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CARBON STARS IN THE GLOBULAR CLUSTERS OF THE MAGELLANIC CLOUDS

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ABSTRACT

Among the red globular clusters of the Magellanic Clouds there are a number with giant branches which reach very red values of $B - V$ at the tip. Results are presented of a spectroscopic survey at the tip of the giant branch in these clusters. Numerous carbon stars were found, whose luminosities place them high on the upper asymptotic giant branch (i.e., well above the first giant branch tip). It is argued that such stars can be produced only by clusters considerably younger than the globular clusters of our Galaxy. The age of the clusters is estimated at 3 billion years (within a factor of 2). Although the Magellanic Cloud globulars split into two groups by color, the existence of a number of intermediate-age clusters in the red group is consistent with a picture of continuous cluster formation in the Clouds. The consequences are discussed of the recent discovery of large numbers of carbon stars in the field of the Clouds.

Subject headings: clusters: globular — galaxies: Magellanic Clouds — stars: carbon — stars: evolution

I. INTRODUCTION

The red globular clusters of the Magellanic Clouds are sometimes thought of as analogous to the halo clusters of our own Galaxy. The similarities extend from the integrated colors to the presence in some clusters of RR Lyrae stars. Further study of their color-magnitude diagrams, however, has sketched a more complex picture. Some of the red clusters seem to lack horizontal branches (Arp 1958*a*, *b*; Gascoigne 1966). Another striking anomaly, which is the subject of this study, is the redward extent of the giant branch in some clusters.

Van den Bergh's (1975) review lists nine clusters whose reddest giants have $B - V > 1.6$. The mean $(B - V)_{\max}$ in these clusters is 2.2, greatly exceeding the maximum color index even in 47 Tuc. Even in this metal-rich cluster the defined giant branch terminates at $B - V = 1.6$ (though some long-period variables almost reach 1.8; Eggen 1972). Although in galactic globulars a redder giant branch tends to indicate higher metallicity, this solution is unlikely for these clusters, as their integrated $U - V$ colors (van den Bergh and Hagen 1968) would suggest $-1.5 < [M/H] < -0.5$, if the calibration of Aaronson *et al.* (1978) were applicable.

The possibility that these red stars are carbon stars was first considered by Arp (1958*b*), but no spectra were obtained until Feast and Lloyd Evans (1973) examined 5 stars in three SMC clusters. They found three carbon stars. As a more complete survey of clusters in both Clouds seems long overdue, we have obtained spectra of 31 stars with $B - V > 1.6$ in nine of the red globular clusters (§ II). A color-magnitude diagram is presented (§ III), and the evolutionary status of these red stars is discussed in § IV, together with the implications for the clusters' ages.

II. CLASSIFICATION FROM VIDICON SPECTRA

For observing red stars with a blue-sensitive SIT tube, the wavelength region 5100–6300 Å was selected as the best compromise. Using grating 35 in the first order gave a dispersion of 100 Å mm^{-1} with an effective resolution (FWHM) of 8 Å. We observed the reddest giants in nine of van den Bergh's clusters (we have added NGC 339 and 1846), supplementing the program with stars from the uncalibrated iris photometry of Hesser, Hartwick, and Uguarte (1976). A total of 31 program stars were observed on 1978 October 6 and 7 with the 4 m telescope at Cerro Tololo.

Each night we observed several standards whose energy distributions have been determined by Stone (1977) and Osmer (1977). The combination of good seeing and transparency with a 2".3 slit ensured that the results were useful for spectrophotometry. The spectra were reduced using the "stellar" software package at La Serena. Gradients in the flat-field response were detected at the 5% level, but not corrected for. Residuals are therefore noticeable on many spectra at 5577 and 5892 Å because of imperfect

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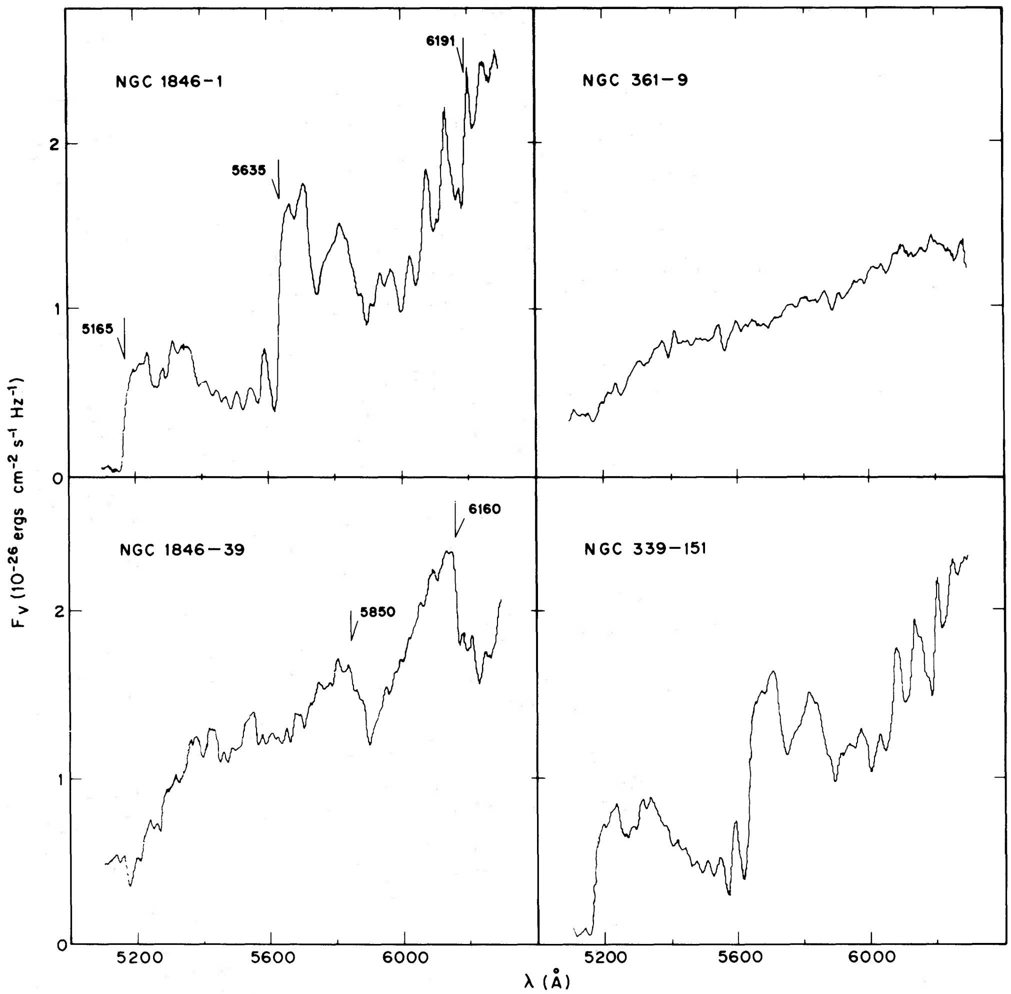

FIG. 1.—Typical spectra from the cluster survey (after five-point smoothing). The types assigned to the four stars are N1846-1 (C), N1846−39 (M), N361−9 (Ctm), and N339−151 (C). TiO and C_2 band heads are identified.

sky subtraction, but are unimportant for the present purposes. Typical spectra are shown after five-point smoothing (15 Å box) in Figure 1.

Features used to classify the program stars were: the Swan bands of C_2 at 5636 and 5165 Å (C stars[4]), and the strong γ' band heads of TiO at 6162 and 5850 Å (M stars). If these features were absent, stars were classified "Continuum" (Ctm) at this resolution, although features such as the Mg *b* lines were frequently noticeable. The ZrO band heads at λλ5304, 5552 best serve to distinguish S stars from M stars in this wavelength region, but no positive identifications were made. These classifications are entered in Table 1 together with the following quantitative measurements based on monochromatic (15 Å) fluxes.

1. *Color* is the continuum gradient in magnitudes over the baseline λλ5350, 6250. For M stars λ6150 is used as the continuum point to estimate this gradient.

[4] Three of the carbon stars identified by Blanco, Blanco, and McCarthy (1978) from their red CN bands were also observed as a control. No differences are evident between these stars and the cluster C stars identified here.

2. *Band strength* is the flux ratio in magnitudes $F_\nu(5615)/F_\nu(5690)$ for C stars and $F_\nu(6250)/F_\nu(6150)$ for M stars. It is a simple matter to obtain a calibration of these indices in terms of spectral type via similar indices measured by Faÿ, Stein, and Warren (1974) and Faÿ *et al.* (1974).

3. *Magnitude* is the monochromatic magnitude at 5556 Å interpolated between the continuum points. In the mean it agrees well with the published *V* magnitudes of the program stars. The large rms deviation of 0.29 mag may be due in part to variability.

III. CLUSTER MEMBERSHIP OF CARBON STARS AND THE H-R DIAGRAM

Surveying the clusters in Table 1, we note that in seven out of nine cases carbon stars have been found on the giant branch. Although numerous carbon stars are present in the field of the Clouds, it is easy to show that their density is greatly enhanced in the clusters and that the program stars must in general be cluster members. The carbon stars detected here all lie within 2′ of their respective cluster centers. A lower

TABLE 1

COLORS AND BAND STRENGTHS OF THE REDDEST GIANTS

Cluster	Star	Reference	Type	Color	Band Strength	Magnitude
L1	64	1	Ctm	0.56	...	16.2
L1	143	1	C	0.55	0.45	16.4
L1	178	1	Ctm	0.56	...	16.4
K3	24	2	C	0.91	1.33	17.0
K3	50	2	Ctm	0.37	...	16.6
K3	54	2	C	0.60	0.87	16.7
N339	151	1	C	1.10	1.43	16.2
N361	9	3	Ctm	0.82	...	16.4
N361	45	3	M	0.86	0.25	16.2
N419	5−3	3	Ctm	0.87	...	16.3
N419	5−7	3	Ctm	0.41	...	16.3
N419	4−133	3	C	0.79	0.74	16.6
N1783	13	4	M	0.73	0.40	16.0
N1783	15	4	Ctm	0.70	...	16.8
N1783	30	4	C	1.29	0.59	15.4
N1783	32	4	M	0.76	0.55	16.2
N1783	39	4	M	0.77	0.17	15.9
N1783	85	4	Ctm	0.90	...	16.3
N1841	142	1	Ctm	0.76	...	16.1
N1846	1	5	C	1.25	1.39	16.2
N1846	21	5	C	1.63	1.07	16.5
N1846	39	5	M	0.87	0.26	16.0
N1846	58	5	M	0.79	0.21	15.9
N1846	1302	6	C	1.64	1.18	16.6
N1846	4403	6	C	1.37	1.68	16.3
N1846	4508	6	C	1.25	1.38	16.3
N1978	1−14	5	M?	0.91	0.18	16.3
N1978	1−18	5	Ctm	0.66	...	16.9
N1978	2509	6	C	0.90	1.29	16.3
N1978	4401	6	C	1.28	1.30	16.2
N1978	4504	6	C	1.76	1.48	17.2

NOTES.—L1 = Lindsay 1; K3 = Kron 3.

REFERENCES.—(1) Gascoigne 1966, 1978; (2) Walker 1970; (3) Arp 1958*a*, *b*; (4) Gascoigne 1962; (5) Hodge 1960*a*, *b*; (6) Hesser *et al.* 1976.

limit on the density in clusters is therefore 500 deg^{-2}. This may be compared with a maximum of 50 deg^{-2} in the "peripheral regions" of the Clouds (Blanco, Blanco, and McCarthy 1978) in which the clusters are located.[5]

The presence of M stars in some clusters should not be overlooked. In NGC 1846 (and perhaps others), M stars and C stars are found side by side. As is also seen in the field, the carbon stars exceed the M stars in number, and the ratio is apparently higher in the SMC. NGC 1783 is exceptional in this regard for it contains three M stars, two of rather late type and one unusual carbon star, twice as bright as any other. The presence of M stars also allows us to discount the theory that the elemental abundance of carbon exceeds that of oxygen generally in the Clouds, as contemplated by van den Bergh (1979).

Figure 2 is a composite color-magnitude diagram for all the program stars. Two things are immediately apparent. First, the carbon stars form a cool extension of the giant branch. Second, the onset of C star characteristics is earlier in the SMC clusters than in the LMC. While it may be contested that the colors measured here are strongly blanketed, so that carbon features rather than low temperatures are implied by red colors, we may reach the same conclusion regarding the location of the C stars relative to the other giants by considering their luminosities.

To determine these luminosities, bolometric corrections for the program stars must be estimated from their colors. Considering the importance we shall ascribe to the luminosities, it would be extremely valuable to measure them directly by infrared photometry to 2.2 μm. Figure 3 is a plot of the bolometric

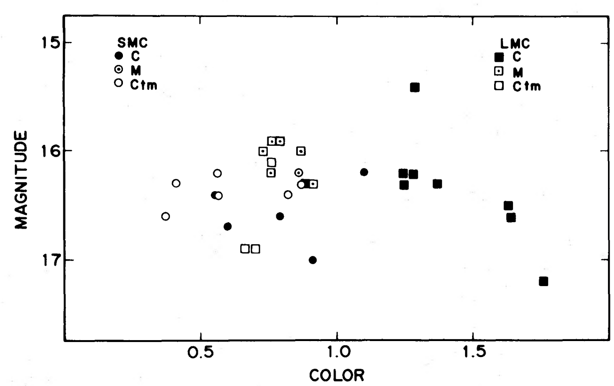

FIG. 2.—Color-magnitude diagram for the program stars in the system described in § 2. The magnitudes are close to visual.

[5] The E region of the LMC has four times this density, but is much closer to the Bar than any of our clusters.

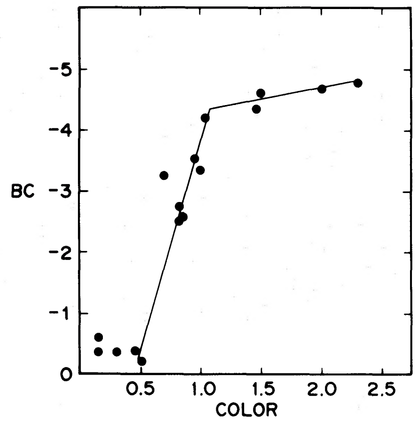

FIG. 3.—Relation between color and bolometric correction (BC) for galactic carbon stars. The two fitted line segments form the adopted relation.

corrections (BCs) obtained for carbon stars by Mendoza and Johnson (1965) against the (5350−6250) color from the scans of Faÿ *et al.* (1974). There are 16 stars in common, and the relation is moderately well determined. Note the very large bolometric corrections implied for the redder carbon stars in the LMC.

For the M and Ctm stars the bolometric corrections are harder to estimate. The (5350−6250) color does not provide a single-valued measure of bolometric correction owing to the effects of TiO blanketing on M stars. The band strengths in Table 1, however, imply that only two stars (NGC 1783−13, 32) are later than M2 (band strength = 0.3 in Faÿ, Stein, and Warren 1974). Hence from the (BC, spectral-type) relation of Johnson (1966), a reasonable estimate of the bolometric correction for the M and Ctm stars is -1.0 ± 0.4.

With distance moduli for the Clouds from Gascoigne (1972) of 18.7 and 19.2, plus the rather crude bolometric corrections described above, and assuming negligible reddening, we obtain the following results. The luminosities of the LMC carbon stars (excluding NGC 1783−30) are in the range $-7 < M_{\rm bol} < -5.5$ with a mean of −6.5. In the SMC the range is $-7 < M_{\rm bol} < -4$ with a mean of −5.1. We are somewhat suspicious of the extreme luminosities of the reddest carbon stars, as their bolometric corrections estimated from a (BC, $B-V$) relation (similar to Fig. 3 but with much more scatter) are considerably lower. With these BCs the mean luminosity of the carbon stars in both Clouds would be $M_{\rm bol} = -5.0$. These uncertainties serve to emphasize the importance of direct luminosity determinations. Nonetheless, the qualitative conclusion is quite clear. The carbon stars in both Clouds attain luminosities well beyond the theoretical limit for the first giant branch (the helium flash) which is in the range $-3.9 < M_{\rm bol} < -3.2$ for $0.2 < Y < 0.3$, $-4 < \log Z < -1$ and $0.7 < M < 1.0$ (Rood 1975). Consequently, they must be on the asymptotic giant branch (AGB). The mean luminosity for the M and Ctm stars, however, is close to that of the first giant branch tip. We obtain −3.8 and −3.4 (with an uncertainty similar to the difference) for the SMC and LMC, respectively.

IV. AGB EVOLUTION AND THE CLUSTER AGES

In this section we seek an explanation of the key question posed by the present results: why are carbon stars found frequently among the Magellanic Cloud clusters, but not among galactic globulars? As we have located these stars on the upper AGB, where Iben (1975) has discovered a mixing mechanism capable of producing carbon stars, we shall attempt to answer this by examining a further question: why are stars on the upper AGB (i.e., above the tip of the first giant branch) found among these MC clusters but not among galactic globulars?

The absence of such stars from the well-studied clusters M3, M13, and M92 has been noted by Cohen, Frogel, and Persson (1978). These clusters, of course, are relatively metal-poor. In 47 Tuc, which must be close to the upper limit for the metallicity of the MC clusters (§ I), the most luminous of the long-period variables (V3) reaches just above the predicted first giant branch tip. But its median luminosity is only $\langle M_{\rm bol} \rangle = -4.2$ (Eggen 1975), well below the mean of the carbon stars in the present study. In these populous clusters the absence of upper AGB stars cannot be due to chance, as Renzini (1977) has pointed out. In 47 Tuc, for example, some 35 such stars are predicted if AGB evolution goes to completion.

It is clear, therefore, that a real cutoff exists on the AGB close to the first giant branch tip and that this cutoff is only weakly sensitive to metallicity. Renzini (1977) has ascribed this cutoff to the effects of mass loss on the stellar envelope and has used it to calibrate the mass-loss scaling laws developed by Reimers (1975) and Fusi-Pecci and Renzini (1975).

If Renzini's hypothesis is correct (and it seems the natural explanation), we must ask how the MC globulars studied here avoid exhausting their envelopes as early as their galactic counterparts. As discussed above, metallicity differences cannot play the key role: the full range of metallicity is spanned by galactic globulars, but they do not populate the upper AGB. We shall argue below that helium abundance differences are relatively unimportant. The most plausible explanation is that our sample of MC globulars is considerably younger than the clusters of the galactic halo. The greater turnoff masses of younger clusters provide larger envelopes for their AGB evolution, which can hold out against mass loss and interior burning until the star reaches a luminosity high on the upper AGB. Little physical basis can be seen for the alternative that mass loss is suppressed in the Clouds in some way. If the formation of planetary nebulae is an important part of the mass-loss process, it is relevant to note that there is apparently no deficiency of planetaries in the Clouds (Jacoby 1978*a*, *b*).

Our argument can be made quantitative with the aid of a mass-loss law which is a function of physical parameters. Reimers (1975) has deduced such a law

($\dot{M} = 4 \times 10^{-13}\, \eta_R L/gR$) with a scaling parameter which lies in the range $\frac{1}{3} < \eta_R < 3$. With $\eta_R = 0.35$, a cutoff on the AGB is produced at the first giant branch tip (Renzini 1977). With this mass-loss law it is possible to derive the final mass on the AGB for a cluster of a given age. For an age of 3 billion years the initial mass on the giant branch at $Y = 0.24$ (Peimbert and Torres-Peimbert 1976) and $-3 < \log Z < -2$ is $1.33 \pm 0.07\, M_\odot$ (Rood 1975). The total mass lost at the end of AGB evolution is $0.60\, M_\odot$ from the geometric mean of the $\eta_R = 0.25$ and 0.5 cases in Figure 2 of Fusi-Pecci and Renzini (1976). The final mass is predicted to be $0.73\, M_\odot$. The corresponding luminosity from Paczynski's (1970) (luminosity, core-mass) relation is $M_{\rm bol} = -5.5$. Similar results are obtained with an alternative mass-loss expression (Fusi-Pecci and Renzini 1975) similarly normalized. ($M_f = 0.70\, M_\odot$ and $M_{\rm bol} = -5.3$).

Higher turnoff masses relative to the galactic halo can also be produced by a lower helium abundance. Pulsation theory (Iben 1974) and horizontal branch counts (Renzini 1977) appear to agree on a value for Y in galactic globular clusters of 0.22. Although there seems to be little scope for reducing this value further, we will consider the case $Y = 0.17$ for a cluster of age 13 billion years and $\log Z = -3$. With the scheme outlined above we obtain $M_f = 0.59\, M_\odot$ and $M_{\rm bol} = -4.3$. This compares with $M_{\rm bol} = -3.9$ for $Y = 0.22$. We conclude that a reduced helium abundance might assist in populating the upper AGB, but it is scarcely likely to manage it alone.

We turn now to Iben's (1975) mechanism for producing carbon stars on the upper AGB. Following strong helium-shell flashes in intermediate-mass, double-shell-source stars, Iben found that a convective zone extends from the surface down to the intershell region containing freshly synthesized helium and carbon. The operation of this mechanism appears to be critically dependent on the core mass (M_c) of the evolving star (Gingold 1977). For core masses larger than Iben's $0.95\, M_\odot$ (Fujimoto, Nomoto, and Sugimoto 1976), the mixing is deep and effective; at $M_c = 0.62\, M_\odot$ Gingold (1975) found no penetration of the intershell zone, and the mechanism failed. Although the critical value for M_c and its possible dependence on other parameters are as yet unexplored, Iben (1976) has noted that observations of LMC carbon stars by Crabtree, Richer, and Westerlund (1976) demand $M_c < 0.74\, M_\odot$. With the mass-loss rate adopted here, a critical core mass close to this value will just permit a cluster of age 3 billion years to show carbon stars at the AGB tip.

Upper and lower limits on this age estimate for the clusters may be obtained as follows. One constraint is provided by their integrated *UBV* colors (van den Bergh and Hagen 1968). Clusters as young as 1 billion years would have markedly bluer colors than observed (Searle, Sargent, and Bagnuolo 1973; Sandage 1963). An upper limit can be derived from Gingold's (1975) lower limit to the critical core mass. A final mass of $0.62\, M_\odot$ corresponds to an age of approximately 6 billion years.

Finally, observational evidence that carbon stars are found in intermediate-age clusters is seen in the open clusters of our Galaxy. Carbon stars are known or suspected to be members of NGC 2477, 2660, and 7789 (Hartwick and Hesser 1973, 1974; Mavridis 1960). Age estimates for the first two clusters are 0.9 and 1.2 billion years; NGC 7789 appears somewhat older (see Burbidge and Sandage 1958). With the exception of one of the two stars in NGC 2477, the luminosities of these stars place them on the upper AGB (see Scalo 1976). It may be objected that six carbon (or CH) stars are now known in ω Cen (McClure and Norris 1977). However, even the three reddest of these stars are fainter than the first giant branch tip (see Wing and Stock 1973). This is confirmed by infrared photometry (Persson *et al.* 1979). Presumably, these carbon stars are produced by a different mixing mechanism. Such a mechanism is also required by the existence of the early *R* stars in the field at $M_{\rm bol} \approx 0$ and the subgiant CH stars (Bond 1975).

V. CONCLUSIONS AND IMPLICATIONS FOR THE EVOLUTIONARY HISTORY OF THE CLOUDS

The hydrogen envelopes of stars on the AGB are consumed by two processes, mass loss acting on the outside and nuclear burning on the inside. When the envelope is exhausted, life on the AGB comes to an end. In the picture we have described the low-mass giants of the old galactic globulars meet an early death, never reaching the upper AGB. More-massive stars, however, in intermediate-age clusters rise high on the AGB and may become carbon stars by Iben's (1975) mixing mechanism.

Specifically, we suggest ages of 3 billion years (within a factor of 2) for a number of the red globulars in the Magellanic Clouds. Lindsay 1, Kron 3, NGC 339, 419 in the Small Cloud and NGC 1846 and 1978 in the Large Cloud fall into this category. The uncertainty in this age estimate is considerable, because of uncertainties in the critical core mass and in mass-loss rates. To the SMC clusters intermediate age has already been tentatively assigned by Gascoigne (1966). This is contested in the case of Kron 3 by Walker (1970). However, for NGC 1978 our assertion is in conflict with the discovery in the cluster of two (possibly four) RR Lyrae stars by Thackeray and Wesselink (1953). Since the presence of RR Lyrae stars is also inconsistent with the horizontal branch morphology suggested by Hesser, Hartwick, and Uguarte (1976), we look for confirmation of the RR Lyraes before discarding the hypothesis of intermediate age for this cluster.

We cannot, of course, make the converse assignment of larger ages to the remaining clusters, as incomplete spectroscopy or photometry may have prevented the detection of carbon stars. In the case of sparse clusters (e.g., many of the open clusters in our own Galaxy) short lifetimes on the AGB may prevent carbon stars from being seen for considerable periods of their evolutionary history.

The presence of M stars in some of the clusters should be taken as evidence of a metal-rich composition. In galactic globulars such stars are found at the tip of the first giant branch for clusters with $[M/H] > -1$ (Mould, Stutman, and McElroy 1979). For intermediate age clusters a similar phenomenon might also be expected. The smaller fraction of M stars in the SMC clusters would be consistent with a lower mean metallicity.

If our interpretation of the presence of carbon stars is correct, the naive analogy of § I between the red globulars of the Clouds and the clusters of the Galactic halo may be quite inadequate. A more correct picture (as discussed by Freeman 1974) is of a broad range of ages for the MC globulars. The clusters with extended red giant branches are intermediate in age between the young blue globulars such as NGC 1831 and 1866 (Freeman and Gascoigne 1971) and proven old globulars such as NGC 2257 (Walker 1972). The separation of clusters into red and blue is thus an artifact of the insensitivity of integrated $B - V$ colors to age for ages exceeding a billion years.

Integrated colors on other systems may well be more sensitive to age. Preliminary results of a program of narrow-band photometry on the Wing (1971) system indicate that carbon stars may be detected in the integrated light of intermediate-age clusters through the CN band at 8120 Å. The very red H-K colors of some MC globulars (Aaronson, Persson, and Frogel 1979) may also be attributed to the presence of carbon stars. If this is the case, an important age constraint for more distant galaxies may be available.

Finally, we consider the presence of large numbers of carbon stars in the Bar of the LMC and SMC (Blanco, Blanco, and McCarthy 1978). This suggests at the very least a large intermediate-age population with characteristics similar to the clusters examined here. Butcher (1977) has suggested from an analysis of the LMC luminosity function that such a population is dominant in the LMC (see also Hardy 1978). We caution, however, that one cannot conclude from the C/M star ratios alone the relative importance of an old population, as the reddest giants in such an old population, if metal poor, would be seen as K stars, not M stars.

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REFERENCES

Aaronson, M., Cohen, J. G., Mould, J. R., and Malkan, M. 1978, *Ap. J.*, **223**, 824.
Aaronson, M., Persson, S. E., and Frogel, J. A. 1979, in preparation.
Arp, H. C. 1958*a*, *A.J.*, **63**, 273.
———. 1958*b*, *A.J.*, **63**, 487.
Blanco, B. M., Blanco, V. M., and McCarthy, M. F. 1978, *Nature*, **271**, 638.
Bond, H. E. 1975, *Ap. J.*, **195**, 95.
Burbidge, E. M., and Sandage, A. R. 1958, *Ap. J.*, **128**, 174.
Butcher, H. R. 1977, *Ap. J.*, **216**, 372.
Cohen, J. G., Frogel, J. A., and Persson, S. E. 1978, *Ap. J.*, **222**, 165.
Crabtree, D. R., Richer, H. B., and Westerlund, B. F. 1976, *Ap. J. (Letters)*, **203**, L81.
Eggen, O. J. 1972, *Ap. J.*, **172**, 639.
———. 1975, *Ap. J.*, **195**, 661.
Faÿ, T. D., Jr., Stein, W. L., and Warren, W. H., Jr. 1974, *Pub. A.S.P.*, **86**, 1054.
Faÿ, T. D., Jr., Warren, W. H., Jr., Johnson, H. R., and Honeycutt, R. K. 1974, *A.J.*, **79**, 634.
Feast, M. W., and Lloyd Evans, T. 1973, *M.N.R.A.S.*, **164**, 15P.
Freeman, K. C. 1974, *ESO/SRC/CERN Conference on Research Programmes for New Large Telescopes*, ed. A. Reiz (Geneva: ESO), p. 177.
Freeman, K. C., and Gascoigne, S. C. B. 1971, *Bull. AAS*, **3**, 26.
Fujimoto, M., Nomoto, K., and Sugimoto, D. 1976, *Pub. A.S. Japan*, **28**, 89.
Fusi-Pecci, F., and Renzini, A. 1975, *Astr. Ap.*, **39**, 413.
———. 1976, *Ast. Ap.*, **46**, 447.
Gascoigne, S. C. B. 1962, *M.N.R.A.S.*, **124**, 201.
———. 1966, *M.N.R.A.S.*, **134**, 59.
———. 1972, *Quart. J.R.A.S.*, **13**, 274.
———. 1978, private communication.
Gingold, R. A. 1975, *Ap. J.*, **198**, 415.
———. 1977, *M.N.R.A.S.*, **178**, 569.
Hardy, E. 1978, *Ap. J.*, **223**, 98.
Hartwick, F. D. A., and Hesser, J. E. 1973, *Ap. J.*, **183**, 883.
———. 1974, *Ap. J.*, **192**, 391.
Hesser, J. E., Hartwick, F. D. A., and Uguarte, P. 1976, *Ap. J. Suppl.*, **32**, 283.
Hodge, P. W. 1960*a*, *Ap. J.*, **132**, 341.
———. 1960*b*, *Ap. J.*, **132**, 346.
Iben, I. 1974, *Ann. Rev. Astr. Ap.*, **12**, 215.
———. 1975, *Ap. J.*, **196**, 525.
———. 1976, *Ap. J.*, **208**, 165.
Jacoby, G. 1978*a*, *Ap. J.*, **226**, 377.
———. 1978*b*, *Bull. A.A.S.*, **10**, 397.
Johnson, H. L. 1966, *Ann. Rev. Astr. Ap.*, **4**, 193.
McClure, R. D., and Norris, J. E. 1977, *Ap. J. (Letters)*, **217**, L101.
Mavridis, L. N. 1960, *Pub. A.S.P.*, **72**, 48.
Mendoza, E. E., and Johnson, H. L. 1965, *Ap. J.*, **141**, 161.
Mould, J. R., Stutman, D., and McElroy, D. B. 1979, *Ap. J.*, **228**, 423.
Osmer, P. 1977, *Ap. J.*, **214**, 1.
Paczynski, B. 1970, *Acta Astr.*, **20**, 47.
Peimbert, M., and Torres-Peimbert, S. 1976, *Ap. J.*, **203**, 581.
Persson, S. E., Frogel, J. A., Aaronson, M., Cohen, J. G., and Matthews, K. 1979, in preparation.
Reimers, D. 1975, *Problems in Stellar Atmospheres and Envelopes*, ed. B. Baschek, W. H. Kegel, and G. Traving (Berlin: Springer-Verlag), p. 229.
Renzini, A. 1977, *Advanced Stages in Stellar Evolution*, ed. P. Bouvier and A. Maeder (Sauverny: Geneva Observatory).
Rood, R. T. 1975, private communication quoted by Renzini 1977.
Sandage, A. R. 1963, *Ap. J.*, **138**, 863.
Scalo, J. M. 1976, *Ap. J.*, **206**, 474.
Searle, L., Sargent, W. L. W., and Bagnuolo, W. B. 1973, *Ap. J.*, **179**, 427.
Stone, R. P. S. 1977, *Ap. J.*, **218**, 767.
Thackeray, A. D., and Wesselink, A. J. 1953, *Nature*, **171**, 693.
van den Bergh, S. 1975, *Ann. Rev. Astr. Ap.*, **13**, 217.

van den Bergh, S. 1979, preprint.
van den Bergh, S., and Hagen, G. L. 1968, *A.J.*, **73**, 569.
Walker, M. F. 1970, *Ap. J.*, **161**, 835.
———. 1972, *M.N.R.A.S.*, **156**, 459.
Wing, R. F. 1971, *Proceedings of the Conference on Late-Type Stars*, ed. G. W. Lockwood and H. M. Dyck (Tucson: Kitt Peak National Obs. Contr. No. 554), p. 145.
Wing, R. F., and Stock, J. 1973, *Ap. J.*, **186**, 979.

Note added in proof.—Frogel (private communication) has recently measured *JHK* magnitudes for many of the carbon stars discussed in this paper. He finds bolometric magnitudes close to -5, considerably fainter than we have obtained from the bolometric corrections in Figure 3 (but close to the values obtained from a [BC, $B - V$] relation [§ IV]). As bolometric luminosities are more directly obtained from infrared photometry, Frogel's values should probably be preferred. We note that the position of our carbon stars on the upper AGB remains secure, and the main conclusions of this paper remain unchanged.

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