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THE ϵ GEMINORUM OCCULTATION: EVIDENCE FOR WAVES OR TURBULENCE

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ABSTRACT

Intercomparison of occultation data from various observatories shows poor correlation of the raw data but apparent correlations in the derived temperature profiles, as discussed in a paper by French and Elliot. We point out that the data are at least as consistent with Martian scintillation as they are with a semiglobal wave model.

Subject headings: occultations — planets: atmospheres — planets: Mars — stars: individual

I. INTRODUCTION

Recently, a number of papers have appeared which claim that the Martian atmosphere has a layered structure, detectable from observations of the ϵ Gem occultation, which is coherent over distances of at least several hundred kilometers. Papers supporting this viewpoint include Elliot *et al.* (1977), French, Goguen, and Duthie (1978), and Wasserman, Millis, and Williamon (1977). The accompanying paper by French and Elliot (1979) presents the case for this hypothesis, which essentially implies that most features in the occultation light curves produced by the Martian atmosphere are caused by such coherent layering. Although the comparison which French and Elliot have carried out is a useful one, we feel that their findings are not as conclusive as claimed, and that a note of caution is appropriate. Our principal points are as follows:

1. The process of inverting light curves to obtain temperature profiles does not eliminate or suppress noise. Rather, the noise appears as correlated fluctuations in temperature profiles, and these fluctuations have a predominantly wavelike character similar to the "waves" seen in the actual temperature profiles. By noise, we mean any fluctuation in the observed light curve, such as shot noise or terrestrial or Martian scintillation, which is not produced by coherent layering in the Martian atmosphere.
2. An Abel inversion of an individual light curve produces a model, spherically layered, refractivity distribution which will reproduce the light curve. If substantially different light curves are produced by essentially the same spherical layers, as evidenced by a lack of correlation between light curves, then the fundamental assumption of the Abel inversion is called into question.
3. The correlation time of fluctuations in individual light curves is more consistent with Kolmogorov turbulence than it is with large-scale atmospheric waves.
4. Measurements of the mean temperature of the Martian atmosphere, when all available occultation data are taken into account, show a greater scatter than is implied by the analysis of French and Elliot.

II. INVERSION OF LIGHT CURVES

In their analysis of light curves, French and Elliot argue that the effect of noise on temperature profiles is to shift the true profile according to some linear displacement, while true temperature waves remain otherwise intact. To test this hypothesis, we have performed the following numerical experiment. We prepared an isothermal light curve running from $\phi = 0.55$ to $\phi = 0.035$ (ϕ is the stellar flux in units of the unocculted flux). On this light curve, we superposed Poisson noise such that the rms fluctuation in ϕ was 0.0447 in a dimensionless time interval $\Delta\tau = 1$. Here τ is related to the actual time t by

$$\tau = v_p t / H_0 , \qquad (1)$$

where v_p is the velocity perpendicular to the limb and H_0 is the scale height of the isothermal atmosphere. For the ϵ Gem occultation, $\Delta\tau = 1$ corresponds to $\Delta t \sim 0.5$ s. The adopted noise level is consistent with that observed by the Texas-Arizona Occultation Group (1977, hereafter Paper I), and includes fluctuations produced by hypothetical Martian scintillations. The latter do not obey Poisson statistics, but over an averaging interval of $\sim$0.5 s, the approximation should be qualitatively valid. The light curve with noise was then inverted to obtain the temperature profile, using standard procedures similar to those of French and Elliot. Next, we Fourier analyzed the temperature profiles from a depth of $3.5H_0$ in the atmosphere (corresponding to $\phi = 0.035$) to a depth of zero (corresponding to $\phi = 0.55$). The results are expressed in the form

$$\frac{\Delta H(h)}{H_0} = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos \frac{2\pi n h}{3.5H_0} + b_n \sin \frac{2\pi n h}{3.5H_0} \right) , \qquad (2)$$

where $\Delta H(h)$ is the fluctuation of the scale height from the true value H_0, h is the depth in the atmosphere, and a_n and b_n are the Fourier coefficients. We then form the rms quantities:

$$\langle c_0{}^2\rangle^{1/2} = \tfrac{1}{2}\langle a_0{}^2\rangle^{1/2}\,,$$
$$\langle c_n{}^2\rangle^{1/2} = \langle a_n{}^2 + b_n{}^2\rangle^{1/2}\,,\quad n \geq 1\,, \qquad (3)$$

where the averages are taken over an ensemble of noisy light curves. The significance of these quantities is that a typical member of the ensemble will have a net fractional shift in H which is approximately equal to $\langle c_0{}^2\rangle^{1/2}$, and a temperature "wave" of wavelength $3.5H_0$ which has random phase and relative amplitude roughly equal to $\langle c_1{}^2\rangle^{1/2}$. There will also be higher frequency waves with amplitudes given by the other $\langle c_n{}^2\rangle^{1/2}$. Figure 1 shows the result of our experiment. For the noise level chosen, we would expect a typical net shift in temperature of about 12 K if the actual temperature is 150 K. More important, we typically expect to see a temperature "wave" with wavelength $3.5H_0$ and an amplitude of nearly 30 K. The rest of the noise gets converted into much lower amplitude "waves" with shorter wavelength. This result agrees with the analysis of Groth *et al.* (1978), who concluded that because of noise in light curves, "large, wavelike features can be erroneously introduced into the [temperature] profiles." Note that in this analysis, we attribute to noise *essentially all* of the light fluctuations except for the background isothermal curve. These fluctuations are primarily due to Martian scintillation and not to terrestrial shot noise or scintillation. The point is that we can easily produce temperature "waves" with Martian scintillation.

Now consider the question of whether it is legitimate to correct for noise in the temperature profile by applying a linear temperature-depth correction. After applying such a correction, can we say anything about a remaining temperature "wave"? If, for example, we know that the light curve contains noise at the level chosen above, we would not attribute significance to any waves in the temperature profile with an amplitude of ~30 K or less. Since a linear temperature-depth relation is composed primarily of low-frequency Fourier components, it is inconsistent to apply such a linear correction to the inversion and still attribute significance to low-frequency Fourier components which are of the same order of magnitude.

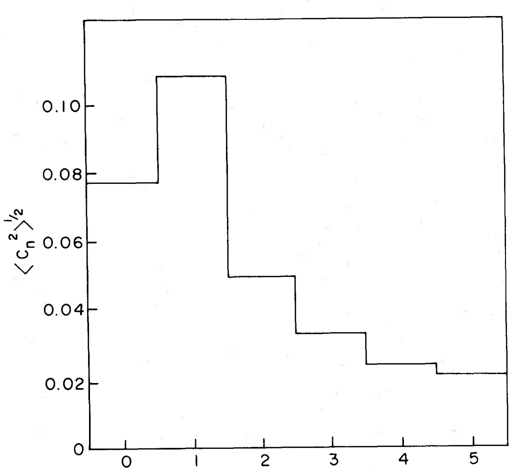

FIG. 1.—Root-mean-square Fourier coefficients for temperature "waves" produced by inversion of noise; see eqs. (2) and (3).

III. ARE THE LIGHT CURVES CORRELATED?

If the processed (i.e., inverted) occultation data seem to show correlated temperature fluctuations, one would expect to see some degree of correlation in the raw data. It is our contention that correlations which appear in the processed data, when there is no evidence for correlation in the raw data, must be regarded with suspicion, particularly in view of the effects already discussed in § II. Previous attempts to discover correlations in occultation data have dealt with the more straightforward problem of cross-correlating the statistically independent data points in the light curve, rather than the nonindependent points in a synthesized refractivity profile. In the case of β Sco occultation by Jupiter, such a statistical test was applied by a group which included Dr. J. L. Elliot (Veverka *et al.* 1972). This group found that "Spike patterns in our occultation curves show no significant correlation between immersion and emersion ... it is therefore improper to speak of global layering." The statistical test was applied to the raw data, rather than to processed data, and although some correlation appeared when visually comparing light curves, it was found to be insignificant when compared with results from nonsense light curves.

Figure 11 of the accompanying paper by French and Elliot convincingly shows that there is no correlation between the emersion light curves. Although reasons other than turbulence can be found to explain this lack of correlation, the need to introduce special explanations for contradictions to a hypothesis can only be damaging to the hypothesis.

Are the immersion fluctuations correlated? At zero time lag, the value of the cross-correlation is hardly more impressive than for emersion. Let us suppose, however, that we may adjust each of the individual half-intensity times by an amount on the order of 0.2–0.3 s in order to create a significant cross-correlation peak at zero lag. Then, in order to be consistent, we must also adjust the depth scale on French and Elliot's Figure 8, which is with respect to the half-light depth. Our Figures 2 and 3 show the result of this adjustment. The correlation of the temperature fluctuations is much less impressive after the light-curve fluctuations have been forced to coincide.

According to the "gravity gradient" model for occultations (Elliot and Veverka 1976), spikes or fluctuations in the occultation shadow pattern are bars of high intensity which are oriented perpendicular to the local "gravity gradient" (by this they mean

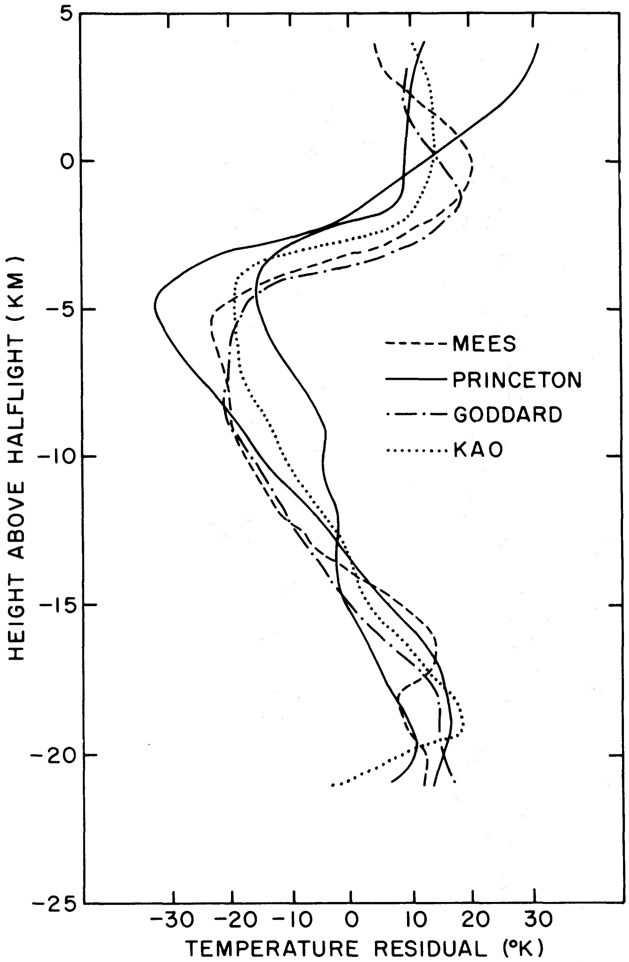

FIG. 2.—Reproduction of French and Elliot's Fig. 10

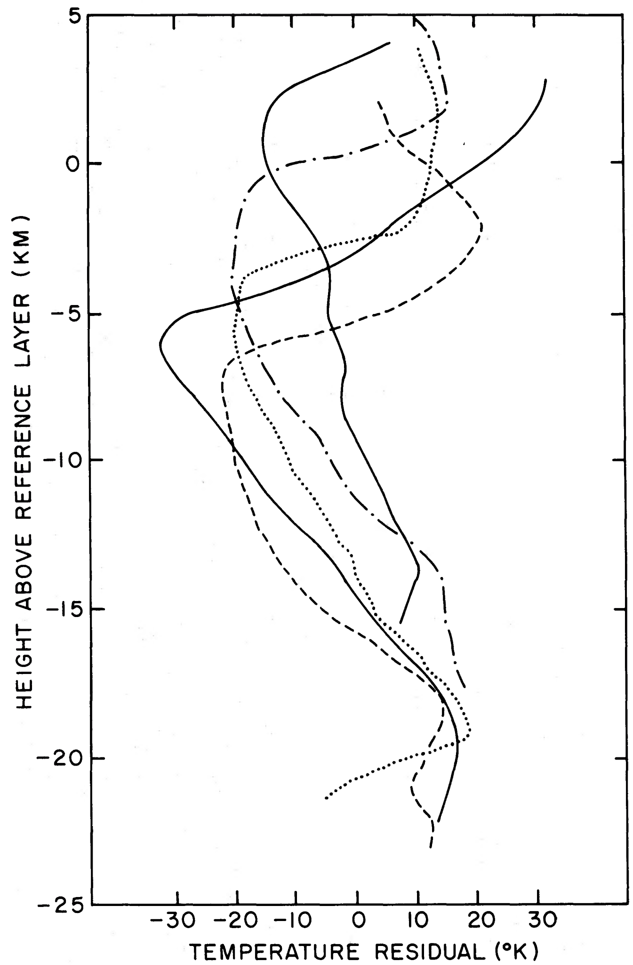

FIG. 3.—Adjustment of French and Elliot's Fig. 10 in altitude by requiring cross-correlation of residuals to be maximum at zero lag.

the gravity vector and not a tensor quantity). According to this model, the limb of the planet lies parallel to such barlike features. At the same time, it is necessary to regard the half-intensity point in the shadow pattern as also lying parallel to the limb. We have just noted that these two conditions cannot be simultaneously satisfied by the ϵ Gem immersion data. Figure 4 shows the geometrical interpretation of this situation. In this figure, we schematically show two rays in an occultation, one corresponding to half-intensity, and the other to a "spike." The rays are shown in their plane of propagation for two neighboring stations A and B. A ray proceeds from the star (at left), is bent by the Martian atmosphere, and then propagates to the observer (at right). We adopt the convention that points which are at equal altitude on the graph are at equal altitude above the limb, and therefore define a perpendicular to the local gravity vector. In Figure 4*a*, the times of half-intensity at stations A and B are forced to coincide, but "spikes" then occur at times separated by Δt at the two stations (this geometry corresponds to Fig. 2). In order to bring the spikes into coincidence, i.e., to form a barlike feature perpendicular to the local gravity vector, we must displace the station B occultation geometry uniformly with respect to station A by a height difference $v_p \Delta t$ (*not* $v_p \phi \Delta t$), as shown in Figure 4*b*. This situation corresponds to Figure 3.

If the light curves are not correlated over limb separations of a few hundred km or less, then the fundamental assumption of the Abel inversion technique is violated. This technique yields a density profile which reproduces the observed light curve, *if the density is a function of planetary radius alone*. For the immersion events, we have shown that the cross-correlation of intensity residuals and the comparison of the temperature residuals yield mutually inconsistent results within the framework of the "gravity gradient" model. If the temperature profiles (and therefore density profiles) are a function of radius only, then the light curves are uncorrelated. If the light curves are correlated, then the temperature profiles cannot be a function of radius only.

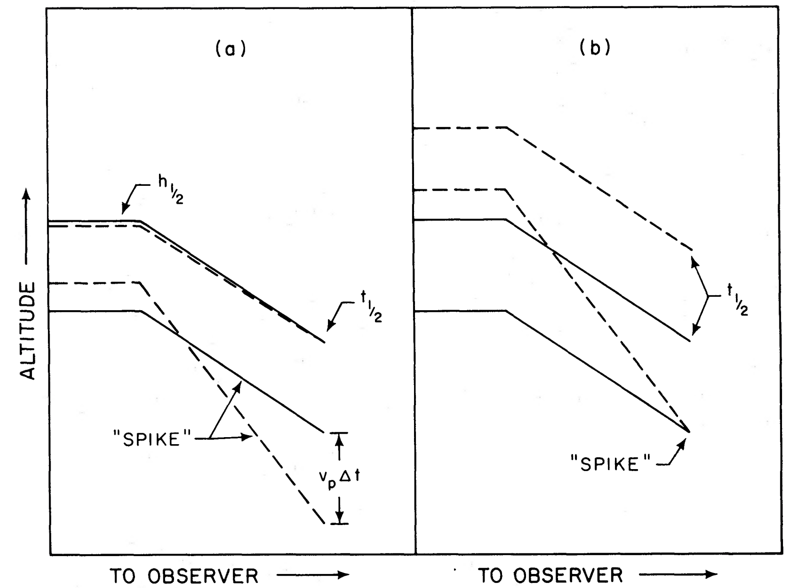

FIG. 4.—Schematic representation of rays in the plane of propagation. *Solid curves*, rays observed from station A; *dashed curves*, rays observed from a neighboring station B.

IV. CORRELATION TIME OF FLUCTUATIONS

Turbulence theory (see Paper I; Hubbard *et al.* 1978) predicts that scintillations will normally have a correlation time on the order of the time required to traverse a Fresnel zone. When the occulted source has a projected diameter which significantly exceeds a Fresnel zone, the correlation time is on the order of the time required to traverse a source diameter; the latter was the case for the ϵ Gem occultation, and the correlation time for the fluctuations observed at McDonald observatory was found to be compatible with the turbulence model. Because of the large angular extent of ϵ Gem, the difference between the correlation times predicted for turbulence and for large-scale atmospheric waves is not extreme, but it is possible to carry out a meaningful test. First we consider an isothermal light curve $\phi_0(\tau)$. We then perturb the refractivity distribution with a temperature wave,

$$T = T_0[1 + A \cos(2\pi h/\lambda)] , \quad (4)$$

where T_0 is the true background temperature, h is the depth in the atmosphere, λ is wavelength of the temperature wave, and A is its relative amplitude. To match the type of temperature waves thought to have been observed, we use $A = 0.08$ and $\lambda = 2H_0$. The corresponding light curve is shown in Figure 5. Next, we autocorrelate the residuals from $\phi = 0.93$ to $\phi = 0.13$. The resulting autocorrelation curve is shown in Figures 6 and 7. Also shown is the predicted autocorrelation curve for isotropic turbulence, as calculated in Paper I for the assumed projected diameter of ϵ Gem. The data plotted in Figures 6 and 7 are cross-correlation curves for the KAO data taken directly from Figure 13 of French and Elliot (1979). The latter, after renormalization to unity at $\Delta t = 0$, should correspond to the autocorrelation function for fluctuations in the light curve produced by the Martian atmosphere (turbulence theory predicts a negligible wavelength dependence for these particular parameters). The comparison of data and theory obviously favors the turbulence model, as it does for the

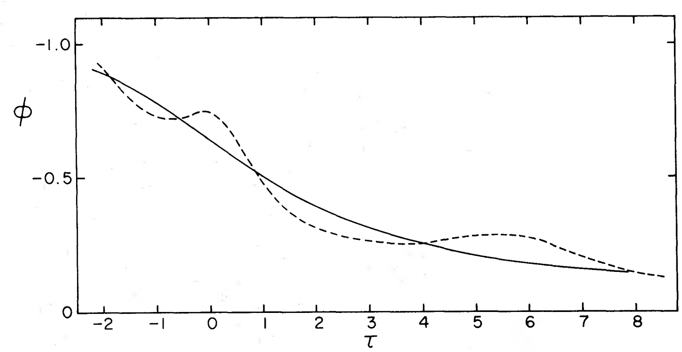

FIG. 5.—An isothermal light curve $\phi(\tau)$ (*solid curve*) perturbed by thermal waves of constant amplitude and wavelength of two scale heights (*dashed curve*).

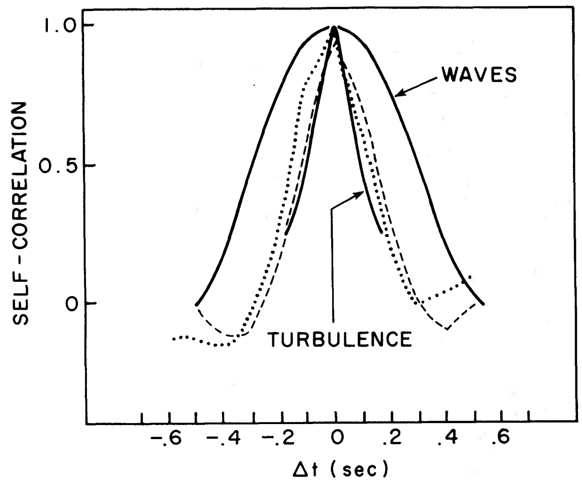

FIG. 6.—Predicted autocorrelation function in time for intensity residuals produced by waves, and by turbulence model discussed in Paper I. Dotted and dashed curves are KAO data taken from Fig. 13 of French and Elliot, for immersion.

McDonald data discussed in Paper I. Note that for this set of parameters, where the projected diameter of the occulted star is quite large (much greater than a Fresnel zone, and nearly equal to a scale height), the comparison between the wave model and the turbulence model is not as dramatic as it would be for a point source. Indeed, we suggest that the large source diameter is partly responsible for the temptation to interpret intensity fluctuations as due to waves. As a further point, we note that for the assumed diameter of ϵ Gem, the intensity fluctuations due to turbulence should tend to be of the same duration regardless of the value of ϕ, while fluctuations caused by density waves will become more "stretched out" in time as ϕ decreases.

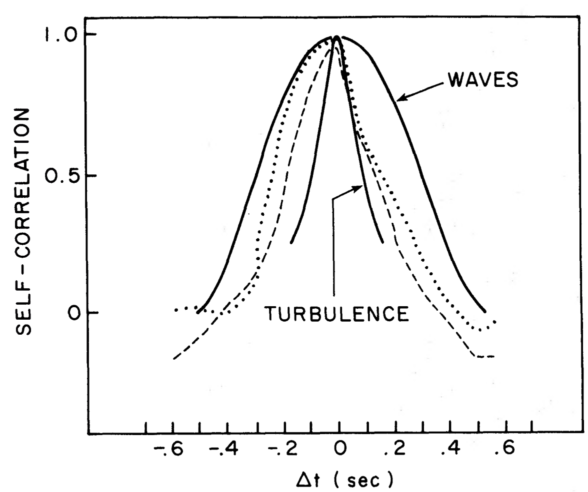

FIG. 7.—Same as Fig. 6, but for emersion

V. TEMPERATURE PROFILES

Figures 8 and 9 show model temperature profiles of the Martian atmosphere obtained from formal inversion of occultation data observed at various sites. All of the curves shown have been published elsewhere, except for the McDonald data. These are all model temperatures in the sense that they would reproduce the observed occultation light curve if the Martian atmosphere were radially symmetric over horizontal distances of at least several hundred kilometers. As is well known, these curves are not unique, particularly in their upper portions, but to minimize confusion we have not exhibited the maximum excursions which would be consistent with the data. We have also avoided plotting the temperature profiles as sequences of data points with individual "error bars," for this gives the misleading impression that the inversion temperatures are real independent measurements, and gives inappropriate significance to apparent correlations between temperature distributions derived at different observing stations.

Of the various curves which are plotted in Figures 8 and 9, either the turbulence model or the semiglobal wave model would predict good correlation between light curves observed at a single station but at different wavelengths (such as the three KAO channels), and between light curves observed from stations separated by less than 1 km (such as the three McDonald telescopes). As has already been discussed, the light curve fluctuations do correlate for these special cases. Otherwise, the temperature distributions show considerable scatter, and even if the McDonald data are excluded as too distant from the other stations, the case for semiglobal layering is not persuasive. Rather, the inverted temperature distributions show a considerable scatter (of $\sim \pm 40$ K) about a mean temperature of about 165 K. This mean temperature is also obtained if the individual scale heights obtained from isothermal fits are simply averaged. The mean temperature obtained from the McDonald data alone is approximately 190 K, regardless of whether the inversion method or the isothermal fitting method is used.

In Figure 8, if one ignores the considerable difference in mean temperatures, there could be a temptation to claim that a similar temperature wave has been detected by all stations. However, as we have just argued, it is difficult to make a self-consistent case for retaining features of the analysis which argue for layering while discarding other features (such as large differences in mean temperature). Note that a common rise in temperature with depth is implied by the three McDonald observations, which is not repeated by any of the other temperature profiles. All of these model temperature profiles could correspond, at least in a rough sense, to actual temperature variations. But, as discussed in Paper I, we recommend a conservative approach to occultation data, at least until a more persuasive case for semiglobal layering has been made. It may be possible to add more evidence after some future Martian occultation

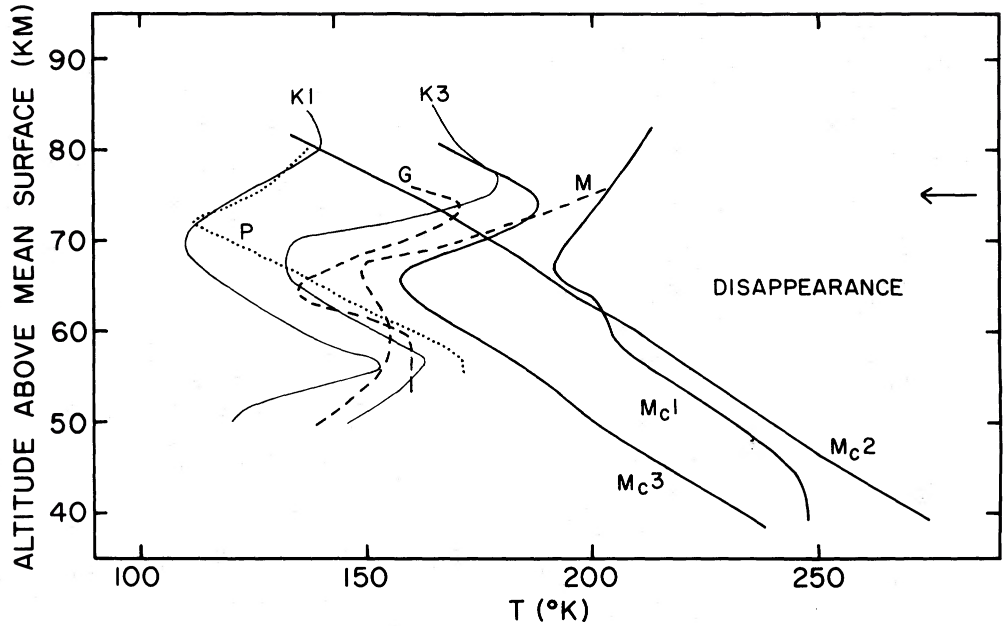

FIG. 8.—Model temperature distributions obtained from immersion occultation data from KAO (*K1*, *K2*, *K3*), Princeton (*P*), Goddard (*G*), Mees Observatory (*M*), and McDonald Observatory (*Mc1*, *Mc2*, *Mc3*). *Arrow*, half-intensity altitude.

when a star of much smaller angular diameter is occulted. In this case, turbulence theory would predict the appearance of prominent, intense spikes due to scintillation in the Martian atmosphere. For a point source, such spikes have a typical spatial width on the order of a Fresnel zone. When the source is extended, convolution with the source distribution produces a lower amplitude spike with a width on the order of a source diameter. Fluctuations with these characteristics *are* a special feature of light transmitted through Kolmogorov turbulence, and are a consequence of the fact that the product of the Fresnel filter function with the Kolmogorov power spectrum tends to favor the production of fluctuations on the scale of a Fresnel zone. A layering model can invoke density fluctuations on any radial scale needed to fit the observations, but the assumption of such a distribution is quite arbitrary and has no predictive power. On the

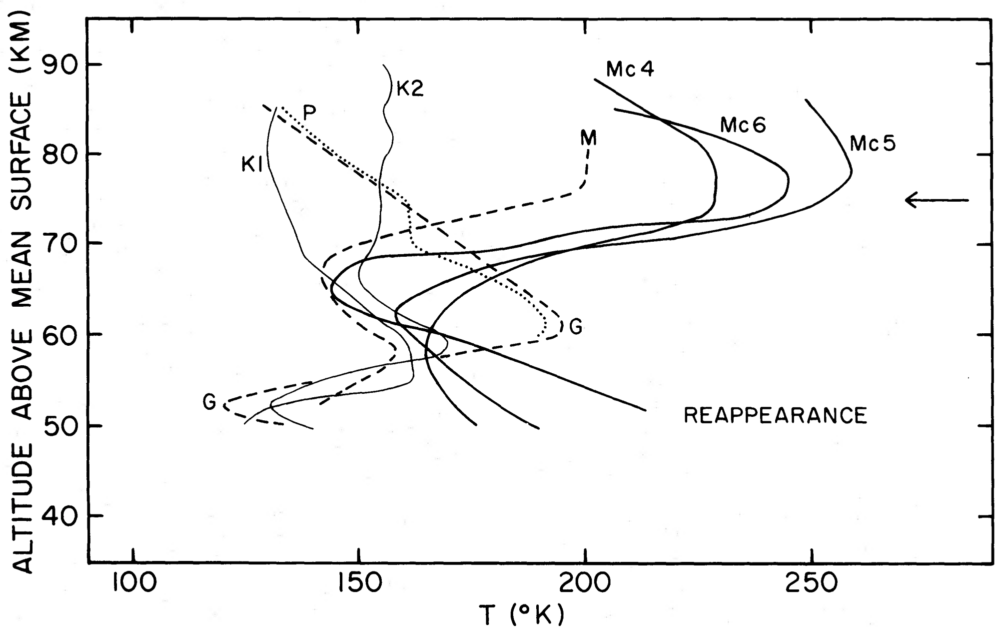

FIG. 9.—Same as Fig. 8, but for emersion. The curves for McDonald Observatory (*Mc4*, *Mc5*, *Mc6*) are denoted in the same way as in Paper I.

other hand, the necessity to invoke density fluctuations on a distance of the order of a Fresnel scale indicates the breakdown of geometrical optics ray-tracing, and strongly favors a turbulence model.

We also maintain that a serious breakdown of the radial layering model is indicated when temperature waves deduced at separate stations exhibit a phase discrepancy of 3 km. This phase shift must be compared not only with the lateral distance between the stations, but also with the radial distance over which the average refractivity changes appreciably, i.e., a scale height. Over a 3 km altitude, the average refractivity changes by a factor of about 1.5, so that the temperature wave detected at one station occurs at a substantially different refractivity for the other station. Such waves could be detected by Abel inversion of occultation data only if the waves are essentially two-dimensional corrugations in the Martian atmosphere, with the long dimension of the corrugations aligned with the line of sight between the star and the observer.

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