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AN H I SURVEY OF NILSON DWARF GALAXIES. II. STATISTICAL PROPERTIES

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ABSTRACT

A statistical analysis of the sample of Nilson dwarf galaxies presented in Paper I together with the DDO objects detected by Fisher and Tully (the whole sample is hereafter called the low-surface-brightness [LSB] galaxy sample) leads to the following conclusions: (1) The Nilson catalog is fairly complete for all galaxies with a Nilson blue diameter larger than 1′. (2) The LSB galaxy sample includes a broad continuum of galaxies, ranging from the "true" dwarfs ($M_{pg} \geq -16$) to more luminous ($M_{pg} < -16$) but very low surface brightness systems. These LSB galaxies have larger $\mathfrak{M}_H/L_{pg}$ and $\mathfrak{M}_H/\mathfrak{M}_T$ ratios than do late-type galaxies. They may be "young" systems experiencing recent bursts of star formation, or systems with older but metal-deficient disk stars. (3) The preponderance of irregular systems and of singly peaked H I profiles, and a $\mathfrak{M}_T/R$ ratio 3–9 times smaller than that of late-type systems of comparable luminosity suggest an inability of the LSB galaxies to maintain a spiral density-wave needed for efficient star formation. (4) To within the accuracy of our data, the ratios $\mathfrak{M}_H/\mathfrak{M}_T$, $\mathfrak{M}_H/L_{pg}$ are independent of M_{pg}. (5) H I-rich LSB galaxies constitute about half of all field galaxies at $M_{pg} = -13$, decreasing to 0.2% at $M_{pg} = -20$. The "iceberg effect" concerning the constancy of the central surface brightness of galaxies with $M_{pg} \leq -19$ appears to be present only at the level of a few percent.

Subject headings: galaxies: structure — line profiles — radio sources: galaxies — radio sources: 21 cm radiation

I. INTRODUCTION

In Paper I (Thuan and Seitzer 1979) we have reported the first results of a neutral hydrogen survey, done with the NRAO[1] 91 m transit telescope, of galaxies classified as dwarf in the Nilson (1973) catalog. In the present paper we use these results in combination with the data which Fisher and Tully (1975, hereafter FT) obtained for the David Dunlap Observatory (DDO) dwarf catalog (van den Bergh 1959, 1966) to investigate the statistical properties of low-surface-brightness systems. In § II we discuss the completeness of the total Nilson dwarf sample and that of the detected sample (ours plus that of FT). A luminosity function for H I-rich low-surface-brightness systems is derived. The inclination corrections are discussed. We show that the selection criteria of "low surface brightness" and "little or no central concentration" isolate a broad continuum of galaxies, from the "true" dwarfs to much more luminous systems. In this paper the following nomenclature shall be adopted throughout: We shall refer to all galaxies in the Nilson and FT sample as "low surface brightness" (LSB) systems. We shall call all systems with $M_{pg} \geq -16$ "dwarf" systems, those with $M_{pg} < -16$ "nondwarf" systems. We summarize our main conclusions in § III.

[1] Operated by Associated Universities, Inc., under contract with the National Science Foundation.

II. STATISTICAL PROPERTIES

a) Completeness Test for the Nilson Sample

In order to evaluate the completeness of the Nilson LSB galaxy sample, we use a modification of the V/V_m method devised by Schmidt (1968). Instead of being an apparent-magnitude–limited sample, the Nilson sample is an angular-diameter–limited sample. We calculate for each LSB galaxy the volume V contained in a sphere whose radius is the distance D to the object. D is related to the angular diameter θ and the linear diameter d by $D = d/\theta$. We also compute the volume V_m contained in a sphere whose radius is the maximum distance D_m the galaxy could have and still be in the sample under study. $D_m = d/\theta_L$, where $\theta_L = 1'$, the limiting diameter of the Nilson catalog. Then

$$V/V_m = (\theta_L/\theta)^3 = (1/\theta)^3 , \quad (1)$$

where θ is in units of arcmin. Note that the ratio V/V_m is independent of the intrinsic linear diameter d. The mean value of V/V_m, which we shall denote by $\langle V/V_m \rangle$, should be 0.5 for objects uniformly dis-

TABLE 1

COMPLETENESS AS A FUNCTION OF ANGULAR DIAMETER FOR ALL NILSON GALAXIES

Blue Nilson Major Diameter (arcmin)	Holmberg Diameter* (arcmin)	$\langle V/V_m \rangle$ D†	$\langle V/V_m \rangle$ T†	Number of Galaxies D†	Number of Galaxies T†
32	35.3	0.387	0.191	2	5
16	18.2	0.248	0.339	4	18
8	9.7	0.236	0.423	12	80
7	8.6	0.278	0.460	14	111
6	7.5	0.359	0.457	18	154
5	6.5	0.413	0.460	25	230
4	5.4	0.462	0.472	39	391
3	4.3	0.475	0.480	65	792
2	3.2	0.511	0.497	157	2156
1	2.2	0.453	0.484	687	12545

* Eq. (1) of Paper I is used to calculate this column.

† D = Dwarf sample; T = all galaxies in Nilson's catalog.

tributed in Euclidean space if the sample is complete. Table 1 gives $\langle V/V_m \rangle$ as a function of observed angular diameter. The diameter used is the blue major diameter given by Nilson (1973). It is evident from Table 1 that when the number of galaxies is large enough ($\theta \leq 5'$), Nilson's catalog of LSB galaxies is fairly complete, as $\langle V/V_m \rangle = 0.453$ for all dwarf galaxies larger than 1′, with no systematic trends toward increasing completeness for smaller galaxies. For comparison, Table 1 also gives $\langle V/V_m \rangle$ as a function of angular diameter for *all* galaxies in the Nilson catalog, irrespective of morphological type. Evidently, this sample is more complete, as $\langle V/V_m \rangle = 0.484$ for all galaxies with diameter greater than 1′. This is not surprising since lower-surface-brightness galaxies are more easily missed. The interesting point is that Nilson is not much more incomplete for LSB galaxies than for normal galaxies.

Starting from a fairly complete sample of LSB galaxies, one may ask how complete the final detected sample (ours plus that of FT) is. This question has to be answered if we want to use the data to derive a luminosity function for the LSB galaxies. We discuss this question in the section on the space density of LSB galaxies (§ II*d*).

b) *Inclination Corrections*

Before we discuss the statistics of masses and luminosities of our sample, it is necessary to correct these quantities for effects due to the inclination i of the galaxy to our line of sight. The luminosities have already been corrected by using Holmberg's (1975) curve as an empirical fit to the variation of $\mathfrak{M}_H/L$ with r_H (Paper I). However, the projected rotational velocity width (Δv) must be corrected to the full width via $\Delta v_0 = \Delta v/\sin i$. We need to know the intrinsic axial ratio (r_0) of the system in order to use the observed axial ratio (r_H) to determine the inclination i via the equation

$$\cos^2 i = (r_H^2 - r_0^2)/(1 - r_0^2) . \qquad (2)$$

The standard technique for determining r_0 is to bin the observed axial ratios of the sample and then compare this distribution with that expected for different values of r_0 (Sandage, Freeman, and Stokes 1970). This has been done for the irregular galaxies in the DDO sample by Hodge and Hitchcock (1966), who concluded that the sample distribution was consistent with $r_0 = 0.25$. Their results are shown as the solid line in Figure 1*a*.

Hodge and Hitchcock measured their own diameters off the Palomar Sky Survey prints. When we use diameters from the Nilson catalog (and convert to Holmberg's system via eq. [1] of Paper I) for the *same* galaxies, we find that the curve (*dashed line*, Fig. 1*a*) is very different from that found by Hodge and Hitchcock: there are many more spherical systems if Nilson diameters are used. This calls into doubt the previously determined value of r_0. There must be a systematic difference between the Nilson (1973) and Hodge and Hitchcock (1966) diameters. We prefer to base our determination of r_0 on the Nilson diameters, which were found to have a very high weight by de Vaucouleurs, de Vaucouleurs, and Corwin (1976).

The distribution of r_H for all the Nilson LSB galaxies is shown in Figure 1*b*, along with the curves expected for a sample of galaxies having $r_0 = 0.25$ (*solid line*) and for a sample of galaxies having r_0 uniformly distributed between 0.25 and 1. It is clear that the observed r_H values are inconsistent with all the LSB galaxies having $r_0 = 0.25$: there is a large excess of spherical systems. When we consider just the detected galaxies (FT's and ours), the situation remains the same (Fig. 1*c*). It appears that the LSB galaxies are not all highly flattened systems, but rather can have a wide range of flattening (r_0 varying between 0.25 and 1).

It is important to note that this conclusion is apparently not due to observational effects. We have found no correlation between the observed axial ratio and (1) surface brightness, (2) apparent diameter, (3) absolute diameter, (4) apparent magnitude, or (5) absolute magnitude.

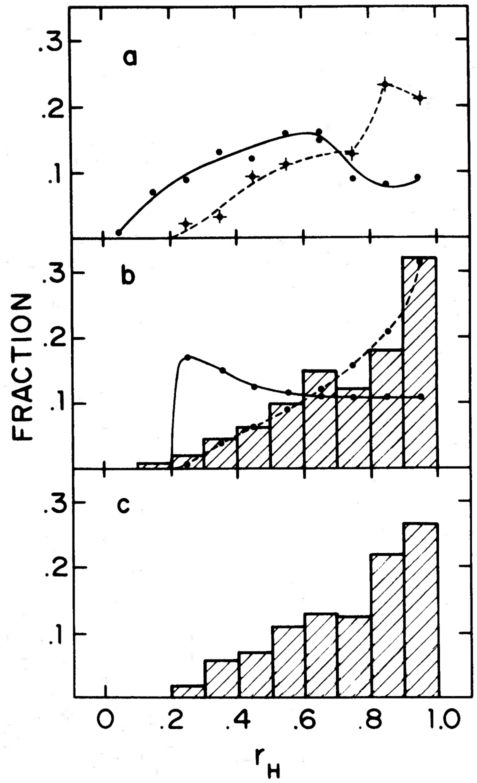

FIG. 1.—Distribution of observed axial ratios for (*a*) irregular DDO galaxies as measured by (*solid line*) Hodge and Hitchcock (1966) and (*dashed line*) Nilson (1973); (*b*) all Nilson dwarfs compared to curves expected for (*solid line*) $r_0 = 0.25$ and (*dashed line*) r_0 uniformly distributed between 0.25 and 1.0; and (*c*) detected dwarfs (FT's plus ours).

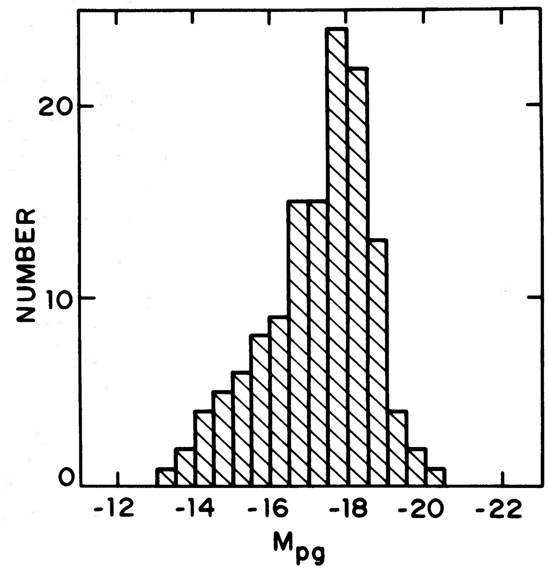

FIG. 2.—Histogram of absolute magnitudes corrected for galactic extinction and inclination effects for all unconfused galaxies in our detected sample.

This result calls into question our using $r_0 = 0.25$ to correct for inclination effects. We have decided, however, to use this value for two reasons. First, we wish to maintain consistency with the previous work of FT. Second, Figure 1*c* shows that there is a cutoff in r_H around 0.25: few, if any, of the detected galaxies are flatter than this. To minimize inclination effects, we will discuss a restricted subset of the detected sample and consider only those galaxies with $i > 45°$ ($r_H < 0.73$) for subsequent analyses.

c) Dwarf and Nondwarf Low-Surface-Brightness Galaxies

Both the Nilson and DDO samples show a long velocity tail, our highest detected velocity being 4236 km s^{-1} (U10903). The latter galaxy has an absolute photographic luminosity of -19.5 and cannot be properly labeled a dwarf galaxy. In fact, the histogram of the distribution of absolute magnitude of the 129 nonconfused galaxies (Fig. 2) shows that the majority (more than 80% of the sample) have an absolute photographic magnitude brighter than -16 mag and are thus not bona fide dwarf galaxies.

The selection criteria of "low surface brightness" and "little or no central concentration" have isolated a broad continuum of galaxies, from the "true" dwarf galaxies to more luminous but very low surface brightness systems. The latter class of galaxies may be particularly interesting, as pointed out by Strom and Strom (1978). In an optical study of 12 such galaxies, Romanishin, Strom, and Strom (cited by Strom and Strom 1978) found that (1) the disk surface brightness in these systems is 2–5 times smaller than that of a typical spiral such as M81, M51, or M101; (2) the disks are unusually blue, with the galaxies of lowest surface brightness the bluest; and (3) their optical spectra usually exhibit emission lines. The low surface brightness of the disk implies that the total number of stars formed during the lifetime of the galaxy is not large, and the colors suggest that star formation is recent (the galaxy is "young") or the disk stars are older but metal deficient. Both indicators suggest that the fraction of gas converted into stars and heavy elements is low and that these systems must have a large ratio of total hydrogen mass $\mathfrak{M}_H$ to photographic luminosity L_{pg} (Strom and Strom 1978). Our observations show that this is the case. Table 2 lists the average value of $\mathfrak{M}_H/L_{pg}$ along with that of other quantities of interest. To minimize the corrections for inclination, we have used only the galaxies in our sample and that of FT with $i \geq 45°$. To use the distances given by the redshifts, we have excluded galaxies with corrected redshift less than 100 km s^{-1}, leaving a total of 134 galaxies. To test the division of the data into a "dwarf" and "nondwarf" sample as proposed by FT, we have divided the galaxies into two categories, those brighter than $M_{pg} = -16$ and those fainter. For comparison, the results of a recent H I survey of late-type galaxies by Shostak (1978) are also given. Shostak's quantities have been corrected so as to be consistent with ours.

Examination of Table 2 reveals that the LSB

TABLE 2

AVERAGES AND DISPERSIONS FOR MEASURED AND DERIVED QUANTITIES FOR THE DDO AND PRESENT DWARF SAMPLE*

PARAMETER (1)	$M_{pg} \geq -16$ (dwarf) (2)	$M_{pg} < -16$ (nondwarf) (3)	FROM SHOSTAK 1978 Sbc (4)	Sc (5)	Scd (6)	Sdc (7)	Sdm–Sm (8)	Im (9)
n	40	94	22	23	34	10	3	2
d (kpc)	5.67 ± 1.9	20.2 ± 9.7	27.4 ± 12.7	23.3 ± 10.8	18.3 ± 6.6	16.5 ± 8.1	12.7 ± 5.3	(13.1 ± 6.5)
a_H'	3.29 ± 1.2	3.80 ± 2.24	...	...	...	...	...	...
r_H	0.51 ± 0.13	0.55 ± 0.13	...	...	...	...	...	...
P†	0.70 ± 0.55	1.16 ± 0.54	...	...	...	...	...	...
S (mag arcsec^{-2})	25.4 ± 0.3	25.2 ± 0.4	...	...	...	...	...	...
$M_{pg,c}$	−15.0 ± 0.71	−17.9 ± 1.1	...	...	...	...	...	...
v_0 (km s^{-1})	495 ± 243	1547 ± 762	...	...	...	...	...	...
Δv_{20}† (km s^{-1})	100 ± 16	177 ± 58	456 ± 131	418 ± 94	310 ± 69	275 ± 56	302 ± 7	(177 ± 96)
F_c ($10^6\ \mathfrak{M}_\odot$ Mpc^{-2})	3.1 ± 3.1	4.56 ± 7.9	...	...	...	...	...	...
$\mathfrak{M}_H$ ($10^9\ \mathfrak{M}_\odot$)	0.12 ± 0.13	1.44 ± 1.14	7.76 ± 5.29	6.41 ± 4.22	3.92 ± 2.18	4.17 ± 2.84	2.81 ± 1.6	(5.35 ± 2.3)
$\mathfrak{M}_T$ ($10^9\ \mathfrak{M}_\odot$)	0.82 ± 0.90	8.84 ± 9.60	136 ± 92	79.3 ± 49.5	33.3 ± 18.2	23.0 ± 12.6	21.6 ± 8.6	(13.0 ± 5.2)
L_{pg} ($10^9\ L_\odot$)	0.17 ± 0.09	3.22 ± 3.6	35.5 ± 23.7	22.9 ± 14.5	11.2 ± 6.9	7.91 ± 4.88	14.7 ± 8.7	(14.3 ± 8.3)
$\mathfrak{M}_H/\mathfrak{M}_T$	0.21 ± 0.19	0.26 ± 0.37	0.07 ± 0.03	0.09 ± 0.04	0.13 ± 0.05	0.18 ± 0.10	0.13 ± 0.05	(0.41 ± 0.01)
$\mathfrak{M}_H/L_{pg}$	0.66 ± 0.43	0.55 ± 0.36	0.24 ± 0.10	0.29 ± 0.14	0.41 ± 0.19	0.52 ± 0.27	0.25 ± 0.14	(0.61 ± 0.19)
$\mathfrak{M}_T/L_{pg}$	4.56 ± 3.40	3.37 ± 2.28	3.75 ± 1.07	3.96 ± 1.78	3.81 ± 1.45	4.69 ± 2.81	2.31 ± 1.31	(1.11 ± 0.31)

* 134 galaxies, $v_0 \geq 100$ km s^{-1} and $i \geq 45°$. The notation is the same as in Table 1 of Paper I. $\mathfrak{M}_H/\mathfrak{M}_T$, $\mathfrak{M}_H/L_{pg}$, and $\mathfrak{M}_T/L_{pg}$ are in solar units.

† From the present sample only (53 galaxies).

galaxies (both dwarf and nondwarf) have indeed a larger $\mathfrak{M}_H/L_{pg}$ than the late-type galaxies listed by Shostak (1978), except perhaps for the Magellanic irregulars, but the statistics in this bin are too poor to be definite. The LSB galaxies also possess a larger ratio of hydrogen mass $\mathfrak{M}_H$ to total mass $\mathfrak{M}_T$, indicating a low total amount of neutral gas that has been converted into stars. In the comparison, we have used $\mathfrak{M}_H/L_{pg}$ and $\mathfrak{M}_H/\mathfrak{M}_T$ calculated for *all* the detected LSB sample rather than for a diameter-limited sample for the reason that Shostak's (1978) sample of late-type galaxies is not strictly a velocity- or diameter-limited sample. The quantities represent the mean of his *detected* galaxies only. Nevertheless, our H I richness statements still hold with a diameter-limited sample. For all galaxies in the detected LSB sample larger than 3′ (Table 4 shows that the sample is fairly complete down to this limiting diameter, V/V_m being equal to 0.503) and with $i \geq 45°$ and $v_c \geq 100$ km s^{-1}, we find $\mathfrak{M}_H/L_{pg} = 0.59 \pm 0.31$ and $\mathfrak{M}_H/\mathfrak{M}_T = 0.27 \pm 0.24$ for the 18 galaxies with $M_{pg} \geq -16$, and $\mathfrak{M}_H/L_{pg} = 0.46 \pm 0.25$ and $\mathfrak{M}_H/\mathfrak{M}_T = 0.28 \pm 0.45$ for the 61 galaxies with $M_{pg} < -16$. These values are not very different from those given in Table 2.

The ratio of total mass to photographic luminosity of the LSB galaxies is comparable to that of the late-type galaxies, with the low amount of light emission compensated for by a low amount of total mass. The low total mass is due primarily to a narrow H I width (177 km s^{-1} versus 302 km s^{-1}), the linear sizes of the LSB and Sm galaxies being very similar. In summary, the LSB galaxies tend to be both less luminous and less massive (by a factor of 3–50) but are enriched in H I (by a factor of 2) compared to the Sm galaxies. Evidently, during their past history the LSB galaxies have had some difficulty converting a significant fraction of their gas into stars. The dominance of irregular systems (for the DDO sample for which morphological types are available, there are only four spirals as against 48 irregulars for $M_{pg} > -17$) and the numerous singly peaked H I profiles in Figure 1 of Paper I suggest random motions comparable to or exceeding ordered motions and an inability to maintain spiral structure in low-mass disk systems. Strom and Strom (1978) have suggested that this may be due to a too low value of $\mathfrak{M}_T/R$, where R is the linear radius. Galaxies having low $\mathfrak{M}_T/R$ are characterized by low values of $\Omega - \Omega_p$, the relative angular speed of the gas with respect to the density-wave, and of $w_{\perp 0}$, the relative velocity of the gas perpendicular to the arms; consequently, they are characterized by weak shocks, weak compression, and weak galactic shock-induced star formation. $\mathfrak{M}_T/R$ is proportional to $(\Delta v)^2$, and our data (Table 2) give a $\mathfrak{M}_T/R$ for the LSB galaxies 3–9 times smaller than for the Sdm–Sm galaxies, consistent with the hypothesis that the LSB galaxies have too small a $\mathfrak{M}_T/R$ to sustain a density-wave necessary for efficient star formation. Strom and Strom (1978) also found a current rate of star formation in the arms of LSB galaxies 2–3 times smaller than in the arms of "normal" spirals by examining ultraviolet plates.

These LSB galaxies may be the quiescent stage of the blue compact galaxies found by Zwicky, Haro, and Markarian and called by Sargent and Searle (1970) "extragalactic H II regions." The idea that blue compact galaxies are LSB galaxies experiencing intense bursts of star formation is also suggested by the great similarity in H I properties between the two classes of galaxies. The origin of such bursts is not understood, but the star formation mode in the blue compact galaxies is certainly very different from the shock-induced star formation mode found in spiral galaxies. It occurs in very dense regions, cannot last very long, and must occur rarely. In a recent study of one such blue compact galaxy, II Zw 70, O'Connell, Thuan, and Goldstein (1978) found that the region of star formation has a density of ~ 400 cm^{-3} compared to the mean H I density of ~ 0.1 cm^{-3}. The rate of star formation is so intense ($\sim 3\, \mathfrak{M}_\odot$ yr^{-1}) that a single burst cannot last for more than $\sim 10^8$ yr before consuming all the gas available. Further work is needed to link the LSB galaxies to the blue compact galaxies and to further understanding of the mode of star formation in these galaxies, where the sustenance of a density-wave does not seem likely.

We should note that the LSB galaxies detected here are different from van den Bergh's (1976) anemic spirals, which share some of the morphological characteristics (low surface brightness and diffuse spiral arms) but show no sign of star formation as evidenced by the lack of H I and the red disk color.

We turn next to the question of the division of the LSB galaxy sample into a dwarf and a nondwarf population. We have chosen to use a division by absolute magnitude ($M_{pg} = -16$) rather than by velocity ($v_c = 1000$ km s^{-1}) like FT because there are many systems with $v_0 \leq 1000$ km s^{-1} but with $M_{pg} \leq -16$. In any case the division is arbitrary and is only used to illustrate our conclusions. Columns (2) and (3) of Table 2 show that the surface brightness is the same for the two populations but that the nondwarf galaxies are, on the average, larger, more massive, more luminous, and farther away than the dwarf galaxies. The increase in light is approximately comparable to the increase in mass, so that the two populations have roughly similar $\mathfrak{M}_H/\mathfrak{M}_T$, $\mathfrak{M}_H/L_{pg}$, and $\mathfrak{M}_T/L_{pg}$ ratios. Figures 3 illustrate the nondependence or very weak dependence of these ratios on the absolute photographic magnitude of the LSB galaxy. The slopes of the least-squares fit of these ratios with M_{pg} are, respectively, (-0.02 ± 0.05), (0.03 ± 0.05), and (0.05 ± 0.05), with correlation coefficients -0.08, 0.21, and 0.23, consistent with *no* dependence on M_{pg}. FT, by binning their data and averaging by morphological type, found a very slight dependence of $\mathfrak{M}_H/L_{pg}$ on M_{pg} (larger $\mathfrak{M}_H/L_{pg}$ for fainter galaxies) at the 1.7 σ level. Table 3 shows the averages of $\log(\mathfrak{M}_H/L_{pg})$ in bins of 1 mag in M_{pg} for the Nilson and FT galaxies with $v_c \geq 100$ km s^{-1} (317 galaxies). The binning of the data shows a slight trend of larger $\mathfrak{M}_H/L_{pg}$ with fainter absolute magnitude, but the dispersions are so large that no firm conclusion can be drawn.

$\mathfrak{M}_T/R$ is 3 times larger for the nondwarf than for the dwarf population, indicating perhaps a greater

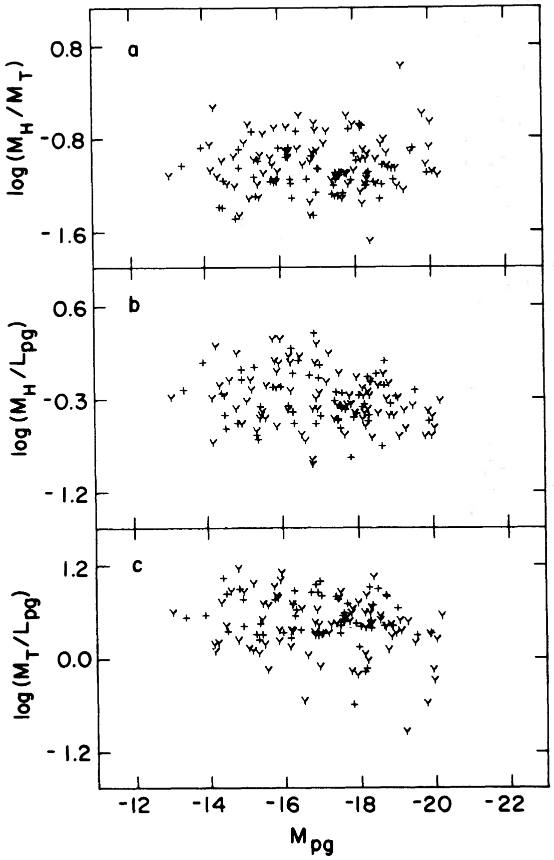

FIG. 3.—The quantities (*a*) log ($\mathfrak{M}_H/\mathfrak{M}_T$), (*b*) log ($\mathfrak{M}_H/L_{pg}$), and (*c*) log ($\mathfrak{M}_T/L_{pg}$) in solar units as a function of absolute magnitude. The systems plotted consist of all galaxies detected by (Y) FT and (+) us with $v_0 \geq 100$ km s^{-1} and $i \geq 45°$, a total of 134 galaxies. Least-squares fits to the data points yield slopes (-0.02 ± 0.05), (0.03 ± 0.05), and (0.05 ± 0.05) and correlation coefficients -0.08, 0.21, and 0.23, respectively.

TABLE 3

$\langle \log \mathfrak{M}_{\rm H\,I}/L_{pg} \rangle$* AS A FUNCTION OF M_{pg} FOR ALL GALAXIES IN FT AND PRESENT SAMPLE WITH $v_0 \geq 100$ km s^{-1}

M_{pg}	n	$\langle \log \mathfrak{M}_{\rm H\,I}/L_{pg} \rangle$	σ
−14........	13	−0.267	0.275
−15........	31	−0.307	0.237
−16........	38	−0.211	0.294
−17........	65	−0.316:	0.296
−18........	101	−0.343	0.215
−19........	52	−0.408	0.197
−20........	13	−0.484	0.207

* In solar units.

ability to maintain spiral structure. The standard deviations about the mean of the quantities listed in Table 2 are very large and reflect the arbitrary division of the LSB sample into two populations. It is probably better to think of the LSB galaxy sample as containing a broad continuum of galaxies varying smoothly in physical properties and extending the Hubble sequence to types later than Sdm, Sm, and Im.

d) *Space Densities*

We have discussed in § II*a* the completeness of the LSB sample (687 galaxies) in the Nilson catalog. We now discuss the completeness of the *detected* sample (FT plus our sample) and restrict the analysis to galaxies having $v_c \geq 100$ km s^{-1} and not in the Virgo cluster (defined by $12^h5^m \leq \alpha \leq 12^h50^m$ and $2°5 \leq \delta \leq 18°$) in order to use the redshift as an indicator of distance. This gives a total of 306 LSB galaxies with known redshifts. In order to calculate their space density, we again used Schmidt's (1968) $\langle V/V_m \rangle$ test. V is, as in § II*a*, the volume of the sphere whose radius is the redshift distance to the object. V_m is the smaller of the two volumes $V_m^{\rm opt}$ and $V_m^{\rm rad}$, where $V_m^{\rm opt}$ is the volume of the sphere whose radius is the

TABLE 4

COMPLETENESS OF THE DETECTED LSB GALAXY SAMPLE AS A FUNCTION OF LIMITING HOLMBERG DIAMETER AND ABSOLUTE MAGNITUDE*

ABSOLUTE PHOTOGRAPHIC MAGNITUDE	LIMITING HOLMBERG DIAMETER							NO. OF GALAXIES ADDED†
	16′	8′	4′	3′	2′5	2′	1′5	
−12............	...	...	...	...	...	...	0.534 (1)	0
−13............	...	...	...	0.655 (1)	0.632 (4)	0.529 (8)	0.224 (8)	0
−14............	...	...	0.511 (3)	0.598 (8)	0.460 (11)	0.389 (16)	0.202 (16)	3
−15............	...	0.629 (1)	0.373 (10)	0.486 (23)	0.385 (29)	0.307 (38)	0.150 (39)	25
−16............	0.812 (1)	0.345 (3)	0.294 (6)	0.462 (19)	0.464 (31)	0.360 (43)	0.185 (45)	19
−17............	...	0.567 (3)	0.473 (17)	0.517 (41)	0.437 (58)	0.334 (78)	0.176 (80)	32
−18............	...	0.607 (1)	0.562 (19)	0.539 (51)	0.422 (69)	0.255 (75)	0.147 (76)	46
−19............	...	...	0.549 (14)	0.435 (24)	0.363 (32)	0.237 (35)	0.134 (35)	30
−20............	...	...	0.627 (3)	0.378 (4)	0.353 (5)	0.242 (6)	0.121 (6)	4
All............	0.812 (1)	0.497 (8)	0.490 (72)	0.503 (171)	0.423 (239)	0.309 (299)	0.165 (306)	159

* The tables gives $\langle V/V_m \rangle$. The number of galaxies on which each value is based is given in parentheses.

† The sample used is all galaxies larger than 2′ (299 galaxies). The last column gives the number of galaxies added to have $\langle V/V_m \rangle = 0.5$ in each interval of absolute magnitude.

maximum distance the galaxy can be at and still have a Nilson blue diameter greater than 1′ and V_m^{rad} is the volume of the sphere whose radius is the maximum distance the galaxy can be at and still have an H I flux greater than 0.045 flux units. Table 4 lists $\langle V/V_m \rangle$ as a function of absolute magnitude for several values of the limiting Holmberg diameter (converted from the Nilson blue diameter by eq. (1) of Paper I) of the sample. $\langle V/V_m \rangle$ should be 0.5 for objects uniformly distributed in Euclidean space. The bottom row of Table 4 shows that the sample of detected LSB galaxies is fairly complete for galaxies larger than 2′.5. The incompleteness is due to the lower limit in H I sensitivity of the survey since Table 1 shows that $\langle V/V_m \rangle$ is still relatively close to 0.5 (equal to 0.453) for galaxies larger than 2′.2 if the *whole* Nilson dwarf sample is considered. If we adopted 20 kpc as the mean linear diameter (Table 2), then a galaxy with a 2′ diameter would be at a systemic velocity of 2600 km s^{-1}, the velocity where the detections begin to taper off.

We now use Table 4 to correct for incompleteness and derive a space density for the LSB galaxies (cf. Huchra and Sargent 1973). Since there are only seven galaxies smaller than 2′ and the incompleteness factor (defined to be $0.5/\langle V/V_m \rangle$) increases from 1.62 to 3.03 by going from a limiting Holmberg diameter of 2′ to one of 1′.5, we restrict the analysis to all galaxies larger than 2′, a total of 299 galaxies. The last column of Table 4 gives the number of galaxies which must be added in order to keep $\langle V/V_m \rangle = 0.5$ in each interval of absolute magnitude. In all, we must add 159 galaxies to the original sample of 299.

The uncorrected space density $\Phi(M_{\text{pg}})$ is obtained from

$$\Phi(M_{\text{pg}}) = \frac{4\pi}{\Omega} \sum_i \frac{1}{V_m^i} \text{ Mpc}^{-3} \text{ mag}^{-1} , \qquad (3)$$

where $\Omega/4\pi$ is the fraction of sky covered by the Nilson catalog and equal to 0.52 ($\delta \geq -2°30'$) and V_m^i is the volume of the sphere whose radius is the maximum distance a galaxy can be at and still be included in the sample (either by being larger than 1′ or by having an H I flux greater than 0.045 Jy). The summation is over all galaxies in any absolute magnitude interval $M_{\text{pg}} \pm$ 0.5 mag. After correction for incompleteness by the numbers given in the last column of Table 4, the results of the space density calculations are given in Table 5 and Figure 4. They are compared to the luminosity function for field galaxies normalized to the Local Supercluster given by Felten (1977) with $H_0 = 75$ km s^{-1} Mpc^{-1}. We have used the local normalization instead of the large-scale normalization (which is about a factor of 2 lower [Felten 1977]) for the luminosity function because the LSB galaxies considered here are for the most part not much farther away than the Local Supercluster. In the comparison, we have neglected the difference of ~0.3 mag between the Holmberg and $B(0)$ magnitude scale used by Felten (1977), this being smaller than the uncertainties in our derived magnitudes. The data at the faint end ($M_{\text{pg}} \geq -16$) are based largely on Holmberg's (1974) work on small groups. The last column of Table 5 shows that the space density of hydrogen-rich LSB galaxies is about half that of all field galaxies at $M_{\text{pg}} = -13$ and decreases to 0.2% at $M_{\text{pg}} = -20$. Some caution must be exercised with these results because (1) they refer only to hydrogen-rich systems and (2) our derived photographic magnitudes are good only to ± 0.5 mag. Nevertheless, it is clear that more than 90% of all hydrogen-rich galaxies with $M_{\text{pg}} \leq -17$ are *not* LSB galaxies. For hydrogen-rich systems, the statement by Freeman (1970) that all nondwarf galaxies have a constant central surface brightness $B_0 = 21.6 \pm 0.3$ B mag arcsec^{-2} appears to be correct to the 2% level at $M_{\text{pg}} = -19$ (i.e., 98% of all galaxies at $M_{\text{pg}} = -19$ do have a constant surface brightness) and to the 0.2% level at $M_{\text{pg}} = -20$. This constancy is not due primarily to selection effects as has been argued by Disney (1976).

TABLE 5

Space Densities of Hydrogen-rich Low-Surface-Brightness Galaxies

M_{pg}	$\log \Phi_{\text{LSB}}$* (M_{pg})	ϵ†	$\log \Phi_{\text{FG}}$* (M_{pg}) (Felten 1977)	$\Phi_{\text{LSB}}/\Phi_{\text{FG}}$‡
−13........	−1.39	0.13	−1.1	0.51
−14........	−1.74	0.10	−1.2	0.29
−15........	−1.45	0.07	−1.3	0.71
−16........	−2.15	0.06	−1.4	0.18
−17........	−2.44	0.05	−1.5	0.11
−18........	−2.84	0.05	−1.6	0.06
−19........	−3.60	0.07	−1.8	0.02
−20........	−4.77	0.15	−2.1	0.002

* Units of Φ in Mpc^{-3} mag^{-1}.

† $\epsilon = \log(1 + 1/\sqrt{N})$, where N is the number of galaxies in each magnitude interval.

‡ FG = field galaxies.

If the blue compact galaxies are LSB galaxies undergoing bursts of star formation, then the comparison of the space densities of the LSB galaxies with those of the Markarian galaxies should give some hints on the duration of the bursts. Unfortunately, the Huchra and Sargent (1973) sample contains only eight Markarian galaxies with $M_{\text{pg}} \geq -16$, so that no meaningful statistical statement can be made.

III. CONCLUSIONS

A statistical analysis of a sample comprising the DDO galaxies and the Nilson low-surface-brightness galaxies (Paper I) led to the following conclusions:

1. The Nilson catalog is fairly complete for all galaxies with a Nilson blue diameter larger than 1′: $\langle V/V_m \rangle = 0.484$. For LSB galaxies, $\langle V/V_m \rangle$ decreases to 0.453.

2. A catalog of objects selected by the criterion of low surface brightness includes a broad continuum of galaxies, ranging from the "true" dwarfs ($M_{\text{pg}} > -16$) to more luminous ($M_{\text{pg}} \leq -16$) but very low-surface-brightness systems. The LSB galaxies have in general larger $\mathfrak{M}_{\text{H}}/L_{\text{pg}}$ and $\mathfrak{M}_{\text{H}}/\mathfrak{M}_T$ ratios than late-type

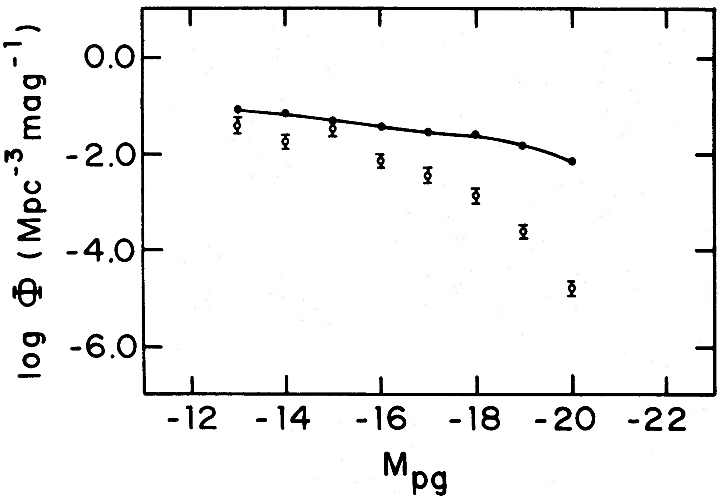

FIG. 4.—Log of the space density versus absolute magnitude (*solid line*) for field galaxies (Felten 1977) and (*dots*) for the LSB galaxies. Error bars represent the square root of the number of systems in each bin. A Hubble constant of 75 km s^{-1} Mpc^{-1} has been used.

galaxies of comparable luminosity, implying that only a small fraction of the total gas has been converted into stars. This is consistent with the blue color of their disks (Strom and Strom 1978), which suggests either recent star formation or the presence of disk stars which are older but metal deficient.

3. The LSB galaxies have a $\mathfrak{M}_T/R$ ratio 3–9 times smaller than that of the Sdm–Sm galaxies. This value may be too small for the LSB galaxies to maintain a density-wave necessary for efficient star formation. The dominance of irregular systems and of singly peaked H I profiles suggests random motions comparable to or exceeding ordered motions.

4. To first order, the LSB galaxies possess $\mathfrak{M}_H/\mathfrak{M}_T$, $\mathfrak{M}_H/L_{pg}$, and $\mathfrak{M}_T/L_{pg}$ ratios which are independent of the absolute magnitude of the galaxy ($-14 \geq M_{pg} \geq -20$).

5. Using the $\langle V/V_m \rangle$ technique to correct for incompleteness, we determine the space density of H I-rich LSB galaxies. Compared to the composite luminosity function given by Felten (1977), the H I-rich galaxies constitute about half of the field galaxies at $M_{pg} \approx -13$, decreasing to 0.2% at $M_{pg} = -20$. The statement by Freeman (1970) that nondwarf galaxies have a constant central surface brightness appears to be accurate to the level of a few percent at $M_{pg} \lesssim -19$.

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