.0	00.0	04.5	00.0	02.5	00.0	00.0	00.0	00.0	00.0	00.0	00.0	16.2	00.0	13.3	00.0	18.4	00.0	00.0
78 VIR	1553	00.0	01.0	04.0	00.0	00.0	00.0	00.0	00.0	00.0	00.0	00.0	00.0	11.6	00.0	10.6	02.5	12.3	00.0	01.0
HR 465(B)	2039	00.0	01.0	11.9	00.0	04.4	00.0	02.5	00.0	00.0	00.0	00.0	01.0	11.2	01.0	11.0	07.8	11.3	00.0	00.0
HR 4816	1969	00.0	02.5	02.5	00.0	00.0	00.0	00.0	00.0	00.0	00.0	00.0	01.0	11.4	00.0	11.7	05.9	09.9	00.0	00.0
HR 4854	1798	00.0	00.0	04.9	00.0	01.0	00.0	00.0	00.0	00.0	00.0	00.0	02.5	09.3	00.0	09.0	04.4	06.8	02.5	01.0
HD 2453	2582	04.6	00.0	07.2	00.0	01.0	00.0	00.0	00.0	00.0	00.0	00.0	04.6	11.5	02.5	09.0	06.5	09.5	00.0	00.0
73 DRA	1844	02.5	00.0	00.0	00.0	00.0	00.0	00.0	00.0	00.0	00.0	00.0	01.0	12.9	01.0	09.7	01.0	09.5	02.5	00.0
HR 8216	1749	01.0	00.0	00.0	00.0	00.0	00.0	00.0	00.0	00.0	00.0	00.0	05.1	14.3	01.0	11.4	05.0	11.3	04.4	04.4
HD 8441	1925	00.0	01.0	00.0	00.0	00.0	00.0	05.6	00.0	00.0	00.0	00.0	00.0	12.7	01.0	10.5	02.5	09.6	01.0	00.0
HD 42616	0755	00.0	00.0	00.0	00.0	00.0	00.0	00.0	00.0	00.0	00.0	00.0	00.0	10.0	00.0	13.5	04.2	10.5	00.0	00.0
HD 216533	1917	00.0	00.0	00.0	00.0	00.0	00.0	00.0	00.0	00.0	00.0	00.0	00.0	12.3	00.0	09.9	06.0	11.0	01.0	00.0
32 AQR	1553	04.9	08.7	10.2	00.0	09.0	05.3	05.2	00.0	04.5	00.0	00.0	02.5	12.7	05.0	07.2	00.0	08.4	00.0	04.1
GAMMA EQU	2023	06.7	07.4	09.1	01.0	09.5	04.3	08.1	00.0	06.1	00.0	00.0	02.5	12.5	06.6	08.4	02.5	08.3	02.5	00.0
33 LIB	2008	03.5	03.8	07.3	01.0	04.2	05.6	07.5	00.0	03.9	00.0	02.5	04.5	07.5	02.5	07.8	02.5	07.7	00.0	00.0
63 TAU	1271	05.3	06.2	10.3	02.5	08.4	02.5	02.5	00.0	01.0	00.0	00.0	01.0	12.4	02.5	07.6	00.0	11.4	00.0	04.3
TAU UMA	1679	05.2	07.0	08.7	02.5	08.3	06.4	06.0	00.0	05.0	00.0	00.0	04.7	13.1	08.6	06.9	01.0	08.0	01.0	04.3
ZETA 1 LYR	0577	05.6	03.6	04.3	00.0	01.0	04.4	00.0	00.0	00.0	00.0	02.5	00.0	13.6	02.5	06.2	00.0	11.7	02.5	02.5
HR 4751	0490	00.0	01.0	02.5	00.0	01.0	01.0	00.0	00.0	00.0	00.0	00.0	01.0	00.0	08.2	00.0	00.0	00.0	00.0	00.0
RR LYN	1059	06.7	07.8	06.2	00.0	05.0	00.0	04.6	00.0	00.0	00.0	01.0	01.0	14.2	02.5	07.3	00.0	11.5	00.0	05.0
TAU 7 SER	0881	05.0	06.0	04.1	00.0	06.5	01.0	00.0	00.0	00.0	00.0	00.0	01.0	15.6	05.7	07.5	00.0	12.6	00.0	04.1
68 TAU	0908	07.9	02.5	01.0	00.0	02.5	00.0	00.0	00.0	00.0	00.0	00.0	02.5	17.3	05.1	11.2	00.0	14.5	01.0	05.6

TABLE 2

PRELIMINARY CLASSIFICATION OF 40 CP STARS BASED ON LANTHANIDE SPECTRA

Star	Class	Symbol	Description of Category
HR 465(a)	A	*	Lanthanide-rich stars. The intermediate and high mass lanthanides spectra such as Dy II - Er II are often identified. La II is usually weak, while Ce II is frequently not the dominant lanthanide. This classification places some weight on the color temperature of the stars. Thus HD 192913 does not have strong lines of Dy II, but because T(eff) is high, a high abundance of Dy II is nevertheless to be inferred.
HR 6958	A	*	
HD 25354(a)	A	*	
HD 25354(b)	A	*	
HD 51418	A	*	
HD 101065	A	*	
HD 192913	A	*	
HD 2003111	A	*	
HD 221568(b)	A	*	
HD 221568(a)	A	*	
10 Aql	B	□	Stars with a Nd-Sm anomaly. La II and Ce II are present and strong, as is Eu II and Gd II in the prototypes, but Nd II and Sm II are weak or absent. In 10 Aql, only Nd II is "weak", 52 Her and RR Lyn show less well defined evidence for anomalous behavior as a function of lanthanide mass.
β CrB	B	□	
52 Her	B	□	
HR 7575	B	□	
HD 71866	B	□	
RR Lyn	B	□	
53 Cam	C_1	O	Iron Group Rich, rare earth moderate stars. In the subgroups C_1, Ce II is usually well represented in the spectrum while the other lanthanide spectra are weaker. This is probably the most common type of Ap star.
49 Cnc	C_1	O	
α^2 CVn	C_1	O	
45 Leo	C_1	O	
41 Tau	C_1	O	
78 Vir	C_1	O	
HR 465(b)	C_1	O	
HR 4816	C_1	O	
HR 4854	C_1	O	
HD 2453	C_1	O	
73 Dra	C_2	X	Subgroup C_2, is best described as a miscellaneous category. Iron group spectra are usually strong, but the lanthanide spectra may be weaker than in "normal" A stars, as in HR 8216. Note that only Gd II is well identified in HD 8441.
HR 8216	C_2	X	
HD 8441	C_2	X	
HD 42616	C_2	X	
HD 216533	C_2	X	
32 Aqr	D	•	In the cooler stars numerous light and intermediate lanthanides are present, and there is little or no evidence for the kind of fractionation that is characteristic of Category B. Lanthanide overabundances are probably not extreme. In the hotter objects, the rare-earth lines are weak, and membership is difficult to assign. HR 4751 has too broad spectral lines to be usefully classified by this method.
γ Equ	D	•	
33 Lib	D	•	
63 Tau	D	•	
τ UMa	D	•	
ζ^1 Lyr	D	•	
τ^7 Ser	D	•	
68 Tau	D	•	
HR 4751	D	•	

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TABLE 3

SUPPLEMENTAL DATA FOR PROGRAM STARS

Star	Sp[a]	$b - y$[b]	$v \sin i$	Plate epoch JD 2,400,000.+	Plate DAO No.
α^2 CVn........	AOp Si Eu Hg	−0.060	≤25[c]	42502.9222	9658
RR Lyn........	A3m	+0.105	10[d]	43725.9611	12406
τ^7 Ser..........	A2m	+0.116	10[e]	43709.7156	12353
68 Tau.........	Al IV	+0.020	< 10[d]	43509.6889	11895

[a] Cowley *et al.* (1969); [b] Lindemann and Hauck (1973); [c] Slettebak (1963); [d] Smith (1971); [e] Abt and Moyd (1973).

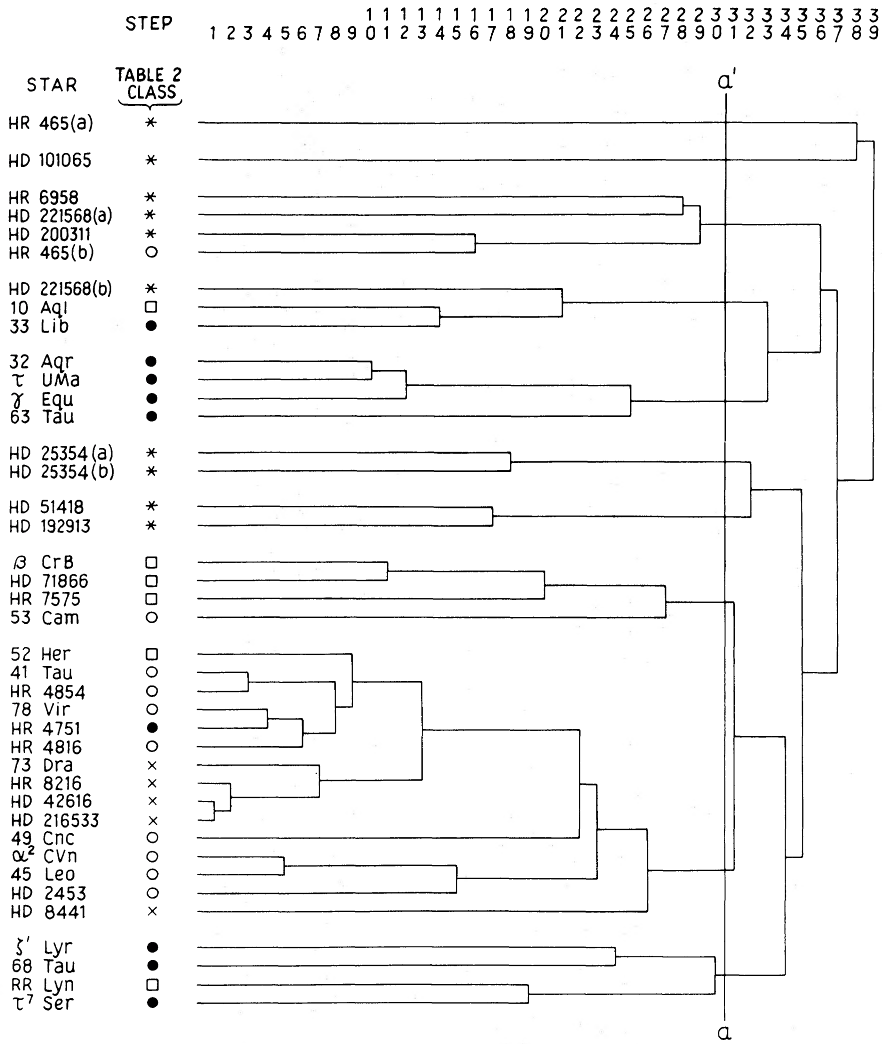

FIG. 1.—Clustering dendrogram based on second spectra of the lanthanides and yttrium. The symbol given beside each star indicates the classification according to Table 2. Clustering proceeds from left to right. At the extreme left, each star is a "cluster" with one member. At each step, indicated along the top of the figure, two clusters are fused as indicated in the figure. The vertical line aa′ is the "cut" referred to in the text. Classification groupings based on this figure are indicated by the spaces between the star names. See text for special remarks concerning α^2 CVn.

The first points clustered are for HD 42616 and HD 216533, for which the separation in v-space was zero because none of the lanthanides (excluding europium, N.B.) gave a significant result; both stars therefore fall at the origin; $S = 0$ for all variables. The tightest clustering of points is for the stars from 52 Her through HD 216533. The common characteristic of these objects is a weakness of the lanthanide spectra.

There is a large category of stars that falls into one group from 52 Her through HD 8441. All but two of these stars were classified C_1 or C_2 in Paper I. All of these objects have the common characteristic of moderate to extremely weak lanthanide spectra.

The reader with some familiarity with Ap stars may be surprised to find α^2 CVn in a group with weak or moderate lanthanide lines, since the star is generally thought to be very rich in rare-earth spectra. The phase of our plate (see Table 3) is 0.16 with respect to the Eu II maximum (Pyper 1969), so we would *not* expect the lanthanide spectra to be generally weak.

In order to clarify this situation we have performed radial-velocity scans to see if the zero points of the rare-earth lines were systematically displaced from the iron-group lines used to determine the radial velocity of the star. The results of these scans are of considerable interest. Highly significant results emerged for Gd II ($S = 6.2$) and Dy II ($S = 6.4$) when these spectra were assumed to be displaced by $+10$ km s^{-1} with respect to the zero of radial velocity that had been adopted earlier. Nd II was identified at good confidence ($S = 4.3$), but with a displacement of $+5$ km s^{-1}. It turned out that Ce II also gave its maximum significance ($S = 8.6$) at $+5$ km s^{-1}.

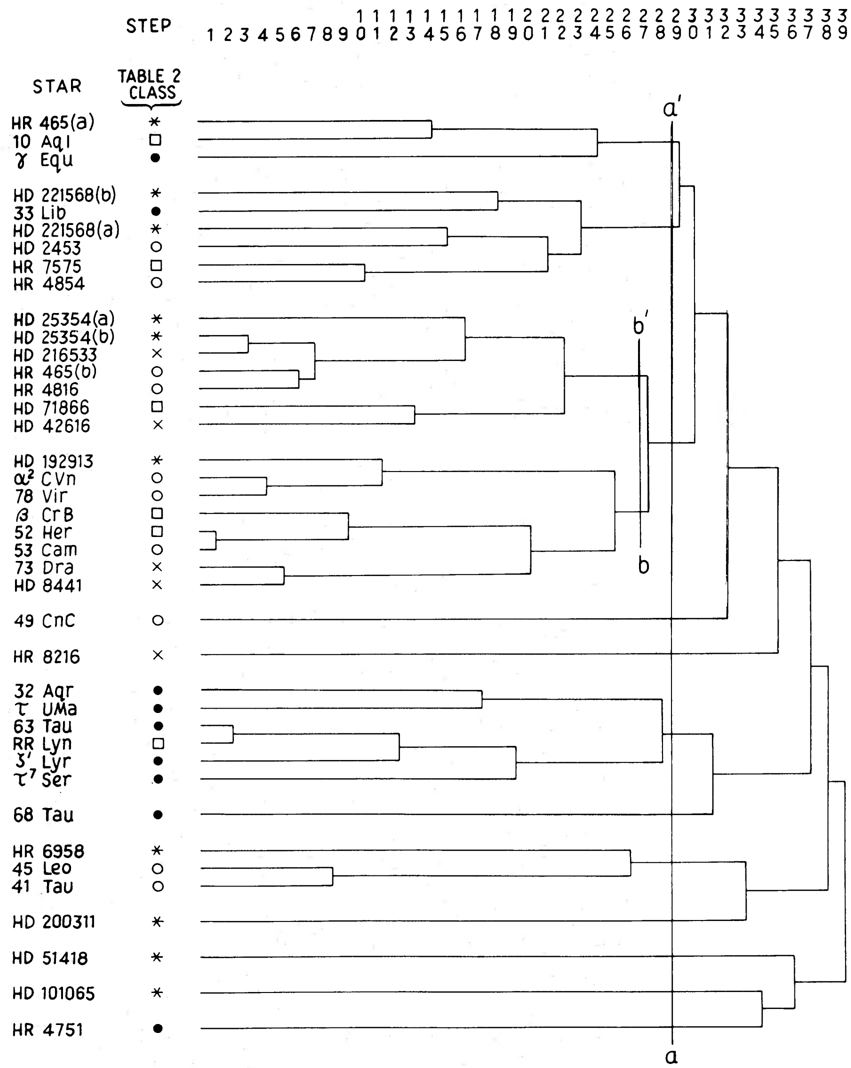

FIG. 2.—Clustering dendrogram based on second spectra of iron group (see the caption to Fig. 1). Note the loss of coherence between the classification of Table 2, which is indicated by the symbols, and that of the present figure. The Am star group (*filled circles*), however, does not dissolve. The classification of Table 4 is made with the help of the "cut" aa′. One large division is subdivided with the cut at bb′.

Only marginal results ($p > 0.01$) were obtained for other lanthanide spectra by means of these scans. The fact that only marginal results were obtained for Sm II is noteworthy. Nd II and Sm II are the outstanding lanthanides in HR 465 at its rare-earth maximum. As far as these atomic spectra are concerned, α^2 CVn is more closely related to β CrB and HR 7575—the stars with Nd–Sm "holes."

There are, however, important similarities between α^2 CVn and the rare-earth maximum of HR 465. In both stars Eu II is very strong, and the third spectrum of dysprosium may be identified with very high confidence ($S = 8.6$, α^2 CVn; $S = 8.0$, HR 465). Dy III is the subject of a recent study by Aikman, Cowley, and Crosswhite (1979).

In view of these results it is difficult to know how to place α^2 CVn among the other Ap stars, and we have decided, arbitrarily, not to use the results of the radial-velocity scans in the present classification work. These scans, which are rather expensive and/or time consuming have not been made for most of our stars. However, we did perform such scans for 73 Dra, a well-known spectrum variable which we have classified here as rare-earth weak. In this case no highly significant coincidences emerged. We note that Leckrone and Snijders (1979) find that the UV spectrum of α^2 CVn is well accounted for by iron-group elements and silicon with little evidence for the rare-earths.

The Am and Fm stars 32 Aqr and τ UMa are quite close to the Ap star γ Equ, in agreement with an assessment of the Jascheks' (1974) that γ Equ resembled the Am stars. The star 63 Tau is, at least formally, a slightly more distant relative. The lanthanide spectra in all four stars are surely similar.

Beta CrB, HR 7575, and HD 71866 were classified together in Paper I as neodymium-samarium anomalous; they are also close together in v-space. The star 10 Aql was assigned to this category with a caveat that only Nd II seemed anomalously weak, while Sm II was present and quite strong. The star RR Lyn is one of four stars that was not discussed in Paper I. We put it in category B, on the basis of its weak Sm II spectrum, but the cluster analysis shows it to be well separated from the prototype objects β CrB, HR 7575, and HD 71866. Neither RR Lyn nor 10 Aql appears to be closely related to the stars with neodymium-samarium anomalies. The star RR Lyn (HR 2291) was studied because of a remark made by the Jascheks (1960) that it might have particularly strong rare-earth lines. Tau^7 Ser (HR 5845), which is close to it in v-space, was observed because the study of Abt and Moyd (1973) showed it to have a low $v \sin i$. The Am or Am-related stars ζ^1 Lyr–τ^7 Ser are well separated from the group 32 Aqr–63 Tau, and although the mean $T_{\rm eff}$ is surely lower in the latter group, it is unlikely that the wide separation may be attributed to ionization alone.

Figure 1 could be used to supplement the classifications of Table 2. We shall not do this here. An iron-group classification based on such a dendrogram will be introduced momentarily, and it will be seen that a completely different grouping of the stars arises. A third classification could be based on the combined data from the lanthanides and the iron peak. It does not seem worthwhile to proliferate these classification schemes. The significant results, for the present, are the interrelationships, many of which are evident simply from an inspection of the figures.

In Figure 2 we show the clustering based on the iron-group spectra. A cut has been made after step number 28, which enables us to classify the 40 stars into the categories summarized in Table 4.

The classifications are all prefixed by "I" for iron peak, to distinguish them from those of Table 2. In order to find the characteristics which distinguish one group from another, we computed mean values of our parameters S for the group members. The descriptions given in the last column of Table 4 refer to these mean values. A number of the stars are spectrum variables, and for these one must keep in mind that the classifications refer to the epochs of observation. However, only one of the spectrum variables for which we have two observations is represented in two groups, HR 465.

The dendrogram of Figure 2, like that of Figure 1, admits one large category of 15 stars which has been arbitrarily divided into IA_1 and IA_2, Mn II strong and weak subcategories. Only eight of the stars are common to the two large groups. Seven of these were classified either C_1 or C_2, while the eighth, 52 Her, was called B, a classification that should surely be questioned in the light of the present work.

It can be seen from Figure 2 and Table 4 that the new classification disperses the groups of Paper I, with the interesting exception of the stars in category D, "rare-earth normal." These stars include, essentially, the Am stars and the possibly intermediate Am–Ap, γ Equ. The latter star is now well separated in v-space from those in category ID, which now contains only Am stars.

Figure 3 shows clustering based on all 19 variables. Coherence with the classifications of Paper I are again obvious. We shall not burden the reader with the tabulation of a third classification, based on this new tree. The figure itself provides the basis for an obvious grouping. The cuts shown again lead to one large category which constitutes 28% of the sample of stars.

V. DISCUSSION

It is clear that a meaningful classification of Ap and Am stars may be given which takes into account much more data than are available on 125 Å mm^{-1} spectrograms. The classifications of Tables 2 and 4 are "satisfactory" in the sense that (1) the number of objects classified is considerably less than the number of categories, and (2) ambiguities of classification are not prohibitive. It is perhaps a weakness of the iron-group classification that the miscellaneous group IF contains so many members, but this does not subtract from the coherence of the other classifications.

It has not been our purpose here to establish a definitive classification of Ap and Am stars. Indeed, the classifications of Tables 2 and 4 are largely incompatible, with the exception of the "Am" stars.

TABLE 4
Classification based on Iron Group Spectra

Star	Class	Description of Category
HD 25354(a)	IA_1	
HD 25354(b)	IA_1	
HD 216533	IA_1	
HR 465(b)	IA_1	Ti II, Cr II Mn II, Fe II very strong.
HR 4816	IA_1	
HD 71866	IA_1	
HD 42616	IA_1	
HD 192913	IA_2	
α^2 CVn	IA_2	
78 Vir	IA_2	Ti II, Cr II, Fe II very strong as with IA_1,
β CrB	IA_2	but Mn II is definitely weaker.
52 Her	IA_2	
53 Cam	IA_2	
73 Dra	IA_2	
HD 8441	IA_2	
HD 221568(a)	IB	
33 Lib	IB	
HD 221568(b)	IB	Ti II, Cr II, Mn II, Fe II strong;
HD 2453	IB	Modest Sc II, V II.
HR 7575	IB	
HR 4854	IB	
HR 465(a)	IC	
10 Aql	IC	Ti II very strong; Cr II, Fe II strong;
γ Equ	IC	Moderate V II, Co II.
63 Tau	ID	
RR Lyn	ID	
ζ^1 Lyr	ID	Ti II, Fe II, very strong, Cr II strong;
τ^7 Ser	ID	V II, Ni II modest; Sc II, Mn II weak.
32 Aqr	ID	
τ UMa	ID	
HR 6958	IE	Ti II, Cr II, Fe II very strong. S parameters
45 Leo	IE	higher than IA_2, hence, separate cluster.
41 Tau	IE	These three stars are all "silicon stars",
		although their classification here is inde-
		pendent of Si I or II.
HD 51418	IM	This is simply a miscellaneous category.
HD 101065	IM	The stars here are generally different
HR 4751	IM	from one another as well as from objects
49 Cnc	IM	in the other categories. Jones et al.
HR 8216	IM	(1974) have pointed out similarities
68 Tau	IM	in the spectra of HD 51418 and HD 101065.
HD 200311	IM	

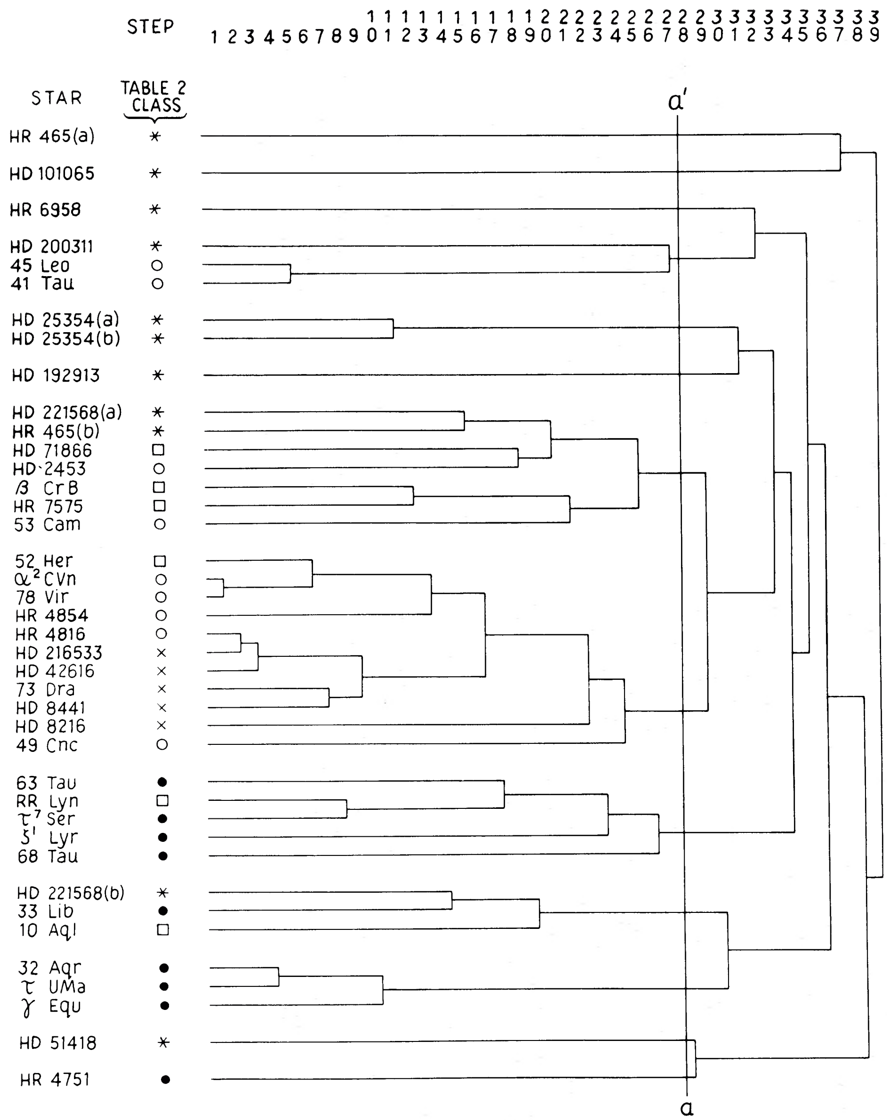

FIG. 3.—Clustering dendrogram based on second spectra of yttrium, the lanthanides, *and* the iron group (see the caption to Fig. 1). A possible classification based on the "cut" aa′ is indicated by the spaces among the star names.

For the present, there is little motivation for attempting to establish a classification based on a combination of lanthanide and iron-peak data as illustrated, for example, in Figure 3. There is reason to suppose that a useful classification could emerge only from a data base considerably larger than the present set of 40 stars. From such a set we might expect to find numerous classes of objects which had similar iron-group *and* lanthanide spectra; at that point, it would be useful to consider a definitive classification scheme. For the present, we must ask ourselves what useful information has been gathered from this very preliminary study. We shall consider as useful any new insight which the numerical taxonomy has given us into the question of the origin of the chemical anomalies of Ap and Am stars.

Let us begin with some generalizations based on Figures 1 and 2.

a) *The cluster analysis properly separates the stars with really peculiar spectra; likewise, stars with similar spectra end up being grouped closely together.* This essentially says that the cluster analysis is functioning as we hoped it would. There have been a few surprises, to be sure, but after we have reviewed the data, these cases no longer conflict with our subjective assessments. We knew, for example, that HR 465(a) and HD 101065 had most unusual spectra, and they are among the last points to cluster in Figure 1. The same is true for HD 101065 in Figure 3, while HR 465(a) does not seem exceptional with regard to its iron-peak spectra, and this is in general agreement with Aller's (1972) abundance study. At first sight, the unusual behavior of the iron-peak spectra in the Am star HR 4751 appears perverse. However, because of the considerable $v \sin i$, only a few hundred lines were measured in this star; the spectrum is certainly unusual *looking*, when compared with our other Am's, whose spectra are sharp lined, but we have no reason to believe this is due to anything *other* than the large $v \sin i$.

Once we have satisfied ourselves that the general procedure is making sense, much valuable insight may be gained by looking at the detailed results. For the most part, this involves the close intercomparison of complex spectra that will be of interest only to a few specialists, and so we shall confine ourselves to one example. Our previous studies appeared to show that γ Equ was closely related to the Am stars, especially 32 Aqr, and we were somewhat puzzled by the asseverations of other spectroscopists familiar with Ap stars that γ Equ and the Ap star 10 Aql were very similar. An immediate resolution of this is suggested by Figures 1 and 2. Our judgment was strongly influenced by the lanthanide spectra, while that of our colleagues was apparently based on the iron group, whose spectra provide the more numerous strong lines. Our attention had been diverted from this by the more exotic lanthanides, and especially the reversal of the line-strength ratios of Nd II to Sm II in the two stars. Other factors, which have been investigated by ourselves as well as others (cf. Adelman 1973*b*; Auer 1964; Cowley *et al.* 1977), indicate that these stars have substantial differences in their spectra of Sc II, Ba II, and possibly U II. Nevertheless, the general notion that γ Equ and 10 Aql have similar spectra is well founded.

b) *Ap stars which appear to be related on the basis of their lanthanide spectra are not closely related by the spectra of their iron-group elements.* Firm conclusions based on this result would surely be premature, but the suggestion is apparent that the mechanism giving rise to the iron-peak anomalies is different in some important way from that giving rise to anomalies in the lanthanide spectra. We find support here for the multiple hypothesis approach of the kind explored by Guthrie (1971), although our results do not appear related to the *details* of his theory.

From the point of view of the diffusion theory (Michaud 1977; Vauclair and Vauclair 1978), it would appear reasonable to explore spectroscopic and mass differences among the iron-group and lanthanide elements as a means of accounting for these observations.

If one attempted an explanation based on nucleosynthesis, it would be natural to consider the addition of material from different supernovae. Mixing of material from shells of the *same* supernova into each Ap star cannot be excluded, but it would be necessary to postulate different kinds of supernovae or to suppose the mixing conditions vary from one event to another in order to account for the lack of coherence between the iron-group and lanthanide spectra.

c) *In both Figures 1 and 2 there is a large grouping of stars comprising roughly one third of the total sample.* If we intercompare the individual stars, there is about a 50% overlap, including the following stars: HD 216533, (C_2), HR 4816 (C_1), HD 42616 (C_2), 78 Vir (C_1), 52 Her (B), 73 Dra (C_2), and HD 8441 (C_2). The classifications of Paper I are either C_1 or C_2 with one exception.

These stars are by no means identical; HD 8441, with its large S parameter for Gd II and *no other lanthanide* is bizarre. Nevertheless, there appears to be some basis for considering these as "typical" Ap stars, the general characteristic being enhanced iron-group elements and modest rare-earth spectra.

d) *Our very limited sample of Am stars shows coherence between the lanthanide and iron-group spectra.* This result, if confirmed by a more extensive data base, could be of great importance. The implication supports an old notion that the Am stars are a less heterogeneous group than the Ap stars, and it would surely follow that less complicated theoretical explanations are required to explain them.

VI. AN ALTERNATE APPROACH

In attempting to understand a complex set of observations such as those described here, one may focus his attention on the objects (stars), and look for interrelationships among them as we have done here. An alternate procedure is to consider the interrelationships among the different *variables*, in this case, chemical elements. This has been done by several workers (cf. Adelman 1971; Khokhlova and Ptitsyn 1976), but apparently without the automated methods of multivariate analysis. Khokhlova and Ptitsyn's study shows a good deal more coherence among the Am stars than among the Ap's, in general agreement with the results of the present work. We have begun work along these lines by using multiple correlation and principal component analyses.

VII. CONCLUSIONS

High-dispersion spectroscopy of Ap and Am stars reveals a wealth of information about these stars that cannot be seen at low dispersion. Although the spectra are complex and variegated, cluster analysis shows that the data are not without structure; star-to-star variations are not random. We have attempted to display this structure by the basic approach of classification. However, if we use all of the information available to us, the sample of stars suitable for study by using the present technique is too small to lead to a definitive classification. If we confine our attention only to the second spectra of the lanthanides, or to iron-group elements, satisfactory classifications do emerge. These provisional classifications limn the observational basis for the problem of the chemical peculiarities in Ap and Am stars.

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