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# WHAT TIDES AND FLARES DO TO RS CANUM VENATICORUM BINARIES*

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## ABSTRACT

We discuss the effects that anisotropic mass loss should have on the orbital and spin states of RS Canum Venaticorum binaries. In the absence of magnetic fields, orbital period changes reported for several RS Canum Venaticorum systems require $dM/dt \sim 10^{-6}\ M_\odot\ \mathrm{yr}^{-1}$. Magnetic braking can lower this required rate if the surface magnetic fields are $\gtrsim 1000$ gauss. However, this requires a method much more powerful than tidal torques to convert spin angular momentum loss to orbital angular momentum loss. This possibility is important when interpreting the complicated light curves of these systems, and may contradict the Hall's "drifting star spot" hypothesis. In addition, large mass-loss rates may result in significant self-absorption of quiescent soft X-rays observed from several of these binaries.

*Subject headings:* stars: eclipsing binaries — stars: flare — stars: mass loss

## I. INTRODUCTION

RS Canum Venaticorum binaries have been drawing attention recently since soft X-rays, nonthermal centimeter radiation, and simultaneous X-ray, Hα, and radio bursts were observed from several of the systems (Walter, Charles, and Boyer 1978a, b). Also, long-period "migrating waves" have been seen in the light curves of many of the systems (Hall 1976). Hall interprets these waves as brightness changes caused by large "star spots" which differentially rotate around a star that is otherwise rotating nearly synchronously with the orbit.

In addition to these features, several RS Canum Venaticorum systems are purported to exhibit large quasi-periodic orbital period variations (Hall 1976; Hall, Richardson, and Chambliss 1976; Arnold, Hall, and Montle 1973; Hall 1972, 1975). For RS CVn, the implied mass-loss or transfer rate is extraordinary: $\gtrsim 10^{-6}\ M_\odot\ \mathrm{yr}^{-1}$. Since the systems are almost certainly detached, Hall (1972, 1975) has proposed that active regions (perhaps flarelike) on one of the components eject the matter. He attempts to correlate the phase of the orbital period variations with that of the photometric "migrating wave," thus establishing a physical link between the slowly drifting star spots and the active region.

In this paper, we argue that large mass-loss rates can indeed be inferred from the purported period changes. Sufficiently strong magnetic braking could in principle lower the requisite mass-loss rates to $\sim 10^{-9}\ M_\odot\ \mathrm{yr}^{-1}$, but only if a torque substantially stronger than the estimated tidal torque exists to couple the stellar spin to the orbital motion. Otherwise, the angular momentum loss would occur in the stellar spin, driving the component out of orbital synchronism. This possibility would contradict the observations of slowly migrating ($\sim 10$ yr) waves in the light curves, which require near synchronism. In any case the implied magnetic fields are large, although this requirement may find support in the reported presence of large starspots in these systems.

Additionally, we show that large mass-loss rates ($\gtrsim 10^{-8}\ M_\odot\ \mathrm{yr}^{-1}$) probably result in self-absorption of the quiescent, or flare-associated, soft X-ray emission. We conclude by calling into question the claim that these systems undergo large orbital period changes.

## II. TIDES

The characteristic time scales for tidal orbit circularization and spin synchronization are, respectively,

$$\tau_{\mathrm{circ}} = \frac{125}{121}(1 - e^2)^5 \left(\frac{A}{a_1}\right)^8 \times \left[\frac{M_1{}^2}{M_2(M_1 + M_2)}\right]\left(\frac{n}{\omega_1}\right)\left(\frac{M_1}{a_1\langle\mu\rangle}\right), \quad (1)$$

$$\tau_{\mathrm{sync}} = \frac{75\alpha}{112}(1 - e^2)^{9/2} \left(\frac{A}{a_1}\right)^6 \left(\frac{M_1}{M_2}\right)^2 \left(\frac{M_1}{a_1\langle\mu\rangle}\right) \quad (2)$$

(Alexander 1973; Zahn 1966). Here $M_1$ and $M_2$ are the masses, where $M_1$ is the mass of the star inside which tidal dissipation is taking place. The star of mass $M_2$ is considered to be a point mass. Henceforth we call the star with mass $M_1$ the "primary." $A$ is the semimajor axis, $a_1$ is the radius of the primary star, $e$ is the eccentricity, $n$ is the orbital mean motion, and

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TABLE 1

Observed Physical Parameters of Sample RS Canum Venaticorum Binaries

| Star | $(A/a_1)$ | $P_{orb}$ (days) | $M_1/M_2$ | $M_1/M_\odot$ | $a_1$* ($10^{11}$ cm) | $A$ ($10^{12}$ cm) |
|---|---|---|---|---|---|---|
| RS CVn.......... | 4.16 | 4.798 | 1.04 | 1.4 | 2.5 | 1.18 |
| Z Her............ | 5.26 | 3.993 | 0.9 | 1.22 | 1.85 | 0.98 |
| MM Her......... | 8.33 | 7.960 | 1.03 | 1.22 | 1.89 | 1.58 |
| AR Lac.......... | 3.12 | 1.983 | 1.01 | 1.32 | 2.04 | 0.64 |
| RW UMa........ | 5.00 | 7.328 | 1.0 | 1.10 | 2.88 | 1.44 |

* Where both component radii are known, the larger radius was chosen as the value of $a_1$.

$\omega_1$ is the spin frequency of the primary. The stellar structure constant $\alpha$ is $I/m_1a_1^2$, where $I$ is the moment of inertia and $\langle\mu\rangle$ is the mean dynamic viscosity given by

$$\langle\mu\rangle \equiv \frac{9}{a_1^9}\int_0^{a_1} \mu(r)r^8dr\,. \quad (3)$$

In the present study, we have chosen five sample systems for which the relevant observational data have been fairly well determined, and which all possess the following characteristics: First, the mass ratio is close to unity; second, the single-component mass is between 1.0 and 1.4 $M_\odot$; and third, the primary component is a subgiant. Characteristics of the five systems are summarized in Table 1. The data were taken from the work of Hall (1976).

Based on the suggestion of Popper and Ulrich (1977), we assumed the stars to be in the post-main-sequence stage. We computed the structure constant $\alpha$ numerically from the evolved models of Iben (1966) (reproduced in tabular form by Novotny [1973, Tables 7-17 and 7-19]). It is easy to see from Table 2 that the value of $\alpha$ is not very sensitive to the expected range of mass and evolutionary state in RS Canum Venaticorum binaries.

There is some question as to the value of the mean viscosity (eq. [3]) appropriate for tidally evolving stars. Subgiants in the mass range 1.0–1.5 $M_\odot$ are, on the average, convective in the outer 10% of their mass. The best estimate we can make of the convective mean viscosity is by way of the local mixing length theory for which

$$\langle\mu\rangle_{conv} \sim \tfrac{1}{3}\rho VH(1-\xi^9)\left(\frac{\tau_T}{\tau_{conv}}\right)^2, \quad (4)$$

where $\rho$ is the density, $V$ the characteristic eddy speed, and $H$ the pressure scale height evaluated in the outer convection zone; $\xi$ is the ratio of the radius of the base of the convection zone to the stellar radius; $\tau_T = \pi/\omega_1$, the period of tides, and $\tau_{conv} \sim H/V$, the eddy turnover time. If $\tau_T > \tau_{conv}$, $\tau_T/\tau_{conv} \equiv 1$ (Goldreich and Nicholson 1977). Using the 1.25 $M_\odot$ subgiant model of Iben (Novotny 1973, Table 7-17) we obtain $H \approx 0.1\,a_1$, $\gamma_{conv} \approx 2.3$ days, and $\langle\mu\rangle_{conv} \approx 10^{12}$–$10^{13}$ g cm$^{-1}$ s$^{-1}$. The exception is AR Lac, for which $\gamma_T = 1$ day, so $\langle\mu\rangle_{conv} \approx 2.5\times10^{11}$–$10^{12}$ g cm$^{-1}$ s$^{-1}$. Actually, this is probably an overestimate for AR Lac: when $\tau_T/\tau_{conv} < 1$, the coriolis force becomes important, further reducing the convective couple (Büsse and Carrigan 1976). Our estimates for $\langle\mu\rangle_{conv}$ should also roughly pertain to the main-sequence stage.

The shearing of the tidal bulge across the effective stellar surface will generate turbulence and hence an additional viscosity (Press, Wiita, and Smarr 1975). Referring to the work of Press *et al.*, we computed values for their structure constant $K_\mu$ (Table 2) and obtained $\langle\mu\rangle \sim 10^{12}\ (\omega_1 - n)/n$ g cm$^{-1}$ s$^{-1}$ for a typical RS CVn physical parameter. There is uncertainty as to how well developed the turbulence is (Lecar Wheeler, and McKee 1976). Therefore, near synchronism, thermal convection almost certainly dominates this coupling mechanism.

Using a convective mean viscosity of $3\times10^{12}$ g cm$^{-1}$ s$^{-1}$ and typical values of $e = 0$, $M_1 = M_2 = 1.25\,M_\odot$, $n = \omega_1$, $\alpha = 0.1$, $a_1 = 2\times10^{11}$ cm (subgiant), and $a_1 = 7\times10^{10}$ cm (main sequence), we have listed the tidal time scales for the sample stars in Table 3 for both the subgiant and main-sequence phases. The results are straightforward: if nothing else disturbs the systems, then most systems of this kind can be first synchronized during the main-sequence stage and then circularized during the subgiant stage.

In addition, we note that the time scale for the damping of "stellar obliquity", i.e., inclination of the stellar spin axis with respect to the orbit axis, should be comparable to the synchronization time (Alexander 1973). Therefore, models used to explain photometric light curve wave migrations and distortions that require sizeable stellar obliquities (see, e.g., Catalano and Rodono 1974) should be regarded with some skepticism in the absence of additional torques that would secularly increase the obliquity.

TABLE 2

Stellar Structure Constants

| Star | $\alpha$ | $K_\mu$ |
|---|---|---|
| 1 $M_\odot$ subgiant.................. | 0.13 | 0.007 |
| 5 $M_\odot$ subgiant.................. | 0.22 | 0.008 |
| 1.25 $M_\odot$ main sequence......... | 0.09 | 0.0004 |
| 7–10 $M_\odot$ main sequence*........ | 0.10 | 0.025 |

* With $n = 1.5$ polytropic envelope (Press *et al.* 1975).

TABLE 3

TIDAL TIME SCALES

| Star | Subgiant | | Main Sequence | |
|---|---|---|---|---|
| | $\tau_{circ}$(yr) | $\tau_{sync}$(yr) | $\tau_{circ}$(yr) | $\tau_{sync}$(yr) |
| RS CVn....... | $6 \times 10^6$ | $5 \times 10^4$ | $6 \times 10^{11}$ | $3 \times 10^8$ |
| Z Her......... | $4 \times 10^7$ | $2 \times 10^5$ | $1 \times 10^{11}$ | $1 \times 10^8$ |
| MM Her...... | $2 \times 10^9$ | $3 \times 10^6$ | $7 \times 10^{12}$ | $2 \times 10^9$ |
| AR Lac....... | $2 \times 10^6$ | $3 \times 10^4$ | $2 \times 10^{10}$ | $4 \times 10^7$ |
| RW UMa..... | $3 \times 10^7$ | $1 \times 10^5$ | $3 \times 10^{12}$ | $1 \times 10^9$ |

## III. FLARES

Most RS Canum Venaticorum systems are probably losing or transferring matter through flare activity or a stellar wind. The purported periodic orbital period variations would require an anisotropic wind. In addition, flarelike phenomena are suggested by the variable emissions of H$\alpha$ and the Ca II cores in the optical, and L$\alpha$ and the Mg II cores in the ultraviolet in most RS Canum Venaticorum systems, as well as the spectacular optical-radio-X-ray event observed in the RS Canum Venaticorum binary HR 1099 (White, Sanford, and Weiler 1978; Weiler *et al.* 1978). Furthermore, Popper and Ulrich (1977) have demonstrated that the evolutionary status of many RS Canum Venaticorum binaries indicates significant mass loss or transfer (for example, as much as 0.2 $M_\odot$ for Z Her). Lastly, emission integrals derived from observations of a quiescent, soft X-ray flux from UX Ari indicate substantial mass flux (Walter *et al.* 1978*a*). Let us call the mass-loss mechanism "flaring" only in the sense that we assume the mass ejection to be anisotropic or localized on the stellar surface. We now discuss how flaring can influence the dynamics of the system, in particular the orbital period and stellar spin rate.

Flare events result in impulsive momentum transfer to both the orbit and spin of the flaring component. The orbital perturbation equations for these impulses are well known (Kopal 1972); it is easy to show that, for near-vanishing eccentricity, the period change due to a tangential impulse is

$$\frac{1}{P}\frac{dP}{dt} = -\gamma\frac{\dot{M}}{M_1}\left(\frac{1}{q}\right), \quad (5)$$

$$\frac{1}{q} \equiv \frac{V_{\rm flare}P}{2\pi A} + \frac{2M_1}{3(M_1+M_2)} \quad \text{(mass loss)},$$

$$\equiv \frac{M_2 - M_1}{M_2} \quad \text{(mass transfer)}, \quad (6)$$

where $\gamma = 3$. Here $M_1$ is the mass of the flaring component, $\dot{M}$ is the mass-loss or transfer rate, $V_{\rm flare}$ is the flare velocity and is positive for flares emitted in a direction opposite to the orbital motion. In Hall's model $V_{\rm flare}$ switches sign periodically owing to the slow migration of the emitting region around the star. From his data we obtain $(1/P)(dP/dt) \approx 5 \times 10^{-13}$ s$^{-1}$ for RS CVn, $1 \times 10^{-13}$ s$^{-1}$ for CG Cyg, $1 \times 10^{-12}$ s$^{-1}$ for SS Cam, and $4 \times 10^{-14}$ s$^{-1}$ for AR Lac. Equation (5), with $V_{\rm flare} = 500$ km s$^{-1}$, gives a mass-loss rate of $2 \times 10^{-6}$ $M_\odot$ yr$^{-1}$ for RS CVn and $2 \times 10^{-7}$ $M_\odot$ yr$^{-1}$ for AR Lac. Even with $V_{\rm flare} = 5000$ km s$^{-1}$, the mass-loss rates are extraordinary. If mass is instead transferred from one component to the other, $\dot{M}$ increases by about an order of magnitude because the mass ratios in these stars are very close to unity.

The foregoing calculation neglected the braking effect of magnetic fields. Let us consider a field line that is assumed to be locked onto the semimajor axis of the system. This situation is tantamount to saying that the stars are rigidly locked in synchronism, so that all angular momentum loss occurs in the orbit, rather than in the spin of the star. Ejected matter will travel along the field line out to the Alfvén radius $r_A$, removing angular momentum at a rate $\omega_1[M_2A/(M_1 + M_2) + r_A^2]\dot{M}$. The period change is then given by equation (5) with

$$\frac{1}{q} = \left(\frac{a_1^2 - r_A^2}{A^2}\right)\frac{M_1+M_2}{M_2} - 2\frac{r_A}{A} + \frac{2M_1}{3(M_1+M_2)}. \quad (7)$$

An accurate determination of the Alfvén radius is impossible. Following standard procedure, we equate the fluid kinetic energy $\frac{1}{2}\rho V^2$ to the magnetic energy $B^2/8\pi$. We assume a highly conductive wind so the field drops off as $r^{-2}$; this assumption is acceptable in that it will contribute to an estimation of an upper limit on the effectiveness of the magnetic braking. The velocity $V$ is the vector sum of the tangential and radial ($V_r$) velocities. In computing $V_r$, we have taken into account the gravitational deceleration of the matter. Finally, the density is set by the continuity equation; it is enhanced by a factor $4\pi/\Omega$ over the spherically symmetric value, where $\Omega$ is the solid angle inside which matter is ejected. We have assumed a value $\Omega/4\pi = 0.1$ in subsequent calculations. The resulting equation for $r_A$ is of fourth order:

$$-\frac{\dot{M}}{r_A^2 V_r}\left(\frac{4\pi}{\Omega}\right)\left[V_r^2 + \left(r_A + \frac{M_2}{M_1+M_2}A\right)^2\omega_1^2\right] = B_0^2\left(\frac{a_1}{r_A}\right)^4,$$

$$V_r^2 = V_{\rm flare}^2 + \frac{2GM_1}{a_1}\left[\left(\frac{a_1}{r_A}\right) - 1\right], \quad (8)$$

where $B_0$ is the surface value of the magnetic field.

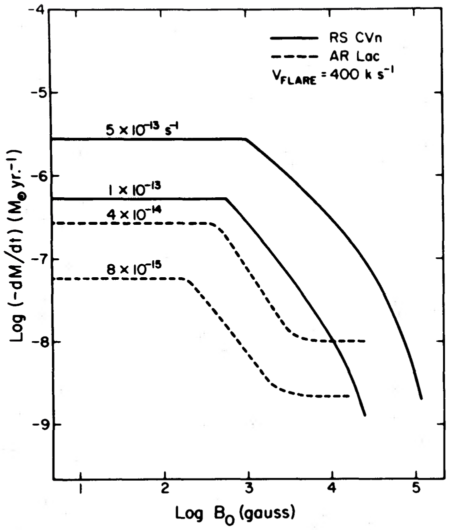


FIG. 1.—Mass-loss rates required to give various values of $\dot{P}/P$ for two sample RS Canum Venaticorum stars versus magnetic field strength. The flare speed at the point of ejection is assumed to be 400 km s$^{-1}$.

The results of solving equations (5), (6), (7), and (8) for two systems, RS Canum Venaticorum and AR Lacertae, are presented in Figures 1 and 2. In Figure 1, the effect of varying $\dot{P}/P$ for an assumed value of $V_{\rm flare}$ is shown. For RS CVn, it is clear that magnetic braking does not become effective until the surface field reaches 800–1000 gauss. A drop in $\dot{M}$ by an order of magnitude requires $B_0 \sim$ 10,000 gauss for the purported value of $\dot{P}/P$. The curves for AR Lac begin to turn over when $B_0$ becomes large. This is an anomaly that is due to the chosen surface flare speed, which is slightly below escape speed for AR Lac. For slightly higher $V_{\rm flare}$, the curves would continue dropping. Figure 2 shows the effect of varying the flare speed. Here we have assumed the purported values for $\dot{P}/P$ for each binary. Raising the flare speed of course increases the rocket effect of the ejection process, lowering the requisite mass-loss rate. However, the onset of magnetic braking still requires $B_0 \gtrsim 1000$ gauss; a substantial decrease in $\dot{M}$ for RS CVn again demands $B_0 \sim 10^4$ gauss even for $V_{\rm flare} = 4000$ k s$^{-1}$. For AR Lac, on the other hand, fields of several thousand gauss would yield substantial orbital braking. If $V_{\rm flare} = 4000$ k s$^{-1}$ for AR Lac, a mass-loss rate of $10^{-9}\, M_\odot$ yr$^{-1}$ could explain the star's purported period changes if the magnetic field were $\sim$5000 gauss.

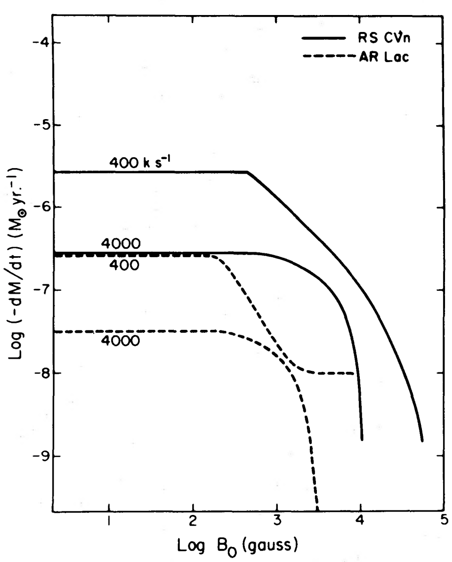


FIG. 2.—Mass-loss rates required to give purported values of $\dot{P}/P$ for two RS Canum Venaticorum stars versus magnetic field strength. Curves corresponding to two initial flare speeds are drawn.

Although large, the magnetic fields derived in the foregoing discussion could not be ruled out, in view of photometric evidence for large star spots on RS Canum Venaticorum systems. Nevertheless, the foregoing model assumed an idealized case in which all the lost stellar spin angular momentum was immediately regained at the expense of orbital angular momentum by a couple between the stellar spin and the orbit. It is interesting to ask whether tidal interactions are capable of providing this couple.

If this couple were infinitely strong, the stars would remain locked in synchronism. For a finite tidal torque, the star spins down below synchronism until the tidal torque on the star balances the magnetic braking torque. The orbital angular momentum loss is limited by the strength of the tidal interaction. Near synchronism, the tidal torque is roughly proportional to $(\omega_1 - n)/\tau_{\rm sync}$. The rate of magnetic braking of the spin is given by equation (5), with $\gamma = 1$ and

$$\frac{1}{q} = [(r_{\rm A}/a_1)^2 - 1]/\alpha \quad (9)$$

(Spiegel 1970). Thus the magnetic torque is proportional to $\omega_1/\tau_{\rm spindown}$, where $\tau_{\rm spindown}$ is the magnetic braking time scale. Hence the deviation of the spin rate from synchronism when the torques balance is

$$\left|\frac{\omega_1 - n}{\omega_1}\right| \sim \tau_{\rm sync}/\tau_{\rm spindown} \,. \quad (10)$$

By the time $\tau_{\rm sync} \approx \tau_{\rm spindown}$, we expect that the previous assumption of a rigidly locked spin breaks down to the extent that the true required $\dot{M}$ is greatly underestimated. Equation (10) holds as long as both the magnetic and tidal torques act on the same stellar moment of inertia. Using ($\dot{M}$, $B_0$) combinations needed to explain the purported orbit period changes, we have computed $\tau_{\rm spindown}$ using equation (9) with $\alpha$ = 0.1. These results are shown in Figure 3 for various values of $V_{\rm flare}$. The curve corresponding to $V_{\rm flare}$ = 40 km s$^{-1}$ was computed neglecting gravitational deceleration just to serve as a comparison against curves computed with higher (more reasonable) velocities.

The tidal synchronization times for the subgiant stages of RS CVn and AR Lac are also indicated.

We immediately notice that when magnetic braking is effective enough, in either star, to decrease substantially the requisite mass-loss rate (to, say, $\lesssim 10^{-8}\ M_\odot$ yr$^{-1}$), then the times for despinning the star are comparable to or less than the tidal spindown times. From the foregoing arguments, we see that this means, first, that the stars should not be rotating synchronously and, second, that the true mass-loss rate is higher than the values indicated in Figure 3. Even when $\dot{M} = 10^{-7}\ M_\odot$ yr$^{-1}$ and $V_{\rm flare} = 4000$ km s$^{-1}$, the spin rate of RS CVn should be well below the synchronous value. This possibility may contradict the "drifting star spot" explanation for the $\sim 10$ yr wave migration seen in the light curves of RS CVn. (See, e.g., Fig. 3 of Hall 1972, which implies $|\omega_1 - n|/\omega_1 \lesssim 10^{-3}$ for RS CVn.) It would also be inconsistent with long-term increases or decreases (also $\sim 10$ yr) in the orbital period, since presumably the emitting region would rotate with the star in a few orbit periods. This consequence would be circumvented if the extraordinary mass loss has been proceeding for a time $\ll \tau_{\rm spindown} \sim 10^5$ yrs, but then one must question the observability of such short-lived phenomena.

There may be some evidence in other stellar systems for the physical processes we have described. For example, a number of the so-called peculiar A stars in close binaries (for example, HR 7113, 41 Tau, and $\kappa$ Cancri) have rotation periods *much longer* than their orbital periods (Preston 1974). On the other hand, the magnetic stars HR 98088 and HR 710 are observed to be rotating synchronously although the estimated tidal interactions are too weak to have induced synchronism.

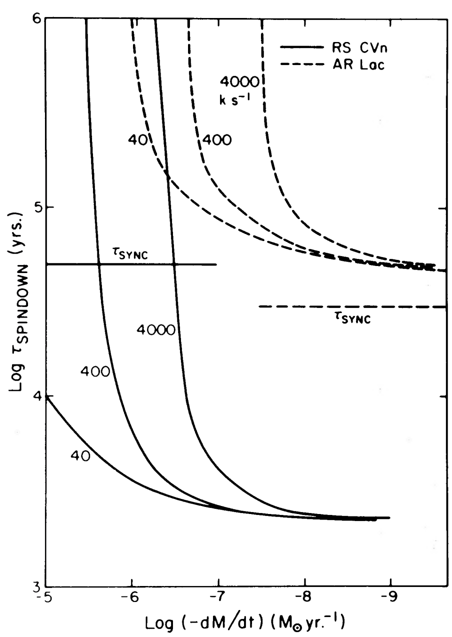


FIG. 3.—Stellar spindown time due to magnetic braking versus mass-loss rate for two sample RS Canum Venaticorum stars. Various initial flare speeds are indicated on the curves. The tidal synchronization time ($\tau_{\rm sync}$) is marked for each star.

This leads us to speculate that another spin-orbit coupling mechanism exists in these stars—for example, the interaction of the magnetic fields of the two components. Bahcall, Rosenbluth, and Kulsrud (1973) have suggested that magnetic reconnection in a non-synchronously rotating magnetic binary could explain the X-radiation from, for example, Cyg X-1. We suggest that this mechanism might explain the synchronous rotation of some peculiar A stars. For RS CVn and AR Lac, the stars would be brought into synchronism in a time $\tau_{\rm lock}$ of order $10^{10}\ B_0^{-2}$ yr. We arrive at this value using equation (1) of Bahcall *et al.* but with $B(r) \propto r^{-2}$ and $\alpha = \beta = 0.3$, $N = 3$ and $P_r = 2\pi/n$. Because to allow low mass-loss rates we require $B_0 \sim$ few $\times 10^3$ gauss, we get $\tau_{\rm lock} \sim 10^3$ yr. Hence this synchronizing magnetic torque may be stronger than the desynchronizing magnetic braking torque (and, incidentally, as much as two orders of magnitude stronger than the tidal torque), allowing large orbital period changes to occur. Some of the x-ray emission might then originate in the region of magnetic reconnection rather than near the stellar surface. The interacting magnetic fields may also be responsible for the development of large star spots and the consequential activity that is normally absent in single subgiants. Unfortunately, the details of the magnetic reconnection mechanism have not been investigated; the uncertainties implied in the work of Bahcall *et al.* may be many orders of magnitude. The present work suggests that a more thorough treatment of this magnetic interaction be made, since in the absence of this torque we have shown that large orbital period changes cannot occur with low mass-loss rates, no matter what the magnetic field strength may be.

## IV. DISCUSSION

The mass-loss rates and mass-ejection process implied by the purported period changes are clearly absurd if one assumes that, instead of being localized on the stellar surface and in orbital phase for many orbit periods (as we have heretofore assumed), the mass ejection takes place randomly over the stellar surface. If ejection takes place in constant time intervals $\Delta t$, then the rms cumulative orbital phase shift is, to first order in $\Delta P/P$,

$$\Delta\theta = \frac{\pi}{P^2}\,\Delta t\{[\tfrac{2}{3}n(n+1)(2n+1)]^{1/2}|\Delta P_1| - n^2|\Delta P_2|\}, \quad (11)$$

where $n$ is the number of flares in a typical span of time (500 orbit periods) and $P$ is the orbit period. $\Delta P_1$ is the period change due to momentum transfer by a single flare, computed from equation (5) and the first term of equation (6), and $\Delta P_2$ is the change due to mass loss, computed using the second term of equation (6). The existence of period *decreases* immediately sets a lower limit on the flare speed. Using equations (6) and (11) for RS CVn, we obtain $V_{\rm flare} \gtrsim 65n$ km s$^{-1}$.

For RS CVn, each flare has a mass of $5.7\dot{M}n^{-1}\ M_\odot$ yr$^{-1}$. For a constant $\dot{M} = 2 \times 10^{-6}$, the reported period changes in RS CVn require $V_{\rm flare} > 500n$ km s$^{-1}$ and flare rate $4.8 \times 10^{-4}\ n$ per day. For any $n$, these results indicate extremely unlikely flare activity.

Even for "coherent" flaring, which we have shown could explain the purported quasi-periodic period changes, a high mass-loss rate $\sim 10^{-6}\ M_\odot$ yr$^{-1}$ is implied by the arguments in § III. We now present a possible argument against large mass-loss rates. We propose that large mass-loss rates are in contradiction with the detection of soft X-rays in RS Canum Venaticorum systems (in particular, from $\alpha$ Aur, UX Ari, HR 1099, and RS CVn; see Walter *et al.* 1978*a*, *b*), if this emission is associated with the mass-ejection mechanism.

A single flux measurement can be used to set an upper limit on the mass-loss rate due to flaring. For example Walter *et al.* (1978*b*) obtain $L_x \sim 6 \times 10^{31}$ ergs s$^{-1}$ in the 0.2–2.8 keV band for RS CVn. However, this luminosity appears to be fairly constant and indicates that associated flare activity is routine. Other RS Canum Venaticorum systems, such as RW UMa and HR 1099 (Walter *et al.* 1978*b*), occasionally show much larger X-ray fluxes which are more properly termed "flare activity." Throughout the following treatment, we shall assume that the quiescent X-ray flux from RS CVn is a consequence of the mass loss responsible for orbital period changes. Since the star is not substantially reddened (Werner 1978), the $V$ magnitude of RS CVn corresponds to a luminosity of about $2 \times 10^{34}$ ergs s$^{-1}$. These luminosities require $\tau \ll 6$ for the emitting region of the star in X-rays. ($L_x$ at the source is, of course, known only within the uncertainties of the interstellar medium (ISM) absorption. However, for RS CVn, $\tau_{\rm ISM} \approx 3.6\ \langle n_{\rm H}\rangle$ (0.2 keV$/E)^3$, where $E$ is the photon energy and $\langle n_{\rm H}\rangle$ is the column-averaged hydrogen atom density. Probably $\langle n_{\rm H}\rangle < 1$ cm$^{-3}$ since RS CVn is out of the galactic plane (see also Cruddace *et al.* 1974), so the present $L_x$ is probably well determined.

The optical depth is related to the mass flow. A simple isotropic constant-velocity flow model gives

$$\begin{aligned}\tau &\approx [(4\pi/\Omega)^{1/2}\dot{m}]/(2\pi a_1 m_{\rm H} V_{\rm flare})\sigma_e(E), \quad V_{\rm flare}\, t_0 \gtrsim \delta \\ &\approx [(4\pi/\Omega)\dot{m}t_0]/(2\pi a_1^2 m_{\rm H})\sigma_e(E), \quad V_{\rm flare}\, t_0 \ll \delta \end{aligned} \tag{12}$$

where $m_{\rm H}$ is the proton mass, $\sigma_e(E)$ the absorption cross section per proton, $t_0$ the flare duration, $\dot{m}$ is the instantaneous mass-loss rate, and $\delta \equiv (\Omega/4\pi)^{1/2}a_1$, the size of the emitting region. Note that $\dot{m} = \dot{M}/Nt_0$, where $\dot{M}$ is the average mass-loss rate per year and $N$ is the number of flares per year; $\dot{m} \geq \dot{M}$, equality holding for a steady stellar wind. For a fully ionized plasma ($T \gtrsim 10^7$ K), $\sigma_e$ is the Thomson cross section. At lower temperatures, photoelectric absorption by heavy elements dominates $\sigma_e$ in the range 0.1–1 keV. Within a factor of 2,

$$\sigma_e(E) \approx 1.3 \times 10^{-19}(0.1\ {\rm keV}/E)^3 \tag{13}$$

in this energy range (Brown and Gould 1970; Cruddace *et al.* 1974). With $\Omega/4\pi = 0.1$ and $V_{\rm flare} = 500$ km s$^{-1}$, we obtain for RS CVn at $E = 1$ keV

$$\begin{aligned}\tau &\gtrsim 1.0 \times (\dot{m}/10^{-6}\ M_\odot\ {\rm yr}^{-1})\,, \quad T \gtrsim 10^7\ {\rm K} \\ &\gtrsim 200 \times (\dot{m}/10^{-6}\ M_\odot\ {\rm yr}^{-1})\,, \quad T \lesssim 10^7\ {\rm K}\,. \end{aligned} \tag{14}$$

Therefore a mass-loss rate $\sim 10^{-6}\ M_\odot$ yr$^{-1}$ is ruled out by any model emitting region other than a fully ionized ($T \gtrsim 10^7$ K) plasma. However, it is unlikely that the ejecta remain fully ionized. Above $10^7$ K, the dominant cooling mechanism is bremsstrahlung emission. The cooling time is

$$\tau_{\rm ff} \approx 1.8 \times 10^{11} T^{1/2} n_e^{-1}\ {\rm s}\,. \tag{15}$$

(Tucker 1975). Let us assume that the emitting region is heated out to a stellar radius above the stellar surface. Matter above this height then cools as it flows outward. With the foregoing flow model and $\dot{M} = 10^{-6}\ M_\odot$ yr$^{-1}$, we obtain for RS CVn, at $r = 3a_1$, $n_e = 2.6 \times 10^{12}$ cm$^{-3}$ and a cooling time of 200 s. However, the dynamical time is at least $a_1/V_{\rm flare} \approx 6.3 \times 10^3$ s for $V_{\rm flare} = 400$ cm s$^{-1}$. Hence the matter most probably cools to temperatures below $10^7$ K followed by rapid recombination of heavy elements. The region is probably transparent to 0.5 keV X-rays only when $\dot{M} \lesssim 10^{-9}\ M_\odot$ yr$^{-1}$, a result which should hold quite generally for RS Canum Venaticorum systems. Of course it is always possible to see X-ray emission if an emitting cone sufficiently narrow is viewed sideways. However, matter traveling not too supersonically would tend to spread out very soon after emission. Another possibility is that the X-ray emission occurs much farther out in radius than the mass ejection site, i.e., it is "secondary" emission or emission from a region of inter-star magnetic activity (§ III). At present we cannot eliminate these possibilities.

Future measurements of the soft X-ray spectrum of RS CVn are crucial. Models fitted to the spectrum should include the possibility of substantial absorption near the emitting region, according to the above argument. Even if such a model can be found, the computed emission integral ($\mathscr{E}$) can be used to check the self-consistency of the assumed $\dot{m}$; using the previous mass-flow model, we find

$$\begin{aligned}\dot{m} &\approx (2\pi)^{1/2} m_{\rm H}(\Omega/4\pi)^{1/2} V_{\rm flare}^{1/2} t_0^{-1/2} a_1 \mathscr{E}^{1/2}\,, \quad V_{\rm flare}\, t_0 \ll \delta \\ &\approx (2\pi)^{1/2} m_{\rm H}(\Omega/4\pi)^{1/4} V_{\rm flare} a_1^{1/2} \mathscr{E}^{1/2}\,, \quad V_{\rm flare}\, t_0 \gtrsim \delta\,. \end{aligned} \tag{16}$$

With $\Omega/4\pi = 0.1$, $a_1 = 10^{11}$ cm, $V_{\rm flare} = 500$ km s$^{-1}$, the latter relation holds when $t_0 \gtrsim 10$ minutes (for a steady wind, $t_0 \rightarrow \infty$). For example, Walter *et al.* (1978*a*) measured $\mathscr{E} = 4 \times 10^{54}$ cm$^{-3}$ for UX Ari, whence $\dot{m} = 6.7 \times 10^{-10}\ M_\odot$ yr$^{-1}$, consistent with their use of an optically thin bremsstrahlung-plus-gaunt factor model for the emission, and with the $10^{-10}\ M_{\odot}\ \mathrm{yr}^{-1}$ rate expected for the KO IV star.

## V. CONCLUSIONS

The main conclusion of this paper is that the period changes reported for several RS Canum Venaticorum binaries should remain highly questionable. Difficulties in inferring orbital period changes from photometric data in close binaries are well documented; the same is true for RS Canum Venaticorum systems in particular (Catalano and Rodono 1974). A spectroscopic determination of these changes is plausible. For RS CVn, a 3% phase shift could be expected in a 2–3 year time interval, using the data of Hall (1975). A long time baseline of complete radial velocity curves would probably enable one to make an accurate assessment of the purported period variability. Correlation of X-ray emission with the phase of the migration wave would indicate a connection between the dark spots on the stars and the emitting region. A detection of blueshifted spectral features would indicate mass flow, as would further X-ray observations. Lastly, stellar rotational velocities should be obtainable, in view of the well-separated radial velocity amplitudes.

Of course our calculations are somewhat uncertain: the details of magnetic braking, magnetic reconnection, and stellar tidal interactions are not well understood. Because RS Canum Venaticorum systems are close but detached binaries, they are possibly good test cases for theories of these physical processes.

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