THE ASTROPHYSICAL JOURNAL, 223: L83–L87, 1978 July 15

DISK ACCRETION BY MAGNETIC NEUTRON STARS*

P. GHOSH
Department of Physics, University of Illinois at Urbana–Champaign
AND
F. K. LAMB†
Department of Physics, University of Illinois at Urbana–Champaign;
and California Institute of Technology
Received 1977 December 12; accepted 1978 April 21

ABSTRACT

We propose a model for disk accretion by a rotating magnetic neutron star which includes a detailed description of matter flow in the transition region between the disk and the magnetosphere. We show that the disk plasma cannot be completely screened from the stellar magnetic field, and that the resulting magnetic coupling between the star and the disk exerts a significant torque on the star. Assuming that the distortion of the residual stellar field lines threading the disk is limited by reconnection, we calculate the total accretion torque on the star. The calculated torque gives period changes in agreement with those observed in the pulsating X-ray sources and provides a natural explanation of why a fast rotator like Her X-1 has a spin-up rate much below the conventional estimate for slow rotators. We show that for such fast rotators, fluctuations in the mass-accretion rate can produce fluctuations in the accretion torque $\sim 10^2$ times larger. For sufficiently fast rotators, or, equivalently, for sufficiently low accretion rates, the star experiences a braking torque even while accretion continues and without any mass ejection from its vicinity.

Subject headings: hydromagnetics — stars: magnetic — stars: neutron — X-rays: sources

I. INTRODUCTION

The qualitative features of disk accretion by rotating magnetic neutron stars were first described by Pringle and Rees (1972) and Lamb, Pethick, and Pines (1973). These authors argued that a slowly rotating neutron star accreting matter from a Keplerian disk should spin up as a consequence of the torque exerted on the star by the accreting matter. Recently, Ghosh, Lamb, and Pethick (1977, hereafter GLP) have given detailed solutions for plasma flow within the magnetosphere and have shown that these solutions can be used to derive bounds on the accretion torque which are quite general and do not require a detailed knowledge of the flow in the transition zone between the disk and the magnetosphere. Although the actual value of the accretion torque cannot be calculated without knowing the structure of the flow in the transition zone, these bounds show that the earlier estimate of the torque on slow rotators is correct if the zone is thin, and that in this case the same estimate is approximately correct even for fast rotators. The problem of disk accretion by magnetic neutron stars has also been studied recently by Scharlemann (1978), who assumed a thin transition zone.

There is now strong observational evidence that many pulsating X-ray sources are rotating magnetic neutron stars which are accreting matter from a binary companion (Lamb 1977). Measurements of secular spin-up time scales (see Rappaport and Joss 1977; Mason 1977) are generally consistent with the above theoretical estimates. However, the observed spin-up time scale in Her X-1 is substantially longer than the slow rotator estimate (Elsner and Lamb 1976), while the sources Her X-1, Cen X-3, and 4U 0900−40 all show episodes of spin-down in addition to their secular trend toward spin-up (Giacconi 1974; Fabbiano and Schreier 1977; Becker *et al.* 1978).

Progress in understanding these observations has been frustrated in part by the absence of a detailed, quantitative model of disk accretion by magnetic neutron stars. The purpose of this *Letter* is to describe such a model, which makes definite predictions regarding the inner radius of the disk, the radial and vertical structure of the transition zone, and the accretion torque. One of our main conclusions is that in disk accretion the transition zone is *not* thin, and recognition of this is an important step in understanding the period behavior of Her X-1, Cen X-3, and 4U 0900−40. A detailed account of these calculations will be given elsewhere (Ghosh and Lamb 1978). Here we restrict ourselves to a description of the basic model and a summary of the accretion torque results.

II. THE MODEL

In roughly spherical accretion by a magnetic neutron star, currents circulating in the transition region between the magnetosphere and the outside plasma com-

* Research supported in part by NSF grant PHY75-08790 at the University of Illinois and NASA contract NAS5-23315 at Caltech.

† Alfred P. Sloan Foundation Research Fellow.

pletely screen the stellar magnetic field from the plasma, confining the field to the interior of the magnetospheric cavity (Lamb, Pethick, and Pines 1973; Arons and Lea 1976; Elsner and Lamb 1977). In accretion from a thin disk, this picture is not self-consistent.

To see this, suppose that current sheets on the surfaces of the disk were initially to screen the disk completely. The magnetic field above and below the disk would then be oppositely directed and of magnitude $\sim B_0$, where $B_0 \sim \mu r^{-3}$ is the strength of the unscreened stellar field, assumed dipolar with moment μ. Such a strong field will rapidly invade the disk as a result of turbulent diffusion, development of the Kelvin-Helmholtz instability, and reconnection to small-scale fields within the disk. Once inside, the stellar field will quickly reconnect through the disk, since the time scale for reconnection, $\tau_r \sim 2h(\xi v_A)^{-1}$, is much smaller than the radial drift time, $\tau_d \sim r|v_r|^{-1}$. Here $v_A = B_r(4\pi\rho)^{-1/2}$ is the Alfvén velocity in the reconnecting field in terms of the disk plasma density ρ, h is the semithickness of the disk, v_r is the radial velocity of matter in the disk, and ξ is a numerical factor $\sim$0.1–1 (Vasyliunas 1975). Thus, for a "standard" disk (Shakura and Sunyaev 1973), one finds $\tau_r/\tau_d \approx 2 \times 10^{-4} \xi^{-1} r_8^{73/40} \mu_{30}^{-1} \dot{M}_{17}^{4/5} (M/M_\odot)^{-11/45}$ near the magnetospheric boundary, where the variables have been scaled in terms of their typical values. Here μ_{30} is μ in units of 10^{30} gauss cm^3, r_8 is the radius in units of 10^8 cm, $\dot{M}_{17}$ is the accretion rate in units of 10^{17} g s^{-1}, and M is the mass of the neutron star. We conclude that the star's magnetic field cannot be completely screened from the disk, and that some stellar field lines connect through it.

The stellar field lines which thread the disk are distorted by the azimuthal and radial motion of the disk plasma. Consider first the azimuthal motion. It generates a toroidal field B_ϕ comparable to the poloidal field B_p in a time $\sim|\Omega_K - \Omega_s|^{-1}$, due to the different angular velocities of the disk plasma and the star. Here $\Omega_K = (GM/r^3)^{1/2}$ and Ω_s is the angular velocity of the star. Were the magnetic field amplification limited only by ordinary or Bohm diffusion, B_ϕ would become extremely large. Before this can happen, the toroidal magnetic field, which is of opposite sign above and below the midplane of the disk, reconnects. Thus, in a steady state the azimuthal pitch, $\gamma_\phi = B_\phi/B_z$, is limited to the value given by the condition that the rates of reconnection and amplification of the toroidal field balance, that is,

$$\xi(\gamma_\phi B_z)(4\pi\rho)^{-1/2}/(2h) = \gamma_0(\Omega_K - \Omega_s) . \quad (1)$$

Here γ_0 is a numerical factor $\sim$1. A similar analysis can be carried through for the distortion caused by the radial motion of the disk plasma. The result is that those stellar field lines that thread the disk are pinched inward at the midplane and stretched in the direction of the disk orbital motion, but on average the rate of slippage of field lines through the disk plasma balances the rate at which the field lines are transported by the plasma.

The picture which emerges from this analysis is as follows. When it is far away, the motion of the accreting matter is determined by the effective viscosity in the disk and is unaffected by the magnetic field of the star. However, as it approaches the magnetosphere it enters a transition region where its motion is determined more and more by magnetic stresses associated with the stellar field and less and less by viscous stresses. The transition region ends and magnetospheric flow begins when the viscous stresses are negligible compared with the magnetic stresses and the flow is field-aligned.

In order to make this picture quantitative, we assume that the dipole moment of the star is nearly aligned with its rotation axis and that the latter is perpendicular to the plane of the disk, and we describe the accretion flow by the hydromagnetic equations, treating the slippage of the field lines through the disk plasma (eq. [1]) by an effective conductivity.

We find that a *boundary layer* exists at the radius

$$r_A = 0.41(\gamma_\phi^{11/27} C_b^{16/27} C_\omega^{-25/27} C_p^{-8/27})^{2/7} r_A^{(0)} , \quad (2)$$

where the variables Ω, v_r, and $\boldsymbol{B}$ change on the length scale

$$\Delta_r = 0.031(\gamma_\phi^{-16/27} C_b^{16/27} C_\omega^{2/27} C_p^{-8/27}) r_A , \quad (3)$$

which is much smaller than r_A. Here $r_A^{(0)} = \mu^{4/7}\dot{M}^{-2/7}(2GM)^{-1/7} = 3.2 \times 10^8 \mu_{30}^{4/7}\dot{M}_{17}^{-2/7} (M/M_\odot)^{-1/7}$ cm is the characteristic Alfvén radius for spherical accretion (see Elsner and Lamb 1977). C_b, C_ω, and C_p are dimensionless constants of order unity whose precise value is not determined by the theory; as usual in boundary layer problems, the solutions are very insensitive to their values. In the boundary layer, which has a width $\delta \equiv (r_0 - r_A) \sim \Delta_r$, currents screen the magnetospheric field by a factor $\sim$10, the toroidal motion changes from Keplerian to corotation, and plasma leaves the disk plane vertically to accrete onto the star. A residual magnetospheric field extends well beyond the boundary layer, creating a broad *outer transition zone* where the physical variables change on a length scale $\sim r$. The slow radial drift of the disk plasma across the residual stellar field in this zone produces a weak screening current which eventually screens the field to zero at a radius $r_s \gg r_A$. The different flow regions are shown schematically in Figure 1.

We describe the unperturbed disk flow by the standard model of accretion disks (Shakura and Sunyaev 1973). In the outer transition zone, we describe the flow by the standard models generalized to include the effect of the magnetic stresses on angular momentum transport and the additional heating due to the dissipation associated with plasma motion across the residual magnetospheric field. The resultant structure of this zone is extremely insensitive to the viscosity parameter, α. In the boundary layer, we solve the hydromagnetic equations numerically. We assume $\gamma_\phi = 1$ in this layer. We describe the vertical motion of the plasma away from the disk plane by the equation $d\dot{m}/dr = 4\pi r\rho c_s g(r)$, scaling the flow velocity out of the

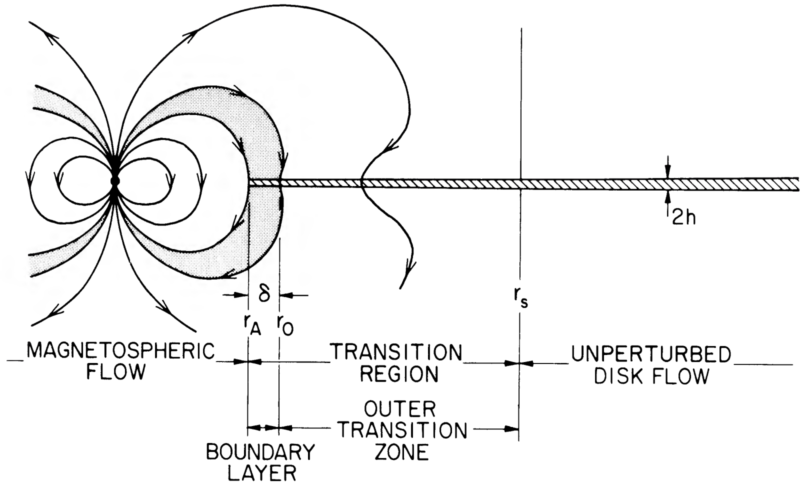

FIG. 1.—Schematic picture of the regions described in the text. Beyond the radius r_s at which the stellar magnetic field is completely screened, the disk flow, of vertical thickness $2h$, is unperturbed by the magnetosphere. In the transition region between r_s and r_A, the disk flow changes into magnetospheric flow. The transition region divides into two parts, an outer transition zone where viscous stresses dominate magnetic stresses, and a boundary layer of width $\delta \ll r_A$ where the magnetic stresses dominate.

disk in terms of the local sound speed, c_s. Here $\dot{m}(r) = 4\pi r h|v_r|\rho$ is the radial mass-flow rate in the disk and $g(r)$ is a "gate" function which describes the radial profile of the mass loss from the disk and which is determined by the shape of the field lines above and below the disk (Scharlemann 1978). We find that the properties of the model are insensitive to the precise form of $g(r)$, and depend only on the facts that $g = 0$ outside r_0, and $g = 1$ well inside the boundary layer. We therefore use a simple mass-loss profile for the results reported here, namely, $g(r) = 1$ for $r < r_0 - \Delta_m$, $g(r) = (r_0 - r)^2/\Delta_m{}^2$ for $r_0 - \Delta_m \leq r \leq r_0$, and $g(r) = 0$ for $r > r_0$, with $\Delta_m = 0.6\,\Delta_r$. The structure of a typical boundary layer solution is shown in Figure 2, where we have taken $C_b = 2$, $C_\omega = 0.5$, $C_p = 1$. Finally, within the magnetosphere the plasma flow is described by the interior flow solutions of GLP.

III. APPLICATION TO PULSATING X-RAY SOURCES

Among the most significant properties of the pulsating X-ray sources is the rate of change of spin, which reflects both the sign and the magnitude of the accretion torque and the moment of inertia of the X-ray star. For example, their secular spin-up rates have been used to identify these sources as neutron stars.

The torque, N, given by the present model of disk accretion can be determined by calculating the inward flux of angular momentum across an imaginary surface enclosing the star-magnetosphere system (see Lamb 1977). In our case the total torque can be written as

$$N = N_0 + \int_{r_0}^{r_s} (\gamma_\phi B_z) B_z r^2 dr\,,$$

where the first term comes from the matter leaving the inner edge of the disk at r_0 and the second term, from

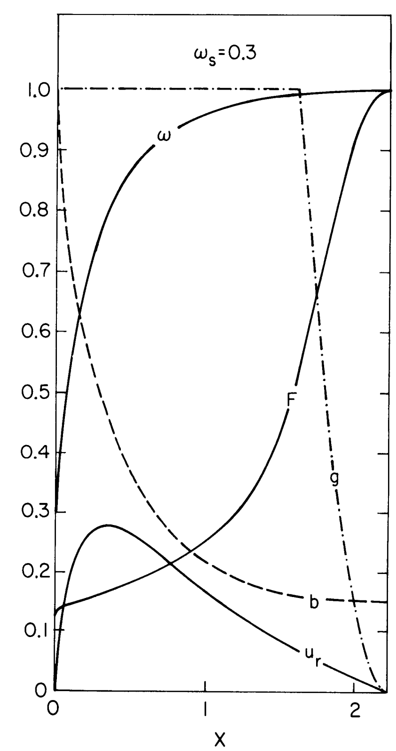

FIG. 2.—The structure of the boundary layer for a star with angular velocity $\Omega_s = 0.3\,(GM/r_A{}^3)^{1/2}$. Shown are the following dimensionless variables: angular velocity $\omega = \Omega/(GM/r_A{}^3)^{1/2}$, radial mass flux $F = \dot{m}/\dot{M}$, poloidal magnetic field $b = B_z/B_z(r_A)$, radial velocity $u_r = |v_r|/(2GM/r_A)^{1/2}(\Delta_r/r_A)^{1/2}$, and the gate function $g(r)$, as functions of the distance $x = (r - r_A)/\Delta_r$ from the Alfvén radius, r_A. In this example $\Delta_r = 0.036\,r_A$ and $\delta = r_0 - r_A = 2.22\Delta_r$. For clarity, only some of the calculated variables are displayed.

the torque on the star due to the twisted field lines threading the disk outside r_0. The latter is calculated from the azimuthal pitch given by equation (1) and the function $B_z(r)$ (see Ghosh and Lamb 1978), which falls from a value $\sim 0.1\ B_z(r_A)$ at r_0 (see Fig. 2) to zero at r_s. Here $N_0 = (GMr_A)^{1/2}$ corresponds to the conventional estimate of the torque on "slow" rotators (see GLP).

For a fixed mass-accretion rate, our solutions show that the torque can be *either* positive (causing spin-up) *or* negative (causing spin-down), depending on the angular velocity of the star, Ω_s. The reason for this behavior is that the pitch of the stellar field lines that thread the disk outside r_0 changes sign at the corotation radius, $r_c = (GM/\Omega_s^2)^{1/3}$. Thus the torque exerted on the star by these field lines is positive when the contribution from the forward-swept field lines inside r_c dominates the contribution from the backswept field lines outside r_c; this is the case for "slow" rotators ($\omega_s \ll 1$), which have $r_c \gg r_A$. This torque is negative when the contribution of the backswept field lines dominates; this is the case for "fast" rotators ($\omega_s \sim 1$), which have $r_c \sim r_A$. Thus, the spin-up torque on slow rotators is $\sim N_0$, whereas that on fast rotators is much smaller; this provides a natural explanation of why Her X-1 has an average spin-up rate much below the slow rotator estimate. For sufficiently fast rotators ($\omega_s \geq 0.4$), the contribution from the backswept field lines dominates all others, and the *total* torque is negative. Finally, for Ω_s larger than a certain critical value, there are no stationary flow solutions, indicating that steady accretion is not possible for such fast rotators.

Alternatively, one can consider the torque on a star of given spin period, P, as a function of the mass-accretion rate, as shown in Figure 3. At high accretion rates the disk penetrates well inside r_c and the backswept field lines contribute little to the torque. At lower rates the disk does not penetrate as far, and the torque due to the backswept field lines becomes more important. At sufficiently low rates the total torque is negative. Finally, at *very* low rates there are no stationary solutions, indicating that steady accretion is not possible.

The braking torque at high angular velocities or low accretion rates given by the present solutions does not involve any mass ejection from the vicinity of the neutron star and therefore contrasts with the braking torques conjectured previously by Davidson and Ostriker (1973) and Illarionov and Sunyaev (1975).

Given a spin period P and luminosity L, and a particular neutron-star model, the present model of disk accretion gives a secular spin-up time scale T_s which depends only on the single parameter μ, the stellar magnetic moment. We have adjusted μ to give the best possible agreement with the value of T_s observed in the eight pulsating X-ray sources for which data are available, and the results are shown in Table 1. In general the model gives satisfactory agreement with the observations, including those of Her X-1, for moments $\sim 10^{30}$ gauss cm^3. An exception is the source 4U 0900−40, for which no agreement is possible unless μ is ~ 10–10^2 times larger. Although our present understanding of neutron-star magnetic moments certainly permits such values of μ, we think it more likely that 4U 0900−40 is not disk-fed, in which case the present theory does not apply.

In addition to their secular spin-up, the most carefully studied sources Her X-1, Cen X-3, and 4U 0900−40 show substantial period fluctuations on time scales of weeks to months, with occasional episodes of spin-down (Giacconi 1974; Fabbiano and Schreier 1977; Ögelman *et al.* 1977). Fluctuations in the accretion torque are one possible explanation for this phenomenon (Lamb, Pines, and Shaham 1976, 1978). Our calculations show that torque fluctuations of the required size can be caused by fluctuations in the mass-accretion rate. For Cen X-3, our model gives relative torque fluctuations $\delta N/N \sim \delta\dot{M}/\dot{M}$. For fast rotators, on the other hand, fluctuations in the mass-accretion rate can produce fluctuations in the accretion torque many times larger. Thus, the model parameters of Table 1 give $\delta N/N = 1.9 \times 10^2\ \delta\dot{M}/\dot{M}$ for Her X-1. If the observed period variations *are* caused by fluctuations in $\dot{M}$, the accretion luminosity should display similar fluctuations, with periods of reduced luminosity associated with periods of slower spin-up or, if the fluctuation is large enough, spin-down.

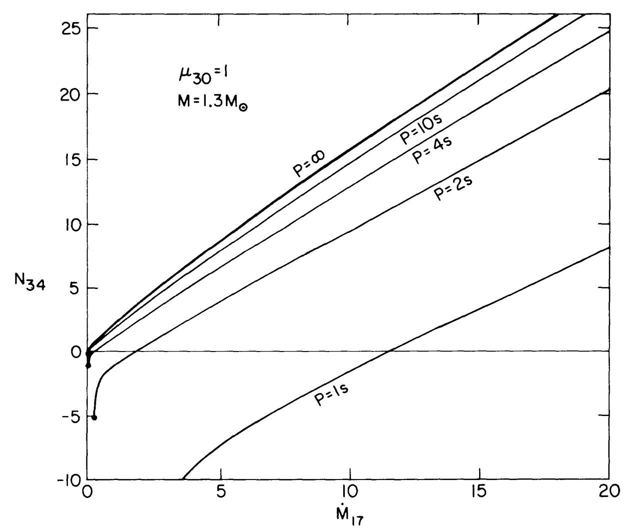

FIG. 3.—The torque N on a 1.3 $M_\odot$ neutron star in units of 10^{34} g cm^2 s^{-2} as a function of the mass-accretion rate $\dot{M}$ in units of 10^{17} g s^{-1}, for four values of the stellar rotation period, P, and $\mu = 10^{30}$ gauss cm^3. The heavy line is the asymptotic behavior obtained in the slow rotation limit; the deviation of the function $N(\dot{M})$ from this curve is less than the width of the curve for $P \gtrsim 100$ s. At high accretion rates, N varies as $\dot{M}^{6/7}$. The torque vanishes at the critical mass-accretion rate, $\dot{M}_c$, which is a function of P, and is negative for rates $\dot{M} < \dot{M}_c$. There is a minimum mass-accretion rate, $\dot{M}_{\min}$, which is also a function of P and below which there are no stationary flow solutions, indicating that steady accretion is not possible for such low rates. The terminations of the curves for $P = 2$ s and $P = 4$ s at the respective $\dot{M}_{\min}$'s are shown by dots in the figure. The termination for $P = 1$ s lies off scale, while that for $P = 10$ s lies too close to the origin to be seen clearly.

TABLE 1

OBSERVATIONAL AND THEORETICAL PARAMETERS OF EIGHT PULSATING X-RAY SOURCES*

Source	P(s)	T_s(years)	μ_{30}	$r_A(10^8$ cm)	ω_s
SMC X-1	0.71	$(1.3\pm0.4)\times10^3$	0.48	0.42	0.18
Her X-1	1.24	$(3.3\pm0.6)\times10^5$	0.40	1.0	0.386
Cen X-3	4.84	$(3.6\pm0.6)\times10^3$	3.65	2.4	0.366
A0535+26	104	29 ± 8	1.3	1.3	6.5×10^{-3}
GX 1+4	121	50 ± 13	0.42	0.73	2.5×10^{-3}
4U 0900−40	283	$(1.0\pm0.4)\times10^4$†	80	37	0.394
GX 301−2	700	120 ± 60	0.11	0.63	3.5×10^{-4}
4U 0352+30	836	$(5.9\pm1.5)\times10^3$‡	5.0	29	0.091

* Observational parameters are from Rappaport and Joss (1977), unless otherwise noted. For each source, the stellar dipole moment μ was adjusted to obtain the best possible agreement between the observed T_s and the calculated T_s for the $M = 1.3M_\odot$ neutron-star model of Pandharipande, Pines, and Smith (1976). As μ is varied, keeping M and the neutron-star model fixed, the calculated T_s for a given source passes through a minimum. If the observed T_s lies above this minimum, there is a value of μ which gives exact agreement between the calculated and observed values of T_s. Exact agreement was achieved for all sources except 4U 0352+30, for which the minimum calculated T_s, 1.3×10^4 years, was slightly above the mean observed T_s, 5.9×10^3 years. The value of μ given for 4U 0352+30 corresponds to the minimum calculated T_s. The parameters r_A and ω_s follow from the inferred values of μ.

† Timing data are from Becker *et al.* (1978) and Ögelman *et al.* (1977).

‡ Timing data are from White, Mason, and Sanford (1977) and Jernigan and Nugent (1978).

The accretion torque predicted by the model can therefore be tested directly by measuring the instantaneous rate of change of the pulsation period as a function of the total luminosity of the source, and comparing with curves like those shown in Figure 3. We note that Fabbiano and Schreier (1977) have reported a possible decrease in the X-ray luminosity of Cen X-3 during the spin-down episode of 1972 September–October.

Finally, we remark that a large fraction of the known pulsating sources have long periods, even though they are, paradoxically, observed to be spinning up on very short time scales, ~50–100 years. It is important to understand how these sources maintain their slow spin rates over long time scales, a problem that has been discussed previously by Fabian (1975), Savonije and van den Heuvel (1977), and GLP. The braking torque found by the present calculation for very low accretion rates may be one way, if the long period sources have low states during which the accretion rate is much reduced.

It is a pleasure to thank Professors D. Q. Lamb, C. J. Pethick, D. Pines, and J. Shaham for stimulating discussions.

REFERENCES

Arons, J., and Lea, S. M. 1976, *Ap. J.*, **207**, 914.
Becker, R. H., Rothschild, R. E., Boldt, E. A., Holt, S. S., Pravdo, S. H., Serlemitsos, P. J., and Swank, J. H. 1978, *Ap. J.*, **221**, 912.
Davidson, K., and Ostriker, J. P. 1973, *Ap. J.*, **179**, 585.
Elsner, R. F., and Lamb, F. K. 1976, *Nature*, **262**, 356.
———. 1977, *Ap. J.*, **215**, 897.
Fabian, A. C. 1975, *M.N.R.A.S.*, **173**, 161.
Fabbiano, G., and Schreier, E. J. 1977, *Ap. J.*, **214**, 235.
Giacconi, R. 1974, in *Astrophysics and Gravitation, Proceedings of the 16th International Solvay Conference* (Brussels: Université de Bruxelles), p. 27.
Ghosh, P., and Lamb, F. K. 1978, in preparation.
Ghosh, P., Lamb, F. K., and Pethick, C. J. 1977, *Ap. J.*, **217**, 578 (GLP).
Illarionov, A. F., and Sunyaev, R. A. 1975, *Astr. Ap.*, **39**, 185.
Jernigan, J. G., and Nugent, J. J. 1978, in preparation.
Lamb, F. K. 1977, *Proc. 8th Texas Symp. Relativistic Ap., Ann. NY Acad. Sci.*, **302**, 482.
Lamb, F. K., Pethick, C. J., and Pines, D. 1973, *Ap. J.*, **184**, 271.
Lamb, F. K., Pines, D., and Shaham, J. 1976, in *X-Ray Binaries, Proceedings of the Goddard Conference*, ed. E. Boldt and Y. Kondo (NASA SP-389), p. 141.
———. 1978, *Ap. J.*, **224**, in press.
Mason, K. O. 1977, *M.N.R.A.S.*, **178**, 81p.
Ögelman, H., Beuermann, K. P., Kanbach, G., Mayer-Hasselwander, H. A., Capozzi, D., Fiordilino, E., and Molteni, D. 1977, *Astr. Ap.*, **58**, 385.
Pandharipande, V. R., Pines, D., and Smith, R. A. 1976, *Ap. J.*, **208**, 550.
Pringle, J., and Rees, M. J. 1972, *Astr. Ap.*, **21**, 1.
Rappaport, S., and Joss, P. C. 1977, *Nature*, **266**, 683.
Savonije, G. J., and van den Heuvel, E. P. J. 1977, *Ap. J. (Letters)*, **214**, L19.
Scharlemann, E. 1978, *Ap. J.*, **219**, 617.
Shakura, N. I., and Sunyaev, R. A. 1973, *Astr. Ap.*, **24**, 337.
Vasyliunas, V. M. 1975, *Rev. Geophys. Space Phys.*, **13**, 303.
White, N. E., Mason, K. O., and Sanford, P. W. 1977, *Nature*, **267**, 229.

P. GHOSH and F. K. LAMB: Department of Physics, University of Illinois at Urbana-Champaign, Urbana, IL 61801