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SILICON LEVITATION IN CHEMICALLY PECULIAR STARS AND THE OBLIQUE ROTATOR MODEL

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ABSTRACT

The radiation force on silicon in chemically peculiar stars has been carefully computed (for $T_{\rm eff}$ = 14,000 K and $T_{\rm eff}$ = 12,000 K) including the effect of Si I. A total of 126 lines have been introduced in the computations, and gathered into 30 groups of similar lines. The equation of transfer has been solved for each group of lines, and non–LTE effects have been taken into account. It is found that in the absence of a magnetic field the radiation force is able to support an abundance of silicon within a factor of 2 of the normal abundance as observed in Hg-Mn stars. However, in the case of horizontal magnetic lines, the diffusion of the ionized silicon is slowed down, especially in the outer parts of the atmosphere. There silicon diffuses upward due to the effect of Si I lines, leading to an overall increase of silicon in the line-forming region. In the lower atmosphere, silicon first falls and leaves an amount of about one-half of the normal value, which is then "aspirated"up to the upper atmosphere. Diffusion processes provide a natural explanation for the silicon overabundance observed in magnetic peculiar stars, and for their range in effective temperature.

Subject headings: stars: abundances — stars: interiors — stars: magnetic — stars: peculiar A — stars: rotation

I. INTRODUCTION

Since the pioneering work of Michaud (1970) the idea of radiative and gravitational element separation in stellar atmospheres has been rather successful in explaining abundance anomalies in the peculiar A and B stars (see, for example, Michaud *et al.* 1976; Vauclair, Vauclair, and Michaud 1978; and references therein). However, a remaining problem has been the overabundance of silicon in magnetic A stars by factors up to 100, and the apparent normal abundance of silicon in the nonmagnetic peculiar stars (Preston 1974). Even in the early calculations Michaud (1970) found that silicon was not supported by the radiation force in the atmospheres of A and B stars, because its large "normal" abundance produces strongly saturated lines. He had to postulate autoionization lines to account for the large overabundances, lines which have not been unambiguously observed.[1] These early computations were, however, somewhat rough in that they treated only ionized silicon and made several other approximations.

In the present paper we report an extensive examination of the behavior of the radiation force on silicon in the late B stars. We have carefully solved the equation of transfer for the 126 most important lines in the first four ionization stages of silicon, including the introduction of departures from local thermodynamic equilibrium (LTE) in Si I. We find that to within a factor of 2 we can account for the normal silicon abundances observed in the nonmagnetic Ap and Bp stars. Moreover, we argue that the radiation pressure support provided to Si I plays a dramatic role in the presence of horizontal magnetic fields. Neutral silicon is strongly pushed upward by the radiation force since, due to its low abundance, its lines are only slightly saturated. On the other hand, the radiation force on both Si II and Si III is slightly less than that of the gravity. When the neutral silicon has to share its radiation force with the entire population of silicon atoms, the radiation force is not strong enough to push them all up. It can just help to balance the gravity and support a normal silicon abundance. However, in the presence of a large horizontal magnetic field, the neutral species is the only one which may move freely in the vertical direction, at least in the upper atmosphere. When silicon ionizes and would normally sink, the magnetic field offsets the excess

[1] See Jamar, Macau-Hercot, and Praderie 1978 for a discussion about the possible Si II autoionization lines.

gravitational forces. Thus, we provide a specific mechanism for producing substantial silicon overabundances in magnetic stars at the places where magnetic lines are horizontal. The location of the silicon-rich spots on the stellar surface will then depend on the exact configuration of the magnetic field.

In § II we discuss the theory of diffusion processes in the case of partial ionization, and how momentum is shared among the various ions. We show how to compute the diffusion velocity when the neutral atoms are the only ones to diffuse as in the case of a large horizontal magnetic field.

In § III we describe our method for computing the radiation forces on each silicon ion by solving the equation of transfer for the lines including departures from LTE where appropriate, and how we compute the radiation force acting on silicon as a whole. Section IV contains the discussion of silicon diffusion across magnetic field lines. There it is shown that the presence of horizontal field lines can lead to substantial apparent silicon overabundances in the range of effective temperatures where Si stars are observed.

Finally, in § V we discuss the temperature boundaries of silicon stars and the observations of other elements in the framework of the oblique rotator model.

II. DIFFUSION PROCESSES IN CASE OF PARTIAL IONIZATION

The problem of diffusion processes in the case of partial ionization has been extensively developed by Montmerle and Michaud (1976), and their method has been used in our computations. We will discuss it here in a slightly different way, which may help illuminate the physics. We neglect for this discussion all second order terms; we deal only with diffusion of test atoms in hydrogen, and we assume no turbulence. In this case the diffusion velocity for an element in a single stage of ionization can be written:

$$v = -D\left[\frac{1}{c}\frac{\partial c}{\partial r} - \frac{m(g_R - g_{GT})}{kT}\right], \qquad (1)$$

where D is the diffusion coefficient, c the concentration of the element, m its mass, g_R the acceleration due to the radiation, and g_{GT} the combined effects of the gravity and the acceleration due to the thermal gradient (Michaud *et al.* 1976).

The diffusion coefficient D is approximately equal to $\frac{1}{3}lC_M$, where l is the mean free path of the particles and C_M their Maxwellian velocity. This can also be written:

$$D = \tfrac{1}{3}t_{\rm col}C_M{}^2 = t_{\rm col}\frac{kT}{m}, \qquad (2)$$

where $t_{\rm col}$ is the time needed for the particles to share their momentum with the surroundings. Except for a very steep concentration gradient of the considered element (which never occurs in the cases discussed here), the first term is negligible in equation (1), which becomes:

$$v = t_{\rm col}g_{\rm eff}, \qquad (3)$$

where $g_{\rm eff} = g_R - g_{GT} \approx g_R - g$ (the thermal diffusion is negligible in the atmosphere). This expression shows that the diffusion velocity is equal to the effective gravity on the ion multiplied by the time during which it can travel before sharing its momentum with the other species.

In the case of a mixture of various ions of the same element, and assuming the same hypothesis (test atoms, no second order terms), the total diffusion velocity of the element is:

$$v_t = \sum_i \frac{N_i v(i)}{N}, \qquad (4)$$

where $v(i)$ is the diffusion velocity for each ion and N_i/N their relative abundance (eq. [5.19] of Montmerle and Michaud 1976 gives the same result as eq. [4] in this case). However, as shown by Montmerle and Michaud (1976), $v(i)$ can be different from equation (3) if the ionization or recombination time for ion i is smaller than $t_{\rm col}(i)$.

An example which will be useful for understanding the case of silicon in peculiar stars is the following. Suppose that the diffusing element is only a mixture of neutral and singly ionized atoms, and that the concentration of the neutral atoms is negligible compared with the ionized atoms. In most cases the ionization time $t_{\rm ion}(n)$ for the neutral atom is smaller than the time $t_{\rm col}(n)$ needed for it to share its momentum with the surroundings, while the reverse is true for the ionized atom: $t_{\rm col}(i) < t_{\rm rec}(i)$.

There are two reasons for this: first, $t_{\rm ion}(n) < t_{\rm rec}(i)$ because of the small abundance of n compared to i. Second, $t_{\rm col}(n) > t_{\rm col}(i)$ because the neutral atom suffers less collisions than the ionized one (which is subject to electrical forces). The element will travel as neutral only during $t_{\rm ion}(n)$, and then it will continue in the ionized state during $t_{\rm col}(i)$. The diffusion velocity of the neutral is

$$v(n) = t_{\rm ion}(n)g_{\rm eff}(n) \qquad (5)$$

while the total diffusion velocity of the element becomes:

$$v_t = \frac{N_n}{N}[t_{\rm ion}(n) + t_{\rm col}(i)]g_{\rm eff}(n) + \frac{N_i}{N}t_{\rm col}(i)g_{\rm eff}(i). \qquad (6)$$

If the ionized element is not allowed to diffuse, for example, in the case of a large horizontal magnetic field, equation (6) reduces to:

$$v_H = \frac{N_n}{N}t_{\rm ion}(n)g_{\rm eff}(n). \qquad (7)$$

If $g_{\rm eff}(n) > 0$, then $v_H > 0$ even if, due to the small abundance of the neutral, $v_t < 0$. The element will be pushed up by the radiation force when the magnetic lines are horizontal, with a diffusion time scale:

$$t_H = h(r)/v_H , \quad (8)$$

where $h(r)$ is approximately the density scale height. This is what may happen to silicon in magnetic B stars, although the real situation is a little more complicated due to the larger number of different ions. In our computations, we have included the effects of all the silicon ions, from Si I to Si V, as described next.

III. COMPUTATION OF THE TOTAL RADIATION FORCE ON SILICON

a) *Radiation Force Received by Each Ion*

For this investigation we have used the ATLAS 5 code (Kurucz 1970) to construct model atmospheres at T_{eff} = 12,000 K and 14,000 K, both at $\log g = 4$. The various model parameters took on the default values as described by Kurucz (e.g., solar abundances) except that a helium abundance of 0.05 by number was adopted and hydrogen blanketing was included.

The acceleration of a particular ion i due to the absorption of photons is

$$g_i = \frac{1}{m}\frac{\pi}{c}\int_0^\infty \kappa_{\nu i} N_i F_\nu d\nu , \quad (9)$$

where $\kappa_{\nu i}$ is the absorption coefficient per ion i and m is its mass. For the atmospheres considered here, absorption in the continua of silicon ions contributes only a negligible amount to g_i. Continuum absorption of neutral silicon can also be neglected for reasons given below. We therefore consider line absorption only.

The line absorption coefficient was represented by a Voigt profile, with the Doppler width caused by thermal motions only. The Lorentz wings are due mainly to radiation damping, but for the neutral lines the quadratic Stark effect was also taken into account. The equation of transfer in the lines was then solved numerically by the methods of Scholz and Traving (1967) and of Unsöld (1955). The continuum was assumed in LTE, as was the absorption by ionized lines. But for the neutral lines, departures from LTE had to be taken into account (see below).

The lines finally used in our computations are listed in Table 1. Before a line was included in this list, its contribution to the radiative force was examined for all depths over a wide range of wavelengths, to determine the extent to which the wings have to be followed (col. [6]). When a line becomes saturated, most of the force is transmitted in the wings, far outside the region contributing to the equivalent width. The extreme case is Si III λ1206.51, where the calculations were extended over 10^4 Doppler widths, $\Delta\lambda_D$. The damping coefficients are, therefore, of utmost importance. We computed the radiation damping for the neutral lines for T = 15,000 K from a new list of *gf*-values of Si I lines kindly provided by Dr. R. L. Kurucz (1977), using only transitions displayed

TABLE 1

ABSORPTION LINES CONTRIBUTING TO THE RADIATION PRESSURE ON SILICON

λ (Å) (1)	χ (eV) (2)	log *gf* (3)	log C_4 (4)	γ_{rad} (5)	$\Delta\lambda_{max}/\Delta\lambda_D$ (6)	*n* (7)
			Si I			
1256.00....	0.01	−0.06	16.35	56.3	10	2
1258.80....	0.03	+0.35	16.35	56.3	10	1
1561.00....	0.02	+0.14	10.00	4.5	100	2
1594.70....	0.03	0.00	9.35	4.3	316	1
1669.00....	0.02	+0.04	11.56	4.1	10	3
1599.00....	0.02	−0.45	11.37	2.3	31.6	18
1649.00....	0.02	−1.00	11.00	2.0	31.6	18
1651.00....	0.02	−1.81	11.0	2.0	10	14
1849.00....	0.02	−0.05	14.1	3.5	3.2	2
1848.00....	0.02	−0.63	14.1	3.4	1	3
1805.00....	0.03	−0.90	13.1	0.7	1	2
2516.11....	0.03	−0.20	14.7	2.0	1	1
2514.11....	0.02	−0.75	14.7	2.0	1	5
2157.00....	0.02	−0.51	15.1	1.3	1	3
2083.00....	0.02	−1.15	15.1	1.3	1	8
2124.12....	0.78	+0.53	15.2	7.4	3.2	1
1870.00....	0.78	−0.19	13.3	2.7	1	2
2881.58....	0.78	−0.14	14.7	2.4	1	1
2000.00....	0.78	−0.97	14.7	2.4	1	9
1795.00....	0.78	−1.71	14.7	2.4	1	14
1310.92....	0.78	+0.14	15.8	3.8	1	1
1391.09....	0.78	+0.74	15.4	4.2	3.2	1
			Si II			
1194.50....	0.02	+0.55	...	50.0	3162	1
1193.00....	0.02	−0.02	...	50.0	3162	3
1263.00....	0.02	+0.32	...	17.2	3162	2
			Si III			
1206.51....	0.00	+0.30	...	31.0	10000	1
1113.20....	6.57	+0.62	...	27.2	1000	1
1109.30....	6.57	+0.21	...	27.2	1000	2
996.09....	6.57	−0.39	...	18.4	316	2
			Si IV			
1396.00....	0.00	−0.10	...	9.2	1000	2

in the Grotrian diagrams of Bashkin and Stoner (1975). Column (5) of Table 1 gives the results (in units of 10^8 s^{-1}). The coefficients C_4 for Stark broadening (col. [4]) were taken from that same list. For the ions we used the *gf*-values given by Kurucz and Peytremann (1975).

We treated the lines in a semistatistical way by representing groups of lines of similar wavelength, excitation, and log *gf* by a single average line, subsequently multiplying the resulting radiative force with the number of lines in that group (col. [7]). Lines were counted as separate if more than 1.4 Å apart, otherwise their *gf*-values were added.

The accuracy achieved that way is conservatively estimated to be better than a factor of 1.5. We did not feel it justified to aim for more in view of the many other uncertainties, such as the unknown effect of blends with lines of other elements, which have been ignored here.

We have found that the ground state of Si I suffers quite large departures from LTE, an effect which is of importance for the calculated radiation force. That Si I was significantly affected by non–LTE effects was initially pointed out by Cuny (1971) with regard to the Sun. Many of her results are applicable in our case.

In calculating these effects we first note that the collisional rates among the ground-state configuration, while relatively weak, are nevertheless large enough to establish an approximate Boltzmann equilibrium. Thus, we can characterize the three levels in the ground state by a single departure coefficient. The error in this procedure is estimated to be less than about 10%.

Furthermore, by examining the various rates for the lowest excited states, we find that the bound-bound collision rates (Seaton 1962) to the next higher levels dominate the other rates by an order of magnitude. Thus, we assume that the excited states are well represented by LTE. These two simplifications allow a rather simple calculation to be performed of an "equivalent" two-level model of the atom by taking the appropriate Boltzmann averages of the radiative and collisional rates that enter the problem. Since we expect to publish this and related calculations in some detail in the near future, we will not go further into the details here.

It might initially appear surprising that an atom like Si I has large departures from LTE in a main-sequence B star. However, the explanation is rather simple. There is quite a strong radiation field at 1500 Å in these stars which tends to strongly photo-ionize the atom. On the other hand, the ground-state configuration lies about 4 eV below the nearest excited levels, and at temperatures approximating 1 eV the collisional coupling is rather weak. Finally, the Si I continua themselves are optically thin and do not self-shield in a significant manner. The result is an underpopulation of the ground state by approximately a factor of 3.

The immediate effect of this is to reduce the contribution of the Si I continua to the overall acceleration from a significant ($\sim 15\%$ of gravity) to an insignificant ($\sim 5\%$) value. The effect on the acceleration contributed by the lines is much less owing to partial saturation.

Departures from LTE are expected to be minor for the higher ionization states since their ground states ionize into the Lyman continuum and the large optical depths there drive the ionization and recombination rates into balance. The large collision rates for ions and the density of states preclude substantial effects in the excited states.

Finally, we should note that apparently Leckrone and Snijders (see Snijders 1977) have independently discovered the importance of departures from LTE in the Si I atom in these stars.

b) Momentum Redistribution among the Ions

The time scale needed for each ion to share its momentum with the surroundings was computed using:

$$t_{\rm col}(i) = mD_i/kT\,. \qquad (10)$$

In the case of ionized atoms, we used the Aller and Chapman (1960) expression for the diffusion coefficient:

$$D = 2 \times 10^9 T^{5/2}\left[N_{\rm H}\left(\frac{m}{m_{\rm H}+m}\right)^{1/2} Z^2 A_{12}\right]^{-1}, \qquad (11)$$

where $N_{\rm H}$ is the number density of hydrogen, Z the charge of the diffusing ion, and A_{12} a slowly varying function of temperature and density (note that the power of $\frac{1}{2}$ on the mass ratio was omitted by Aller and Chapman). The diffusion coefficient for neutral atoms has been discussed by Ratel (1976) and Michaud, Martel, and Ratel (1978). They showed that the usual expression obtained for hard sphere collisions could give an overestimate of D by a factor 5. The major collision effect in the case of neutral atoms diffusing in an ionized medium is due to the polarization of the neutral which leads to a dipolar interaction. In this case the expression of D is:

$$D = 0.144\frac{kT}{\alpha_A^{1/2}q}\left(\frac{mm_{\rm H}}{m+m_{\rm H}}\right)^{-1/2}\frac{1}{N_{\rm H}}, \qquad (12)$$

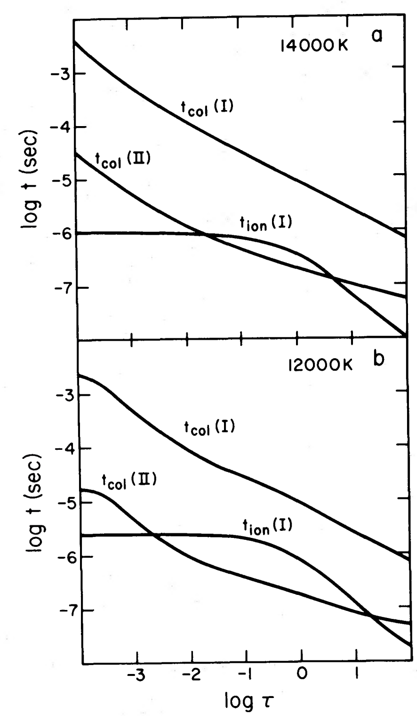

FIG. 1.—Shown are the collisional times for Si I [$t_{\rm col}$(I)] and Si II [$t_{\rm col}$(II)] to share their momenta in model atmospheres with $\log g = 4$ and $T_{\rm eff}$ = (*a*) 12,000 K and (*b*) 14,000 K. Note that since neutral atoms are not subject to Coulomb forces, $t_{\rm col}({\rm I}) \gg t_{\rm col}({\rm II})$. The collision times for the higher ionization states basically scale like $t_{\rm col}({\rm II})/Z^2$. We also show the ionization time, $t_{\rm ion}$(I), for Si I, which is always shorter than the collision time by at least an order of magnitude. Ionization times for the higher ionization states exceed their collision times by several orders of magnitude, and are not shown. The optical depth scale is at λ = 4000 Å.

where α_A is the polarizability of the atom and q the charge of the induced dipole. The resulting values for t_{col} are plotted in Figure 1 versus depth for the two first stages of ionization, for both $T_{eff} = 14{,}000$ K and $T_{eff} = 12{,}000$ K.

The ionization rate $1/t_{ion}(\text{I})$ for neutral silicon is available directly from the non-LTE calculations. We should note that these hydrogen-line-blanketed models account for the dominant source of line blanketing. Comparison with Kurucz's (1977) statistical metal-line-blanketed models shows that for a given Balmer discontinuity (i.e., similar gas temperatures) the emergent radiation fields in the relevant portions of the ultraviolet for the two sets of models are within about 10% of each other. Thus, we expect that the ionization rates (and, incidentally, the non-LTE effects) are correctly calculated to that level of accuracy. The time scales for Si I ionization are plotted in Figure 1. We see that they are typically 30 times shorter than the collision times, which clearly demonstrates their importance.

For Si II and the higher ionization states, the ground states ionize at such short wavelengths that the photoionization rates are very small. For example, at $\log \tau = -1$ in the $T_{eff} = 14{,}000$ K model, the photoionization rate for Si II from the ground state is about 10^{-4} that of Si I. The comparison is even more extreme for the 12,000 K model. That, coupled with the much higher collision rates for the ions, allows us to ignore ionization effects in calculating their velocities.

c) The Total Radiation Force

The total radiation force on silicon was computed according to both equations (6.15) and (6.16) of Montmerle and Michaud (1976). For the use of equation (6.15) we computed separately the diffusion velocity of each ion and carefully derived the mean diffusion coefficient. We found that the two equations gave the same result within 1%. The total radiation force can be given by:

$$\langle g_R \rangle = \frac{\sum N_i D_i g_i'}{\sum N_i D_i} , \tag{13}$$

where g_i' is the radiation force effectively acting on the ion. For the ionized atoms $g_i' = g_i$, while for the neutral atoms

$$g_I' = g_I \frac{t_{ion}(I)}{t_{col}(I) + t_{ion}(I)} . \tag{14}$$

The results for the total radiation force on silicon in the atmospheres of stars of effective temperatures 12,000 K and 14,000 K are given in Figure 2, as well as the contribution of each ion. The quantity g_I is plotted in Figure 2*a*. In Figures 2*b* and 2*c*, $\langle g_R \rangle$ and the contributions $N_i D_i g_i' / \sum N_i D_i$ for each ion are given. A comparison of Figure 2*a* with Figures 2*b* and 2*c* shows that, when Si I shares its momentum with all the silicon ions, its effective radiation force is decreased by at least four orders of magnitude.

The radiation force shown in Figure 2 is directly comparable with the gravity, $\log g = 4$. In large parts

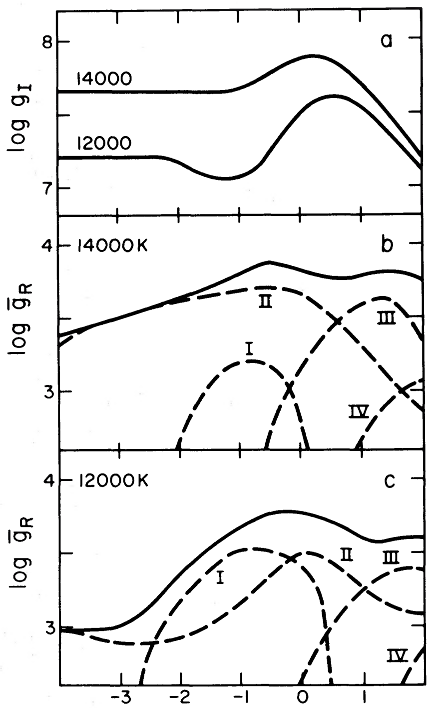

FIG. 2.—(*a*) The radiation force provided to Si I (in units directly comparable to gravity, $\log g = 4$) is shown for the two model atmospheres. While not sufficient to levitate all of silicon, this force clearly overcomes gravity when only Si I is allowed to diffuse. The depression at $\log \tau = -1$ for the 12,000 K model results from the onset of saturation just where the Si I ionization fraction peaks. (*b*) This shows the radiation force on the total silicon population in the $T_{eff} = 14{,}000$ K model and the contributions from the individual ionization states when the forces are shared (as in the absence of a horizontal magnetic field). Note, by comparison with Fig. 2*a*, the dilution by four orders of magnitude of the contribution from Si I. (*c*) As in Fig. 2*b*, only here for the $T_{eff} = 12{,}000$ K model. Note that although the radiation force on Si I is less for 12,000 K compared to the 14,000 K model, due to desaturation, its contribution to the total radiation force is larger for the cooler model.

of the atmospheres, and especially in the regions between $\log \tau = -1$ and $\log \tau = 0$, the radiation force is found to be within a factor of 2 of the gravity. (H. P. Gail kindly informed us [1976] that the much larger radiation forces found by Gail and Sedlmayr 1976 were due to a programming error.) In the absence of a magnetic field, the abundance of silicon initially decreases by diffusion. However, since the lines are saturated, the radiation force on each atom increases when their number decreases; the silicon abundance does not decrease by more than a factor of 2. This result explains why the silicon abundance is found to be normal in most Hg-Mn stars (Preston 1974).

IV. DIFFUSION ACROSS THE MAGNETIC LINES

It is well known that magnetic fields cannot produce any observable migration of elements unless large magnetic gradients are hypothesized (Babcock 1967).

The Zeeman splitting of lines could have an effect on the radiation force, but this effect is small (Shore and Adelman 1974). *The basic effect of the magnetic field is the trapping of charged particles along the field lines.*

The details of the diffusion of charged particles across a magnetic field in a partially ionized medium is now being studied (Theys 1978). In the "free path approximation" (Chapman and Cowling 1970; Cowling 1945), the diffusion velocity of a particle i of charge e (in electrostatic units) is slowed down by a factor $(1 + \omega_i^2 t_i^2)$ compared to the nonmagnetic case, with

$$\omega_i = \frac{eH}{m_i c}, \tag{15}$$

and

$$t_i = \frac{m_H N_H + m_i N_i}{m_H N_H} t_{col}(i), \tag{16}$$

(and not $[(m_H + m_i)/m_H]t_{col}(i)$ as erroneously written in Chapman and Cowling below eq. [19.31, 3]). For test atoms or for electrons, equation (16) reduces to

$$t_i = t_{col}(i) = \frac{m_i D_i}{kT}. \tag{17}$$

Equation (6) becomes:

$$V_t = \frac{N_n}{N}\left[t_{ion}(n) + \frac{t_{col}(i)}{1 + \omega^2 t_{col}^2(i)} g_{eff}(n)\right] + \frac{N_i}{N}\frac{t_{col}(i)}{1 + \omega^2 t_{col}^2(i)} g_{eff}(i), \tag{18}$$

which reduces to equation (7) in case of a large magnetic field. We find from equation (18) that the upward diffusion of Si I is preponderant as soon as

$$\omega^2 + \frac{1}{t_{ion}(n) t_{col}(i)}\left[\frac{N_i g_{eff}(i)}{N_n g_{eff}(n)} + \frac{t_{ion}(n)}{t_{col}(i)} + 1\right] > 0. \tag{19}$$

Table 2 gives the limiting intensity of the magnetic field for which $V_t = 0$, as a function of the optical depth. Equation (19) has been used, with values averaged over Si II, Si III, and Si IV for $N(i)$, $g_R(i)$, and $1/t_{col}(i)$. Due to saturation effects on Si I lines, the radiation force decreases when the silicon abundance increases. We find that a magnetic field equal to 1.7 times these values can support a factor 10 overabundance, and a magnetic field of 3 times these values can support a factor 100 overabundance (the radiation force approximately decreases like $N^{-1/2}$, and the limiting magnetic field approximately increases like $g_R^{-1/2}$). These values are large compared to the observed ones, of only a few kilogauss (Babcock 1960; Preston 1971*a*, *b*; Borra and Landstreet 1977; Landstreet and Borra 1977; etc.). However, our theoretical values concern the horizontal surface magnetic field in some spots, while the observed values usually refer to the longitudinal component over the stellar surface. Also, as developed below, the diffusion of the ionized silicon does not have to be stopped below $\tau = 1$ to produce a significant overabundance of silicon in the atmosphere.

TABLE 2

LOCAL MAGNETIC FIELD NEEDED TO CREATE AN UPWARD SILICON DIFFUSION

	H (kilogauss)	
log τ	12,000 K	14,000 K
−3............	2.6	8.1
−2............	3.2	6.0
−1............	3.0	24
0............	30	82
+1............	940	1700

In Figure 3 we have plotted the "diffusion flux of silicon" for a normal silicon abundance (log Si = 7.5 in the log H = 12 scale) with the assumption that the neutral is the only one to diffuse:

$$\Gamma(Si) = v_H N(Si), \tag{20}$$

where v_H is given by equation (7). When $d\Gamma(Si)/d\tau < 0$, diffusion leads to a silicon depletion, while it accumulates when $d\Gamma(Si)/d\tau > 0$. Silicon diffusion in case of horizontal magnetic lines leads to a silicon accumulation primarily above log $\tau = -2$. Also plotted in Figure 3 is the diffusion time scale across the magnetic lines given by equation (8), with $h(r) = 2 \times 10^8$ cm, for a normal Si abundance (i.e., at the onset of diffusion). As diffusion proceeds, the radiation force on Si I decreases at log $\tau < -1$ due to saturation (as the Si abundance increases). On the other hand, as silicon becomes depleted at log $\tau > -1$, the radiation force per atom *increases.* As soon as the silicon abundance is $\frac{1}{2}$ to $\frac{1}{4}$ of the normal value, *the ionized silicon begins to rise*, and diffusion processes are accelerated. In fact, the magnetic field only has to initiate the upward silicon diffusion in the upper atmosphere. Suppose, for example, that a magnetic field of 10 kilogauss exists in some spot in the atmosphere of a 12,000 K star. Silicon will diffuse upward above $\tau \approx 0.3$, and it will diffuse downward below this depth (see Table 2). However, the radiation force per atom will increase when the abundance decreases, and silicon will stop diffusing downward as soon as it attains the "equilibrium abundance" for which $g_R = g$. Meanwhile, as it diffuses upward above $\tau \approx 0.3$, the abundance at this depth will decrease under the "equilibrium value." As soon as this happens, the radiation force per atom becomes larger than g, and silicon diffuses upward. The same effect is created underneath and proceeds deeper and deeper in the atmosphere. The net balance of this effect is the following: at $\tau < 0.3$, the silicon goes up and accumulates around $\tau = 10^{-2}$ to 10^{-3} with a time scale of $\sim 10^4$–10^5 yr. At $\tau > 0.3$ the silicon excess (which cannot be supported by the radiation force) falls with an original time scale $\lesssim 2 \times 10^5$ years (original diffusion time scale for the ionized silicon

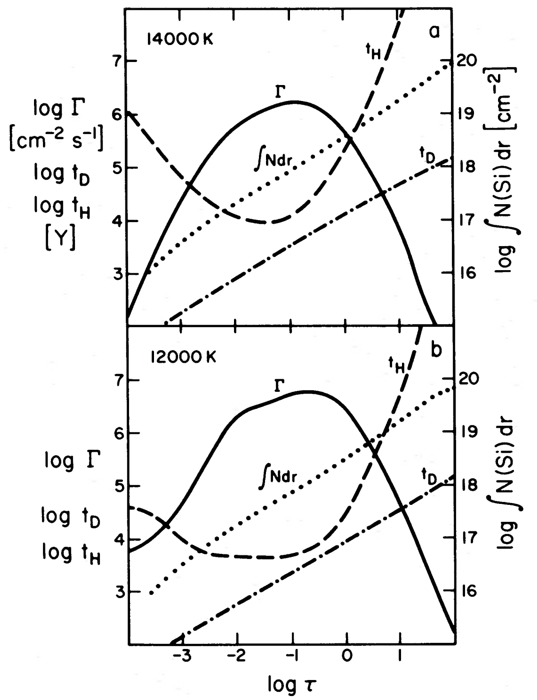

FIG. 3.—For the (*a*) 14,000 K and (*b*) 12,000 K models we show the "silicon diffusion flux" Γ, which is the diffusion velocity times the silicon number density, for normal Si abundance (log Si = 7.5), with the assumption that the diffusion of the ionized silicon is completely stopped (Si I is the only atom to diffuse). In this case, silicon will accumulate where the diffusion flux decreases with decreasing τ. The quantity t_H represents the original diffusion time scale (computed for a normal silicon abundance) *with the same assumption*, while t_D represents the original diffusion time when the ionized silicon is free to diffuse (*left-hand scale*). If the magnetic field stops the ionized atoms down to $\tau = 0.3$ (see text), silicon will go up above this depth and accumulate at $\tau < 10^{-2}$. Below this depth, the excess silicon, that the radiation force cannot support ($\frac{1}{2}$ to $\frac{3}{4}$ of the normal abundance), will go down. The remaining silicon will be "aspirated up" to the accumulation region in a self-accelerating process. The dotted line shows the initial column density (cm^{-2}) of silicon nuclei integrated up to $\log \tau = -4$ (*right-hand scale*). Since they accumulate at $\log \tau < -2$ where initial column densities are approximately 10^{17}, and that more than 10^{19} atoms cm^{-2} can be levitated up to these depths, we can account for enhancements of 100 in the magnetic spots of Si stars.

at $\tau = 100$). Then the silicon which can be supported by the radiation force (the "equilibrium abundance") is "aspirated up" to the upper atmosphere in a self-accelerating process. Although an exact evaluation of the time scale of the whole process would need step-by-step computations of the abundance variations in the atmosphere, we can infer that it is compatible with the mean age of Ap stars of 10^6–10^7 years (Abt 1978).

The dotted line in Figure 3 represents the total abundance of silicon in a column of unit cross section, arbitrarily integrated up to $\log \tau = -4$, for a normal silicon abundance. Since the silicon equilibrium abundance is as large as $\frac{1}{2}$ to $\frac{1}{4}$ of the normal value, there is enough "reservoir" above $\tau = 100$ to create an overabundance as large as 100 in the upper atmosphere.

V. DISCUSSION

The diffusion theory leads to a natural explanation of the silicon overabundance in magnetic B stars while it is normal in Hg-Mn stars. In the absence of magnetic lines, or when the magnetic lines are radial, the radiation force is able to support an abundance of Si within a factor 2 of the normal abundance. When the magnetic lines are horizontal, the diffusion of the ionized atoms is slowed down, and the upward motion of the neutral silicon leads to an overall increase of Si in the line-forming region. Note that in the general case, spots of horizontal magnetic lines may be found anywhere on the stellar surface, and even close to the magnetic poles in case of a displaced dipole (Michaud 1978). The range in effective temperature of silicon stars is well explained in this frame. At high effective temperatures the onset of Si II ionization forces the abundance of Si I to such small values that it cannot produce an observable overabundance of Si in the stellar lifetime. At low effective temperatures the abundance of Si I becomes so large that the saturated lines are unable to support Si I itself. An exact theoretical determination of the boundaries of Si stars will be given by further computations at various T_{eff} (in preparation).

Other elements will probably follow the same pattern as Si if the radiation force on the ionized atoms is of the same order as the gravity while the radiation force on the neutral atoms is much larger than the gravity. Good candidates are Mg and Fe, which have normal abundances of the same size as silicon (7.4 and 7.6, respectively, compared with 7.5 for Si) and first ionization potentials just a little less than Si I (7.85 and 7.87 compared with 8.15 for Si I). However, predictions cannot be made before computing precise radiation forces on these elements. In the framework of our discussion we expect helium to sink at places where silicon is being pushed up, as is observed, for example, in CU Vir (Hardorp and Megessier 1977). Since most of the helium is in the neutral state, it can sink through the horizontal field lines. The behavior of the rare earths may vary according to the strength of the magnetic field below $\tau = 10^{-1}$; they can diffuse upward and accumulate in phase with Si if the magnetic field below $\tau = 10^{-1}$ allows ionized atoms to diffuse and if the magnetic field above this depth stops them. In this case large overabundances of rare earths can be seen in the Si spots. If the magnetic field is strong enough to stop completely the diffusion of ionized atoms down to large optical depths ($\tau > 1$), no rare-earth abundance will be seen in the Si spots. However, they will still diffuse when the magnetic lines are radial. This could create variations in opposite phase with Si as observed in α^2 CVn (Pyper 1969). In any case careful computations of the radiation forces are needed before giving precise predictions on their abundance variations.

In summary, the theory of diffusion processes provides a natural explanation of the silicon overabundance in silicon stars in the frame of the oblique rotator model, while it is normal in nonmagnetic peculiar stars. The occurrence of Si stars and Hg-Mn stars at the same effective temperatures is very simply related to the presence of a magnetic field in the former.

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