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A CORRELATION BETWEEN SiO AND STELLAR LUMINOSITIES IN LONG-PERIOD VARIABLES AND THE NATURE OF THE SiO MASER PUMP MECHANISM

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ABSTRACT

Luminosities of and distances to SiO–emitting long-period variables have been determined. The good correlation obtained between the SiO and stellar luminosities has the following consequences: the SiO masers are (1) radiatively pumped by the direct stellar radiation, (2) saturated, and (3) occur at roughly the same distance from the stellar center. In addition, (4) the pumping scheme proposed by Kwan and Scoville may provide the basic explanation for the inversion but has to be modified in some way to increase the SiO power it predicts. The tight correlation obtained for SiO is not expected for the other circumstellar masers, OH and H_2O.

Subject headings: masers — stars: circumstellar shells — stars: long-period variables

I. INTRODUCTION

The period-luminosity–spectral type scheme for Miras (Cahn and Wyatt 1978*a*) has led to two successful correlations. In the first, the velocity of expansion of the circumstellar shell of a Mira was found to be well correlated with stellar luminosity (Cahn and Wyatt 1978*b*). In the second, a preliminary analysis (Cahn 1977*b*) showed that the luminosity and derived distance of the Mira determined a close relation between the SiO maser luminosity and the stellar luminosity of the Mira, thus clarifying an earlier correlation (Cahn 1977*a*) between SiO maser luminosity and spectral type. Like the period, spectral type alone is not unique in determining luminosity.

We were thus encouraged to look more closely into the SiO maser luminosity–stellar luminosity relation in an effort to determine the physical parameters responsible for the maser phenomenon. In § II we determine the radius, temperature, and luminosity of each long-period variable and then calculate the quantity $W = 4\pi r_*^2 B_\nu(T_{\rm eff})$ where B_ν is evaluated at the effective temperature $T_{\rm eff}$ of the star at the frequency of the $v = 0$ to $v = 2$ vibrational transition and r_* is the mean stellar radius. It is the quantity W which is compared with the SiO maser output and which is shown in § III to be of significance in the radiative pumping scheme of Kwan and Scoville (1974, hereafter KS). Further, in § III appropriate modifications to the KS radiative pumping model are suggested based on our actual correlation of SiO output with stellar luminosity.

II. OBSERVATIONAL MATERIAL AND THE CORRELATION

The long-period SiO maser stars which have been included have been selected according to the following: (i) the mean spectral type at maximum is known so that the luminosity can be determined, and (ii) 1.04 μm photometry is available so that accurate distances (at the 20% level) can be determined (Cahn and Wyatt 1978*a*). In order that the IR pump power may be estimated, we need in addition the stellar radius r_* and effective temperature $T_{\rm eff}$, for which Wood and Cahn (1977) provide the formalism. With the exception of W Hya, R Leo, and χ Cyg, all the stars in this study are M-type Mira long-period variables and meet the above criteria.

In the case of R Leo, we find that its somewhat smaller amplitude makes it comparable to W Hya, a semiregular long-period variable. An analysis as yet unpublished shows a first-overtone domain of semiregularity immediately adjacent to and above the Mira domain (see Wood and Cahn 1977) which is characterized by somewhat higher masses and luminosities for the same period as the corresponding Miras. The deduced luminosity, and thus the distance, for R Leo is 147 pc, in excellent agreement with Eggen (1971). We also use these unpublished results to obtain the luminosity and distance of W Hya, classified as SRa. For the S star χ Cyg, the Cahn-Wyatt period-luminosity–spectral type scheme does not apply; it is necessary to use the luminosity and distance found by Eggen (1971). For all the stars in Table 1 except χ Cyg, distances were determined from 1.04 μm photometry following the analysis in Cahn and Wyatt (1978*a*). The mean stellar luminosities and derived quantities, mean radius and effective temperature, at optical depth $\frac{2}{3}$, are recorded in Table 1. The corresponding distances in parsecs are listed in column (5) of Table 1.

We adopt the point of view of Lépine and Paes de Barros (1977) in correcting the observed SiO maser fluxes to zero phase. For example, in the case of the time-varying flux of R Cas reported by Spencer and Schwartz (1975), we find that measurements at phases 0.15, 0.76, and 0.88 agree to within 25% when following the Lépine and Paes de Barros prescription.

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TABLE 1

SUMMARY OF PARAMETERS OF SiO MASERS

Name (1)	log $(\bar{L}/L_\odot)$ (2)	T_{eff} (K) (3)	r_* (cm) (4)	D (pc) (5)	Φ_{SiO} (s^{-1}) (6)	Ref. (7)	$\bar{\Phi}_{SiO}$ (s^{-1}) (8)	W (ergs s^{-1} Hz^{-1} sr^{-1}) (9)
R Aql	3.812	2848	2.30 (13)	261	1.7 (43)	1	2.6 ± 1.2	1.615 (22)
	...	...	...	...	3.4	5	...	...
RR Aql	3.967	2754	2.94	375	10.4	5	...	2.487
RT Aql	3.872	2811	2.53	629	7.4	2	...	1.909
R Aur	4.094	2680	3.60	466	13.3	2	...	3.540
TX Cam	4.200	2619	4.25	696	93.9	3	...	4.749
R Cnc	3.925	2779	2.75	339	2.8	4	4.0 ± 1.6	2.212
	...	...	...	...	5.1	3	...	...
R Cas	4.038	2712	3.29	266	17.7	5	18.0 ± 11.9	3.029
	...	...	...	...	30.0	6	...	...
	...	...	...	...	6.2	7	...	...
Y Cas	4.011	2728	3.15	686	8.3	2	...	2.810
T Cep	3.975	2749	2.98	240	3.3	4	7.4 ± 5.8	2.542
	...	...	...	...	11.5	3	...	...
S CrB	3.925	2779	2.75	467	5.8	5	...	2.212
χ Cyg	3.38	3127	1.16	89	0.044	5	...	0.483
U Her	4.000	2735	3.10	397	9.9	5	...	2.727
RU Her	4.134	2657	3.83	734	25.4	4	...	3.956
W Hya	4.07:	2693	3.46:	135	40.6	1	23.5 ± 16.2	3.310
	...	...	...	...	21.3	5	...	...
	...	...	...	...	8.5	7	...	...
R Leo	3.93	2776	2.29	147	15.3	5	8.2 ± 7.2	2.243
	...	...	...	...	8.6	7	...	...
	...	...	...	...	0.83	1	...	...
R Peg	3.900	2711	2.64	499	6.2	3	...	1.955
IK Tau	4.081	2687	3.52	400	15.9	5	21.3 ± 13.0	3.324
	...	...	...	...	36.1	7	...	...
	...	...	...	...	11.9	1	...	...

REFERENCES.—[1] Balister *et al.* 1977; [2] Dickinson *et al.* 1978; [3] Spencer *et al.* 1977; [4] Blair and Dickinson 1977; [5] Snyder and Buhl 1975; [6] Spencer and Schwartz 1975; [7] Snyder *et al.* 1978.

After correcting the SiO maser flux to zero phase, the total maser photon rate is determined by using the calculated distance to the star. The results for the $v = 1$, $J = 1 \rightarrow 0$ measurements are listed in Table 1 in column (6). In column (8) are given the averages and standard deviation where more than one measurement is available. For a number of stars, the data show a rather large range which is marked in Figure 1. The reason for these big differences between measurements is not at all clear, but they may reflect differences in telescopes, or real variations in maser output. We show data from Parkes (Balister *et al.* 1977), Haystack (Dickinson *et al.* 1978; Snyder *et al.* 1978), Maryland Point (Spencer and Schwartz 1975; Spencer *et al.* 1977), and Kitt Peak (Snyder and Buhl 1975; Blair and Dickinson 1977) at different phases and separated by more than one period. We also find in some cases that the power in the weak pedestal (Snyder *et al.* 1978) is higher than that reported for the total power. Clearly, more measurements at high resolution will be needed to clarify these points.

We estimate that the error in Φ_{SiO} consists of (1) the error in the individual power measurement, (2) the error due to zero phasing, and (3) the error in distance. The first is negligible compared with the latter two, which are shown as an error box in Figure 1 and are clearly small compared with the differences between some individual measurements.

Guided by the KS calculations, we anticipate the inversion to be caused by cascades from $v = 2$. We therefore look for a correlation with the available IR radiation for pumping into the $v = 2$ level. To that end, we estimate the value of $W = 4\pi r_*^2 I(7.44 \times 10^{13}$ Hz) for the $v = 2$ vibrational level by evaluating the blackbody approximation $4\pi r_*^2 B_\nu$ for this frequency. Thus, for each stellar temperature and radius, we compute W; it is given in column (9) of Table 1. It should be noted that the value of W is dependent on both r_* and T_{eff}, whose independent variation is evident from columns (3) and (4). The correlation of the total photon output rate with pumping power is shown in Figure 1. The straight line represents a least-squares power law

$$\Phi_{SiO} = 2.54 \times 10^{-25}[4\pi r_*^2 B_{7.44\times 10^{13}}(T_{eff})]^{3.06}\ s^{-1}\ . \quad (1)$$

The correlation coefficient for this best fit was $r = 0.97$. The fit is influenced by the strongest and weakest SiO emitters (TX Cam and χ Cyg, respectively) and omitting them leads to a spectral index of 2.31 with $r = 0.89$. Thus there seems to be some uncertainty in the exact value of the correlation spectral index. This is not that important, however, since the significant point essentially is that the spectral index is different from 1, the *a priori* preferred value (see below). Attempts to force a linear fit through the points lead to a correlation coefficient smaller than 0.1.

A notable exception to the stars listed in Table 1 is Mira itself. Mira is well studied and has all the

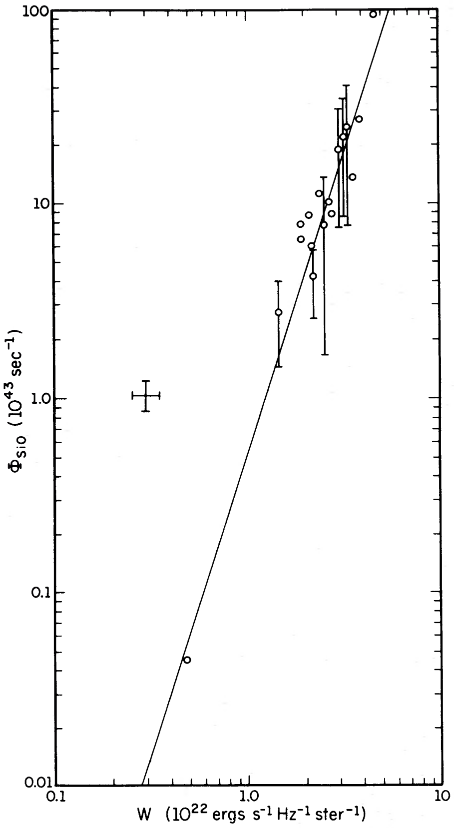

FIG. 1.—Total photon luminosity in the $v = 1, J = 1 \to 0$ transition of SiO as a function of stellar output in the $v = 0$ to $v = 2$ transition for long-period variables. The error box corresponds to errors in zero phasing and distance. When an error is given on a point, it represents the range when several measurements are available.

qualifications for inclusion in this study. Unfortunately, the SiO luminosity of Mira is extremely low, and forced us to determine that this is due to Mira being a member of a visual binary system, the other member of which is a reasonably active white dwarf accreting mass from Mira (Warner 1972). The probable effect on the SiO maser is a sizable reduction of SiO number density due to the UV flux of the white dwarf.

A similar analysis has also been performed for the $v = 2, J = 1 \to 0$ maser transition. While this maser also shows the same correlation between pumping power and SiO photon output, we have less than half the number of cases, so the statistics are not convincing enough to justify their inclusion.

III. DISCUSSION

The good correlation between the IR and SiO fluxes demonstrated in Figure 1 provides a strong case for radiative pumping being the cause of the maser inversion. An almost equally good correlation is obtained for an exponential fit and one could therefore argue for unsaturated maser emission. However, the resulting exponential fit provides too small amplification gains and requires an unrealistically high spontaneous emission rate. In addition, the time stability of the SiO maser emission also argues against an unsaturated maser, and this interpretation must therefore be rejected.

In the case of a saturated maser, the photon emission rate is given by

$$\Phi = \tfrac{1}{2}\Delta P \cdot V \cdot \frac{\Omega}{4\pi}, \qquad (2)$$

where V is the emission volume, Ω is the solid angle into which the maser radiation is beamed, and ΔP is the difference in pump rate per unit volume into the two maser levels. For pumping of an optically thick line at a frequency ν by the direct stellar radiation, the pump rate $P(\nu)$ is given by (KS)

$$P(\nu) = \frac{I(\nu)}{hc}\left(\frac{\pi r_*^2}{r^2}\right)\frac{\partial v}{\partial r}, \qquad (3)$$

where $I(\nu)$ and r_* are the stellar intensity and radius, respectively; r the radius at the maser region; and $\partial v/\partial r$ the radial-velocity gradient. If the SiO masers around Miras are located at roughly the same distance from the star (where the velocity gradient is probably also roughly the same), the only dependence on the stellar properties is through the quantity $I(\nu)\ r_*^2$. According to KS, the inversion of the $v = 1$ rotation levels is due to cascades from the $v = 2$ state and ΔP is proportional to $I(\nu_{02})$. The SiO maser luminosity would therefore vary only with $I(\nu_{02})r_*^2$, which is precisely what is demonstrated by Figure 1. The small scatter in points (1) indicates that the masers do indeed occur at the same distance from the star, (2) strengthens the case for saturation, since only in that case does the SiO number density drop out of the calculation, and (3) argues for direct pumping by the stellar radiation. In the case of OH maser radiation from OH/IR stars, for instance, one would not expect such a tight correlation, since the grains transform the direct stellar radiation into far-IR pump photons (Elitzur, Goldreich, and Scoville 1976). The pump rate therefore depends also on the grain properties in that case and one would expect a larger scatter than for SiO.

In the calculations of KS only rotation states with $J \leq 5$ and only direct pumping from the ground state into $v = 2$ were considered. One may expect, however, that the $v = 2$ level is populated by cascading from higher vibrational levels as well. It is also likely that many more rotational states are involved. We therefore also checked the correlation with the total number of photons available shortward of 4.1 μm and it was found to be very similar to that demonstrated in Figure 1.

The difference in pumping rates into the maser levels, ΔP, can be written as

$$\Delta P = \eta P, \qquad (4)$$

where $\eta = \Delta P/P$ is the pumping inversion efficiency.

substituting this form in equation (2),[1] it follows from equation (1) that $\eta \sim W^2$, whereas *a priori* one may expect η to be independent of the pump rate. The dependence of η on the pump rate is already indicated by the KS calculations. Using their explicit analytic expressions for the level populations, one finds a relative inversion for the $v = 1$, $J = 1 \to 0$ transition of about 10^{-3} and no inversion at all for $v = 1$, $J = 3 \to 2$. On the other hand, their explicit numerical calculations, which include saturation effects, show that in a saturated maser both transitions are inverted with an efficiency of about 15% for $J = 1 \to 0$ and 2% for $J = 3 \to 2$.[2] Hence η is clearly dependent on W.

The last point to check is the question of the capability of the stellar IR radiation to produce the observed SiO output. The number of photons emitted by the star shortward of either 8.2 or 4.1 μm is of order 10^{49} s^{-1}. One expects a few vibrational transitions to be optically thick and at least a few hundred rotational transitions for each of them. With bandwidth $\Delta\nu/\nu \approx 10^{-5}$ one therefore expects about 10^{-2}–10^{-3} of all the photons to be available for pumping. The observed SiO photon rates of 10^{44}–10^{45} s^{-1} thus require a conversion efficiency of at most 10%, which is quite reasonable. The model calculated by KS corresponds to $W = 5.2 \times 10^{22}$ ergs s^{-1} Hz^{-1} sr^{-1} and $\Phi_{\rm SiO} = 2 \times 10^{43}$ s^{-1} and falls below our curve by about a factor 40. The efficiency of the KS model clearly has to be increased to conform with the data. One way which perhaps could lead to an increase in the efficiency is the inclusion of more rotation levels. As mentioned already, KS considered only $J \leq 5$. The rotation constant of SiO corresponds to only 1 K and the temperature in the maser emitting region of the KS model is ~300–400 K. One therefore expects all the rotation levels up to about $J \sim 300$ to be significantly populated. The excited vibrational-rotational states are populated by direct transitions from the ground state and this leads to a significant population for them. If rotational cascades within the excited vibration states become as important as vibrational decays, the pumping efficiency is enhanced significantly. If so many rotational states have to be included, one should of course adopt different methods to treat the pump calculations.

The assumption about cascades within the vibrational states does not hold in general but may be valid in the present case due to effects of optical depth and saturation. If it is proved incorrect, it is hard to see what other modifications of the KS calculations could increase the inversion efficiency. The solution in that case may be in adding collisions to the basic radiative pump. In fact, KS have presented calculations with collisions included which led to an increase in the SiO flux. However, because of the unrealistic cross-section law used by KS, these results cannot be considered reliable.

In summary—our analysis shows that the SiO masers are (1) saturated, (2) occur at approximately the same distance from the star, and (3) are pumped by the direct stellar radiation. The inversion efficiency varies with the pump rate, and the mechanism suggested by KS is quite probably the correct one but has to be modified in some way to increase its efficiency in order to explain the observed SiO output.

Independent arguments, both theoretical and observational, support the case for a radiative pump. If the SiO maser occurs at a radius of a few stellar radii, as suggested by most observations, it is quite easy to show that the rate for radiative excitations is much higher than the collisional excitation rate. Observationally, the recent detection of maser emission in the $v = 3$ level (Scalise and Lépine 1978) which lies 5260 K above the ground state almost certainly eliminates collisional pumps and indicates that the basic pumping of SiO masers is done by radiation.

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[1] Strictly speaking, when using eq. (4) to evaluate ΔP one should include direct pumping into $v = 1$ in P. We have checked that this does not change any of the discussion.

[2] The predicted inversion of the $v = 1$, $J = 3 \to 2$ transition is in agreement with observations (Davis *et al.* 1974).

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