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DIFFUSION TIME SCALES IN WHITE DWARFS

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ABSTRACT

The time scales of a few representative elements diffusing in hydrogen- and helium-rich white-dwarf envelopes are presented. The dependences on depth, the stellar mass, and effective temperature are investigated. It is shown that diffusion processes are sufficiently rapid that the observed monoelemental character of white-dwarf spectra can be explained. Even deep convective mixing, present in the surface layers of cooler white dwarfs, cannot bring back to the photosphere heavy elements that have diffused downward. The present calculations lead to estimates of diffusion time scales that are so short that no trace element should be seen in the spectra of white dwarfs. Mechanisms such as accretion competing with diffusion may have to be invoked to explain the small but measurable abundances of heavy elements in the spectra of these objects. For the vast majority of white dwarfs these conclusions are not affected by our having neglected radiative forces or by inaccuracies in the transport coefficients. However, improvements in the estimates of transport coefficients in dense plasmas are needed.

Subject headings: convection — stars: interiors — stars: white dwarfs

I. INTRODUCTION

Following Schatzman's (1958) early recognition that white-dwarf spectra present large and genuine abundance anomalies, a wealth of high-quality observational data have been accumulated, and increasingly sophisticated model-atmosphere analyses have confirmed those results. It is now generally believed that, as Schatzman suggested, diffusion plays an essential role in the evolution of the superficial chemical compositions of white dwarfs. A systematic study using fairly realistic envelope models has yet to be done, however, and in this paper we address ourselves to some of the problems involved.

At least two other mechanisms are thought to modify the surface composition of white dwarfs. Accretion of interstellar matter onto a star has often been invoked, but little is known about the details of the process, and detailed calculations remain to be done. We also have much to learn about potential convective mixing phases which could possibly bring to the surface elements that have been processed in the core of the star. In both cases, however, the large underabundances of heavy metals that are seen in white-dwarf spectra remain unexplained.

In the present work, we estimate the diffusion time scales of helium, carbon, oxygen, and calcium in white-dwarf envelopes. While a few estimates of the diffusion velocities under white-dwarf conditions are available in the literature, a detailed discussion of diffusion processes in both hydrogen- and helium-rich envelopes is still lacking. In this paper we partly fill the need for such a discussion and show that the large underabundances of heavy elements in white-dwarf spectra are consistent with the diffusion model.

After briefly reviewing the evidence, both observational and theoretical, favoring element segregation in the atmospheres of white dwarfs (§ II), we present the hypotheses that are made as well as the methods used to estimate diffusion time scales (§ III). Our results are presented in § IV and we then discuss their implications (§ V). We finally summarize our findings in § VI.

II. EVIDENCE IN FAVOR OF ELEMENT SEPARATION IN THE ATMOSPHERES OF WHITE DWARFS

a) *Observational Evidence*

Several important research efforts have been made in order to understand the optical spectra of white dwarfs and derive their atmospheric composition. Photometric data in both the broad-band and Strömgren systems, as well as photographic and scanner spectra, have been analyzed with the help of model-atmosphere techniques. A list of the results of several recent analyses for various spectral types is presented in Table 1. (We also refer the reader to Weidemann 1975.) In the first column we give the spectral type. This is followed by the hydrogen-to-helium number ratio as inferred from the analyses, and the number fraction of heavy elements as compared with the Sun. The type of observational data used is then listed and references to the original papers are finally given. From these results, giving more weight to the newer, more detailed analyses, it appears that all white dwarfs are extremely metal-deficient as compared with the Sun. Atmospheres of spectral type DA are very hydrogen-rich, but it is generally difficult, from optical observations alone, to derive a stringent limit on the hydrogen-to-helium ratio (see, however, Auer and Shipman 1977). Such is

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TABLE 1

INFERRED ATMOSPHERIC COMPOSITIONS OF WHITE DWARFS FROM OPTICAL OBSERVATIONS

Spectral Type	n_H/n_{He}	n_{metals}/n_{metals} (⊙)	Data	Reference
DA	~ 5.7	< 1	*UBV*	1
	~ 6.7	< 1	*UBV*	2
	~ 6.5	< 1	*uvby*	3
	> 99	< 1 $< 10^{-2}$–10^{-1} (Ca)	Scans	4
	~ 9	$< 10^{-2}$	*uvby*	5
	No He (assumed)	$\sim 2 \times 10^{-4}$ (Ca)	Scans	6
	$\gtrsim 10^3$	No Metals (assumed)	Scans	7
DB	$< 10^{-5}$–10^{-4}	$< 10^{-3}$–10^{-1}	*UBV*	8
	$< 10^{-4}$	$< 10^{-3}$ ($n_C/n_{He} < 5 \times 10^{-3}$)	*UBV*	9
	$< 10^{-4}$	$< 10^{-2}$ (Ca)	Scans	4
	$< 10^{-5}$	$\sim 10^{-3}$ (Ca)	IPCS	10
DA–B	$\sim 10^{-5}$–10^{-4}	...	ITS	11
DC	$< 10^{-3}$	< 1	*uvby*	12
	$< 10^{-4}$	$< 10^{-4}$ ($n_C/n_{He} < 10^{-3}$) ($n_C/n_O < 10$)	Scans	13
λ4670	$< 10^{-5}$–10^{-4}	$< 10^{-4}$ ($10^{-3} < n_C/n_{He} < 10^{-2}$) ($n_C/n_O > 10$–$10^2$)	Scans	13
DF, DG	$< 10^{-4}$	$\sim 10^{-5}$	Photographic spectra	14
	$< 10^{-4}$	$\sim 10^{-5}$–10^{-4}	Scans	15
	$< 10^{-4}$	$< 10^{-5}$–10^{-4}	Scans	13
	$\sim 3 \times 10^{-4}$	...	ITS	16

REFERENCES.—[1] Weidemann 1963. [2] Terashita and Matsuhima 1969. [3] Wickramasinghe 1972. [4] Shipman 1972. [5] Wehrse 1975. [6] Shipman 1977. [7] Auer and Shipman 1977. [8] Bues 1970. [9] Strittmatter and Wickramasinghe 1971. [10] Shipman *et al.* 1977. [11] Wickramasinghe and Whelan 1977. [12] Baglin and Vauclair 1973. [13] Bues 1973. [14] Wegner 1971. [15] Grenfell 1972. [16] Liebert 1977*a*.

not the case for the non-DA stars, and the analyses then lead to atmospheres made up of almost pure helium. Finally, the number ratios n_C/n_{He}, and especially n_C/n_O, seem to be important quantities for the DC and the λ4670 stars, which are believed to be helium-rich stars of intermediate temperatures.

Recent observations at nonoptical wavelengths have also added more weight to these conclusions and, in some cases, have been used to put more stringent limits on the possible chemical composition of the atmospheres of a few white dwarfs. For instance, the young DA white dwarf HZ 43 has been detected in both the EUV (Lampton *et al.* 1976) and in the soft X-ray ranges (Hearn *et al.* 1976; Margon *et al.* 1976*c*). These detections have been interpreted by Margon *et al.* (1976*b*), Durisen, Savedoff, and Van Horn (1976), and in more detail by Auer and Shipman (1977) as thermal emission from a hot atmosphere. The latter authors have assigned an effective temperature of $T_e \sim$ 55,000–70,000 K to HZ 43 and have shown that a model atmosphere consistent with the observations would have $n_H/n_{He} \gtrsim 10^3$ with negligible metal content. Shipman (1976) has also proposed a thermal origin for the soft X-rays detected by the *ANS* satellite (Mewe *et al.* 1975*a*, *b*) from the Sirius system. In this model, thermal emission from deep transparent layers of the DA white dwarf Sirius B is responsible for the X-rays. This model is consistent with the *Copernicus* observations of Savedoff *et al.* (1976) in the far-UV. Pursuing Shipman's idea, Shipman *et al.* (1977) used the *Apollo-Soyuz* EUV upper limits to derive rather stringent conditions on the atmospheric parameters of the white dwarf: $T_e \sim$ 32,000–32,500 K $n_H/n_{He} \sim 0.5$–1.0×10^4 with no metals. (Until recently, this suggestion was quite attractive, since the alternative model of a nonthermal source produced by a hot corona around Sirius B does not seem to work [Fontaine 1977]. However, Cash, Bowyer, and Lampton [1978] have argued against the Shipman model on the basis of more recent EUV rocket observations.) Finally, another hot DA white dwarf (Feige 24) has also been detected in the EUV (Margon *et al.* 1976*a*) and UV (Holm 1976), and the origin of these emissions is again explained (Margon *et al.* 1976*a*) in terms of thermal emission from a helium-poor and metal-deficient atmosphere. In fact, it now seems likely (Shipman 1976; Wesemael 1978*a*) that hot DA's could be significant contributors to the diffuse, galactic soft X-ray background radiation. A thermal origin for these X-rays requires a minimal X-ray opacity, and since the opacity is provided mainly by heavy elements, these stars should have $n_H/n_{He} \gtrsim 10^3$ and essentially no heavier elements.

While white dwarfs, as a class, are characterized by remarkably pure (monoelemental) atmospheres, the trace element content can vary by large amounts from one star to another. This is probably due to different past histories (e.g., accretion and convective mixing phases), masses, states of rotation, and ages

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(Weidemann 1975). For instance, the DB star GD 40 is the only one of its spectral type showing metallic lines (Ca II H and K lines, according to Wickramasinghe *et al.* 1975). Shipman, Greenstein, and Boksenberg (1977) have assigned an effective temperature of $T_e \sim$ 14,000 K to that star and have derived a calcium-to-helium ratio three orders of magnitude smaller than the solar value. Though small, this ratio is still three orders of magnitude greater than the value derived by Grenfell (1972) for the cool ($T_e \sim 5800$ K) DG white dwarf Van Maanen 2. Another white dwarf, Stein 2051B (spectral type DC), is even more deficient in H, Ca, Fe, and Mg than Van Maanen 2, according to Liebert (1976).

Several other stars have spectra worth mentioning: Gr 346 is the hottest object showing only Ca II lines in its spectrum (Greenstein 1975); it has an estimated effective temperature $T_e \sim 12{,}000$ K and, although lower than the solar value, its atmospheric calcium content is the highest so far observed in a white dwarf (Greenstein 1976*a*). Three white dwarfs are unique in showing both hydrogen and helium lines in their spectra: Feige 7, EG 153, and Gr 295 (Liebert and Strittmatter 1977), belonging to the so-called DA–B spectral type. These all have helium-dominated atmospheres and their importance lies in the fact that hydrogen-to-helium number ratios can be derived instead of obtaining only upper limits for helium-rich white dwarfs. For EG 153, Wickramasinghe and Whelan (1977) obtain $n_{\rm H}/n_{\rm He} \sim 10^{-5}$–$10^{-4}$. According to Liebert (1977*a*), Ross 640 could be a cool counterpart of these objects. This DFp star shows weak hydrogen lines, and Liebert has derived a value of $n_{\rm H}/n_{\rm He} \sim 3 \times 10^{-4}$. Very cool stars have also been discovered recently: Greenstein (1976*b*) mentions that Gr 302 and Gr 382 could very well be extremely metal-deficient DG's, while Dahn *et al.* (1977) and Cottrell, Bessell, and Wickramasinghe (1977) found the coolest degenerate star so far: LP 701-29. This star has an estimated effective temperature of $T_e \approx 4000$ K, its atmosphere is hydrogen-dominated, and the metals are underabundant. Very cool, helium-rich, and metal-deficient stars have also been discussed by Mould and Liebert (1978). At the other end of the scale, Liebert (1977*b*) has identified HZ 21 as a hot helium-rich white dwarf ($T_e \approx 70{,}000$ K; Shipman 1972).

In short, the observations indicate that all white dwarfs are metal-deficient with respect to the Sun by factors ranging from 10^2 to 10^5 and possibly higher. Both the hydrogen- and helium-rich sequences appear to span the observed temperature range. For helium-rich stars, the nonvisibility of hydrogen lines coupled with the relatively high opacity of hydrogen with respect to helium lead to a stringent criterion: $n_{\rm H}/n_{\rm He} < 10^{-4}$. For DA's, we do not have generally such strong conditions on $n_{\rm H}/n_{\rm He}$ based on optical observations. However, the existence of hot DA's as soft X-ray sources implies $n_{\rm H}/n_{\rm He} \gtrsim 10^3$–$10^4$. Finally, the CNO elements appear to be strongly underabundant in the atmospheres of white dwarfs except for the λ4670 stars where the carbon-to-helium number ratio is normal. From this brief survey of the observational data, we therefore find that a white-dwarf atmosphere is primarily characterized by an almost pure chemical composition. Such a degree of extreme purity has not yet been found in other stars, and we take this as a very strong indication that diffusion has occurred in the outermost layers of white dwarfs.

Several other authors have also invoked diffusion to qualitatively explain the observations. Among them, let us mention Strittmatter and Wickramasinghe (1971), Shipman (1972), Wegner (1972), Grenfell (1974), Hintzen and Strittmatter (1975), Greenstein (1976*a*), Margon *et al.* (1976*b*), and Shipman (1976). It is now generally accepted that diffusion is the principal factor in the formation of almost pure white-dwarf atmospheres.

b) *Theoretical Evidence*

White dwarfs are characterized by large surface gravities, which strongly suggests that gravitational sorting of the elements could be operative in the atmospheres and envelopes of these stars. This was first put forward by Schatzman (1958), who showed that a normal-composition atmosphere would be sorted over relatively short time scales under the influences of the gravitational and electric fields. While white dwarfs are probably not born with normal-composition atmospheres (Weidemann 1975), Schatzman's calculations indicate that gravitational settling can be an effective process in these objects.

Diffusion velocities under white-dwarf conditions have been estimated by Baglin (1971, 1974) and Vauclair and Reisse (1977). These calculations indicate that heavy elements sink relatively rapidly below the photosphere if diffusion is allowed to take place. Since young, hot white dwarfs have radiative envelopes (Fontaine and Van Horn 1976), the suggestion is that element separation occurs early in the lifetime of a star provided that other competing mechanisms (e.g., accretion, meridional circulation) are not important. This is compatible with the existence of hot DA white dwarfs as soft X-ray sources (Margon *et al.* 1976*b*; Vauclair and Reisse 1977), the high hydrogen-to-helium number ratio in DA's, and the absence of metallic lines in white-dwarf spectra above $T_e \sim$ 14,000 K.

As a white dwarf cools down, recombination of the main constituent (hydrogen or helium) leads to the formation of a convection zone which grows inward from the surface, following the region of partial ionization, until the base of the zone becomes degenerate (Fontaine *et al.* 1974; Fontaine and Van Horn 1976). The base of the convection zone then retreats outward toward the surface with a further decrease of the luminosity as pressure ionization and electron conduction tend to suppress convection. For a typical white-dwarf gravity of $g \sim 10^8$ cm s^{-2}, we find that surface convection sets in at $T_e \sim 17{,}500$ K for a hydrogen-rich atmosphere, and at $T_e \sim 60{,}000$ K for a helium-rich model. Below these temperatures, convective mixing can dredge up some of the heavy elements that were sinking down, dilute them in the convection zone,

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and bring them back into the line-forming region. While the details of this mechanism are still unclear (we address ourselves to some of these problems in the present paper), this qualitative theoretical picture has been invoked by several authors to explain the presence and faintness of some metallic lines in the spectra of relatively cool white dwarfs (Weidemann 1975; Hintzen and Strittmatter 1975; Greenstein 1976*a*).

III. METHODS OF COMPUTATIONS

a) *Motivations and Hypotheses*

In this paper, we present computations of diffusion time scales of a few representative elements in the envelopes of white dwarfs. We wish to determine how these time scales vary with depth, mass, and effective temperature (or, equivalently, age). In particular, time scales are evaluated at mass shells corresponding to maximum depths reached by convection zones in white-dwarf models. By comparing them with estimates of stellar ages, we can verify whether or not some heavy elements have had the time to diffuse below, out of reach of the convection zone. We also want to study the relative importance of gravitational settling with respect to thermal diffusion in these models. Finally, since nonideal effects are important in white-dwarf envelopes, we would like to see how the element separation process is affected in a dense plasma.

In the present calculations, we assume that *trace elements* diffuse in a medium dominated by one species (either hydrogen or helium). The diffusion problem in presence of two elements in roughly the same proportions is highly complex. Some recent progress has been made (see, e.g., Montmerle and Michaud 1976) for the hydrogen-helium case, but we have not used the results here, since other uncertainties appear to be more important. The mechanical structure of a white-dwarf model is then essentially specified by the dominant species via the equation of state. On the other hand, the thermal structure depends on the opacity which can be affected by variations of even the trace element content. In principle, therefore, evolutionary calculations that take into account opacity changes due to a variable trace element content (both with time and depth) are required. Such calculations are beyond the scope of this work. Fortunately, however, significant opacity differences due to a different heavy-element content occur only at low enough temperatures. It is only when the main constituent is not ionized that this could happen, and since we are dealing mostly with relatively large depths where the temperature is fairly high, our approximation is probably quite acceptable. In fact, even near the photosphere, Fontaine *et al.* (1974) have shown that the thermal structure of a helium-rich white-dwarf model ($\log g = 8$) is quite insensitive to variations of the heavy-element content for $T_e \gtrsim 22{,}000$ K. In short, we completely decouple the diffusion problem from the envelope-modeling problem.

The white-dwarf envelope models have been computed by using the same input physics as in Fontaine and Van Horn (1976), except that we have employed an improved version of the equation of state (Fontaine, Graboske, and Van Horn 1977). In our calculations, we have also assumed that the amount of mass of the main constituent is larger than the maximum possible mass contained in the convection zone. This prevents mixing of that constituent with underlying layers, thus preserving the initial character of the envelope (either hydrogen- or helium-rich). If, for example, all trace elements have had the time to sink below this boundary, convection will obviously not be able to dredge them up. Another approach is to assume that the amount of mass of the main atmospheric constituent is so small that effectively the settling process has been completed. In that case, it is no longer proper to speak of a trace element diffusing in a main constituent; one should instead envision a stratification with a chemical composition discontinuity. In that transition zone, both elements have comparable abundances, as they have reached their equilibrium profiles (the relative diffusion velocity is zero). If the transition zone is above the depth at which the potential convection zone is found, mixing generally occurs. This approach has been followed by Vauclair (1974), Koester (1976), and Vauclair and Reisse (1977) for the hydrogen-helium case. The helium-carbon case is currently being investigated by Vauclair and Fontaine (1979). Finally, except for convection, we assume that the envelope is stable enough so that diffusion operates without competition from other possible mechanisms.

b) *Element Separation*

i) *Radiative Forces*

The acceleration exerted on a given atom in the stable envelope of a star is due to three terms. The usual gravitational acceleration (pressure gradient) plus a thermal diffusion term (temperature gradient) must be compared with a radiative force that tends to push the atom upward. An additional term comes from the concentration gradient, but it is negligible here, since equilibrium is generally not approached (but see also Appendix B). One can obtain from equation (1) of Michaud *et al.* (1976) that even for $T_e = 10^5$ K the radiative acceleration exerted on the atmospheric material (ionized hydrogen assumed) is only $\sim 10^5$ cm s^{-2} [use $K_\nu(A) = K_\nu$, $X(A) = 1$, $\bar{K} \approx 0.4$ for electron scattering and integrate over $u \equiv h\nu/kT$], which is much less than a typical white-dwarf surface gravity of $g \sim 10^8$ cm s^{-2}. Radiation pressure cannot lead to dynamical instabilities and mass loss of the Lucy and Solomon (1970) type at the surfaces of white dwarfs.

On the other hand, *selective* radiative forces can be exerted on the atoms of a given species by bound-bound and bound-free transitions (Michaud 1970). The largest possible radiative acceleration can be transmitted to *unabundant* elements under the following favorable circumstances: the lines are *unsaturated*, and the transitions must have *large f-values* near the maximum of the monochromatic flux distribution.

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Under these conditions, Michaud *et al.* (1976) have shown that

$$g_R \approx 1.7 \times 10^{-4} \frac{T_e^4 R^2}{ATr^2} \text{ cm s}^{-2}, \quad (1)$$

where T_e is the effective temperature, R the radius of the star, A the atomic mass of the element under consideration, and T and r, respectively, the temperature and radius at the depth of interest. When the above conditions are not met, equation (1) is a gross overestimate of the radiative acceleration.

Thus, under favorable conditions, the radiative acceleration on a *trace element* could become larger than $g = 10^8$ cm s^{-2} in the *atmospheres* of white dwarfs with $T_e > 13{,}400$ K (helium), $> 19{,}400$ K (carbon), and $> 28{,}000$ K (calcium). However, below $T_e \approx 60{,}000$ K, the atmosphere of a 0.612 $M_\odot$ helium-rich white dwarf is convective. The convective zones are very effectively homogenized. The radiative acceleration must then be compared with gravity not in the atmosphere but below the convection zone. Taking into account the T^{-1} dependence of equation (1), *trace abundances could appear in the atmosphere*, supported by the radiative acceleration below the convection zone, only for $T > 29{,}000$ K(C) and $T > 38{,}000$ K(Ca) in a helium-rich star. Furthermore, it is most important here to estimate the radiative acceleration at the depth convection will ultimately reach in a stellar model: we want to know whether diffusion can empty that whole region so that no element can be brought back up to the surface as the star cools. We find that the radiative acceleration is always small at large depth. For example, the convection zone of a 0.612 $M_\odot$ helium-rich model reaches a maximum depth of $\Delta M/M \approx 10^{-6.2}$ during its cooling sequence (see below). At $T_e = 10^5$ K, the lighter elements (C, N, O) are completely ionized at that depth; then they have no bound-bound transitions and the radiative force on them is very small. Because of the T^{-1} dependence, equation (1) gives a maximum possible radiative acceleration of 4×10^7 cm s^{-2} on Ca, which is too small to counter gravity, especially when the effect of thermal diffusion is taken into account (see below). By the time the stars cool down to $T_e = 20{,}000$ K, most elements have partially recombined at $\Delta M/M \approx 10^{-6.2}$, but the maximum radiative acceleration would be only $\sim 3 \times 10^5$ cm s^{-2} on carbon, and $\sim 10^5$ cm s^{-2} on calcium.

Selective radiative forces on certain heavy elements could be important to keep them from settling below the visible region of the atmosphere in hot white dwarfs ($T_e > 29{,}000$ K for C and $> 39{,}000$ K for Ca in helium-rich models). But they will not play a significant role at lower effective temperatures. We will therefore not study radiative forces quantitatively in this paper.

ii) *Gravitational Settling and Thermal Diffusion*

We will not repeat here the derivations that lead to estimates of diffusion time scales. These can be found in Aller and Chapman (1960), Michaud (1970), and Michaud *et al.* (1976). We will simply indicate how these results can be used in the present calculations. The relative diffusion velocity w_g of a trace element 2 in a main constituent 1 due to gravitational settling alone (the pressure gradient term) is given by

$$w_g = D_{12}\left[\frac{m_2}{m_1}(1 + Z_1) - Z_2 - 1\right]\frac{\rho g}{P}, \quad (2)$$

where g is the local gravity, ρ the density, P the pressure, m_i the mass of an atom of element i, Z_i the average charge of element i ($Z_i \leq$ atomic number), and D_{12} the diffusion coefficient expressed by (Chapman and Cowling 1970)

$$D_{12} = \frac{3(2kT)^{5/2}}{16[(\pi m_1 m_2)/(m_1 + m_2)]^{1/2} n_1 Z_1^2 Z_2^2 e^4 A_1(2)}. \quad (3)$$

In this expression k is Boltzmann's constant, T the temperature, n_1 the number density of the main species, e the electric charge unit, and $A_1(2)$ a logarithmic term given by

$$A_1(2) = \ln(1 + x_D^2) \quad (4)$$

with

$$x_D^2 = \frac{16k^2T^2\lambda_D^2}{Z_1^2 Z_2^2 e^4}; \quad (5)$$

λ_D is the Debye length defined by:

$$\lambda_D = \left(kT\Big/4\pi e^2 \sum_i n_i Z_i^2\right)^{1/2}, \quad (6)$$

where the summation is taken over all charged particles. These results apply for a binary gas mixture sufficiently dilute for binary collisions to dominate and for particles interacting via the Coulomb force. The temperature gradient in the envelope of a star also leads to a diffusion term. The total relative diffusion velocity can be written as:

$$w_t = w_g(1 + f), \quad (7)$$

where the factor f expresses the relative importance of thermal diffusion to gravitational settling. This quantity is discussed in Appendix A.

Once the diffusion velocity is obtained at a given depth in the envelope of a star, an estimate (see Michaud *et al.* 1976) of the mass abundance at that depth of the trace element 2 goes as:

$$X_2 = X_2(0)\exp(-t/\theta), \quad (8)$$

with the *e*-folding time θ given by

$$\theta = \frac{g}{4\pi G}\frac{(\Delta M/M)}{\rho w_t}, \quad (9)$$

where G is the gravitational constant, g the local gravity, and $\Delta M/M$ the fractional mass of the layers above the point under consideration. The diffusion

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time scale due to gravitational settling alone is, of course, given by

$$\theta_g = \theta(1 + f) . \tag{10}$$

When diffusion occurs below a convection zone with fractional mass $\Delta M/M$, equation (8) gives an exact solution of the time evolution of the abundance in the convection zone so long as equilibrium is not approached (see Appendix B). Here, we will calculate the diffusion time scales not only at the base of the convection zone but also for deeper regions. At any given point we then approximate the time evolution of the abundance by the time evolution that would prevail if all the mass above the point of interest were in a convection zone. By comparison with time-dependent solutions in main-sequence stars, we have found this approximation always to give the correct order of magnitude of the abundance variation until equilibrium was approached (see Appendix B). We feel that a detailed time-dependent solution of the diffusion equation will be warranted only when better values are available for α_{12}' and the diffusion coefficient.

It should be stressed here that the above expressions hold only for particles interacting via a *pure* Coulomb potential. The evaluation of both D_{12} and α_{12}' then involves an integral over the impact parameter that has the familiar long-range divergence of $1/r^2$ forces. This divergence is removed by introducing an arbitrary long-range cutoff distance which is usually taken as the Debye radius λ_D (Chapman and Cowling 1970). This leads to the logarithmic terms encountered above (eqs. [4] and [A4]). The quantities D_{12} and α_{12}' then remain only weakly dependent on the exact value of the cutoff distance as long as these logarithmic terms are large enough. Otherwise our results will be underestimates of the diffusion time scale. The $A_1(2)$ term of equation (4) can be written as

$$A_1(2) = \ln\left[1 + \left(\frac{4\lambda_D}{\lambda_C}\right)^2\right] , \tag{11}$$

where λ_C is the classical distance of closest approach. In contrast to what happens in main-sequence stellar atmospheres, the ratio λ_D/λ_C is no longer very large in the dense plasmas encountered in cooler white-dwarf envelopes. The quantity $A_1(2)$ may assume relatively small values, but it is difficult here to obtain a limit below which the diffusion time scale is substantially underestimated. As a reasonable guess, we will use $A_1(2) = 3$ corresponding to $\lambda_D/\lambda_C \approx 1$. The line where this occurs is shown, for example, in Figures 1, 2, and 6. For $A_1(2) < 3$, our results are less accurate, and for values of $A_1(2)$ much smaller than this limit, they may even be large underestimates of the diffusion time scale. There exists no reliable estimate of D_{12} nor of α_{12}' below this line. An accurate evaluation of these terms is important but difficult in white dwarfs, since it involves triple integrals that must be obtained numerically for *screened* Coulomb potentials. Preliminary calculations of transport coefficients by Fontaine and Michaud (1979) involving the Debye-Hückel potential indicate that the present formulae may underestimate θ by about a decade when $A_1(2) \approx 0.1$ and that for $A_1(2) = 3$ the results agree within 30%.

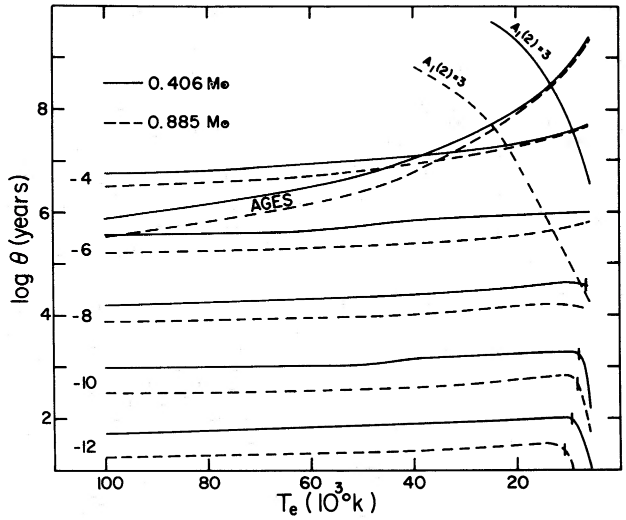

FIG. 1.—Time scale versus effective temperature for helium diffusing in hydrogen-rich envelopes. The continuous lines refer to models with $M/M_\odot = 0.406$, the dashed lines to models with $M/M_\odot = 0.885$. The curves are parametrized by the logarithmic value of the mass depth log q. The two curves labeled "AGES" are the estimated cooling ages. The small vertical bars indicate the effective temperature for which convection just reaches that mass depth. To the right of the lines $A_1(2) = 3$, the time scale is somewhat underestimated.

IV. RESULTS

We have computed envelope models for two chemical compositions: Iben I($X_H = 0.999$; e.g., Cox and Stewart 1970) and Iben V($X_{He} = 0.999$). The masses and temperatures considered are $M/M_\odot = 0.406$, 0.612, and 0.885, and $T_e(10^3$ K$)$ = 6, 10, 15, 20, 25, 30, 40, 50, 70, and 100. Once the physical conditions at a given depth are known, the degrees of ionization of various trace elements can be estimated and diffusion time scales computed.

Figure 1 refers to the diffusion of helium in a hydrogen-rich environment. It shows the helium e-folding time θ (see eq. [9] above) in terms of the effective temperature T_e at various mass depths $q \equiv \Delta M/M$ ($\log q = -12, -10, -8, -6, -4$). The continuous lines show the results for $M/M_\odot = 0.406$, and the dashed lines refer to $M/M_\odot = 0.885$. A qualitatively very similar diagram is shown in Figure 2 where, this time, the diffusion of carbon in helium-rich envelopes is considered. Except for the cooler models, the characteristic time scales are only weakly dependent upon the effective temperature. At lower luminosities, the main constituents begin to recombine at the surface, and this leads to the formation of superficial convection zones whose thicknesses increase with a further decrease of the luminosity. In Figures 1 and 2, the small vertical bars indicate the effective temperatures for which convection just reaches that mass depth. For the hydrogen-rich models, convection

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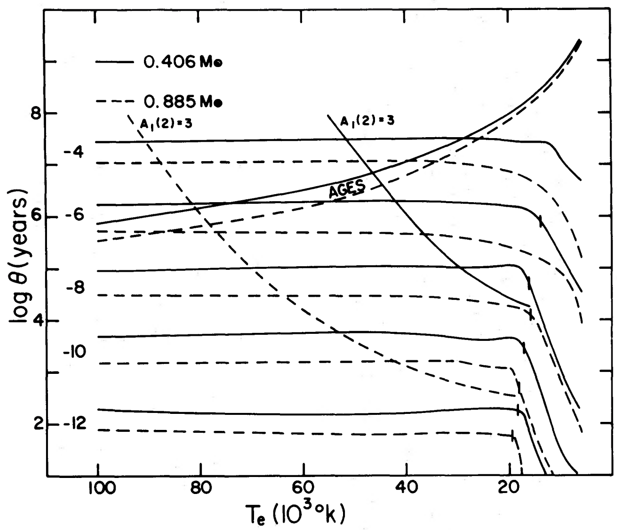

FIG. 2.—Same as Fig. 1, but for carbon diffusing in helium-rich envelopes.

reaches down to q(max) $\approx 10^{-7.0}$, $10^{-7.7}$, and $10^{-8.2}$ for $M/M_\odot = 0.406$, 0.612, and 0.885, respectively, at the smallest effective temperature considered here (T_e = 6000 K). In the case of helium-rich envelopes, the convection zones reach a maximum depth and then retreat toward the surface with further lowering of the effective temperature. We find that q(max) $\approx 10^{-5.2}$ at $T_e \approx$ 8500 K, q(max) $\approx 10^{-6.2}$ at $T_e \approx$ 9500 K, and q(max) $\approx 10^{-7.2}$ at $T_e \approx$ 10,700 K, for $M/M_\odot = 0.406$, 0.612, and 0.885, respectively. [These values of q(max) are somewhat smaller than those of Fontaine and Van Horn (1976) as a consequence of our use of an improved equation of state.] Since turbulent convective mixing will completely destroy the concentration gradient, the computed diffusion time scales are physically meaningless in a convection zone. We note, however, that in Figure 2, θ decreases rapidly at the low-temperature end of the sequence even for depths larger than q(max), where convection does not reach. This is due to a second effect that is related to the divergence of our formulae when the plasma becomes too dense. As discussed above, our results become inaccurate for too small a value for $A_1(2)$ ($\ll 3$) and the diffusion time scale is underestimated. The effect is not seen in the case of helium diffusing in hydrogen for two reasons: (a) the plasma is less dense (λ_D is larger), and (b) the particles are less charged (λ_C is smaller). Thus, in the most extreme case considered here, where T_e = 6000 K, $q = 10^{-4.0}$, and $M = 0.885\, M_\odot$, $A_1(2) \approx 0.4$ for helium diffusing in hydrogen but can be as small as ~ 0.004 for carbon diffusing in helium. In the former case, the diffusion time scale is probably accurate within factors of 2 or 3, but in the latter one the underestimation is more substantial. Preliminary results of the computations of Fontaine and Michaud (1979) indicate that the rapid decrease of θ versus T_e shown in Figure 2 is not correct. The curve more likely stays flat even at the lowest effective temperatures. In short, we find that diffusion time scales are generally only weakly coupled to the effective temperature.

Figures 1 and 2 also show that the diffusion time scales are smaller for the higher masses. This, of course, is a gravity effect which does act not only via the gravitational force ($w_g \propto g$) but also in a more complicated way via the envelope stratification. It is indeed well known that more massive white dwarfs are denser than others at comparable depths. Very roughly, we find that the net effect is $\theta \propto g^{-0.6}$; this approximate relationship gives a reasonable estimate under most conditions.

Thermal diffusion plays an important role in white dwarfs. Its importance relative to gravitational settling is measured by the quantity f which mostly depends on the factor $(Z_2/Z_1)^2$ in the α_{12} term. When radiative equilibrium prevails, the main constituent is essentially completely ionized (Z_1 = constant). Under those conditions, Z_2 is already large enough and $(Z_2/Z_1)^2$ does not change rapidly as a function of q or, when the star evolves, as a function of T_e. The quantity f is then relatively constant. This is illustrated in Figure 3 where we plot both the diffusion time scale θ and the gravitational settling time θ_g as functions of depth for carbon diffusing in helium-rich envelopes parametrized by $M/M_\odot = 0.612$, $T_e(10^3\text{ K}) = 100$ (*continuous lines*), and 20 (*dashed lines*). Similar results are obtained for other masses. For the hot model, the results are shown from the photosphere ($q \approx 10^{-14.3}$) down to $q \approx 10^{-4}$; the envelope is entirely radiative and helium is completely ionized throughout. For the cool model, diffusion starts below the superficial convection zone that exists as a result of partial ionization of helium in the outermost layers. Below $q \approx 10^{-12.6}$, helium is ionized and radiative equilibrium prevails. On the other hand, the trace element carbon undergoes ionization of its K shell in both models, as is illustrated by the inset plot of the degree of ionization versus mass depth. (The degree of ionization α is defined here as the average number of free electrons contributed by an atom of a given element divided by its atomic number.) We note that in the regions where the ionization of carbon does not change, the factor f is essentially constant with depth. In the outermost layers of both models where carbon is mostly in the form of C V, f is approximately equal to 0.4, and therefore thermal diffusion accelerates the element separation by about 29% in these regions. Deeper in the envelopes, however, where carbon is completely ionized, thermal diffusion dominates the process as the diffusion time scale is reduced by a factor ~ 4 as compared with gravitational settling acting alone. Thus we find the important result that thermal diffusion can play an essential role in the element separation processes acting in white-dwarf envelopes.

Both in the case of carbon in helium (Fig. 3) and in that of trace helium diffusing in hydrogen-rich envelopes, the diffusion time scale increases monotonically with depth following approximately the

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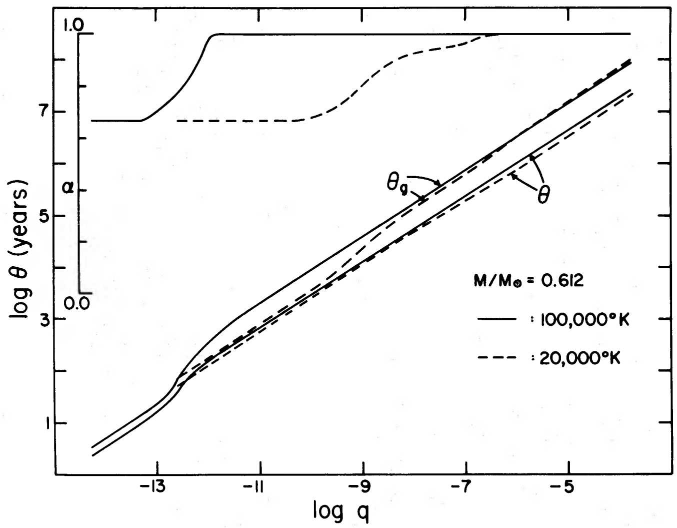

FIG. 3.—Diffusion time scale versus mass depth for carbon diffusing in helium-rich envelopes specified by $M/M_{\odot} = 0.612$ and $T_e(10^3\ \mathrm{K}) = 100$ (*continuous lines*) and 20 (*dashed lines*). The gravitational settling time θ_g and the net diffusion time θ are indicated. The inset plot shows the average degree of ionization of carbon α as a function of depth. It is seen that carbon loses its last two electrons with increasing depth.

relation $\theta \propto q^{0.65}$. In that latter case, when both hydrogen and helium are ionized, the quantity f again becomes essentially independent of depth and effective temperature. Because of the smaller charges, however, thermal diffusion decreases the characteristic time scale by only 30%, a result similar to the case of C V diffusing in He III because the ratio $(Z_2/Z_1)^2$ is the same in both cases. Conversely, for more highly charged ions (e.g., heavy metals), thermal diffusion may completely dominate the process.

So far, we have considered only three elements: hydrogen, helium, and carbon. The average charges Z_i of these elements can be obtained from our dense plasma equation of state tables (see Fontaine, Graboske, and Van Horn 1977), and it is then a relatively simple matter to estimate diffusion time scales. Trace elements are also present in the Iben I and Iben V compositions, and they were properly included in the ionization equilibrium calculations. However, the free energy minimization technique used by Fontaine, Graboske, and Van Horn (1977) is purely numerical and does not make use of Saha equations. As a consequence, the average charge Z_2 of each individual trace element in the mixture is not explicitly computed. For these elements, therefore, a procedure such as the one used by Michaud *et al.* (1976) must be employed. It is possible to estimate the ionization potential a trace element must have to be about half-ionized at a given temperature and density. From the Saha equation, assuming that the electron density is specified by the main constituent (i.e., all free electrons come from that element), assuming that the internal partition functions of the two stages of ionization are equal, and asking that the ratio of the number densities of the ions in the two stages of ionization be equal to 2, we obtain, for the ionization potential,

$$\chi(\mathrm{eV}) = 1.987 \times 10^{-4}T \times \left(-8.392 - \log\rho + \tfrac{3}{2}\log T - \log\frac{Z_1}{A_1}\right), \quad (12)$$

where χ is in electron volts, T is the temperature, ρ the density, and Z_1 and A_1 the average charge and atomic mass, respectively, of the main constituent. This estimate is quite insensitive to assumptions made about the partition functions. Comparing χ with tables of ionization potentials (e.g., Allen 1973) then leads to Z_2.

This technique works well for main-sequence stars in which the gas is sufficiently dilute for the ideal gas equation of state and Saha equation to hold. For dense gases, however, characteristic of white-dwarf envelopes, equation (12) may lead to complete recombination. As is well known, this nonphysical asymptotic behavior is removed through the phenomenon of pressure ionization. In the absence of detailed calculations of ionization equilibria for nonideal, multi-component gases, phenomenological models of pressure ionization must be used. We have employed here a model that goes from the Debye-Hückel correction to the ion-sphere limit following Stewart and Pyatt's (1966) prescription. An additional term is also introduced following Rouse's (1964) suggestion (see Böhm 1970). This model leads to a correction term which,

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for example, has the following form in a helium-rich medium:

$$\Delta\chi(\mathrm{eV}) = 2.718 \times 10^{-5}\frac{\rho T}{Z_2^{3}} + \text{smaller of}\begin{cases}1.982 \times 10^4 Z_2 \dfrac{\rho^{1/2}}{T^{1/2}}\,, \\ 23.26\rho^{1/3}(Z_2^{2/3}+0.95)\,,\end{cases} \quad (13)$$

where Z_2 is the charge of the trace ion. Insofar as comparisons are possible with the few detailed calculations that we know of, we find that this model leads to degrees of ionization which are generally in good agreement with more exact results. For highly charged particles, however, the agreement is poorer and the uncertainties are substantial. We will investigate below the effects of these uncertainties of the degree of ionization on the diffusion time scales.

As representative cases, we have considered oxygen and calcium diffusing in helium-rich envelopes. Since we wish to know whether or not heavy elements have had time to diffuse below the level where convection ultimately can reach during the cooling of the star, we have computed diffusion time scales at these depths only. A brief survey of published models by other authors (Sweeney 1973; Baglin and Vauclair 1973; D'Antona and Mazzitelli 1975*a*; Muchmore and Böhm 1977) as well as our own results given above indicate that the maximum value of the fractional mass contained in a helium-rich white-dwarf convective envelope is perhaps uncertain by a factor 8–10. In addition, as indicated by Fontaine *et al.* (1974), the dependence on the heavy-element content seems fairly weak. Allowing for possible overshooting, we thus consider it unlikely that convection in a homogeneous helium-rich envelope can reach down below $q(\max) = 10^{-4.0}$, $10^{-5.0}$, and $10^{-6.2}$ for $M/M_\odot = 0.406$, 0.612, and 0.885, respectively.

Our results are summarized in Figure 4, where we plot the diffusion time scales of carbon, oxygen, and calcium in helium-rich envelopes in terms of the effective temperature. We note again the relative insensitivity of θ upon the effective temperature except below $T_e \approx 25{,}000$ K, where our estimates become inaccurate as $A_1(2)$ can become much smaller than our limit of 3. For a 0.612 $M_\odot$ model with $T_e = 10{,}000$ K, $A_1(2)$ is equal to 0.3 for carbon and equal to 0.1 for calcium. As discussed above, such small values lead to underestimates of the diffusion time scale, and it is more likely that the curves of θ versus T_e stay flat. We also find that diffusion time scales are substantially smaller for calcium than carbon, a result that was qualitatively expected. In the entire temperature range shown in the figure, carbon is completely ionized ($Z_2 = 6$), and, according to our model of pressure ionization, oxygen goes from $Z_2 = 8$ at high temperatures to $Z_2 = 6$, and calcium recombines from Ca XIX to Ca XI. The relatively high charges of the ions imply that thermal diffusion completely dominates the segregation process. For instance, for the model parametrized by $M/M_\odot = 0.885$, $T_e = 50{,}000$ K, the quantity f equals ~ 15.5 for calcium ($Z_2 = 18$), ~ 4.6 for oxygen ($Z_2 = 8$), and ~ 3.8 for carbon ($Z_2 = 6$) at a depth of $q = 10^{-6.2}$. As noted above, our model of pressure ionization is somewhat uncertain for heavy ions and it is of interest to compare the results obtained for different degrees of ionization. As a typical example, let us consider a helium-rich envelope specified by $M/M_\odot = 0.612$ and $T_e = 20{,}000$ K. At a depth of $q = 10^{-5}$, carbon is completely ionized, and our model of pressure ionization predicts that $Z_2 = 6$ for oxygen and $Z_2 = 10$ for calcium. As a test, under the same physical conditions, we have computed diffusion time scales for calcium assuming degrees of ionization in the range $Z_2 = 6$ (a definite lower limit) to $Z_2 = 20$ (complete ionization). The results are shown in Figure 5, where we plot the gravitational settling time θ_g and the diffusion time scale θ as a function of the ionic charge Z_2. If gravity alone were to operate, more highly charged calcium ions would take longer to diffuse in helium. This is due to the effect of Z_2 on the diffusion coefficient D_{12}. Note, however, that even for complete ionization θ_g is always smaller for heavier elements. The strong increase of thermal diffusion as Z_2 increases more than compensates the increase of θ_g with Z_2. If a calcium atom were in fact completely ionized, thermal diffusion would dominate gravitational settling by more than a factor of 78. The uncertainty of the degree of ionization leads to an uncertainty of about a decade in the characteristic time scale over a range of values $Z_2 =$ 6 to 20. This wide range of values is ultraconservative and an uncertainty of a factor of ~ 3 in θ is probably much more reasonable.

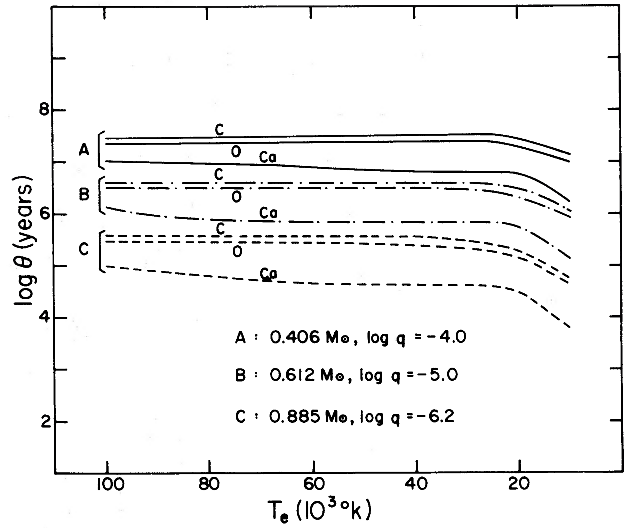

FIG. 4.—Diffusion time scale versus effective temperature for carbon, oxygen, and calcium diffusing in helium-rich envelopes. For each mass, the time scales are given at a depth where convection never reaches. *Continuous*, *dotted-dashed*, and *dashed lines*, $M/M_\odot = 0.406$, 0.612, and 0.885, respectively.

V. DISCUSSION

We now wish to compare our diffusion *e*-folding times with estimated ages of white dwarfs. Several

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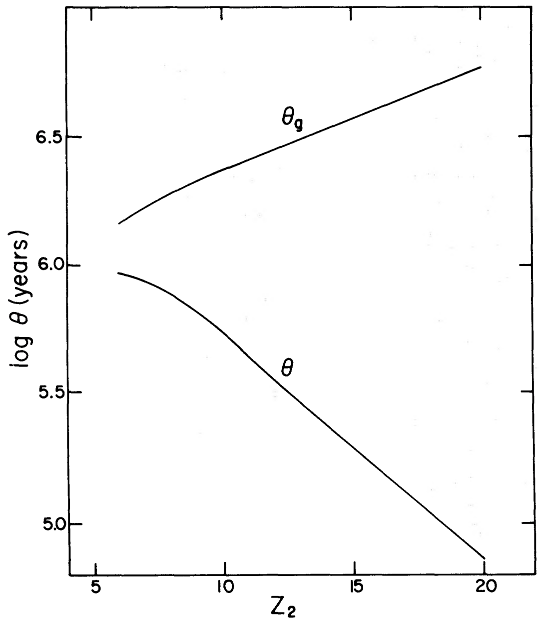

FIG. 5.—Time scale versus ionic charge for calcium diffusing in a helium-rich envelope. The parameters of the envelope are $M/M_{\odot} = 0.612$, $T_e = 20{,}000$ K, and the depth considered is $q = 10^{-5.0}$. The gravitational settling time θ_g increases with Z_2, whereas the net diffusion time scale θ decreases as a result of thermal diffusion.

authors have published evolutionary models of these objects for various masses and chemical compositions. We have interpolated among the results of Koester (1972), Sweeney (1973), Lamb (1974), Lamb and Van Horn (1975), and Kovetz and Shaviv (1976). The derived ages should be uncertain by a factor of no more than 2, except at relatively high luminosities ($L/L_{\odot} \gtrsim 10^{-1}$) where neutrino cooling processes are important and few models have been computed. White-dwarf cooling times are shown in Figures 1 and 2 as functions of the effective temperature and for two masses $M/M_{\odot} = 0.406$ and 0.885. We take the lifetimes to be the same for both hydrogen- and helium-rich envelopes, as the effects of changing the chemical composition of the envelope are small on the central temperature of white dwarfs (Fontaine and Van Horn 1976).

From Figure 1, it is immediately evident that helium sinks relatively rapidly from the outermost layers of a hydrogen-rich white-dwarf envelope. For models which are completely radiative ($T_e \gtrsim 17{,}500$ K for a 0.612 $M_{\odot}$ star), helium should completely disappear from the photosphere because its diffusion time scale is much smaller than the age of the star (cf. eq. [8]). Since heavier elements will settle down even more quickly, our diffusion model predicts pure hydrogen atmospheres for hot nonconvective DA white dwarfs. This is in qualitative agreement with the existence of hot DA's as EUV and soft X-ray sources. Further, any small amount of hydrogen originally mixed in a helium-dominated envelope would very quickly diffuse upward and contaminate the atmosphere. The small amount of hydrogen seen in a few DA–B's (Feige 7, EG 153, Gr 295; see § II) could be the remnant of such a diffusion process according to our model. A more important result, however, is that, by the time an outer convection zone reaches down to its maximum depth, all trace elements have had time to diffuse below that level. *Thus, in the present model, the diffusion process is so efficient that it leads to the formation of pure hydrogen or pure helium atmospheres in the entire temperature range* $6 \leq T_e(10^3\ \mathrm{K}) \leq 100$. This important conclusion is not likely to be changed by the results of more detailed calculations of transport coefficients in dense plasmas. Trace element abundances in the atmospheres of white dwarfs can be non-negligible (see eq. [8]) only if the present estimates of the diffusion time scales were too small by some two or three orders of magnitude for the lighter elements (C, O) and by more than 3 decades for heavy metals. So far, the preliminary calculations of Fontaine and Michaud (1979) indicate that the accuracy of the present estimate is sufficient.

Perhaps a more transparent way of presenting the same results is Figure 6, where we consider a typical case of carbon diffusing in a helium-rich envelope for a star with $M = 0.612\ M_{\odot}$ ($g \approx 10^8$ cm s^{-2}). The evolution is approximated here by a series of static models: the envelope is initially radiative until, around $T_e \approx$ 60,000 K, convection develops near the photosphere. With further cooling, the top of the convection zone always stays near the photosphere at the mass depth indicated in the figure. On the other hand, the base of the convection zone passes through a maximum depth around $T_e \approx 10{,}000$ K, a feature that was often referred to above.[1] Lines of constant diffusion times are also shown as a function of depth and effective temperature; the dashed line defines the cooling age of the star. From the figure, let us try to maximize the diffusion time scales so as to create an optimum situation. First, we assume that, due to uncertainties in envelope modeling and perhaps overshooting, the convection zone could extend down to $q(\max) \approx 10^{-5}$ near $T_e \approx 10{,}000$ K, i.e., at a depth where the mass of the envelope is more than 15 times larger than the actual value predicted by the model. Second, we also suppose that diffusion times at given mass depths are essentially independent of the effective temperature. We use the larger value of θ, about 5 times that of the present estimate where $A_1(2) \approx 0.3$. Under these optimum conditions, we have compared (in Table 2) the diffusion time scales of some heavy elements diffusing below the convective zone with the age of the star. Similar results for $M/M_{\odot} = 0.406$ and 0.885 are also tabulated. Even under optimum conditions, the characteristic diffusion time scales are very short compared with the lifetimes of the stars, and these elements should not be seen in the spectra of white dwarfs. (Still shorter time scales would be obtained for hydrogen-rich envelopes.) Thus the present calculations

[1] This characteristic is also present in hydrogen-rich envelopes, but the effective temperature for this maximum to arise is smaller than $T_e = 6000$ K (see Fontaine and Van Horn 1976).

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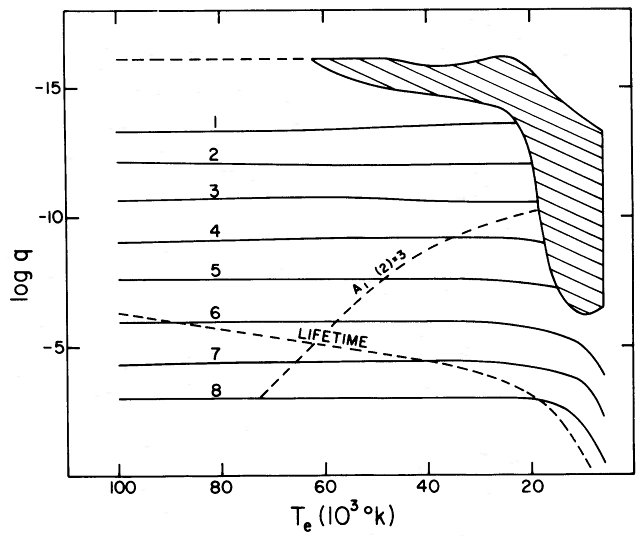

FIG. 6.—Mass depth versus effective temperature for helium-rich envelopes with $M/M_\odot = 0.612$. The profile of the helium convection zone is shown by the shaded area. Above $T_e \approx$ 60,000 K, the envelope is completely radiative. Lines of constant diffusion times of carbon in these envelopes are parametrized by log θ (years). They indicate how the time scales depend on depth and effective temperature. The dashed line labeled "LIFETIME" calibrates the effective temperature scale with cooling ages. *Dotted line*, locus of points where $A_1(2) = 3$. Below this line, the results become underestimates of the diffusion time scale.

are compatible with the fact that white-dwarf atmospheric compositions are essentially mono-elemental.

The diffusion time scales evaluated here are so short that the abundances of heavy elements in the atmospheres of white dwarfs should be vanishingly low (see eq. [8] above). However, observations of several cool white-dwarf spectra reveal metallic lines. This suggests that a competing mechanism may be operative in the envelopes of white dwarfs to reduce the effects of diffusion. It is then of interest to discuss briefly a few possibilities.

In outer stellar atmospheres, magnetic fields can appreciably slow down diffusion (Michaud 1970). However, only a few percent of all white dwarfs have magnetic fields exceeding 10^4–10^5 gauss (Angel 1977). Since this is already too small to have any significant effect on diffusion here, magnetic fields may be safely neglected.

Turbulence could also enter in competition with diffusion (Vauclair, Vauclair, and Michaud 1978). It is difficult here to see how such turbulence could be generated in white-dwarf envelopes except perhaps through some instabilities triggered by meridional circulation. In the absence of detailed calculations, we can only suggest that the observed rotational states of white dwarfs (Greenstein and Peterson 1973; Greenstein *et al.* 1977) lead to a regime in which rotationally induced turbulence competes with diffusion via a turbulent diffusion coefficient, so as to reduce considerably the element segregation process. If this is true, this leads to certain predictions which can be checked against observations. Clearly, turbulence will always tend to increase diffusion time scales for all elements, so that relative diffusion time scales should be approximately the same as those found here. Since it is now generally believed (Weidemann 1975) that a typical white-dwarf core consists of the products of helium burning, heavy metals should not have been affected by nucleosynthesis. It is difficult to estimate the diffusion time scale of iron in helium, say, and compare it with that of calcium because of the uncertainties of pressure ionization models. However, it is clear that a heavier atom diffuses downward more rapidly than a lighter one. *This implies that the abundance ratio of a heavy-to-light metal must be less than the solar value.* Unfortunately, very few white dwarfs show spectra with lines of two or more metals, and the relative abundances are thus difficult to obtain. We know of only two stars for which relative metal abundances have been derived: Van Maanen 2 is a cool DG (helium-rich) white dwarf which was first analyzed by Weidemann (1960), who obtained essentially solar ratios for magnesium/calcium/iron. However, more

TABLE 2

DIFFUSION OF HEAVY ELEMENTS IN HELIUM-RICH ENVELOPES

Parameters	Age (years)	Diffusion Time Scales (years): Carbon	Oxygen	Calcium
$M/M_\odot = 0.406$, T_e = 9000 K, q(max) = $10^{-4.0}$	8.9×10^8	3.2×10^7	2.5×10^7	7.0×10^6
$M/M_\odot = 0.612$, T_e = 10,000 K, q(max) = $10^{-5.0}$	5.6×10^8	4.0×10^6	3.1×11^6	7.1×10^5
$M/M_\odot = 0.885$, T_e = 11,000 K, q(max) = $10^{-6.2}$	4.5×10^8	3.8×10^5	2.7×10^5	4.5×10^4

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recent analyses by Wegner (1972) and Grenfell (1974) suggest that magnesium and calcium are perhaps overabundant with respect to iron by factors of ~ 3 and ~ 1.5, respectively, as compared with solar ratios. For the cool DF (helium-rich) star R640, Wegner (1972) has found that the ratios magnesium/iron and calcium/iron are larger than the solar value by factors of ~ 45 and ~ 4, respectively, if we adopt Allen's (1973) universal cosmic abundances. While the uncertainties can be substantial, we find that these results are in qualitative agreement with our diffusion model; the degree of turbulence must certainly be a variable from one star to another.

If, on the other hand, turbulence should turn out to be unimportant in white dwarfs, the presence of metallic lines in cooler white dwarfs could still be explained by accretion. For example, let us assume a steady state, in which the amount of calcium that accretes,

$$\dot{N}(\text{acc}) = \frac{dM}{dt}\frac{X_{\text{Ca}}}{40m_p}, \quad (14)$$

equals the amount that diffuses below the convection zone,

$$\dot{N}(\text{dif}) = 4\pi r^2 \rho \frac{\bar{X}_{\text{Ca}}}{40m_p} w_{\text{CaH}}, \quad (15)$$

where X_{Ca} is the mass fraction of Ca in the interstellar matter (assumed solar) and $\bar{X}_{\text{Ca}}$ the mass fraction in the convection zone. We evaluated w_{CaH}, ρ, and r at the bottom of the convection zone of a typical cool hydrogen-rich model to obtain, for the accretion rate,

$$\frac{dM}{dt} \approx 5 \times 10^{-12} \frac{\bar{X}_{\text{Ca}}}{X_{\text{Ca}}} M_{\odot}\,\text{year}^{-1}. \quad (16)$$

This estimate is uncertain by perhaps an order of magnitude, but, for a value $\bar{X}_{\text{Ca}}/X_{\text{Ca}} = 2 \times 10^{-4}$ appropriate to DA white dwarfs (Shipman 1977), it is comparable to the Hoyle-Bondi result ($dM/dt \approx 10^{-16} M_{\odot}\,\text{year}^{-1}$) obtained under typical white-dwarf conditions (Bondi 1952; Wesemael 1978*b*). Smaller accretion rates would suffice to maintain the trace abundances of lighter elements, as they sink less rapidly. The problem that remains, however, is how to maintain helium-rich white-dwarf atmospheres when the accreted matter is hydrogen-rich (Sion 1973; D'Antona and Mazzitelli 1975*b*; Koester 1976; Wesemael 1978*b*). We cannot yet present a definitive solution to this problem; however, we are investigating (Michaud and Fontaine 1979) whether the electric field present in a helium-rich atmosphere (Montmerle and Michaud 1976) may not render proton accretion impossible. Crucial to this suggestion is the conclusion of Bally and Harrison (1978) that the electric field lines would close only in intergalactic space. The implied electron transport mechanism seems to require a hot corona.

For elements such as carbon and oxygen the situation is further complicated, since their abundances can be altered by past nuclear processes. Since the core composition of a typical white dwarf is expected to be mostly carbon and oxygen, the (unknown) amounts of helium and hydrogen in the external envelope become critical factors. If, for instance, the envelope is thin enough, convective mixing can reach into the C–O core and bring to the surface these elements. Such a possibility is currently being investigated by Vauclair and Fontaine (1979). The present diffusion model, however, gives a carbon-to-oxygen diffusion time scale ratio of ~ 1.3 in helium-rich envelopes, only weakly dependent on mass, effective temperature, and depth. Since the degrees of ionization of these elements should be fairly reliable, we find an abundance ratio C/O $\approx$ 3.7 compared with a solar value of 0.5 if we neglect alterations due to past nuclear burning. This result is consistent with the limit obtained by Bues (1973) for DC (helium-rich) stars but is in conflict with the ratio (~ 10–10^2) required to explain the λ4670 phenomenon. The carbon seen in the spectra of λ4670 stars is therefore probably not primeval; mixing, accretion, or some other causes may have to be invoked.

VI. CONCLUSIONS

We have computed diffusion time scales of representative *trace* elements in hydrogen- and helium-rich white-dwarf envelopes. These layers were assumed stable enough so that, except for convection, diffusion was allowed to proceed freely without competition from other mechanisms. Diffusion time scales at a given mass depth in a star were found to be almost independent of the effective temperature, that is, of evolutionary times. Because of the higher gravity, elements diffuse more rapidly in more massive stars according approximately to the relation $\theta \propto g^{-0.6}$. Moreover, we found that time scales increase with depth roughly as $\theta \propto q^{0.65}$. This last relation is particularly appropriate when ionization is complete.

An important result is that thermal diffusion can play an essential role in element separation processes in white dwarfs. As the dominant term of the thermal diffusion coefficient goes as $\sim (Z_2/Z_1)^2$, elements (such as metals) that are easily ionized have time scales that are considerably shortened as compared with the situation in which gravitational settling acted alone. In particular, time scales of various elements do *not* scale as the inverse of the atomic mass, as has been assumed by several authors.

By comparing with published evolutionary time scales, we have found that our diffusion model is in qualitative agreement with inferred surface chemical compositions of white dwarfs, in the sense that all elements heavier than the main constituent are extremely underabundant. However, our time scales are so short that the model predicts atmospheres made up of *pure* hydrogen or helium. Elements have had the time to diffuse below the base of the convection zone. Since the observations indicate the presence of trace elements in the atmospheres of white dwarfs, this implies that diffusion probably competes with another mechanism so that diffusion time scales can be lengthened considerably. We suggest that either

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accretion or turbulence could reduce the effect of element segregation, but this must be confirmed by detailed calculations.

In the presence of turbulence, we still find that a heavier element will sink more rapidly than a lighter one, so that the heavy-to-light element abundance ratio must be *less* than the solar value in the atmosphere of a white dwarf. So far, the observations do not contradict this prediction. The case of the λ4670 stars is interesting in this context. Bues (1973) requires a ratio C/O $\sim$ 10–10^2, which is indeed much larger than the solar value, but it may even be too large for the diffusion model. They presumably have experienced some nuclear processing. On the other hand, it may be worth mentioning that Bues's results for the C/O ratio in λ4670 stars were not confirmed by Grenfell (1974). Further oxygen abundance determinations for these objects would be useful to test the diffusion model.

Finally, we have indicated several areas where research should be directed. In particular, the physics of collisions in a dense plasma should be improved. The present diffusion coefficients become too large at high densities where nonideal effects are important; this circumstance affects almost all white dwarfs. Calculations of appropriate accretion rates as well as detailed evolutionary calculations with realistic surface compositions also remain to be done. Further, the effects of turbulence on the diffusion process must be studied. Evidently, much work remains to be done in order to understand the complicated interplay of diffusion, accretion, and convective mixing at the surfaces of white dwarfs.

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After this research was started, we learned that S. and G. Vauclair and J. L. Greenstein were attacking a similar problem.

APPENDIX A

THERMAL DIFFUSION

The ratio of thermal diffusion to gravitational settling is given by

$$f = \left\{\alpha_{12}' \Big/ \left[\frac{m_2}{m_1}(1 + Z_1) - Z_2 - 1\right]\right\} \frac{\partial \ln T}{\partial \ln P} . \tag{A1}$$

The exact value of the term α_{12}' is still debated (Aller and Chapman 1960; Burgers 1960; Montmerle and Michaud 1976; Noerdlinger 1978). For reasons given below, we here suggest the following expression:

$$\alpha_{12}' = \left[\alpha_{12} \Big/ 1 + \frac{15\sqrt{(2m_e)}}{4Z_1\sqrt{(m_1)}}\right] - \left(\frac{Z_2^2}{Z_1} - Z_2\right)\frac{\alpha_{1e}}{1 + Z_1} , \tag{A2}$$

where α_{1e} is the thermal diffusion coefficient of electrons with respect to element 1 and is given by (Chapman 1958):

$$\alpha_{1e} = -\frac{3(Z_1 + 1)}{2.6 + 2.828\bar{\bar{A}}/Z_1} , \tag{A3}$$

with

$$\bar{\bar{A}} = 0.4\left(1 - \frac{y_D^2}{(1 + y_D^2)\ln(1 + y_D^2)}\right) , \tag{A4}$$

where

$$y_D^2 = Z_2^2 x_D^2 . \tag{A5}$$

The thermal diffusion coefficient α_{12} is given to first order by (Chapman 1958; note that his eqs. [11], [12], and [13] are erroneous: the exponents of M_2 in the numerator are incorrect)

$$\alpha_{12} = \frac{3[\![(Z_2/Z_1)^2\{1/[\sqrt{(2)}\bar{A}]\}[4\bar{A}(1 - M_2)M_2^{3/2} - 3(1 - 2M_2)M_2^{3/2}] + M_2 - 1]\!]}{6M_2^2 + 2.6(1 - M_2)^2 + 8\bar{A}(1 - M_2)M_2} , \tag{A6}$$

where

$$\bar{A} = 0.4\left[1 - \frac{x_D^2}{(1 + x_D^2)A_1(2)}\right] , \tag{A7}$$

and

$$M_2 = \frac{m_2}{m_1 + m_2} . \tag{A8}$$

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To arrive at equation (A2), we have rederived Burgers's (1960) results for the diffusion of test particles in a plasma. We started from his equations (5-32) and (5-35), but keeping terms in $(m_e/m_1)^{1/2}$. To compare with the results of Burgers (1960) and those of Aller and Chapman (1960), we now specialize to protons ($Z_1 = 1$), and assume that $m_2 \gg m_p(M_2 \to 1)$ and that the logarithmic terms in equations (A4) and (A7) are infinite ($\bar{A} = \bar{\bar{A}} = 0.4$). In this limit, equation (A2) becomes

$$\alpha_{12}' \to 3.15 Z_2^2 - 0.804 Z_2 \,. \tag{A9}$$

We thus find that Burgers's 3.45 multiplying Z_2^2 now becomes 3.15. This compares with the 2.65 of Aller and Chapman (1960) but agrees with and explains the numerical result of Noerdlinger (1978). It appears likely to us that the remaining difference with Aller and Chapman (1960) comes from their also neglecting terms in $(m_e/m_p)^{1/2}$. Burgers (1960) does not obtain an analytic expression for the variation of thermal diffusion with the ratio m_2/m_1 of the masses of the diffusing particles nor with the ratio of their charges. We then note numerical equalities between some of the terms of Burgers and some of Chapman's (1958), and we suggest that the better expression to use at the moment would be the one given by equation (A2).

APPENDIX B

EQUILIBRIUM ABUNDANCES

Throughout this paper we have neglected one physical property of the diffusion equation: diffusion is driven by the tendency of each type of ion to get to its own equilibrium in the existing gravitational, electric, and radiative fields, and diffusion stops locally when this equilibrium is reached locally. That diffusion stops when equilibrium is achieved leads to a concentration gradient term in the diffusion equation (see Michaud 1977),

$$w = D_{12}\left[\left[\frac{\partial \ln c_2}{\partial r} + (1+f)\left\{\left[\frac{m_2}{m_1}(1+Z_1) - Z_2 - 1\right]\frac{\rho g}{P}\right\}\right]\right], \tag{B1}$$

where

$$c_2 \equiv n_2/n_1 \,, \tag{B2}$$

with the assumption that the element of interest is a trace element, that is,

$$n_2 \ll n_1 \,. \tag{B3}$$

The other terms were defined after equation (2). When the concentration gradient term equals the gravitational settling and thermal diffusion terms, diffusion stops. Equilibrium has been obtained at least locally.

To evaluate when the density gradient term starts influencing diffusion calculations, we now give a few examples of underabundance factors that are reached in the atmospheres of white dwarfs when equilibrium is obtained. We make one approximation. We assume that the equilibrium has been obtained between the stellar surface (or the bottom of convection zone) and the point where the diffusion time scale equals the white-dwarf age. At that point we assume that the abundance of the element of interest is normal. Above that point the abundance is obtained everywhere by solving equation (B1) with $w = 0$. We make one further approximation: we use for Z_1 and Z_2 the largest values obtained in the region of interest, and obtain the values listed in Table 3. We so *underestimate* the factor at equilibrium. The values listed in Table 3 should be smaller, the true anomaly being larger. Clearly, the abundances that equilibrium predicts are much smaller than those observed. Using equation (8) at the bottom of the convection zone would, in some cases, lead to even smaller abundances than equilibrium gives here. However, it hardly matters, since the values given by equilibrium are already so small.

When better estimates of the diffusion coefficient are available, a detailed solution of the diffusion equation will be justified. It will be difficult to obtain, since numerical instabilities have a tendency to appear in the time evolution

TABLE 3

RELATIVE ATMOSPHERIC ABUNDANCES AT EQUILIBRIUM*

$T_e(10^3$ K)	$n/n_\odot$(C)	$n/n_\odot$(Ca)
100	3.6–55	3.4– 693
50	1.4–70	1.0–1483
20	4.7–68	1.2– 786
10	1.3–39	2.6–1024

NOTATION—$a \times 10^{-n} \equiv a - n$.

* Helium-rich 0.612 $M_\odot$ model.

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after the abundances have changed by about 2 decades. The detailed time-dependent calculations that have been done in main-sequence stars (Vauclair, Michaud, and Charland 1974; Vauclair, Vauclair, and Michaud 1978) generally had to be stopped for that reason after the abundances had changed by approximately 2 decades. In all the cases studied up to now, the approximation given by equation (8) turned out to be a good approximation to gravitational settling and thermal diffusion when the numerical solution applied. The estimates given here and in the text are then reliable.

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