MASS LOSS AND OH MASER EMISSION FROM MIRA VARIABLES

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Received 1978 March 31; accepted 1978 June 7

ABSTRACT

We have estimated the mass, radius, and luminosity of 26 Mira variables that are known OH sources of radio emission at 1612 MHz. The time-independent solution of Salpeter's stellar wind equation and a period-density relation are used to solve for basic stellar parameters, with the aid of the terminal expansion velocity of the OH maser cloud. Masses obtained from these calculations are consistent with other estimated values for masses of Mira variables. Good agreement is obtained when comparing the rate of mass loss as determined from Reimers's semiempirical relation to estimates of the mass loss rate as deduced from theoretical models involving radiation pressure on grains. These calculations suggest a strong correlation between the mass loss rate and the pulsation period. Arguments concerning the general properties of silicate grains from radiation-pressure-driven stellar wind equations are discussed.

Subject headings: masers — stars: circumstellar shells — stars: long-period variables — stars: mass loss

I. INTRODUCTION

Microwave emission at 1612 MHz from the OH molecule has been observed in association with a large number of long-period variable stars. Mira variables comprise a large fraction of OH emitting objects in which the emission originates from an extended masing region that surrounds a central star (cf. Litvak and Dickinson 1973). The signature of the radio emission consists usually of two narrow emission spikes that are separated by Doppler velocity differences that range from 2 to 30 km s⁻¹ (cf. Bowers and Kerr 1977; Lepine and Barros 1977). Typical dimensions associated with OH masers are $\sim 10^{16}$ cm (Reid *et al.* 1977; Schultz, Sherwood, and Winnberg 1978).

Mira variables are also known to be losing mass at typical rates $\sim 10^{-6} M_\odot$ yr⁻¹, as evidenced by the infrared silicate emission in the 10 μm to 20 μm region (Gehrz and Woolf 1971). Accordingly, the appearance of circumstellar shells in the infrared, and the detection of OH emission from regions more removed from the central star, probably reflects the general state of the long-period variable at a particular phase in its course of evolution along the giant branch. In this paper we wish to explore the likelihood of a relationship that exists between the dynamics of the masing cloud, as reflected in the Doppler separation of the 1612 MHz OH emission peaks, and the general properties of the central star.

II. FORMULATION

Approximately 40 long-period variables are presently identified as OH maser sources. We have consid-ered the list of OH maser stars recently published by Bowers and Kerr (1977). Their data are principally of Mira-type regular variables with visual and IR periods in excess of 250 days. We have excluded the very-long-period infrared variables from our analysis for reasons which will be apparent further on in this paper (see discussion at end of this section). The stars analyzed are all visual variables, with well-defined periods and spectral classifications.

In our analysis we assume that the high- and low-velocity components of the 1612 MHz emission originate from a shell that expands with a terminal velocity u_∞. Accordingly, the velocity separation ΔV_{OH} of the peaks here is equal to the difference of velocity between the near side of the shell, which is approaching, and the far side of the shell, which is receding, that is observed in our line of sight. It follows that the velocity expansion of the OH emitting region is $u_\infty = \Delta V_{OH}/2$. We consider only those stars that show a definite two-peak signature in their radio spectrum.

Salpeter (1974) formulated the mechanism of radiation-pressure (on grains) -driven mass loss by assuming that grains are formed close to the photosphere of the star. Mass loss occurs provided the luminosity L_* of the star exceeds a critical luminosity L_C, given as

$$L_C = 4\pi cGM_*\kappa^{-1}, \qquad (1)$$

where c is the speed of light, G is the gravitational constant, and κ is the opacity due to grains and gas combined. Upon integrating the time-independent hydrodynamic equation (Marlborough and Roy 1970),

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assuming supersonic flow and under reasonable conditions, Salpeter (1974) finds that the terminal velocity of an expanding shell is approximately

$$u_\infty = u_{es}[L_*/L_C - 1]^{1/2} , \qquad (2)$$

where u_{es} is the velocity of matter escaping from the star. Here we assume that grains form near the stellar surface, so u_{es} is essentially the stellar escape velocity, which is

$$u_{es} = (2GM_*/R_*)^{1/2} , \qquad (3)$$

where M_* and R_* (asterisked in cgs) are the mass and radius, respectively. Since the luminosity is given as

$$L_* = 4\pi R_*{}^2 \sigma T_e{}^4 , \qquad (4)$$

where σ is the Stefan-Boltzmann constant, it is possible to solve for R_* in equation (2) by eliminating M_* from u_{es} and L_C. In order to accomplish this, we now assume that the period-density relation for Cepheids is applicable for long-period variables (Kafatos, Michalitsianos, and Vardya 1977), so that

$$\Pi(\rho_*/\rho_\odot)^{1/2} = Q , \qquad (5)$$

where Π is the pulsation period (days), ρ_* and $\rho_\odot$ are the mean density of the star and Sun, respectively, and $Q = 0.03$, which is a value adopted as a lower limit for stars with highly condensed cores appropriate for red giants (Cox and Giuli 1968). Accordingly, the mass can be written in terms of the radius and the period of pulsation as

$$M_* = \tfrac{4}{3}\pi R_*{}^3 \frac{Q^2 \rho_\odot}{\Pi^2} , \qquad (6)$$

where $\rho_\odot \approx 1.41 \text{ g cm}^{-3}$.

Substituting equation (6) into equations (1) and (3), and substituting the resultant expressions together with relation (4) in equation (2), we obtain a quadratic equation in R_*, which is

$$R_*{}^2 - \left(\frac{\sigma T_e{}^4 \Pi^2 \kappa}{cG\beta}\right) R_* + \frac{u_\infty{}^2 \Pi^2}{2G\beta} = 0 , \qquad (7)$$

where $\beta = \tfrac{4}{3}\pi Q^2 \rho_\odot$. Equation (7) can now be solved for R_* in terms of the observed pulsation period, the mean effective temperature, and the terminal velocity. The long-period IR variables observed by Bowers and Kerr (1977) were not included in this analysis since an accurate estimate of the temperature from visual spectral data is not available.

III. STELLAR PARAMETERS AND MASS LOSS

a) Correlation of Stellar Parameters and Mass Loss

Equation (7) has been evaluated using the long-period variables observed by Bowers and Kerr (1977). From the observations we specify the pulsation period, the mean effective temperature as determined from the spectral classification, and terminal velocity $\Delta V_{OH}/2$. The only remaining quantity is κ, which was varied as
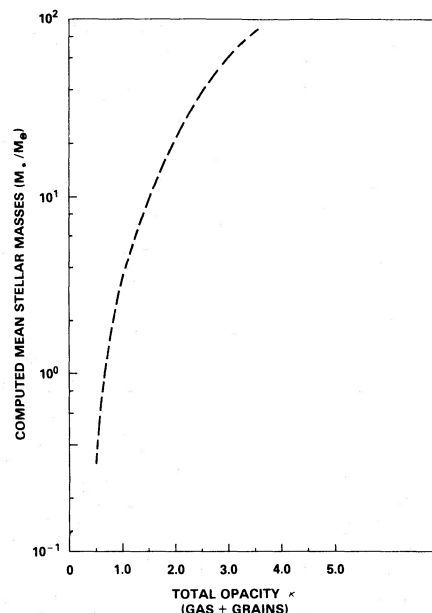

FIG. 1.—Mean computed mass versus total envelope opacity (gas + grains combined).

an open parameter. Since $M_* \propto \rho_* R_*{}^3$, where ρ_* is the mean density of the star, the stellar mass is critically dependent on the value of R_* that is obtained from equation (7). As evident from the solution of the quadratic equation (7), R_* is roughly proportional to κ. Since M_* depends critically on κ, and if $\kappa \approx 1$, we obtain the most reasonable values of $M_* \gtrsim M_\odot$. As evident in Figure 1, where we plot the mean stellar mass for these stars against the corresponding value of κ, increasing or decreasing κ by even a factor of 2 or 3 produces stellar masses that are unreasonably too large or too small, respectively.

In Figure 2 the resultant mass distribution for $\kappa = 1$ of these stars is shown. One star (UX Cyg) was found to have an unrealistically large mass of $\sim 30\ M_\odot$. However, the majority of stars tend to have approximately $1\ M_\odot \lesssim M_* \lesssim 2\ M_\odot$. These values seem appropriate for the expected masses of Mira variables (cf. Wood and Cahn 1977). The values of R_* and L_* obtained in this manner are shown in the H-R diagram (Fig. 3). The points plotted lie in the long-period variable star region of the diagram, and have radii in the range $10^2 \lesssim R_*/R_\odot \lesssim 10^3$, with luminosity an increasing function of mass (IK Tau being one notable exception). This range of stellar radii is consistent with the results of Nather and Wild (1973), considering the range of uncertainty in the distance, who find from lunar occultation measurements that the estimated size of R Leo (OH-M type, $\Pi = 313$ days, distance 250 pc) is $R_*/R_\odot \approx 1800$. Lepine and Barros (1977) adopt distances to this star based on V and I magnitudes of 238 to 129 pc, respectively, which if correct would bring the physical size of this star in better agreement with the characteristic radii determined here. Since appropriate values of R_*, M_*, and L_* have been obtained from our relations, the general region in
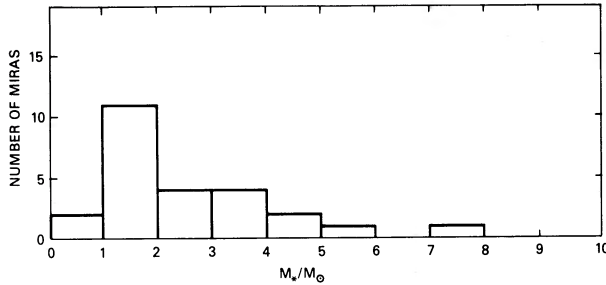
FIG. 2.—Mass-number distribution of OH Mira variables

which these stars lie in the H-R diagram provides a form of self-consistent check of these computations. An average luminosity that corresponds to $\kappa \approx 1$ is $L_* \sim 3 \times 10^4 L_\odot$, and is also a value generally consistent with the range of luminosities obtained by Cahn and Wyatt (1978). Furthermore, the values of the stellar parameters computed here yield surface escape velocities for these Miras of ~ 30 km s^{-1}, that are in agreement with the general values for Doppler measurements of chromospheric spectral lines observed in early M giants (Reimers 1977b).

Since Dickinson, Kollberg, and Yngvesson (1975) have shown that the pulsation period is correlated with $\Delta V_{\rm OH}$(1612 MHz), it would be of interest here to investigate whether u_∞ is related to M_* and R_*, upon which Π is fundamentally dependent through the period-density relationship. We found that a plot of mass against u_∞ showed no strong correlation, although the stellar radius and luminosity exhibited some indication of a direct dependence on u_∞.

b) Mass Loss Rate, Expansion Velocity, and Pulsation Period

Having derived M_* and L_*, it is now possible to examine the dependence that u_∞ has on the basic stellar parameters in combination. Since the mass loss
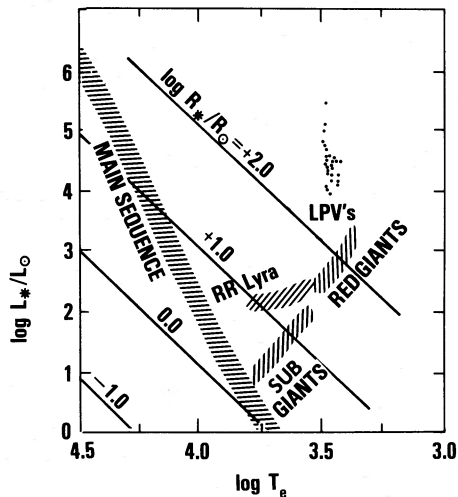

FIG. 3.—H-R diagram showing the computed luminosities of OH Miras (dots) in the long-period variable star region.

rate $\dot{M}$ appears to be an integral factor in determining the evolutionary course of an M giant in the giant branch (cf. Reimers 1977b), we can examine the relation between $\dot{M}$ and u_∞ in order to determine whether OH maser observations of long-period variables are indicative of the general dynamic state of the red-giant envelope.

The rate of mass loss $\dot{M}$ is obtained from Reimers (1975, 1977a, b), where

$$\dot{M} = \frac{A(L_*/L_\odot)(R_*/R_\odot)}{(M_*/M_\odot)}, \qquad (8)$$

and $A = 1.4 \times 10^{-13} M_\odot$ yr^{-1} (Reimers 1977a). The estimated values of $\dot{M}$ computed in this manner ($\sim 10^{-6} M_\odot$ yr^{-1}) are consistent with the general values of $\dot{M}$ obtained from Gehrz and Woolf (1971) in the infrared, and from Reimers (1977a) from Doppler measurements of circumstellar absorption lines. We found that if $\dot{M}$ is plotted against the outward expansion velocity u_∞, a general tendency for $\dot{M}$ to increase with $\Delta V_{\rm OH}/2$ is suggested. Unfortunately, the observations provide us with only a few stars with $\Delta V_{\rm OH}/2 > 10$ km s^{-1}, where most of the OH sources have values of $\Delta V_{\rm OH}/2 \approx 3.5$ km s^{-1}. It would not be surprising if u_∞ did increase with $\dot{M}$, since a greater expansion velocity is most likely indicative of the energy flux required to drive the wind, which in the model assumed here is derived from radiation pressure associated with highly luminous stars. However, more observations are required of OH Miras that have large velocity separations between the 1612 MHz peaks, in order to verify this particular dynamic property of maser shells.

Kafatos, Michalitsianos, and Vardya (1977) have suggested that $\dot{M}$ is correlated with Π in Miras. Since the pulsation period is one of the best-determined quantities of Mira variables, we might also investigate the dependence which $\dot{M}$ has upon Π from these calculations. Figure 4 shows a tendency for $\dot{M}$ to increase overall with the period of pulsation, and supports the conclusions of Kafatos, Michalitsianos, and Vardya, who find that infrared observations of silicate emission in red giants exhibits a similar dependence on the pulsation period.

Equation (8) for $\dot{M}$ is essentially derived from dimensional considerations, for which the magnitude of $\dot{M}$ is also dependent upon the value of the constant A. We can examine the validity of equation (8) by evaluating another relationship for $\dot{M}$ obtained from arguments involving the effect of radiation pressure on grains (Salpeter 1974). In the notation adopted here, Salpeter finds that

$$\dot{M} \approx \frac{\tau_s}{2u_{\rm es}c} [L_C(L_* - L_c)] \qquad (9)$$

(see eq. [11] of Salpeter) where τ_s is the optical depth at 1 μm at the sonic point in the atmosphere where the flow begins. We have evaluated both equations (8) and (9) assuming $\tau_s = 1$ for these Mira variables, and plotted the results against luminosity in Figure 5. Both equations show very close agreement for estimating
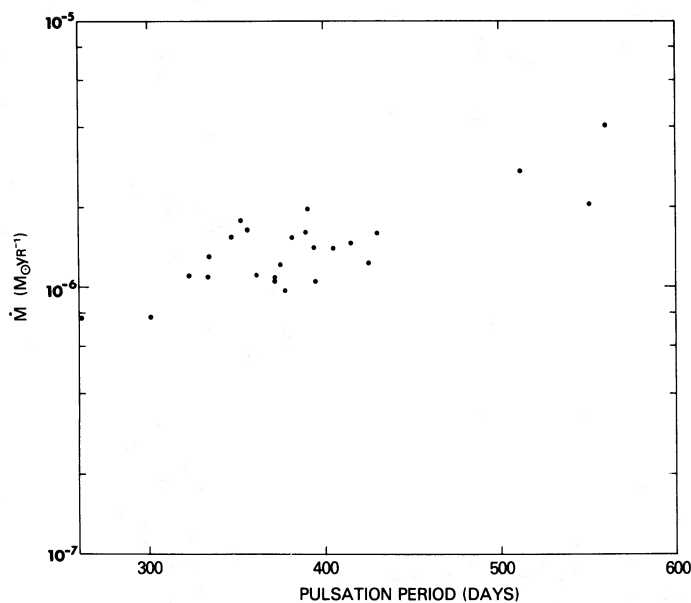
FIG. 4.—Mass loss rates (from eq. [8]) plotted against the visual pulsation period of OH Mira variables

$\dot{M}$, with possibly a linear dependence on luminosity. For other values of τ_s the results of equation (9) would only be shifted either upward or downward in Figure 5, thus preserving the slope of the dependence. Estimates for the mass loss rates obtained from equation (9), therefore, provide confidence in our use of Reimers's dimensional relation for $\dot{M}$. The values obtained for M_*, R_*, and L_*, and the close agreement between equations (8) and (9) when $\kappa = 1$, lend great support to the mechanism of Salpeter (1974) for radiation-pressure-driven stellar winds. We also note that $\tau_s \approx 1$ and $\kappa \approx 1$ imply densities in the stellar photosphere of $10^{-13} \geq \rho \geq 10^{-14}\,\mathrm{g\,cm^{-3}}$, for path

lengths of $10^2 \lesssim R_*/R_\odot \lesssim 10^3$. These values are certainly in agreement with the cool giant atmosphere models of Johnson (1974), and with the observed size of OH emitting regions.

c) Grains in Atmospheres of Cool Giants

It is interesting to examine the implications that $\kappa \approx 1$ has for the general properties of grains in the atmospheres of cool giants. The opacity due to grains and gas combined is given by the expression (cf. Salpeter 1974)

$$\kappa = f\frac{3}{4}\frac{Q_{\mathrm{ext}}}{a\rho_g}, \qquad (10)$$

where f is the mass fraction of material condensable into grains, a is the radius of a grain particle, ρ_g is the density of grain material, and Q_{ext} is the momentum transfer efficiency factor at $\lambda \approx 1\,\mu\mathrm{m}$, where the maximum photon emission of the star occurs. Q_{ext} is identical to the extinction factor at $\lambda \approx 1\,\mu\mathrm{m}$, which can be computed from Mie scattering theory or from laboratory measurements of the complex index of refraction m for the grain material (Wickramasinghe 1972). The density ρ_g here was set equal to $3\,\mathrm{g\,cm^{-3}}$, appropriate for grains.

We have considered silicate grains, which is consistent with the detection of silicate emission around late-type stars (Woolf and Ney 1969). Graphite grains are not considered here since the majority of Mira variables of $\sim 1\,M_\odot$ that are losing mass at high rates are oxygen-rich, i.e., have high O/C ratio (cf. Zuckerman et al. 1978). Following Wickramasinghe (1972) we adopt the complex index of refraction $m = 1.66 - 0.005i$ at near-infrared wavelengths. The extinction factor is then computed for different values of a (see Table 1).
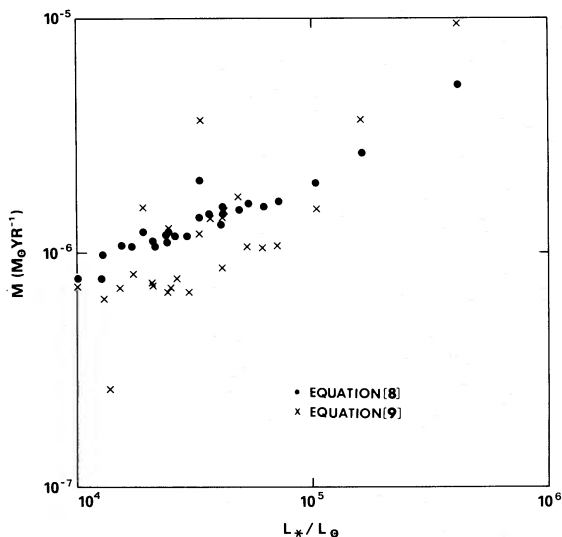

FIG. 5.—Mass loss rates computed from two different equations plotted against luminosity. Reimers's semiempirical relation (8) (dots) and Salpeter's theoretical equation (9) (crosses).

TABLE 1*

PHYSICAL PARAMETERS FOR GRAIN SIZES

a (cm)	Q_{ext}	f	Percentage of Condensable Material†
5×10^{-7}...	2.8×10^{-4}	7.1×10^{-3}	43‡
10^{-6}...	5.6×10^{-4}	7.1×10^{-3}	43
5×10^{-6}...	6.3×10^{-3}	3.2×10^{-3}	19
8×10^{-6}...	2.8×10^{-2}	1.2×10^{-3}	7
10^{-5}...	6.2×10^{-2}	6.4×10^{-4}	4

* Q_{ext} is computed for five different values of a. Then, using the computed value of Q_{ext} for a, and $\rho_g = 3 \, \mathrm{g \, cm^{-3}}$ and $\kappa = 1$, f is obtained.

† This percentage is computed for a combined abundance by mass of O, Mg, and Si of 1.7×10^{-2} (Cameron 1968).

‡ For smaller values of a, $Q_{ext} \propto a$, and therefore f remains constant.

For different values of a we compute Q_{ext}. Equation (10) can then be used to obtain the required mass fraction f in order to yield $\kappa \approx 1$. The computed mass fraction f is shown in Table 1. With the combined abundance by mass of oxygen, magnesium, and silicon relative to hydrogen of 1.7×10^{-2} (Cameron 1968), the last column of Table 1 gives the percentage of condensable material available for grain formation. We note that the range of grain sizes assumed here is consistent with the size distribution of interstellar grains computed by Mathis, Rumpl, and Nordsieck (1977). Additionally, Jura and Jacoby (1976) obtained $a \lesssim 0.05 \, \mu m$ for grains in reflection nebulae around supergiants, which is a value consistent with this suggestion.

We conclude that the size of silicate grains should be in the approximate range $0.1 \, \mu m$ to $0.01 \, \mu m$, which accounts for the opacity required in our analysis. A value of between 4% to 40% of the available heavy elements would be required to be condensable into grains for this range of sizes.

IV. SUMMARY AND CONCLUSIONS

We have estimated the mass, radius, and luminosity of a group of Mira variables that are known OH sources of emission at 1612 MHz. Combining the time-independent solution of the stellar wind equation and the period-density relation, and using the observed pulsation period, spectral temperature, and terminal expansion velocity indicated by the OH velocity, we are able to estimate the basic parameters for each star. The masses obtained in this manner generally lie in the range $1 \, M_\odot \leq M_* \leq 2 \, M_\odot$ and are in good agreement with the expected values for Mira variables. The Doppler velocity separation of the OH emission peaks is possibly correlated with luminosity, and a dependence on the mass loss rate is suggested.

The mass loss rate obtained from Reimers's (1977a) semiempirical relation shows a good correlation with the period of pulsation of Mira variables, and is consistent with previous findings from infrared silicate emission in circumstellar shells. A comparison of the estimated values of $\dot{M}$ obtained from Reimers's equation and the mass loss rate derived from Salpeter's (1974) mechanism shows close agreement.

The size of the grains as deduced from our relations places them generally in the approximate range of $0.1 \, \mu m$ to $0.01 \, \mu m$, which is consistent with independent estimates for the dimensions of silicate particles in the photospheres of cool giants and in the interstellar medium. A general value for the grain opacity $\kappa \approx 1$ provides the most appropriate range of stellar masses and radii of M type OH Mira variables studied here.

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