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THE LOCATIONS OF X-RAY SOURCES IN GLOBULAR CLUSTERS*

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ABSTRACT

We have used the *SAS 3* X-ray observatory to measure the positions of the X-ray sources in the globular clusters NGC 1851, 6441, 6624, 6712, and 7078. The derived positions have error circles with 90% confidence radii of 20″ or 30″ which, in each case, include the optical center of the cluster. The results of a statistical analysis demonstrate that the X-ray sources are more concentrated toward the cluster centers than the visible stars and are therefore more massive than those stars.

Subject headings: clusters: globular — X-rays: sources

I. INTRODUCTION

Seven variable high-luminosity ($L_x \gg L_0$) X-ray stars have been associated with globular clusters on the basis of position determinations which were sufficiently accurate to justify the conclusion that they are members of those clusters. Three of the associations suggested by Giacconi *et al.* (1974) on the basis of the *Uhuru* survey, namely, 3U 1820−30/NGC 6624, 3U 1746−37/NGC 6441, and 3U 2131+11/NGC 7078(M15), were subsequently confirmed by Clark, Markert, and Li (1975) in observations with *OSO 7*. The *OSO 7* data also revealed two more, MX 0513−40/NGC 1851 and MX 1746−20/NGC 6440 (Markert *et al.* 1975). A sixth association, A1850−08/NGC 6712, was suggested by Seward *et al.* (1976) on the basis of *Ariel 5* observations, and confirmed by Cominsky *et al.* (1977) with *Uhuru* data, by Grindlay *et al.* (1977) with *ANS* and by Doxsey *et al.* (1977) with *SAS 3*. The seventh association is between the unique rapid burster MXB 1730−335, discovered by Lewin *et al.* (1976) with *SAS 3*, and a previously unknown and highly obscured cluster, LI, found by Liller (1977) within the X-ray error box.

Sources of slowly recurrent bursts, designated type I by Hoffman, Marshall, and Lewin (1978), have been found at positions with error boxes that include 3U 1820−30/NGC 6624 (Grindlay *et al.* 1976), MX 0513−40/NGC 1851 (Forman and Jones 1976; Clark and Li 1977), and MXB 1730−335/LI (Hoffman *et al.*). Two very brief outbursts which may have been type I bursts were observed from a source at a position with an error circle that includes 3U 1746−37/NGC 6441 (Li and Clark 1977). In the case of 3U 1820−30/NGC 6624 an apparent correlation between the occurrence of bursts and a low level of persistent flux from 3U 1820−30 has been cited as evidence that the burst source is, in fact, identical with 3U 1820−30 (Grindlay *et al.* 1976; Clark *et al.* 1977). In the case of MXB 1730−335, an apparent correlation between the bursts of type I and the rapid bursts of type II indicates that the bursts of both types come from the same source in LI.

The discovery of high-luminosity variable X-ray stars in globular clusters has raised a number of interesting problems concerning the nature and origin of these objects and their relation to the late stages of dynamical evolution in clusters. Not only are they more frequent by two orders of magnitude among globular cluster stars than among stars of the whole Galaxy (Katz 1975; Clark 1975), but they are also found preferentially in clusters with high central densities (Clark 1975; Bahcall and Ostriker 1975; Bahcall and Hausman 1977) indicative of an advanced state of dynamical evolution and possible core collapse, the only exception being A1850−08/NGC 6712. It is generally assumed that these and all other high-luminosity X-ray stars derive their energy from accretion of matter onto compact objects, and that in most or all cases the matter for accretion is supplied by a nuclear burning companion in a close binary system. Within a globular cluster the available channels for the evolution of a primordial binary into a presently active binary X-ray source are narrow or nonexistent. Thus, to explain the presence of any such sources in globular clusters, and, even more, their high relative occurrence rate and strong preference for centrally condensed clusters, most speculations about the nature of the globular cluster sources have been based on the assumption that the conditions which are peculiar to the condensed cores result in the formation of X-ray binaries through capture of field stars by neutron stars or black holes (Clark 1975; Fabian, Pringle, and Rees 1975; Hills 1975), or give rise to more or less massive black holes which accrete the debris from surrounding stars (Bahcall and Ostriker 1975; Grindlay and Gursky 1976).

In all theories of globular cluster X-ray sources, the hypothetical objects are substantially more massive than the visible stars and are therefore expected to lie close to the cluster centers. Since the expectation distance of an X-ray source from the center of its cluster must be a function of its mass, measurements of the radial distances of several X-ray sources can, in principle, provide a statistical measure of the mean mass of the sources.

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We call $G_3(q, r)dV$ the probability of finding an object of mass $m_x = qm_*$ in a volume element dV at a space radius r from the center of a spherically symmetric cluster of stars which are all of mass m_*. We assume the clusters are dynamically relaxed, so that $G_3(q, r)$ is proportional to the Boltzmann factor, $\exp[-q\psi(r)/\langle\Delta v^2\rangle]$, where $\psi(r)$ is the gravitational potential at r and $\langle\Delta v^2\rangle^{1/2}$ is the radial-velocity dispersion of the field stars in the core region. (Chandrasekhar 1942; Bahcall and Wolf 1976). The density of the stars, which we call $f(r)$, is proportional to the Boltzmann factor with q replaced by unity. Thus we obtain the relation,

$$G_3(q, r) \propto [f(r)]^q .$$

For an isothermal distribution $f(r)$ is approximated by the function (King 1972),

$$f(r) \propto [1 + (r/r_c)^2]^{-3/2} , \quad r < r_t ,$$

where r_c is the core radius and r_t the tidal radius. The measured quantity is the projected distance, r', of the source from the center. We therefore require an expression for the two-dimensional probability density which we call $G_2(q, r')$. Projecting $G_3(q, r)$, one finds,

$$G_2(q, r') \propto [1 + (r'/r_c)^2]^{-(3q-1)/2} .$$

The expection value of the projected position of the X-ray source can now be computed according to the formula

$$\langle r\rangle = \int_0^{r_t} G_2(q, r')r'^2dr' \Big/ \int_0^{r_t} G_2(q, r')r'dr' .$$

In the limit as $r_t/r_c \to \infty$ the quotient approaches the quantity,

$$\langle r\rangle \approx \frac{2r_c}{\sqrt{\pi}} \frac{\Gamma(3q/2 - 3/2)}{\Gamma(3q/2 - 1)} \quad \text{for } q > 11/6 .$$

This is a good approximation for $\langle r\rangle$ in the case of the X-ray globular clusters for which $r_t \gg r_c$. Figure 1 shows a plot of $\langle r\rangle/r_c$ versus q. For $q \gg 1$ the expression for $\langle r\rangle$ approaches the one derived previously by Bahcall and Wolf (1976), which is

$$\langle r\rangle \approx 2r_c\left(\frac{2}{3\pi}\right)^{1/2} q^{-1/2} \approx 0.9r_c q^{-1/2} .$$

Objects with masses that are only a few times larger than the average mass of the visible stars are concentrated in the core region. Thus, X-ray position measurements with 1 σ error radii of the order of the core radii can provide significant information about the masses of globular cluster X-ray sources.

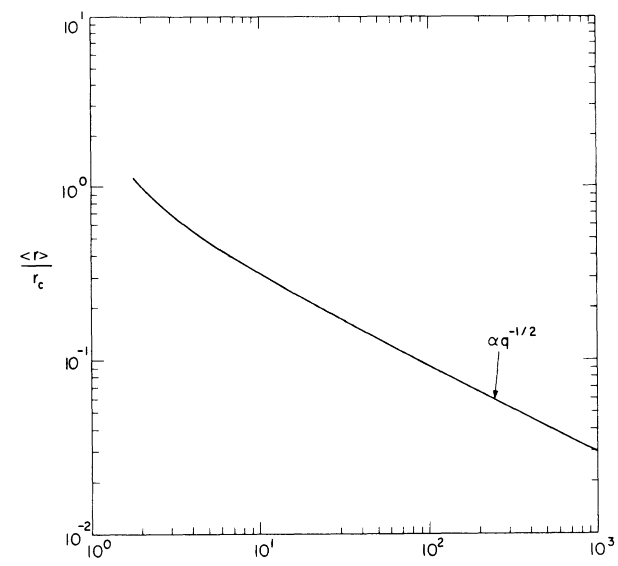

FIG. 1.—The normalized projected radius, $\langle r\rangle/r_c$, as a function of the normalized mass of the X-ray source, $q = m_x/m_*$. For large q the asymptotic limit of $\langle r\rangle$ is proportional to $q^{-1/2}$.

In this *Letter* we report our measurements of the positions of five of the X-ray sources which are associated with globular clusters, and the implications which these measurements have for the masses of the sources.

II. OBSERVATIONS

Two or more independent position measurements were made of each source with the two rotating modulation collimator detectors which have been described previously by Doxsey *et al.* (1976). Each independent observation had an effective exposure of approximately 10^5 s. In each case the X-ray source was detected at a sufficiently high level of significance that the error in the position is dominated by systematic effects and not by photon counting statistics. In Table 1 we list the final positions and the 90% confidence (2.15 σ) error radii. The 90% confidence error circles are shown on photographs of the clusters in Figure 2 (Plate L4). The results were calculated according to the procedure described by Doxsey *et al.* (1977). This work was part of a *SAS 3* program to measure the positions of galactic X-ray sources (Bradt *et al.* 1977).

III. DISCUSSION

Each of the 90% error circles lies entirely within the tidal radius of the respective cluster, and includes the cluster center. We estimate the probability of this being an accidental coincidence by multiplying the probabilities of a globular cluster center occurring by chance within each of the five error circles. As in a previous and similar discussion (Clark, Markert, and Li 1975), we take for the probability the product of the error circle solid angle and the average density of globular cluster centers at the angular distance from the galactic center of the given error circle. The total probability is much less than 10^{-4}. The smallness of this number further strengthens the evidence that all five sources are members of the clusters with which they have been previously associated.

To compare the positions of the sources with the distributions of the visible stars, we computed the likelihood function for the projected radial displacement of each source from its cluster center according to the formula,

$$L_i(r') \propto \int_{\phi=0}^{\pi} \exp\left[-(r'^2 + s_i^2 - 2r's_i\cos\phi)/2\sigma_i^2\right]d\phi \propto I_0(s_i r'/2\sigma_i^2)\exp(-r'^2/2\sigma_i^2) ,$$

where s_i and σ_i are the measured displacement and standard deviation of the ith source from its cluster center, respectively, and I_0 is the modified Bessel function of zeroth order. The results are shown in Figures 3*a*–3*e* along with the normalized integrated distributions of stars calculated from the functions given by King (1962) with the measured core and tidal radii listed in Table 1. In each case the likelihood is at or near its maximum value at $r' = 0''$. The radii at which the likelihood functions have fallen to 10% of their maximum values are listed in Table 1. In each case the fraction of visible stars within the 10% radius is less than one-half. If X-ray sources in globular clusters were actually distributed like the visible stars of clusters the chances of all five lying at less than the median radius would

TABLE 1

X-RAY AND OPTICAL DATA ON X-RAY SOURCES IN GLOBULAR CLUSTERS

Globular Cluster X-Ray Source Designations	Measured X-Ray Position* 90% Error Radius $(2.15\sigma_i)$	Optical Center Position*†	Optical X-Ray Difference s_i	r_c r_t	Radius for $L(r_i)/L_{MAX}=0.1$	Fraction of Stars inside r_i	Observed Flux Densities‡ (2-11) keV	Dates (mo./yr)
NGC 1851/	$5^h12^m28^s.7, -40°05'53''$	$5^h12^m27^s.7, -40°06'03''$		8″[1]				
M X0513−40/	78°.1196, −40°.0981	78°.1154, −40°.1008	15″		32″	0.38	4 μJy	12/76
2S 0512−400......	20″			22′				
NGC 6441/	17 46 48.7, −37 02 17	17 46 48.8, −37 02 25		9″[2]			13	6/75
4U 1746−37/	266.7029, −37.0381	266.7033, −37.0403	8		30	0.48		
2S 1746−370......	30			8′			13	6/77
NGC 6624/	18 20 28.4, −30 23 14	18 20 27.7, −30 23 11		8″[3]			68	5/75
4U 1820−30/	275.1183, −30.3872	275.1154, −30.3864	9		25	0.36	256	6/75
2S 1820−303......	20			13′			210	7/75
NGC 6712/	18 50 21.9, −8 45 54	18 50 20.8, −8 46 01		49″[1]			4	5/77
A1850−08/	282.5913, −8.7650	282.5867, −8.7669	18		43	0.21		
2S 1850−087......	30			≥8′				
NGC 7078/	21 27 34.2, 11 56 51	21 27 33.7, 11 57 03		10″[4]			6	12/75
4U 2131+11/	321.8925, 11.9475	321.8904, 11.9508	14		30	0.37		
2S 2127+119......	20			15′				

REFERENCES.—[1] Peterson and King 1975. [2] Illingworth and Illingworth 1976. [3] Bahcall 1976. [4] Bahcall, Bahcall, and Weistrop 1975.

* Epoch 1950.

† Estimated centroid of an isophotal contour at several core radii from center.

‡ 1.0 μJy is equivalent to 2.2×10^{-11} ergs $^{-1}$ cm^{-2} in the energy range from 2 to 11 keV. $I_{crab} = 1060$ μJy (see Bradt *et al.* 1977).

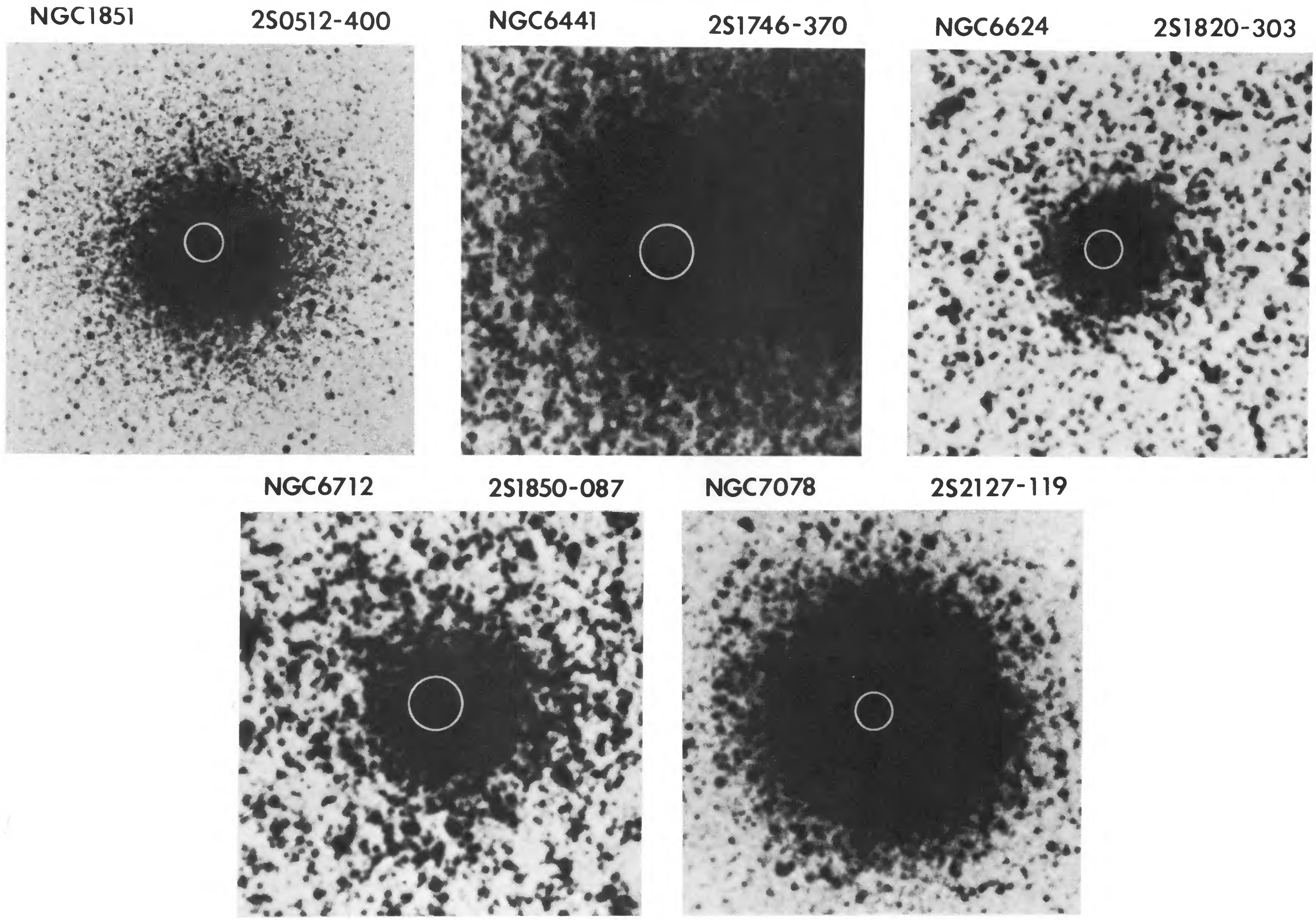

FIG. 2.—X-ray error circles (90% confidence) superposed on optical photographs of globular clusters. The photograph of NGC 1851 is from the ESO quick survey. All other photographs are taken from red plates of the Palomar Sky Survey (© National Geographic Society). All the photographs are displayed with the same scale. All circles have diameters of 40″ or 60″. For each globular cluster the 2S designation for the error circle is indicated. North is up and east is to the left. The west edge of the photograph of NGC 6441 is part of the image of a 3d mag star, SAO 209318. Note that in all five cases the tidal radius would lie outside the boundary of the photograph. (See Table 1.)

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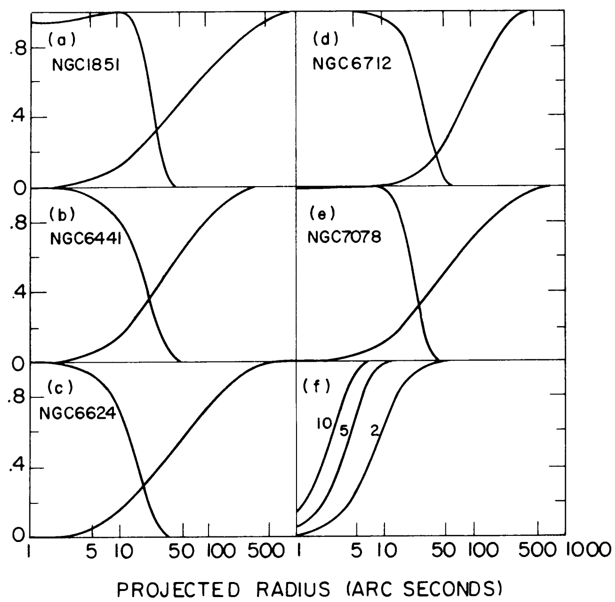

FIG. 3.—(*a*)–(*e*) The likelihood functions for the projected radial displacement of each source from its cluster center and the normalized integrated distributions of stars. The maximum likelihood is normalized to unity in each case. The values of the projected radii at which the likelihoods fall to 10% are listed in Table 1. The integrated distributions increase to 1 as the projected radii increase to the tidal radii. (*f*) Probability that an X-ray source of mass 2, 5, or 10 times m_* will lie within a given projected radius for the case of NGC 7078.

be 2^{-5}. The product of the actual fractions is 5×10^{-3}. These results indicate that the sources are more concentrated toward the cluster centers than the visible stars. We show in Figure 3*f* the probability that an X-ray source of mass 2, 5, or 10 times m_* is located within a given projected radius for the case of NGC 7078.

To judge what this concentration implies about the masses of the X-ray sources, we employed another likelihood analysis. The likelihood function for the mass ratio ($q = m_x/m_*$) of the ith X-ray source is

$$M_i(q) \propto \int_0^{r_t} L_i(r')G_2(q, r')r'dr' \Big/ \int_0^{r_t} G_2(q, r')r'dr' .$$

If we now make the additional assumption that q is the same for all the cluster sources, then the composite likelihood is,

$$M(q) \propto \prod_{i=1}^{5} M_i(q) .$$

Figure 4 is a plot of $M(q)$ normalized to its value for large q. We note that the relative likelihood falls to a value

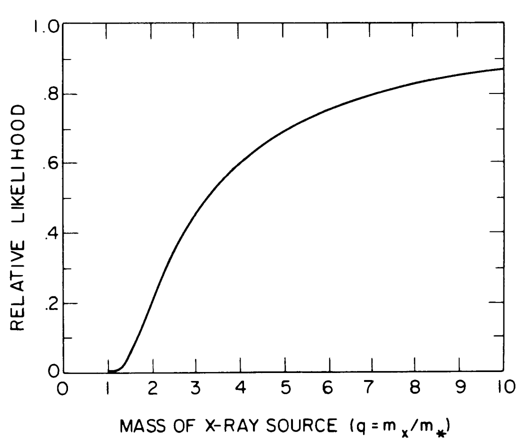

FIG. 4.—Relative likelihood, $M(q)$, as a function of the mass of the X-ray source in units of m_*. This function is computed assuming that q is the same for all five globular cluster X-ray sources.

of 10% at $q \approx 1.6$. This supports the conclusion that the X-ray sources are heavier than the visible stars, the masses of which are less than $\sim 0.8\ M_{\odot}$. It is evident, however, that more accurate X-ray positions and optical center positions are required to decide the question of whether the X-ray sources are low-mass binaries or massive black holes.

In conclusion, we note that the large core radius of NGC 6712 makes that object especially promising for future measurements of the X-ray source position with higher-resolution instruments.

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REFERENCES

Bahcall, J. N., Bahcall, N., and Weistrop, D. 1975, *Ap. Letters*, **16**, 159.
Bahcall, J. N., and Ostriker, J. P. 1975, *Nature*, **256**, 23.
Bahcall, J. N., and Wolf, R. A. 1976, *Ap. J.*, **209**, 214.
Bahcall, N. A. 1976, *Ap. J. (Letters)*, **204**, L83.
Bahcall, N. A., and Hausman, M. A. 1977, *Ap. J.*, **213**, 93.
Bradt, H. V., *et al.* 1977, *Nature*, **269**, 21.
Chandrasekhar, S. 1942, *Principles of Stellar Dynamics* (Chicago: University of Chicago Press).
Clark, G. W. 1975, *Ap. J. (Letters)*, **199**, L143.
Clark, G. W., and Li, F. K. 1977, *IAU Circ.*, No. 3092.
Clark, G. W., Li, F. K., Canizares, C., Hayakawa, S., Jernigan, G., and Lewin, W. H. G. 1977, *M.N.R.A.S.*, **179**, 651.
Clark, G. W., Markert, T. H., and Li, F. K. 1975, *Ap. J. (Letters)*, **199**, L93.
Cominsky, L., Forman, W., Jones, C., and Tananbaum, H. 1977, *Ap. J. (Letters)*, **211**, L9.
Doxsey, R., Apparao, K. M. V., Bradt, H. V., Dower, R. G., and Jernigan, J. G. 1977, *Nature*, **269**, 112.
Doxsey, R., *et al.* 1976, *Ap. J. (Letters)*, **203**, L9.
Fabian, A. C., Pringle, J. E., and Rees, M. J. 1975, *M.N.R.A.S.*, **172**, 15P.
Forman, W., and Jones, C. 1976, *Ap. J. (Letters)*, **207**, L177.
Giacconi, R., Murray, S., Gursky, H., Kellogg, E., Schreier, E., Matilsky, T., Koch, D., and Tananbaum, H. 1974, *Ap. J. Suppl.*, **27**, 37.
Grindlay, J., and Gursky, H. 1976, *Ap. J. (Letters)*, **205**, L131.
Grindlay, J. E., Gursky, H., Parsignault, D. R., Cohn, H., Heise, J., and Brinkman, A. C. 1977, *Ap. J. (Letters)*, **212**, L67.
Grindlay, J., Gursky, H., Schnopper, H., Parsignault, D. R., Heise, J., Brinkman, A. C., and Schrijver, J. 1976, *Ap. J. (Letters)*, **205**, L127.
Hills, J. G. 1975, *A.J.*, **80**, 1075.
Hoffman, J. A., Marshall, H. L., and Lewin, W. H. G. 1978, *Nature*, **271**, 630.
Illingworth, G. D., and Illingworth, W. L. 1976, *Ap. J. Suppl.*, **30**, 227.
Katz, J. L. 1975, *Nature*, **253**, 698.
King, I. 1962, *A. J.* **67**, 471.
King, I. R. 1972, *Ap. J. (Letters)*, **174**, L123.
Lewin, W. H. G. *et al.* 1976, *Ap. J. (Letters)*, **207**, L95.
Li, F., and Clark, G. 1977, *IAU Circ.*, No. 3095.
Liller, W. 1977, *Ap. J. (Letters)*, **213**, L21.
Markert, T. H., Backman, D. E., Canizares, C. R., Clark, G. W., and Levine, A. M. 1975, *Nature*, **257**, 32.
Peterson, C. J., and King, I. R. 1975, *A.J.*, **80**, 427.
Seward, F. P., Page, C. G., Turner, M. J. L., and Pounds, K. A. 1976, *M.N.R.A.S.*, **175**, 39P.

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